Shape Variables and Power Corrections

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Shape variables and QCD

Shape variables in e^+e^- annihilation are the simplest contest where we can study perturbative QCD.

For example, thrust:

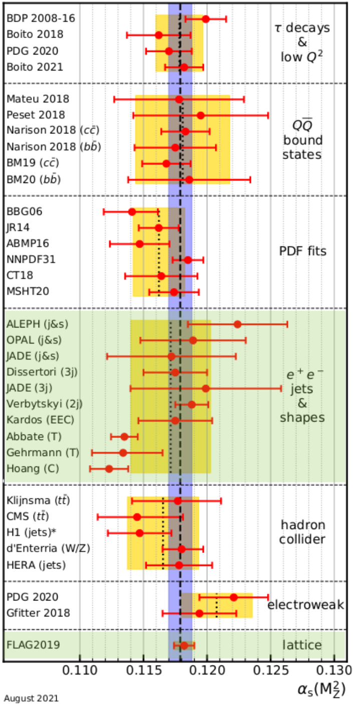
$$T = \max_{\vec{t}} \frac{\sum |\vec{p}_i \cdot \vec{t}|}{\sum |\vec{p}_i|}$$

equals 1 for two narrow back-to-back jets, and $2/3 < T < 1$ for three narrow jet.
Thus in the region $2/3 < T < 1$ the thrust distribution is proportional to α_s, and can be used for its determination.

On the other hand, the thrust distribution is sensitive to non-perturbative hadronization effects.

For example, the emission of a soft hadron with momentum 500 MeV, perpendicular to the thrust direction, affects the thrust by an amount $0.5/91 \approx 0.005$ on the Z peak. The average value of $1 - T$ is of order $\alpha_s \approx 0.1\%$, so this shift in T can affect the determination of α_s by an amount of the order of 5%.

In practice non-perturbative corrections can reach the 10% level, and can affect at the same level the extracted value of α_s.



α_s determinations (PDG)

Determination of α_s from the first seven rows of the jets & shapes cathegory (highlighted in green) use Monte Carlo models to correct for non-perturbative effects.

The following three lines (Abbate, Gehrmann, Hoang) are based upon analytic modeling of non-perturbative effects.

Status

- The use of Monte Carlo modeling for hadronization corrections is not totally satisfying, since it lacks a sound theoretical basis.
- Analytic models seem to favour a too low value of α_s as compared to the world average and to the precise lattice determination.
- No bridge between MC and analytic models
- It is disturbing that we do not fully understand the role of non-perturbative effects at least in the simplest context where they can be studied.
- Understanding non-perturbative effects can have important consequences also for precision physics at hadron colliders, where linear power corrections can play an important role.

There are two broad classes of analytic methods:

- those based upon the so called "Dispersive approach", Based upon work of Dokshitzer, Webber, Marchesini, Salam and others. It is based upon the computation of the emission of a very soft gluon, with an associated non-perturbative coupling. The reference to Gehrmann in the previous slide refers to this method.
- Those based upon factorization, that separates the QCD calculation into a perturbative and non-perturbative contribution (a so called Shape Function), based upon work of Collins, Soper, Korchemsky, Sterman, and followed by a vast literature (Hoang, Stuart, Thaler, Mateu, Bauer, Schwartz and many others) using SCET. The references to Abbate and Hoang refer to this method.

These methods have however a common feature: the non-perturbative correction is computed in the two-jet limit, and then it is extrapolated to the three-jet region, where the measurement is performed.

Recent progress

There have been recently new findings regarding the structure of linear power corrections in collider observables:

- In ref. Eur.Phys.J.C 81 (2021), (Luisoni, Monni, Salam) it was shown that linear power corrections to the C parameter in the 3-jet symmetric limit are about 1/2 of those in the two jet limit.
- In ref. JHEP 01 (2022) 093, (Caola, Ferrario-Ravasio, Limatola, Melnikov, P.N.) it was demonstrated that linear power corrections are absent in sufficiently inclusive observables, in a variety of processes, in a model theory (large n_f QCD) that shares some properties with the full theory. These findings confirmed previous results obtained at the numerical level JHEP 06 (2021) 018, (Ferrario-Ravasio,Limatola,P.N.).
- The same findings opened the possibility to compute linear power corrections to shape variables in the 3-jet configuration arXiv:2204.02247, (Caola,Ferrario-Ravasio,Limatola,Melnikov, Ozcelik,P.N.)

