

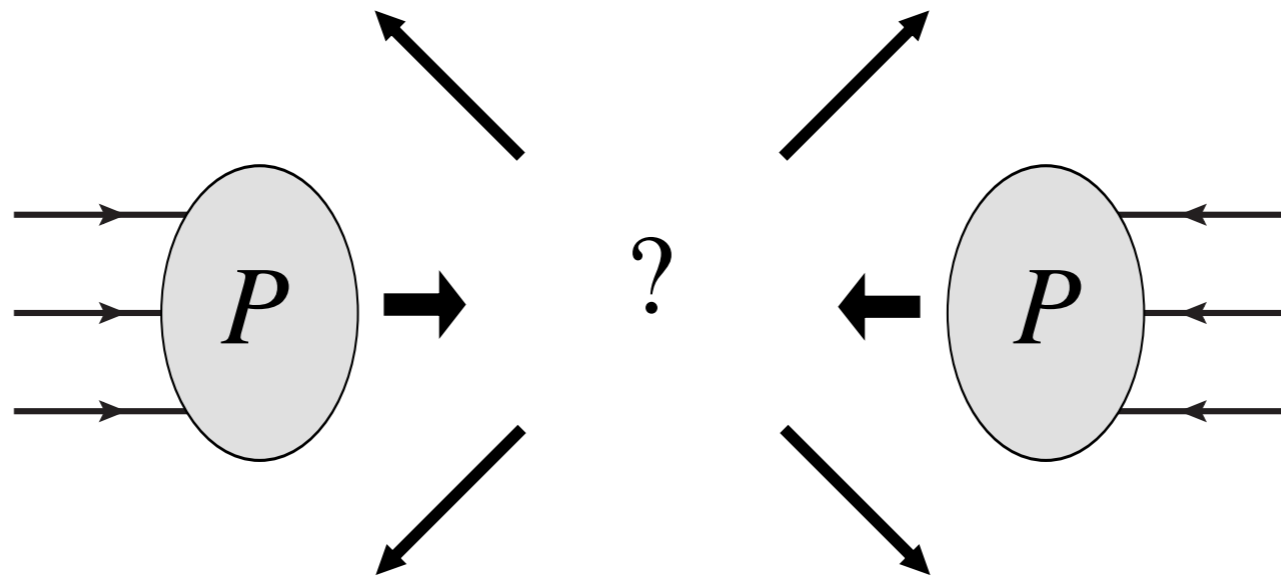
A dress of **flavour** to suit any jet

Rhorry Gauld

Workshop on Tools for High Precision LHC Simulations  
Castle Ringberg (03/11/22)



MAX-PLANCK-INSTITUT  
FÜR PHYSIK



$$PP \rightarrow f + X$$

$f$  composed of :

- leptons
- hadrons
- photons
- missing  $E_T$
- jets
- ...

$$d\sigma_{PP \rightarrow f+X}^{\text{meas.}} \quad \text{vs} \quad d\sigma_{PP \rightarrow f+X}^{\text{theory}}$$

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Focussing on IRC (InfraRed and Collinear) safe observables:

- Those not impacted by collinear splitting(s) or emission(s) of soft particles
- ➔ Can (reliably) use fixed-order perturbation theory

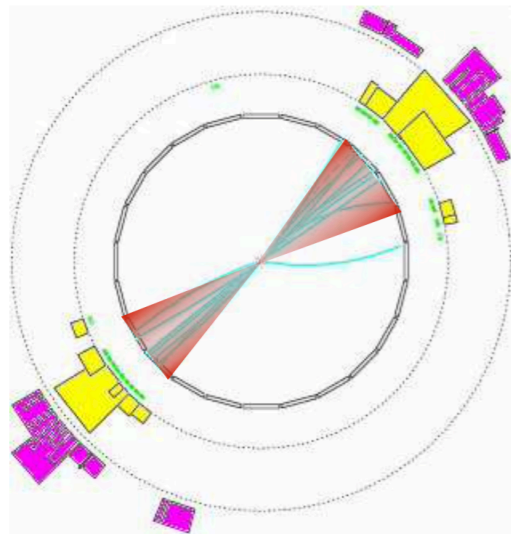
KLN theorem: (Kinoshita '62, Lee & Nauenberg '64)

