



NLL-accurate PanScales showers for hadron collisions

Silvia Ferrario Ravasio

CERN, TH department

Workshop on Tools for High Precision LHC Simulations.

Ringberg Castle, Germany

3rd November 2022

Based on:

“PanScales showers for hadron collisions: a fixed-order study” [arXiv:2205.02237],

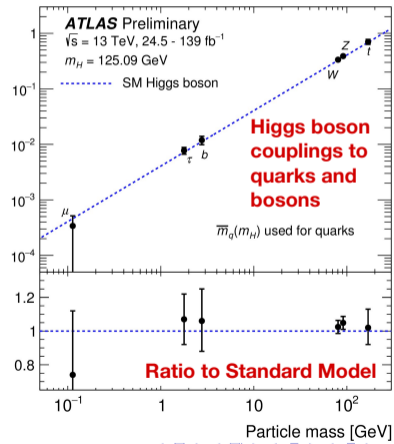
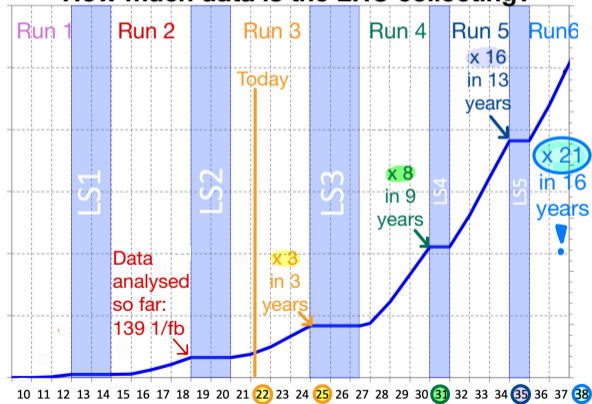
“PanScales showers for hadron collisions: all-orders validation” [arXiv:2207.09467],

M. van Beekveld, S.F.R., K. Hamilton, G. Salam, A. Soto Ontoso, G. Soyez, R. Verheyen

LHC: future prospects

- The LHC aims at $\times 20$ its current statistics
- More precise measurements e.g. in the **Higgs** sector: success of the **Standard Model** or hints of **new physics**

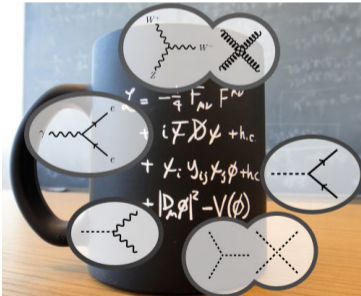
How much data is the LHC collecting?



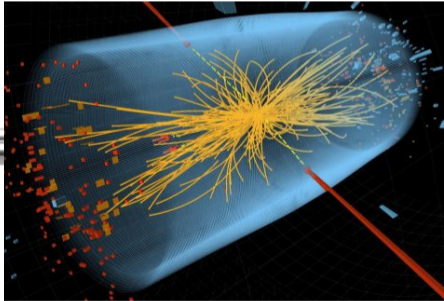
LHC: future prospects

- The LHC aims at $\times 20$ its current statistics
- More precise measurements *e.g.* in the **Higgs** sector: success of the **Standard Model** or hints of **new physics**
- This also requires **accurate theoretical predictions** ...

and a **connection** between **theory** & **experiment**!



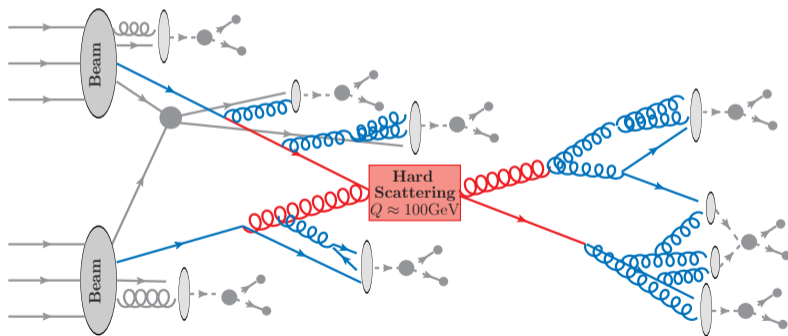
Ideal world



Reality

Shower Monte Carlo Generators and Parton Showers

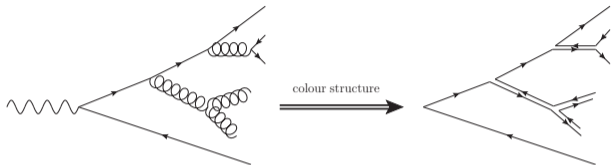
Shower Monte Carlo generators describe complex collider events, which are characterized by a **large number of particles**. **Parton Showers** (PS) are the core of SMC. They evolve the **hard system** from a hard scale $Q \sim 100 - 1000 \text{ GeV}$ to hadronic scales $\Lambda \sim 1 \text{ GeV}$, adding softer and softer partons (**quarks** and **gluons**), which are later-on converted into **hadrons**.



This evolution generates large **logarithms** of the scale ratios which are resummed by the PS.

Dipole Showers in a nutshell

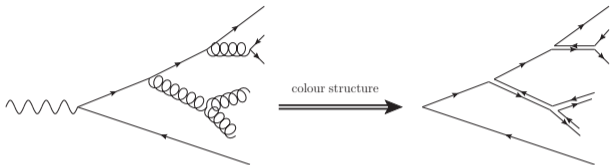
- **Parton showers** describe the **energy degradation** of fast-moving partons via **softer and softer emissions**



- **Dipole showers** are the most popular PS: available in Pythia8, Herwig7, Sherpa2
- Each **dipole** emits a parton independently

Dipole Showers in a nutshell

- **Parton showers** describe the **energy degradation** of fast-moving partons via **softer and softer emissions**



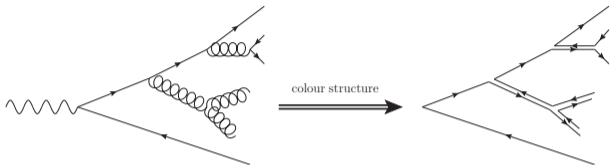
- Momentum conservation is as **local** as possible: the dipole leg **closer in angle in the dipole frame** to the emitted parton takes the transverse momentum recoil.