The results of Luisoni, Monni, Salam are based upon the so called "dispersive approach", where one assumes that the strong coupling at low energy can be given by an effective coupling

The results of Caola et al. and Ferrario-Ravasio et al. are obtained from the study of IR renormalons. (These, in turn, can be shown to be related to calculations with a massive gluon: the presence of effects linear in the mass signals the presence of linearly suppressed power corrections due to renormalons.)

The two approaches are deeply related.

How does it work

Assume that radiation kinematics, $\tilde{p}, k$ (k is the soft gluon momentum) can be given as a function of a Born kinematics p and of k, $\tilde{p} = \tilde{p}(p, k)$, and this function in linear in the components of k for small k. We found (Caola etal.) that the integral of the amplitude at fixed p

$$\int [\mathrm{d}k] M(\tilde{p}(p,k),k)\,[1, k\cdot r]$$

for a gluon with mass λ does not have any terms linear in λ for small λ. Then, the cumulant of a shape variable is

$$\int \mathrm{d}M(\tilde{p},k)\Theta(v - V(\tilde{p},k)) = \int \mathrm{d}M(\tilde{p},k)\delta(v - V(p))\,[V(p) - V(\tilde{p},k)] + \int \mathrm{d}M(\tilde{p},k)\Theta(v - V(p))$$

The second line does not lead to linear λ terms. But now the square bracket on the first line suppresses the amplitude, so, in order to get a linear term, we only need to consider the soft-divergent part of the amplitude.

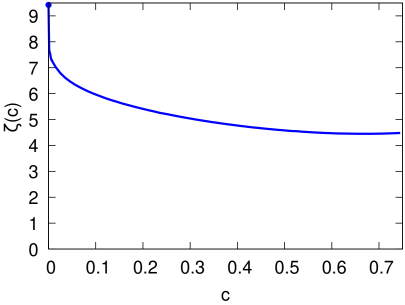
How does it work

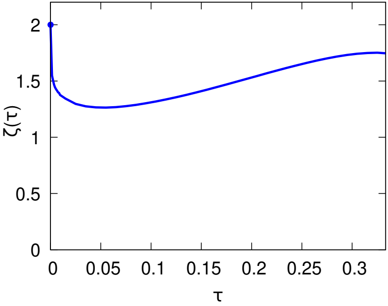
- In the two-parton case, the Born kinematics is fixed, so $V(p)$ is fixed. So, the previous argument works with no need of further assumptions.
- $C(p)$ in the 3-jet symmetric limit depends only weakly on p. Thus, it does not matter what mapping one uses, $C(p)$ is essentially fixed (Luisoni et al.)
- In the generic case, it is essential to use the fact demonstrated by Caola et al., that in suitable recoil schemes recoil effects cannot generate linear power corrections

Non-perturbative corrections can be parametrized by a shift in the perturbative cumulant distribution:

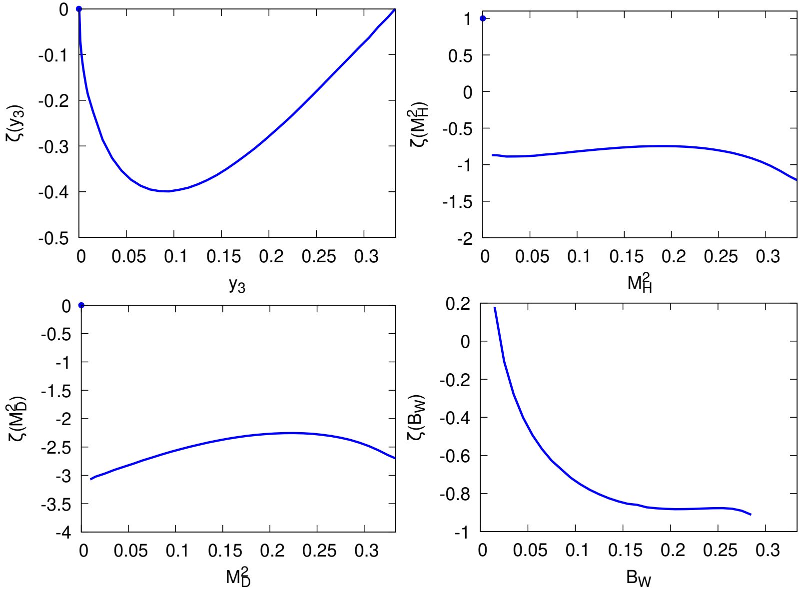
$$\Sigma(s + H_{\mathrm{NP}}\zeta(s)) - \Sigma(s) \approx \frac{\mathrm{d}\sigma}{\mathrm{d}s} H_{\mathrm{NP}}\zeta(s), \qquad \Sigma(s) = \int \mathrm{d}\sigma(\Phi)\theta(s - s(\Phi))$$

and $H_{\mathrm{NP}} \approx \Lambda/Q$ is a non-perturbative parameter that is fully calculable in the large n_{f} approximation but must be fitted to data in real QCD.