- For such observables, a cancellation of IRC divergences between virtual and real emissions is ensured (order-by-order)

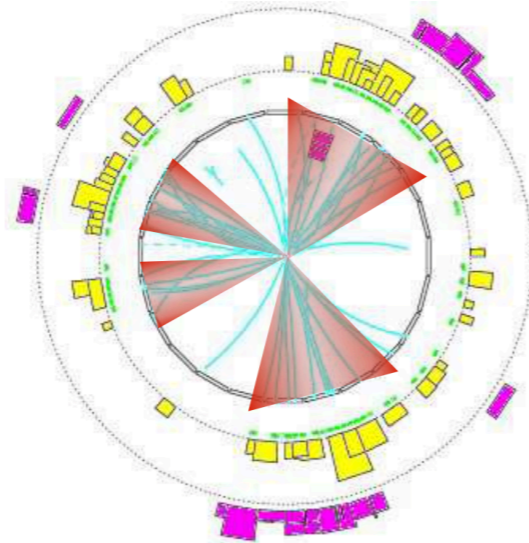
Comments:

- IRC unsafe observables can of course be defined, but then all order-resummation is required (e.g. PDF evolution, obs. dependent resummation)

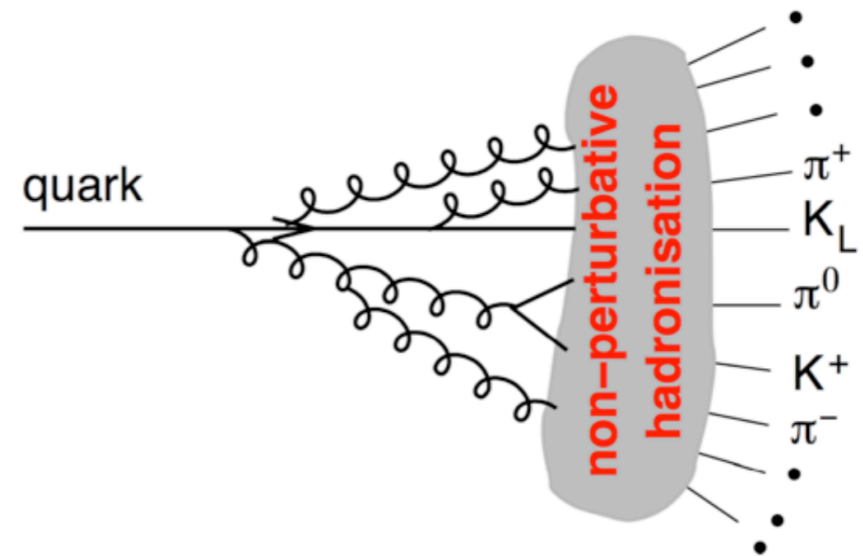
# Jet algorithms



2 clear jets



3 jets?  
or 4 jets?



## Experimentally:

- Apply an algorithm to particle flow objects (Kaons, Pions,...)  
(e.g. ATLAS arXiv:1703.10485, CMS arXiv:1706.04965, LHCb arXiv:1310.8197)
- ➔ Reconstruct a hadronic jet (~collimation of hadronic radiation)

## Theoretically:

- If IRC safe, can be applied to parton-level fixed-order predictions

$$d\sigma_{PP \rightarrow f+X}^{\text{meas.}} \quad \text{vs} \quad d\sigma_{PP \rightarrow f+X}^{\text{fixed-order}}$$

# The anti- $k_T$ algorithm

(Cacciari, Salam, Soyez arXiv:0802.1189)

Initialise a list of particles (pseudo jets)

Introduce distance measures between particles (pseudo jets) and a Beam:

$$d_{ij} = \min \left( k_{Ti}^{2p}, k_{Tj}^{2p} \right) \frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = k_{Ti}^{2p}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$