- **Dipole showers** are the most popular PS: available in Pythia8, Herwig7, Sherpa2
- Each **dipole** emits a parton independently



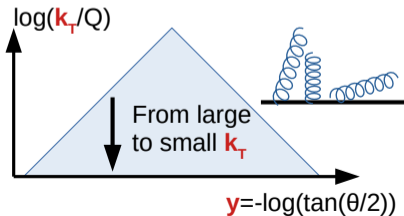
Dipole Showers in a nutshell

- **Parton showers** describe the **energy degradation** of fast-moving partons via **softer and softer emissions**



- **Dipole showers** are the most popular PS: available in Pythia8, Herwig7, Sherpa2
- Each **dipole** emits a parton independently

- Momentum conservation is as **local** as possible: the dipole leg **closer in angle in the dipole frame** to the emitted parton takes the transverse momentum recoil.

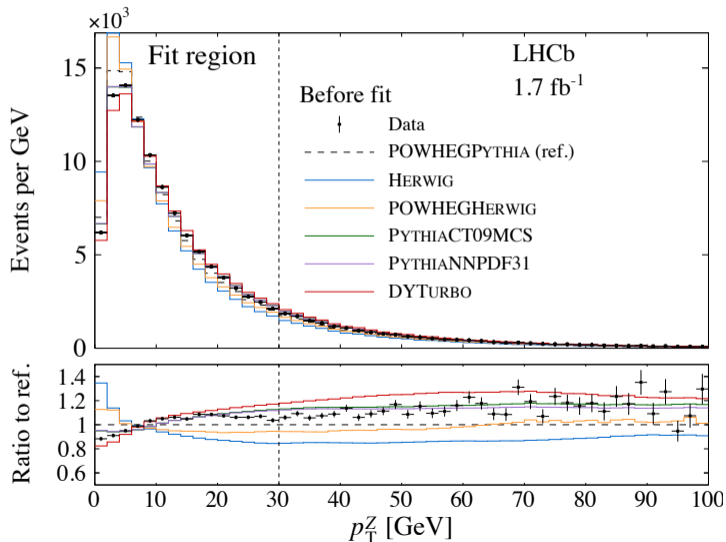


- emissions are ordered in **transverse momentum k_t** (except *Deductor*, which is "virtuality" ordered)
- "easy" to **match/merge** with F.O. calculations because one needs to correct only the first emissions

Why controlling the formal accuracy of parton showers?

W-boson mass measurements

[LHCb, 2109.01113]



$p_{T,Z}$, which is measured very precisely, is used to calibrate $p_{T,W}$ in W boson mass extractions.

$$m_W = 80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \text{ MeV}$$

The envelope in the different PS predictions causes the **dominant theory uncertainty (11 MeV)** in m_W

How to assess the accuracy

- **Tame the PS accuracy** to exploit the potential of colliders!
- The event evolves from **hard** to **soft** energies: **large logarithms** appear

How to assess the accuracy

- **Tame the PS accuracy** to exploit the potential of colliders!
- The event evolves from **hard** to **soft** energies: **large logarithms** appear

Logarithmic counting to define the accuracy!

- From **analytic resummation**

$$\Sigma(\underbrace{V}_{\text{obs}} < Q \underbrace{e^{-L}}_{\text{large log}}) = \exp(\underbrace{L g_{\text{LL}}(\alpha_s L)}_{\text{leading log}}) + \underbrace{g_{\text{NLL}}(\alpha_s L)}_{\text{next-to-leading log}} + \dots)$$

$\alpha_s L \sim 0.55$ if $Q = 100$ and $v = 1$ GeV: **NLL** are $\mathcal{O}(1)$!

How to assess the accuracy

- **Tame the PS accuracy** to exploit the potential of colliders!
- The event evolves from **hard** to **soft** energies: **large logarithms** appear

Logarithmic counting to define the accuracy!

- From **analytic resummation**

$$\Sigma(\underbrace{V}_{\text{obs}} < Q e^{-\underbrace{L}_{\text{large log}}}) = \exp(\underbrace{L g_{\text{LL}}(\alpha_s L)}_{\text{leading log}}) + \underbrace{g_{\text{NLL}}(\alpha_s L)}_{\text{next-to-leading log}} + \dots)$$

$\alpha_s L \sim 0.55$ if $Q = 100$ and $v = 1$ GeV: **NLL** are $\mathcal{O}(1)$!

PanScales criteria for assessing NLL

- 1 Behaviour of the exact **amplitudes** in singular limits [Dasgupta et al., JHEP **09** (2018), 033]
- 2 Logarithmic **resummation** results [Dasgupta et al., Phys. Rev. Lett. **125** (2020) no.5, 052002]

How to assess the accuracy

- **Tame the PS accuracy** to exploit the potential of colliders!
- The event evolves from **hard** to **soft** energies: **large logarithms** appear

Logarithmic counting to define the accuracy!

- From **analytic resummation**

$$\Sigma(\underbrace{V}_{\text{obs}} < Q \underbrace{e^{-L}}_{\text{large log}}) = \exp(\underbrace{L g_{\text{LL}}(\alpha_s L)}_{\text{leading log}}) + \underbrace{g_{\text{NLL}}(\alpha_s L)}_{\text{next-to-leading log}} + \dots)$$

$\alpha_s L \sim 0.55$ if $Q = 100$ and $v = 1$ GeV: **NLL** are $\mathcal{O}(1)$!

PanScales criteria for assessing NLL

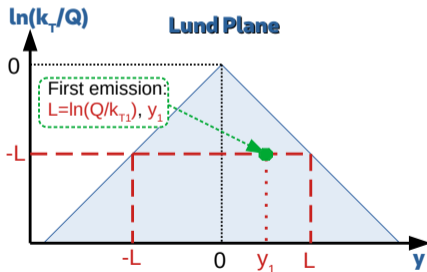
- 1 Behaviour of the exact a
- 2 Logarithmic **resummatio**

Also the **CVolver** (Platzer, Foreshaw, Holguin et al.), **Deductor** (Nagy & Soper) and **Alaric** (S. Hoche et al.) collaborations addressed this topic, but here I focus on **PanScales**

- Case of study: emission of two soft gluons, well separated in **rapidity** are **independent**

$$e^+e^- \rightarrow q\bar{q} : \quad dP_2 = \frac{C_F^2}{2!} \prod_{i=1}^2 \frac{2\alpha_s(k_{T,i})}{\pi} \frac{dk_{T,i}}{k_{T,i}} dy_i \quad y_i = -\log\left(\tan\left(\frac{\theta}{2}\right)\right) \text{ and } k_{T,i} = \text{transverse mom}$$