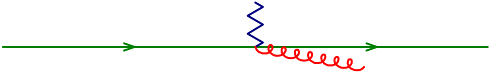
The dot in the plots represents the constant value that was used in earlier studies. Notice that the value of ζ at the symmetric point is about one/half of the value at $c = 0$, consistent with Luisoni etal.



(G.Zanderighi, P.N.) In some cases the ζ function is negative!

Rapid variations near $v = 0$

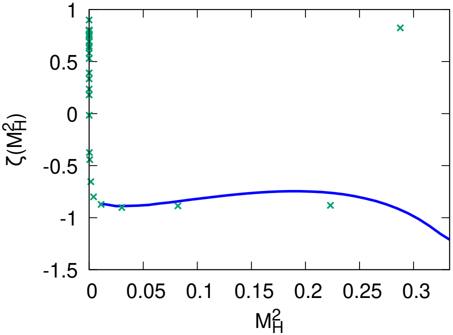
Near $v = 0$, the Born amplitude is dominated by the soft-collinear region.



$$\text{radiation} = \frac{C_A}{2} M_{\bar{q}g} + \frac{C_A}{2} M_{qg} + \left(C_F - \frac{C_A}{2} \right) M_{q\bar{q}}$$

but $M_{qg} \approx 0$, $M_{\bar{q}g} \approx M_{q\bar{q}}$, so the total is $\approx C_F M_{q\bar{q}}$.

Our $\zeta(v)$ functions, for $v \to 0$ MUST approach the 2-jet limit value; but up to single logs!, i.e. terms of relative order $1/|\log(v)|$.



Insist on $v \to 0$ (quadruple precision, log scale histogram).
Two-jet limit reached, but subleading logs are extremely important!

Implications

- There is a clear indication that the non-perturbative correction in the two jet limit cannot be safely extrapolated in the region where α_s is fitted.
- Indication that the two-jet limit is more tricky than thought before: subleading logs can be extremely important.
- It is likely that this is not the whole story, and more needs to be understood before these findings can be safely used.

Does it fit data?

It is interesting to ask whether current data favours the newly computed power corrections. (Zanderighi,P.N., in preparation)
In the following I will illustrate preliminary results obtained by fitting ALEPH data.
The non-perturbative shift for C and t are available from arXiv:2204.02247, Caola et al..
In Zanderighi,P.N. we also computed it for the y_3 in the Durham scheme, the Heavy jet mass M_h^2 and the heavy-light mass difference $M_h^2 - M_l^2$ and the broadening of the wide jet B_W.
We computed the shape variables at NNLO, and implemented the power correction. No other ingredient (i.e. resummation) was included. On the other hand, the fit ranged were chosen far from the two jet region and before the 3-parton kinematic limit.

Mass corrections

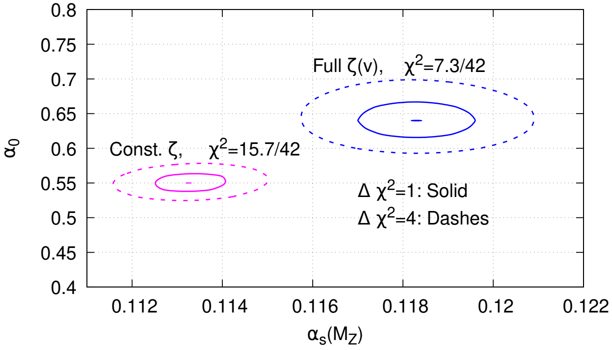
- Shape variables are defined for massless partons, and the analytic models refer to the "massless" definition.
- Final state hadrons are massive; so the definition of the shape variables must be extended to massive objects. This leads to ambiguities in the definition.
- This problem has been extensively studied in JHEP 05 (2001) 061, (Salam, Wicke). Three mass schemes where proposed:
  - the p scheme, where the energy of a particle is set equal to the modulus of the 3-momentum;
  - the E scheme, where the modulus of the momentum is set equal to the energy;
  - the D scheme ("Decay scheme"), where massive hadrons are decayed isotropically into a pair of fictitious massless particles before the shape variable is computed.