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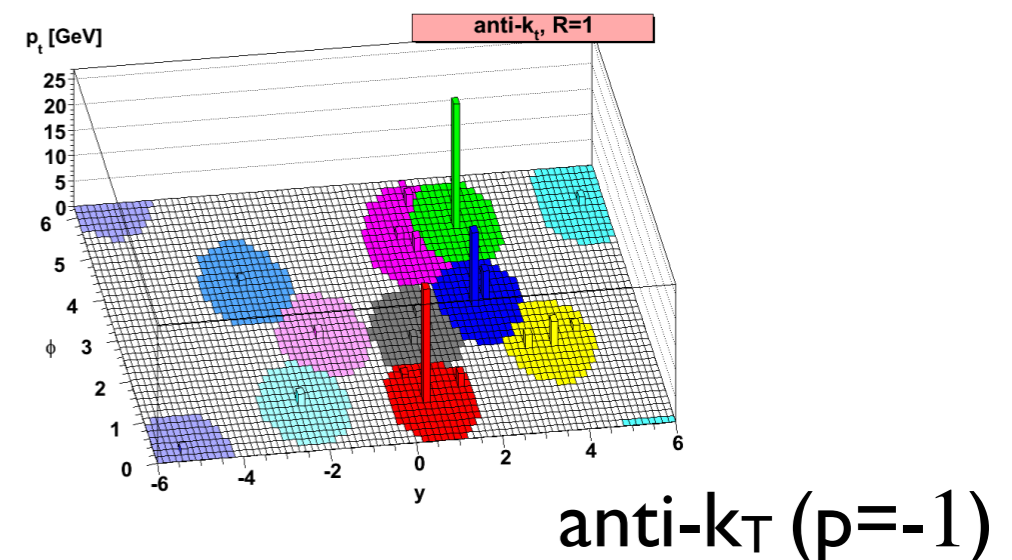
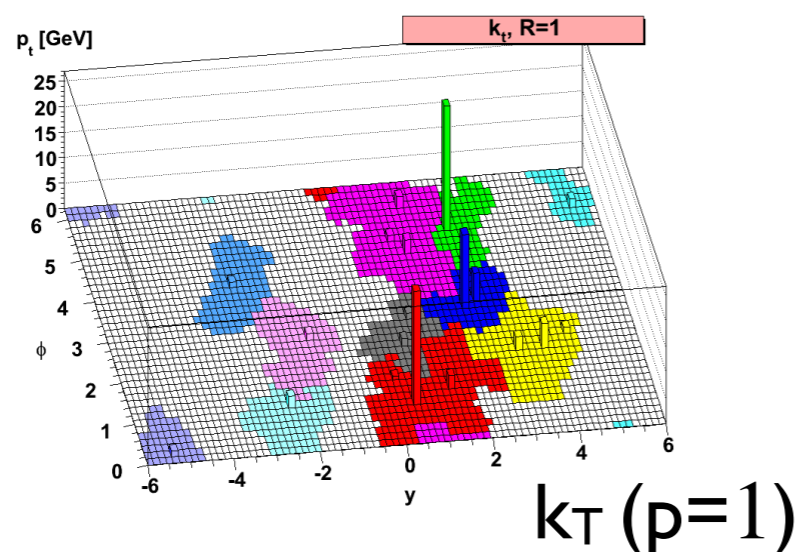
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(Inclusive) clustering proceeds by identifying the min. distance:

- If it is  $d_{ij}$  combine particles  $ij$  (update list to contain combined particle)
- If it is  $d_{iB}$ , identify  $i$  as a jet and remove from list

[repeat until list is empty]