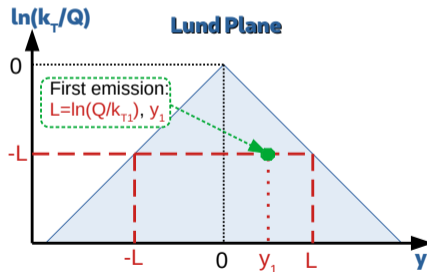
- Lund plane** = phase space available to an emission in terms of $\log Q/k_T = L$ and y



- Case of study: emission of two soft gluons, well separated in **rapidity** are **independent**

$$e^+e^- \rightarrow q\bar{q} : \quad dP_2 = \frac{C_F^2}{2!} \prod_{i=1}^2 \frac{2\alpha_s(k_{T,i})}{\pi} \frac{dk_{T,i}}{k_{T,i}} dy_i \quad y_i = -\log\left(\tan\left(\frac{\theta}{2}\right)\right) \text{ and } k_{T,i} = \text{transverse mom}$$

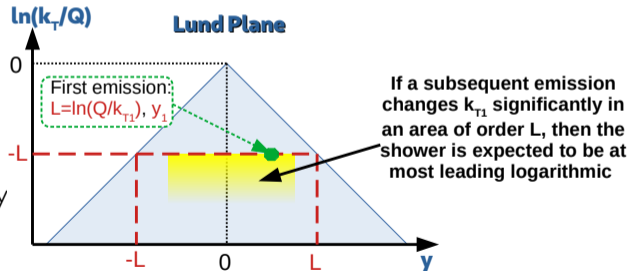
- **Lund plane** = phase space available to an emission in terms of $\log Q/k_T = L$ and y
- If a second emission disturbs a first one in an area $\mathcal{O}(L^2)$, it is not **LL**!



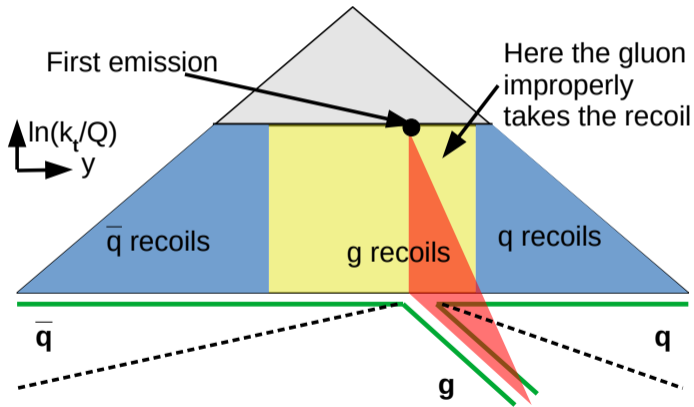
- Case of study: emission of two soft gluons, well separated in rapidity are **independent**

$$e^+e^- \rightarrow q\bar{q} : \quad dP_2 = \frac{C_F^2}{2!} \prod_{i=1}^2 \frac{2\alpha_s(k_{T,i})}{\pi} \frac{dk_{T,i}}{k_{T,i}} dy_i \quad y_i = -\log\left(\tan\left(\frac{\theta}{2}\right)\right) \text{ and } k_{T,i} = \text{transverse mom}$$

- Lund plane** = phase space available to an emission in terms of $\log Q/k_T = L$ and y
- If a second emission disturbs a first one in an area $\mathcal{O}(L^2)$, it is not **LL**!
- If the rapidity range in which a subsequent emission affects the first one grows linearly with $L = -\log(k_{T,1}/Q)$, then the shower cannot be **NLL**!

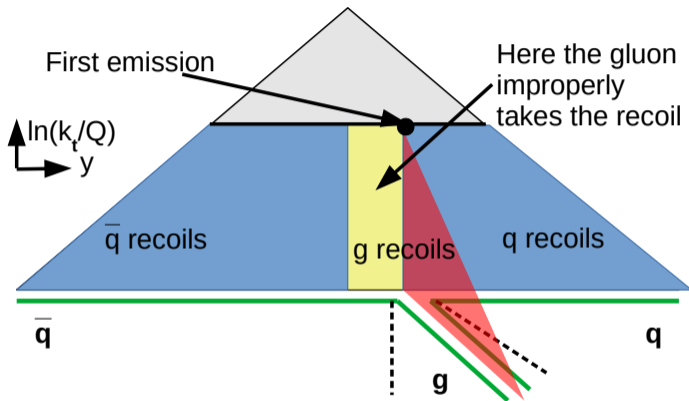


- State-of-the-art dipole showers, which are LL, are ordered in **transverse momentum** $v = \kappa_t$;
- Momentum conservation is fully local, the parton closer in angle to k **in the dipole frame** takes the **transverse momentum recoil**



- Issues due to how k_{\perp} is **redistributed** can be seen already from the second emission (from $e^+e^- \rightarrow qq\bar{q}$)

- Defining y in the **event-frame** partially improves but does not solve the issue



- Defining y in the **event-frame** partially improves but does not solve the issue

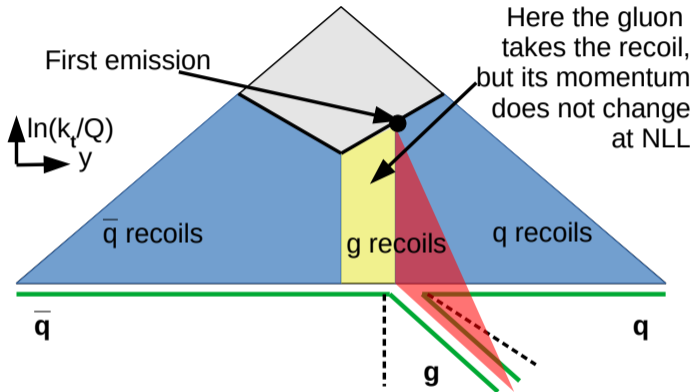
- But we can choose the **ordering variable** such that when the gluon improperly recoils,

$$\vec{k}_{T,1} \gg \vec{k}_{T,2}, \text{ so}$$

$$\vec{k}_{T,1} \rightarrow \vec{k}_{T,1} - \vec{k}_{T,2} \approx \vec{k}_{T,1}$$

$$v^2 \sim (\mathbf{k}_t^2)^{1-\beta} \underbrace{(q^2)^\beta}_{\text{virtuality}} \quad 0 < \beta < 1$$