- The perturbative theoretical errors were estimated with a 3-point scale variation $\mu_R/Q = 0.25, 0.5, 1$. An estimate of the error on the non-perturbative component was also included and added in quadrature.
- Diagonal terms of the covariant matrix were computed by summing in quadrature the systematic statistical and theoretical errors. The off-diagonal terms were computed as $E_{ij} = \min(\delta\sigma^2_{\mathrm{syst},i}, \delta\sigma^2_{\mathrm{syst},j})$ (the so called minimal-overlap model).
- We adopted the E scheme as our default treatment of hadron masses. We computed the associated bin migration matrix using Pythia8. Using Herwig7 we obtain compatible results with a slightly worse χ^2.

PRELIMINARY RESULTS

Simultaneous fit to C, t and y_3, both for our newly computed $\zeta(v)$, and, for comparison, with $\zeta(v) \to \zeta(0)$ (traditional method for the computation of power corrections).

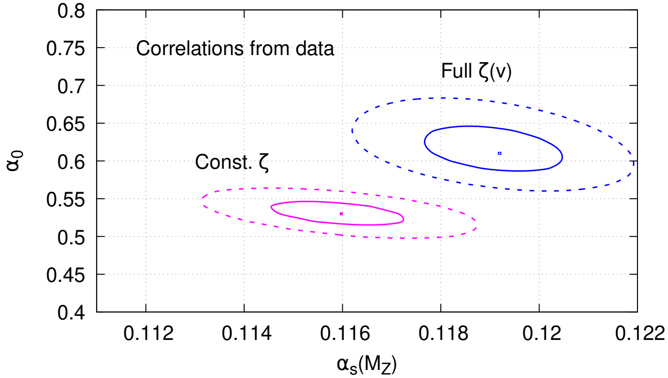
(we excluded variables with "bizarre" behaviour near the 2-jet limit)



The central value is at $\alpha_s(M_Z) = 0.1182$, $\alpha_0 = 0.64$. The "traditional" method leads to smaller values of α_s.

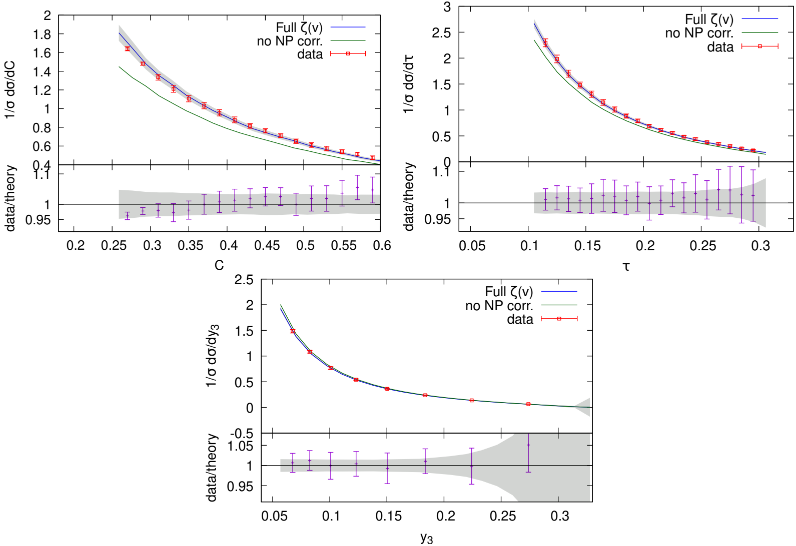
PRELIMINARY RESULTS

Different correlations computation, using correlation data not publicly available (thanks to Hasko Stenzel).

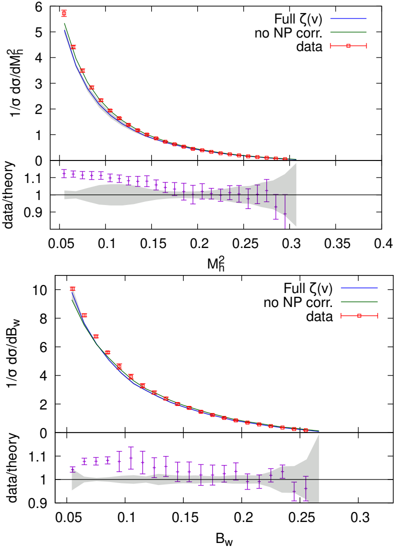


The central value is at $\alpha_s(M_Z) = 0.1192$, $\alpha_0 = 0.61$. The "traditional" method leads to smaller values of α_s, but higher than the previous case.