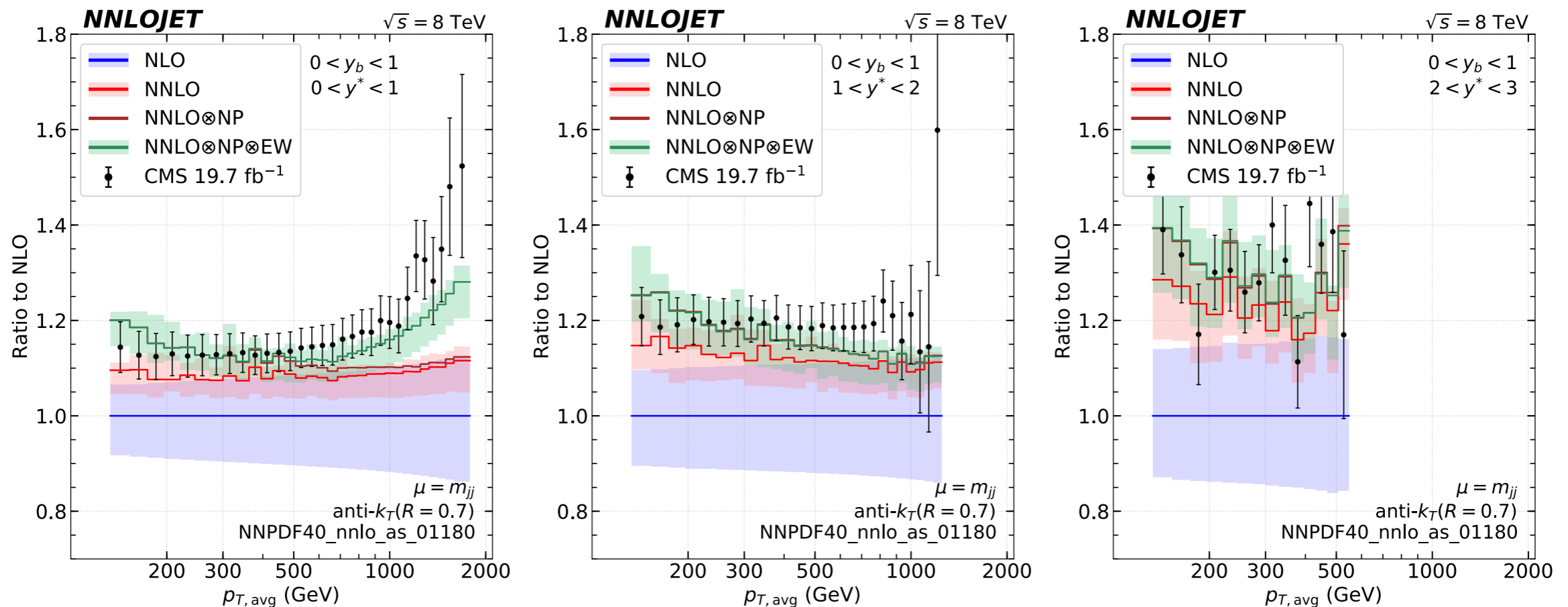


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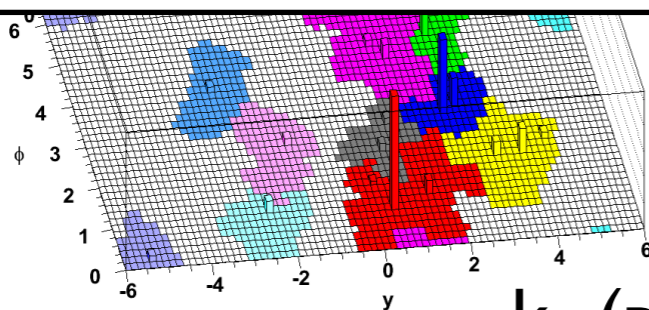
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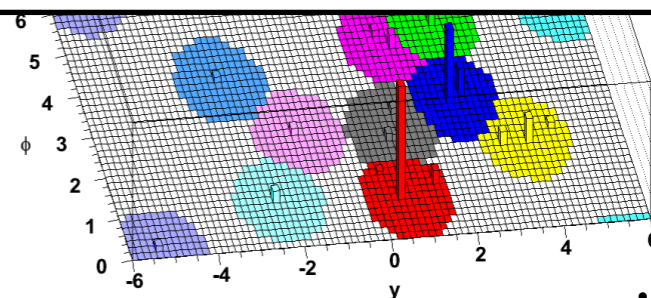


Chen et al. from arXiv:2204.10173, comparing to CMS dijet data



$k_T$  ( $p=1$ )