- We call this shower **PanLocal**



- Defining y in the **event-frame** partially improves but does not solve the issue

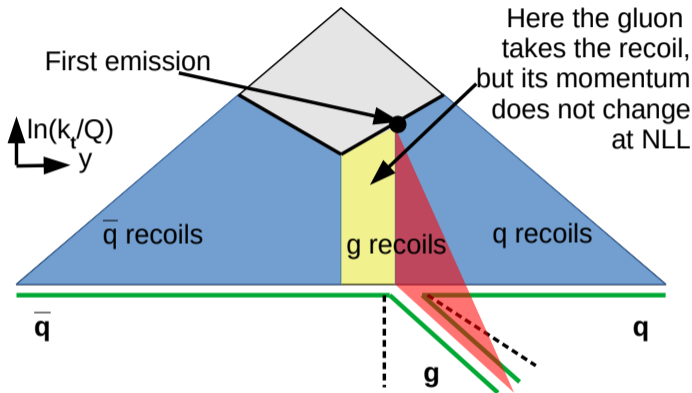
- But we can choose the **ordering variable** such that when the gluon improperly recoils, $\vec{k}_{T,1} \gg \vec{k}_{T,2}$, so $\vec{k}_{T,1} \rightarrow \vec{k}_{T,1} - \vec{k}_{T,2} \approx \vec{k}_{T,1}$

$$v^2 \sim (\mathbf{k}_t^2)^{1-\beta} \underbrace{(q^2)^\beta}_{\text{virtuality}} \quad 0 < \beta < 1$$

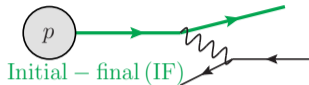
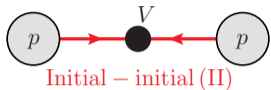
- We call this shower **PanLocal**

- To implement **transverse-momentum ordered** showers then one needs to redistribute the transverse momentum recoil globally \rightarrow **PanGlobal**.

All the particles are boosted to ensure full-momentum conservation. The boost mainly affects hard particles, leaving **soft** ones **unchanged**.

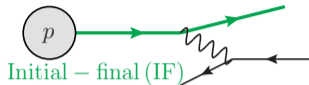
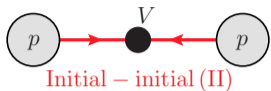


Initial-state radiation in state-of-the-art Dipole showers



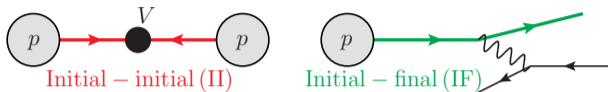
- In **hadron collider** processes, a dipole can comprise partons in the **initial-state**, which must be aligned with the **beams**

Initial-state radiation in state-of-the-art Dipole showers



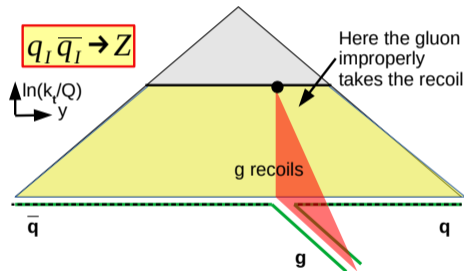
- In **hadron collider** processes, a dipole can comprise partons in the **initial-state**, which must be aligned with the **beams**

Initial-state radiation in state-of-the-art Dipole showers



- In IF dipoles the final-state leg recoils also for ISR. In DY, the Z boson recoils only after the first emission! But resummation tell us low- k_T region is dominated by emissions with opposite \vec{k}_\perp which cancels in the sum!

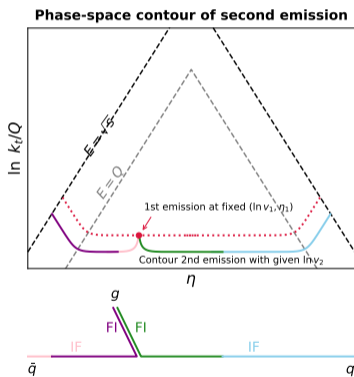
- In **hadron collider** processes, a dipole can comprise partons in the **initial-state**, which must be aligned with the **beams**



- To remedy this [Platzer and Gieseke \('09\)](#) proposed to give the p_\perp recoil to the incoming partons and then boost to realign it with the beams (option available in [Dire](#), [Höhe](#), [Prestel '15](#)): this renders DY “not worse” than the case with partons only in the final state (left plot)

Two-emission contours for state-of-the-art dipole showers

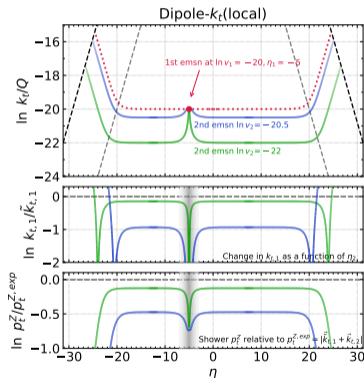
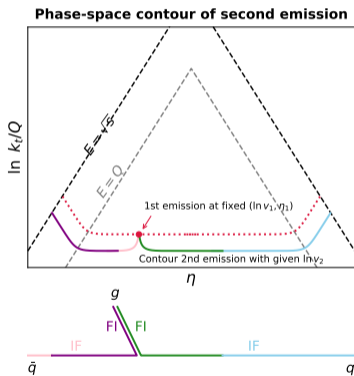
What happens to the **first gluon** and to the Z **boson** transverse momentum after a **second emission** is added for state-of-the-art dipole showers?



van Beekveld, SFR, Salam, Soto-Ontoso, Soyez, Verheyen, [arXiv:2205.02237]

Two-emission contours for state-of-the-art dipole showers

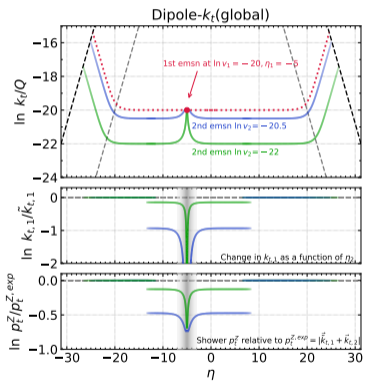
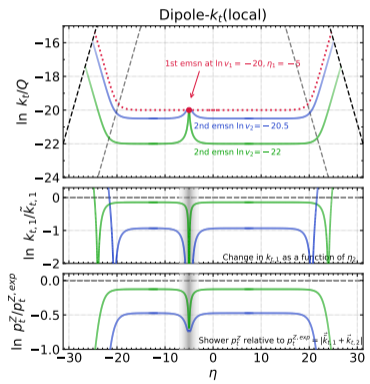
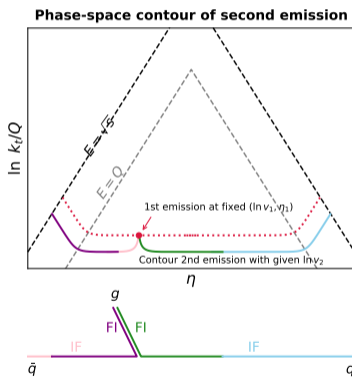
What happens to the **first gluon** and to the Z **boson** transverse momentum after a **second emission** is added for state-of-the-art dipole showers?