Quality of the fit for C, τ and y_3, using the new calculation of the non-perturbative effect (i.e. the full $\zeta(v)$ dependence.)

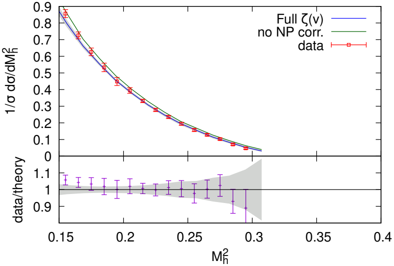


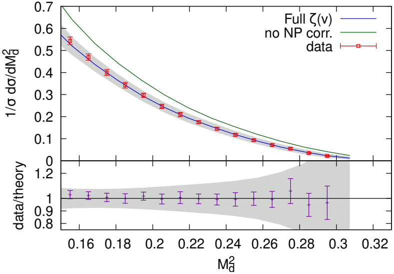
Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

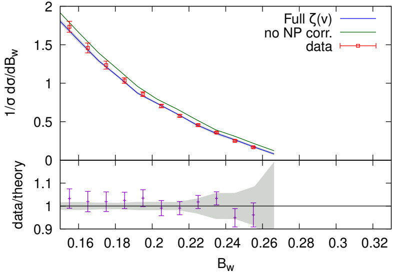


Good fit far away from the two jet region.

Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

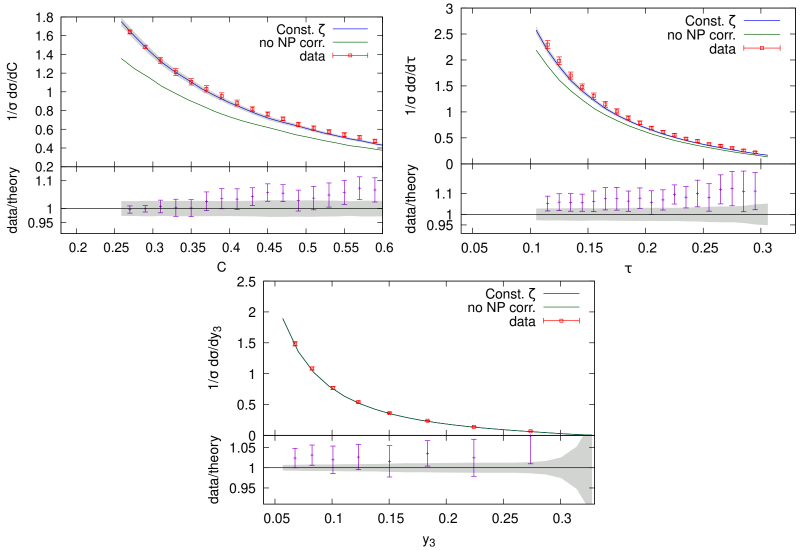




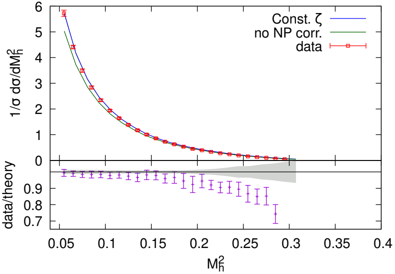


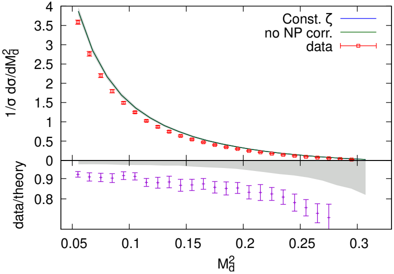
Good fit far away from the two jet region.

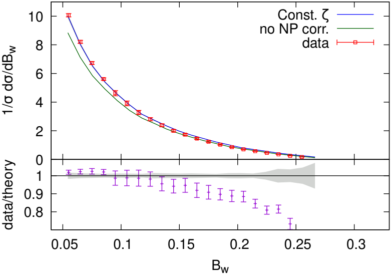
Quality of the fit for C, τ and y_3, obtained setting $\zeta(v) = \zeta(0)$.



Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.