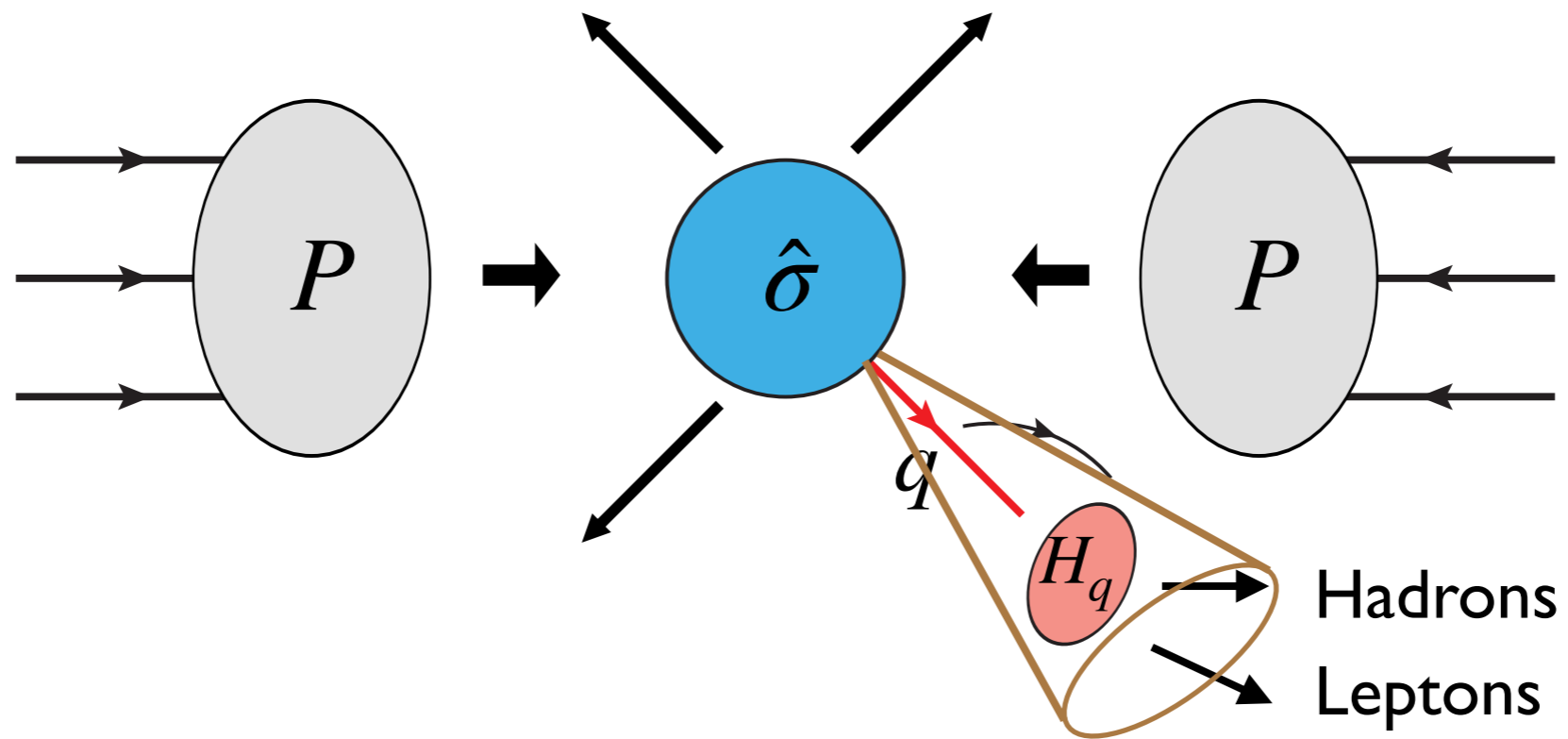
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anti- $k_T$  ( $p=-1$ )

# The (heavy) flavour of jets

(so far we focussed only on the momentum of the jets)