van Beekveld, SFR, Salam, Soto-Ontoso, Soyez, Verheyen, [arXiv:2205.02237]

Two-emission contours for state-of-the-art dipole showers

What happens to the **first gluon** and to the **Z boson** transverse momentum after a **second emission** is added for state-of-the-art dipole showers?



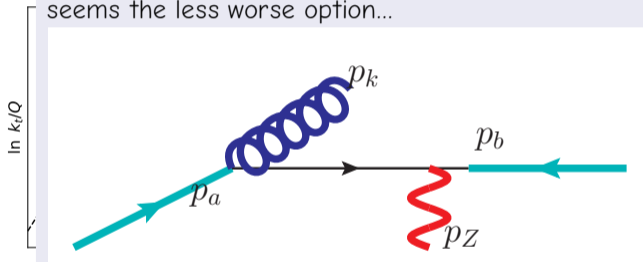
van Beekveld, SFR, Salam, Soto-Ontoso, Soyez, Verheyen, [arXiv:2205.02237]

Two-emission contours for state-of-the-art dipole showers

When
em

PanLocal starting point

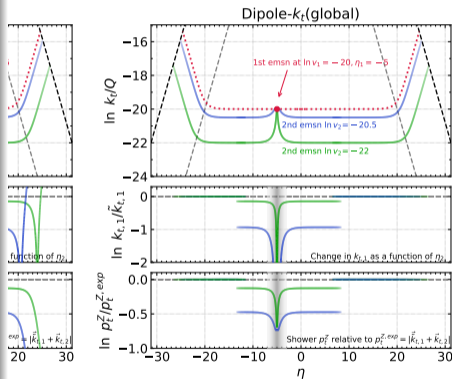
Giving transverse momentum recoil to the incoming parton and then applying a global boost seems the less worse option...



To get PanLocal:

- 1 Measure the rapidity in the Z boson rest frame
- 2 Ordering variable $v^2 = \sqrt{k_t^2 q^2}$

transverse momentum after a **second**



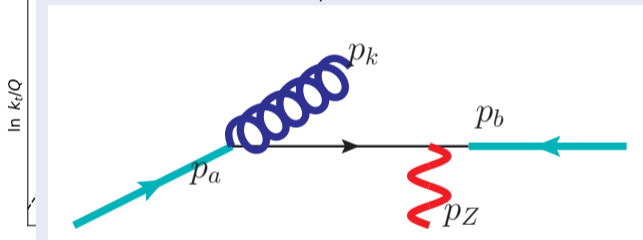
VAN DEEKVELD, ZEK, SUIJTH, SUTO-ORRISO, Soyez, Verheyen, [arXiv:2205.02237]

Two-emission contours for state-of-the-art dipole showers

When
em

PanLocal starting point

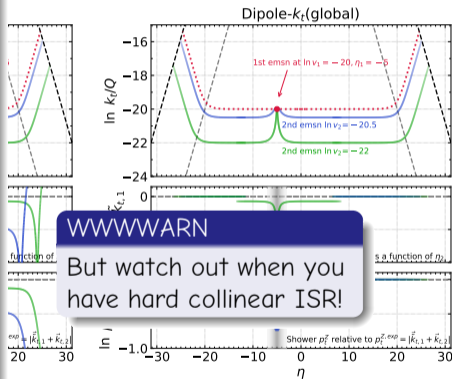
Giving transverse momentum recoil to the incoming parton and then applying a global boost seems the less worse option...



To get PanLocal:

- 1 Measure the rapidity in the Z boson rest frame
- 2 Ordering variable $v^2 = \sqrt{k_t^2 q^2}$

transverse momentum after a **second**



WWWARN

But watch out when you have hard collinear ISR!

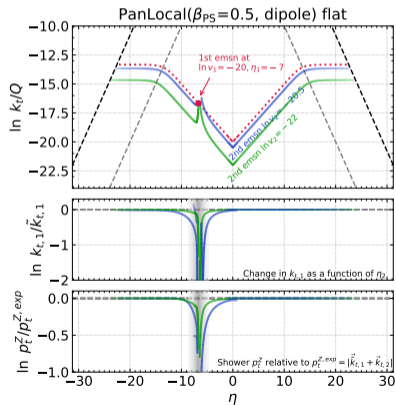
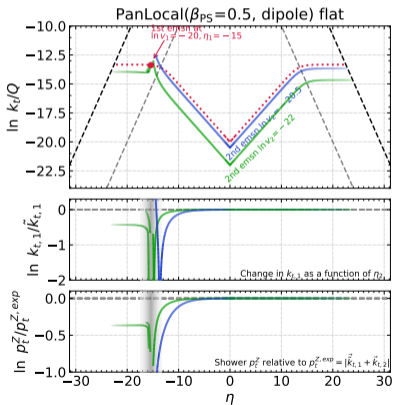
VAN DEEKVELD, ZFK, SUIJTH, SUTO-ORITOSO, SoyeZ, Verheyen, [arXiv:2205.02237]

PanLocal for hadron collisions

We use $\mathbf{v} \approx k_t e^{-|\eta|/2}$ for soft-collinear emissions (like for FSR) and restore **transverse momentum** conservation for very collinear emissions.