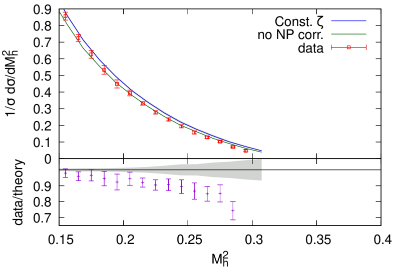


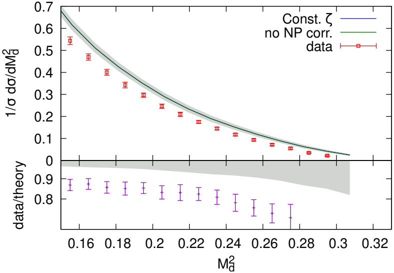


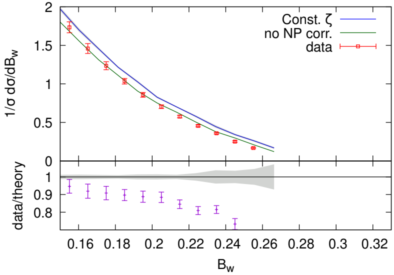


No Good fit in any region.

Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.







No Good fit in any region.

- The heavy jet mass, mass difference and broadening are well fitted far enough away from the two jet region with the newly computed ζ functions.
- On the other hand, it seems impossible to fit them using a constant, two-jet limit ζ.

Variation	α_s	α_0	χ^2	$\chi^2/N_{\rm deg}$
Default setup	**0.1182**	**0.64**	**7.3**	**0.17**
Renormalization scale $Q/4$	0.1202	0.60	9.1	0.21
Renormalization scale Q	0.1184	0.68	8.7	0.20
NP scheme (B)	0.1198	0.77	7.0	0.16
NP scheme (C)	0.1206	0.80	5.4	0.12
NP scheme (D)	0.1194	0.66	5.8	0.13
P-scheme	0.1158	0.62	10.7	0.24
D-scheme	0.1198	0.79	5.7	0.13
no scheme	0.1176	0.58	9.2	0.21
No heavy to light correction	0.1186	0.67	6.8	0.16
Herwig 6	0.1180	0.59	15.9	0.36
Herwig 7	0.1180	0.60	12.0	0.27
Ranges (2)	0.1174	0.62	12.7	0.23
Ranges (3)	0.1188	0.69	2.7	0.08
Replica method (around average)	0.1192	0.61	7.0	0.16
Replica method (around default)	0.1192	0.61	7.0	0.16
y_3 clustered	0.1174	0.66	8.2	0.19

We have considered several variations of the methods. They lead substantially to the same picture, with a spread in the value of α_s of the order of 2%.

Hadron mass effects are particularly important ...

Conclusions?

- New results cast doubts on the "traditional" implementation of power corrections. For observables like the heavy jet mass and jet mass difference, these doubts also concern the use of the traditional method in the resummed part of the cross section.
- Our "close your eyes and do it" approach turned into a mixed success. The new result seems to be favoured by data when staying far away from the two-jet region, but we are unable to obtain good fits in a reasonably extended three-jet range for all variables considered.
- It is customary, in e^+e^- shape variable studies, to include resummation effects in the whole 3-jet range, and sometimes also beyond. These effects increase the cross section also in the 3-jet region, and lead to a reduction in the extracted value of α_s. In our N^{3}LO fit we do not seem to need this increase to get values of α_s near the workd average.

- The interplay of the new results with soft gluon resummation needs further investigation. Our results are based upon the assumption that effects associated with higher order radiation remain effectively of higher orders, i.e. that power corrections to real and virtual soft gluon corrections cancel among each other. One can argue that such cancellation must occour between the virtual correction to $q\bar{q}$ production and the $q\bar{q}g$ cross section when g becomes unresolved. One can hope that the same happens in the $q\bar{q}g$ production.

The two-jet limit case

We can guess that the correction of order λ, up to orders α_s, has the form

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}v}\right|_{\lambda} = N\lambda\left[\delta'(v)\zeta_c + (\delta'(v)V_1 + \delta(v)V_2)\alpha_s + \frac{\mathrm{d}}{\mathrm{d}v}\left(\frac{\mathrm{d}\sigma_{q\bar{q}g}}{\mathrm{d}v}\zeta(v)\right)\right]$$

where we assume some regularization for the last term. But the whole thing must integrate to 0 (no λ term in total cross section). Thus, divergences in the middle and last term must cancel, and the α_s correction must be a hard correction.