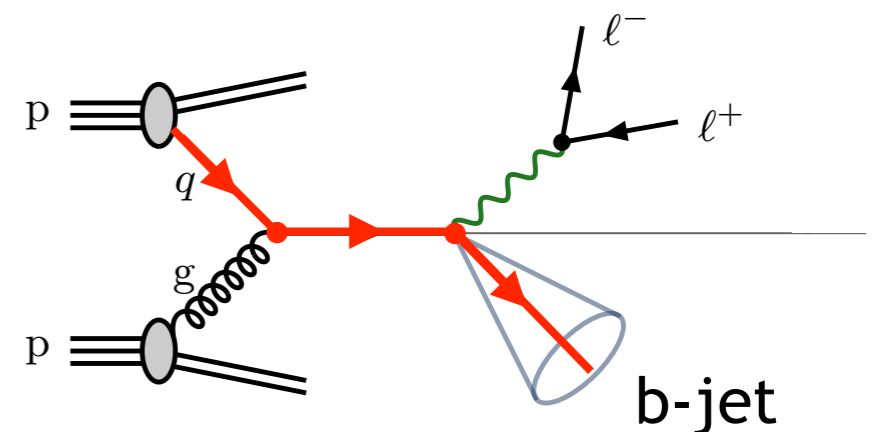
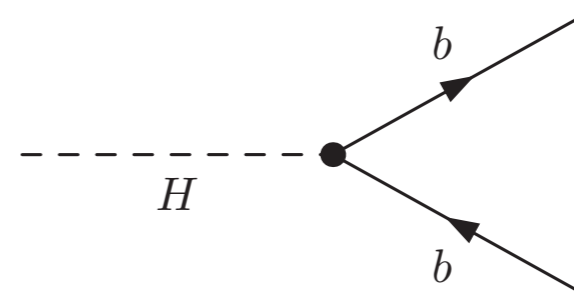


(i) Higgs physics (hadronic decays)

(ii) Top-quark physics ( $|V_{tb}| \sim 1$ )

(iii) New physics searches (f-jet +  $E_T^{\text{miss}}$ )

(iv) Gauge-boson + heavy-flavour





# The (heavy) flavour of jets

Examples of experimental approaches of defining jet flavour:

ATLAS arXiv:1504.07670, CMS arXiv:1712.07158, LHCb arXiv:1504.07670

Generally (at level of published data/truth level):

i) First identify flavour-blind anti- $k_T$  jets in a fiducial region

ii) Tag these jets with flavour by the presence of 1 or more D/B hadrons

$$\Delta R(j, D/B) < 0.5$$

iii) [ATLAS/LHCb] Additionally make a  $p_T$  requirement on the D/B hadrons

$$p_T^{D/B} > 5 \text{ GeV}$$

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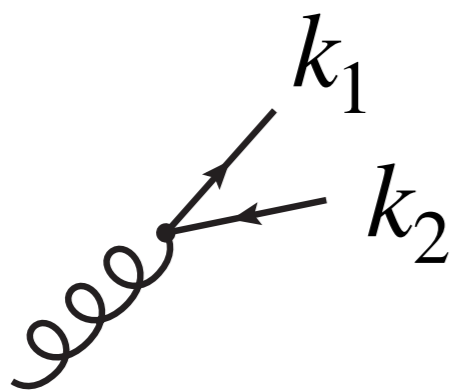
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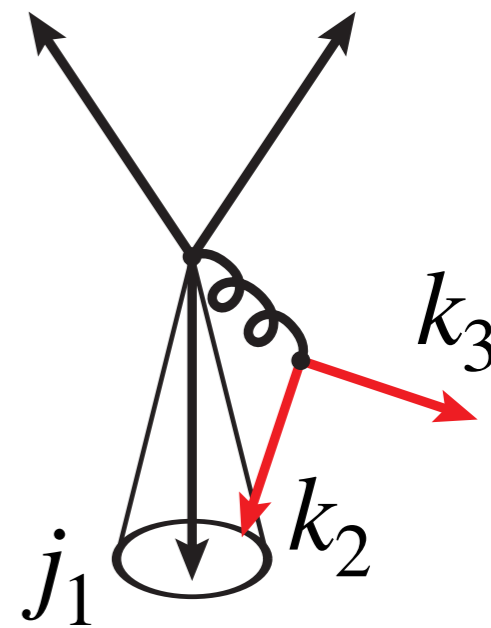
Many IRC problems...