$$\text{ISR: } \kappa_{t,\text{shower}}^2 = \frac{|k_{\perp}^2|}{(1 + a_k)^{\frac{2}{1+\beta}}}$$

$$\text{FSR: } \kappa_{t,\text{shower}}^2 = |k_{\perp}^2|$$

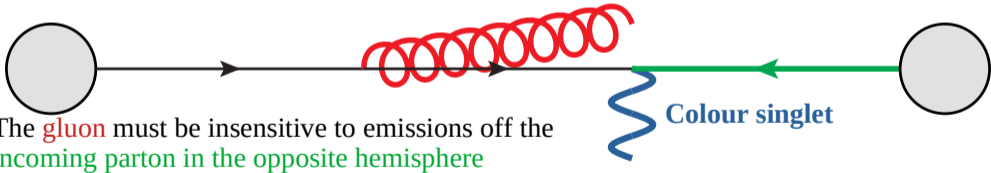


PanGlobal for hadron collisions

- To have $\beta = 0$ (i.e. k_t -ordering) we cannot conserve the **transverse momentum** locally.
- In the $Z \rightarrow q\bar{q}$ variant of **PanGlobal**, the **whole final-state is boosted** to absorb the transverse momentum of the emission, and the hardest partons (typically the original $q\bar{q}$ pair) takes the recoil;

PanGlobal for hadron collisions

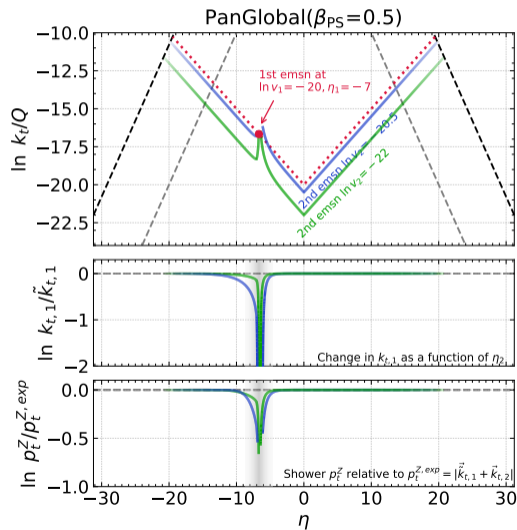
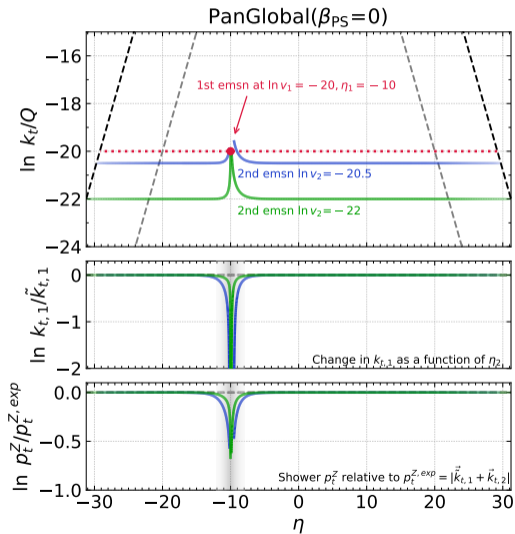
- To have $\beta = 0$ (i.e. k_t -ordering) we cannot conserve the **transverse momentum** locally.
- In the $Z \rightarrow q\bar{q}$ variant of **PanGlobal**, the **whole final-state is boosted** to absorb the transverse momentum of the emission, and the hardest partons (typically the original $q\bar{q}$ pair) takes the recoil;
- For $q\bar{q} \rightarrow Z$, boosting the whole final-state is dangerous, because we can have very energetic partons produced from the backward evolution of the incoming partons that should not be affected by emissions well-separated in rapidity (interesting solution by [Nagy, Soper '09](#), that however works only for $\beta > 0$)



- We boost only the Z boson to absorb the recoil (and rescale the beams to ensure momentum conservation)

Two-emission contours for PanGlobal

With this map, we can build an NLL shower ordered with several ordering scales



All-order tests: general strategy

- We want to compare against the analytic NLL result

$$\Sigma(O < e^L) = \exp(Lg_{\text{LL}}(\alpha_s L) + g_{\text{NLL}}(\alpha_s L) + \alpha_s g_{\text{NNLL}}(\alpha_s L) + \dots)$$

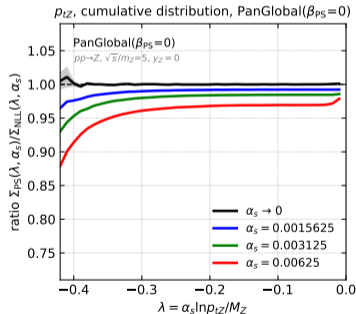
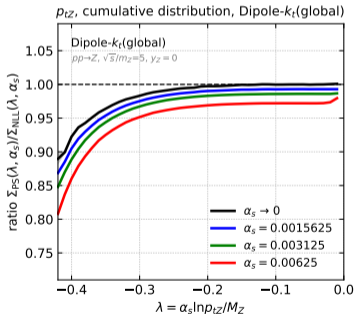
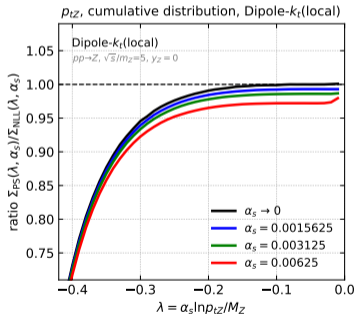
van Beekveld, SFR, Hamilton, Salam, Soto-Ontoso, Soyez, Verheyen, [arXiv:2207.09467]

All-order tests: general strategy

- We want to compare against the analytic NLL result

$$\Sigma(O < e^L) = \exp(Lg_{\text{LL}}(\alpha_s L) + g_{\text{NLL}}(\alpha_s L) + \alpha_s g_{\text{NNLL}}(\alpha_s L) + \dots)$$

- We want to be sure higher order corrections do not pollute our comparison: we need to extract $\Sigma_{\text{PS}}/\Sigma_{\text{analytic}}$ for $\alpha_s \rightarrow 0$ at fixed value of $\lambda = \alpha_s L$