the 'even tag'



collinear 'cutout'



soft 'pollution'

# An elegant solution (flavour $k_T$ algorithm)

(Banfi, Salam, Zanderighi hep-ph/0601139)

A flavour dependent jet algorithm (i.e. flavoured particle inputs)

1) Flavour number assignment:

$$q = +1, \quad \bar{q} = -1$$

2) Flavour dependent distance measures (and hence clusterings)

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}{R^2} \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}, k_{tj})^\alpha & \text{softer of } i, j \text{ is unflavoured.} \end{cases}$$

3) Rapidity-dependent Beam distances (differentiates soft vs. initial collinear)

$$d_{fB} = \max \left( p_{T,f}, p_T^B(y) \right)^\alpha \min \left( p_{T,f}, p_T^B(y) \right)^{2-\alpha}$$
$$p_T^B(y) = \sum_i p_{T,i} \left( \Theta(y_i - y) + \Theta(y - y_i) e^{y_i - y} \right)$$

Never adopted by experiment (jet calibration, flavour tracking, ...)

# An elegant solution (flavour $k_T$ algorithm)

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A flavour dependent jet algorithm (i.e. flavoured particle inputs)

I) Flavour number assignment:

In the last months, there has been a lot of activity on this topic:

- (i) Soft Drop grooming approach, [Caletti et al. 2205.01109](#)
- (ii) Winner-Takes-All approach, [Caletti et al. 2205.01117](#)
- (iii) Flavoured anti- $k_T$ , [Czakon et al. 2205.11879](#)
- (iv) Successive iterations of flavour- $k_T$  and anti- $k_T$ , [Caletti et al. 2108.10024](#)
- (v) Jet angularities & primary Lund jet plane, [Fedkevych et al. 2202.05082](#)

...

$$p_T^B(y) = \sum_i p_{T,i} (\Theta(y_i - y) + \Theta(y - y_i)e^{y_i - y})$$

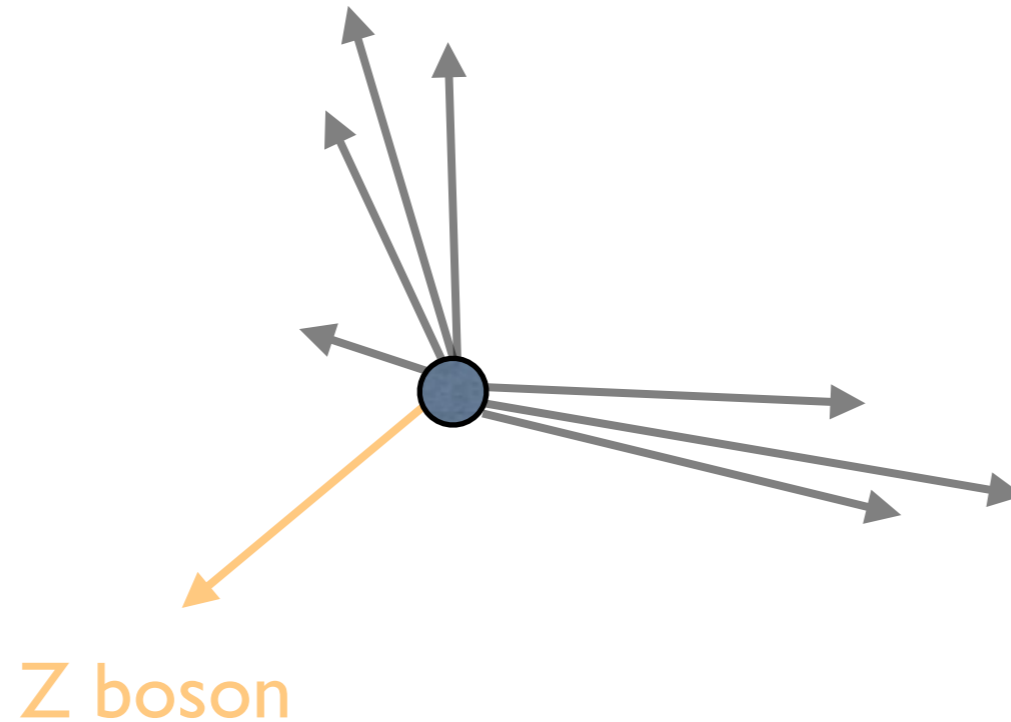
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# A dress of **flavour** to suit any jet