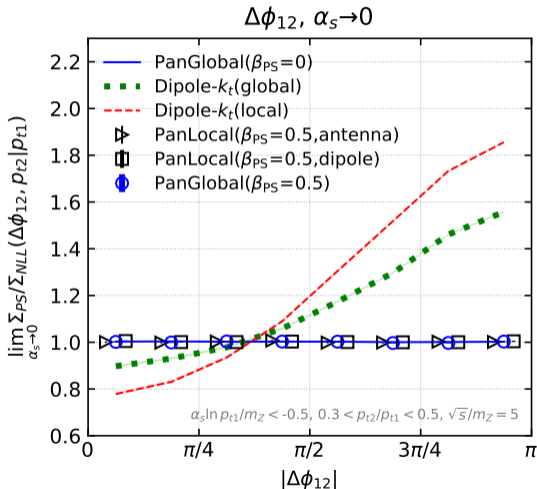
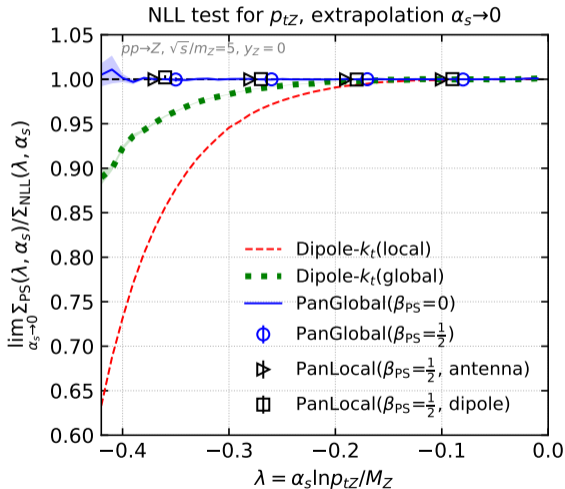


van Beekveld, SFR, Hamilton, Salam, Soto-Ontoso, Soyer, Verheyen, [arXiv:2207.09467]

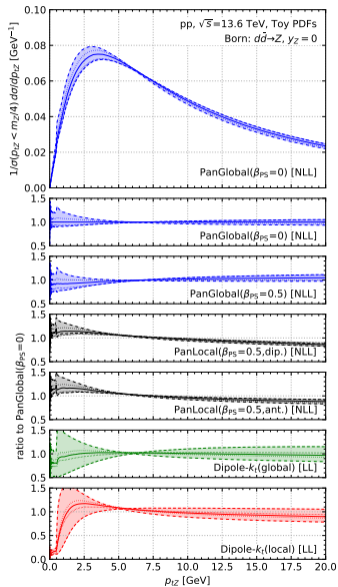
All-order tests: p_{tZ} and leading jets $\Delta\Phi_{1,2}$

Z boson p_T

$\Delta\Phi_{1,2}$ between the two leading (C/A, $R = 1$) jets



Exploratory pheno with p_{TZ}



- We vary by a factor 2 the scale used to evaluate the PDF's

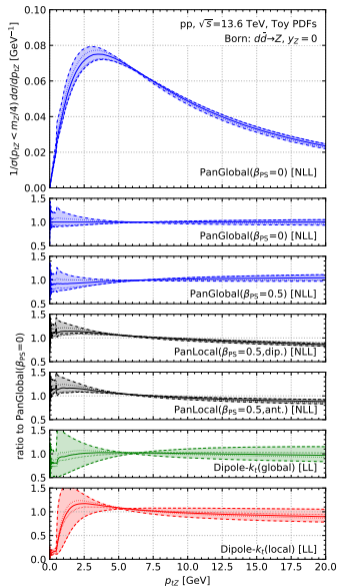
$$\mu_F = x_F \mu_F^{\text{central}}, \quad x_F = 1/2, 1, 2$$

- We vary by a factor 2 the scale used to evaluate α_s , adding a compensation factor for soft emissions in NLL showers

$$\mu_R = x_R \mu_R^{\text{central}}, \quad x_R = 1/2, 1, 2 \quad \alpha_s(\mu_R) \left(1 + \frac{K}{2\pi} \alpha_s(\mu_R) + \underbrace{2(1-\xi)b_0 \alpha_s(\mu_R) \log x_R}_{\text{In the soft limit evaluated at } k_t} \right)$$

(Similar to [Mrenna, Skands \[1605.08352\]](#))

Exploratory pheno with p_{TZ}



- We vary by a factor 2 the scale used to evaluate the PDF's

$$\mu_F = x_F \mu_F^{\text{central}}, \quad x_F = 1/2, 1, 2$$

- We vary by a factor 2 the scale used to evaluate α_s , adding a compensation factor for soft emissions in NLL showers

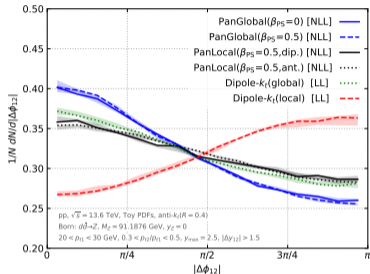
$$\mu_R = x_R \mu_R^{\text{central}}, \quad x_R = 1/2, 1, 2 \quad \alpha_s(\mu_R) \left(1 + \frac{K}{2\pi} \alpha_s(\mu_R) + \underbrace{2(1-\xi)b_0 \alpha_s(\mu_R) \log x_R}_{\text{In the soft limit evaluated at } k_t} \right)$$

(Similar to [Mrenna, Skands \[1605.08352\]](#))

- Scale variations do not capture the differences among the NLL showers: open question on realistic estimate of **shower uncertainties**
- Except in the very small p_T regions, LL and NLL showers yield similar results (with the NLL ones having much smaller scale uncertainty) → **Does NLL matter?**

Exploratory pheno with $\Delta\Phi_{12}$

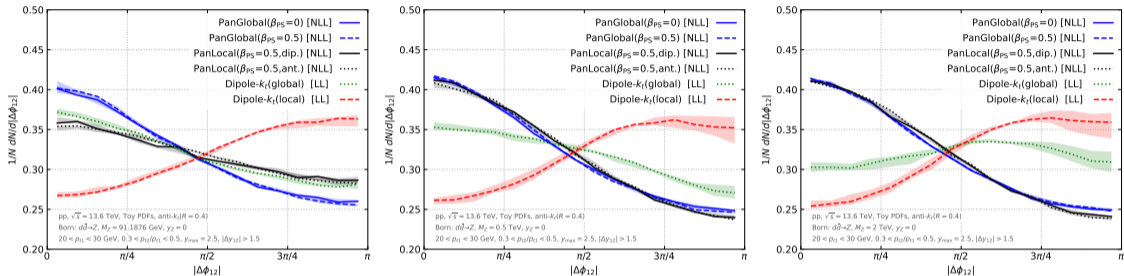
Azimuthal correlations between the two leading (C/A, $R = 1$) jets, requiring $p_{T,1} \sim 25$ GeV, $p_{T,2} \sim 10$ GeV, $\Delta y > 1.5$ for Drell-Yan, with $y_Z = 0$ and $m_Z/\text{GeV} = 91$



- For **onshell Z production**, the “best” LL shower is undistinguishable from the NLL, and scale variations are much smaller than recoil scheme variations within NLL showers. **Can we tune to get the correct picture at the Z pole?**

Exploratory pheno with $\Delta\Phi_{12}$

Azimuthal correlations between the two leading (C/A, $R = 1$) jets, requiring $p_{T,1} \sim 25$ GeV, $p_{T,2} \sim 10$ GeV, $\Delta y > 1.5$ for Drell-Yan, with $y_Z = 0$ and $m_Z/\text{GeV} = 91, 500, 2000$



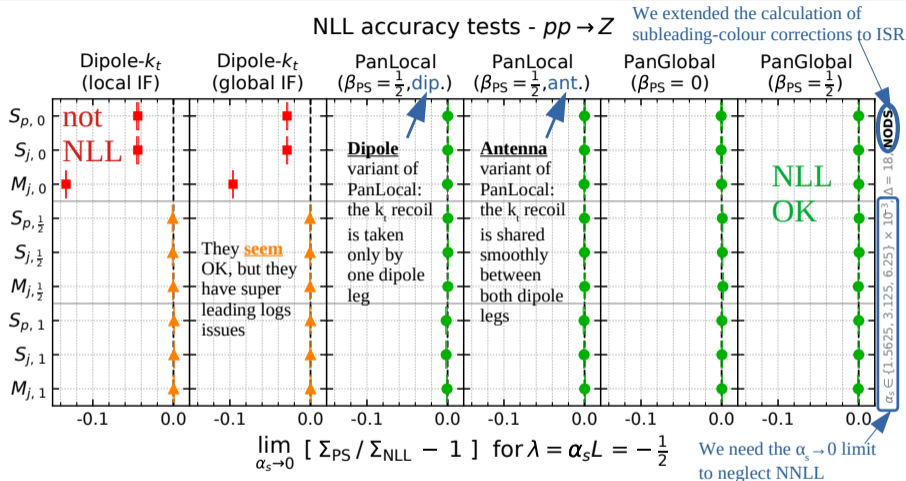
- For **onshell Z production**, the “best” LL shower is undistinguishable from the NLL, and scale variations are much smaller than recoil scheme variations within NLL showers. **Can we tune to get the correct picture at the Z pole?**
- For **very large mass $M_Z \geq 500$ GeV**, the LL showers lead to clear distortions wrt the NLL ones, scale variations are smaller but of the same order of magnitude of NLL shower differences: **Tuning will not help if you want to be predictive across several \sqrt{s} , you need NLL!**

Conclusions and Outlook

- **Parton Showers** are employed in almost every analysis from the LHC experimental collaborations: indispensable for **collider phenomenology**!
- The **accuracy** of Parton Showers is very **rough** compared to state-of-the-art analytic calculations, which in turn however have limited applicability (only few observables, joint resummation very difficult, analytic hadronization models not so advanced ...)
- We can learn from analytic resummation how to build a **next-to-leading-logarithmic** shower!
- Several NLL showers for all the most relevant LHC processes are around the corner, in particular PanScales is working hard towards having a **public code** usable for phenomenology at the LHC (ongoing work on matching, masses, processes with generic jets and more).
- Long-standing issue: **shower uncertainties**. Having several and not just 1 NLL shower will help.

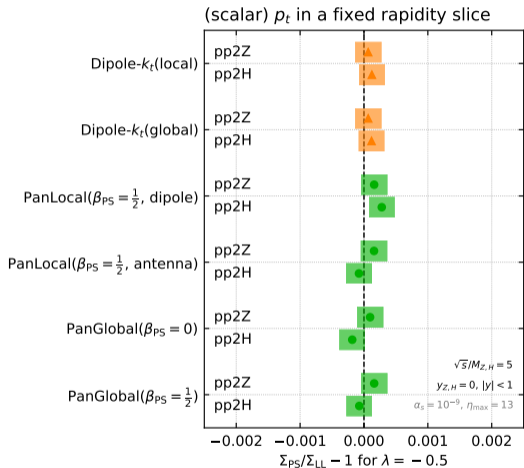
BACKUP

All-order tests: global event shapes



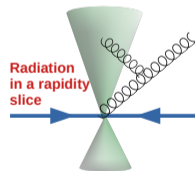
$$M_{j,\beta} = \max_{j \in \text{jets}} (|p_{\perp,j}| e^{-\beta|\eta_j|}), \quad S_{j,\beta} = \sum_{j \in \text{jets}} |p_{\perp,j}| e^{-\beta|\eta_j|}, \quad S_{p,\beta} = \sum_{p \in \text{partons}} |p_{\perp,p}| e^{-\beta|\eta_p|}$$

All-order tests: non-global observables



- **Non-global QCD observables** are characterised by a sensitivity to the full angular distribution of soft radiation emitted coherently in hard scattering processes.
- **Dipole showers** can also describe non-global observables, such as $S_{p,0}$ (=scalar sum of \vec{p}_\perp) for emissions in a rapidity slice

$$\Sigma(S_{p,0} < e^L; |y| < y_{\text{cut}}) = \exp(g_{\text{LL}}(\alpha_s L) + \alpha_s g_{\text{NLL}}(\alpha_s L) + \dots)$$



Superleading logs in dip showers [arXiv:2002.11114, Dagupta et al. '20]

$$\log(\Sigma) = Lg_{LL}(\alpha_s L) + g_{NLL}(\alpha_s L) + \alpha_s g_{NNLL}(\alpha_s L) + \dots = \sum_{i=1}^{\infty} (\alpha_s L)^i [c_{i,LL} L + c_{i,NLL} + c_{i,NNLL} L^{-1} + \dots]$$

If we ignore the running of α_s , $b_0 = 0 \rightarrow g_{LL} = 0$, so at order α_s^n there cannot be more than n powers of L . If we find terms $\alpha_s^n L^m$ with $m \geq n + 1$ (or $n + 2$ if we include the running) those are **superleading logarithms**, which should not present (here is $e^+e^- \rightarrow q\bar{q}$ at leading colour)

$$\lim_{L \rightarrow \infty} \frac{[\log(\Sigma_{PS}) - \log(\Sigma_{NLL})]_{\alpha_s^n}}{L^n} = \begin{cases} 0 & \text{the shower is NLL} \\ \text{const} & \text{the shower is not NLL} \\ cL^i \text{ with } i \geq 1 & \text{there are superleading logs} \end{cases}$$