(**RG**, Huss, Stagnitto arXiv:2208.11138)

**Our motivation:** A well defined flavour algorithm applicable to anti- $k_T$  jets  
(actually, any jet)

Toy event



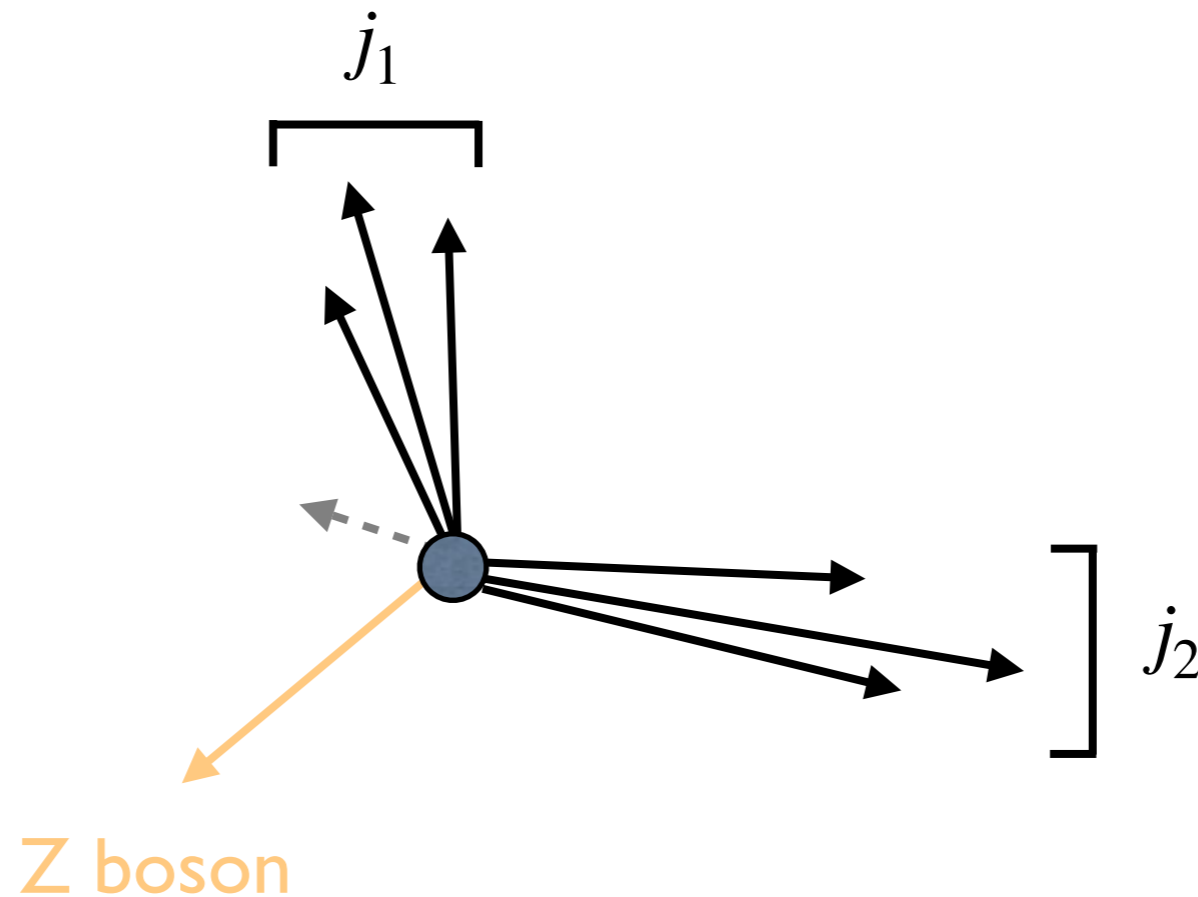


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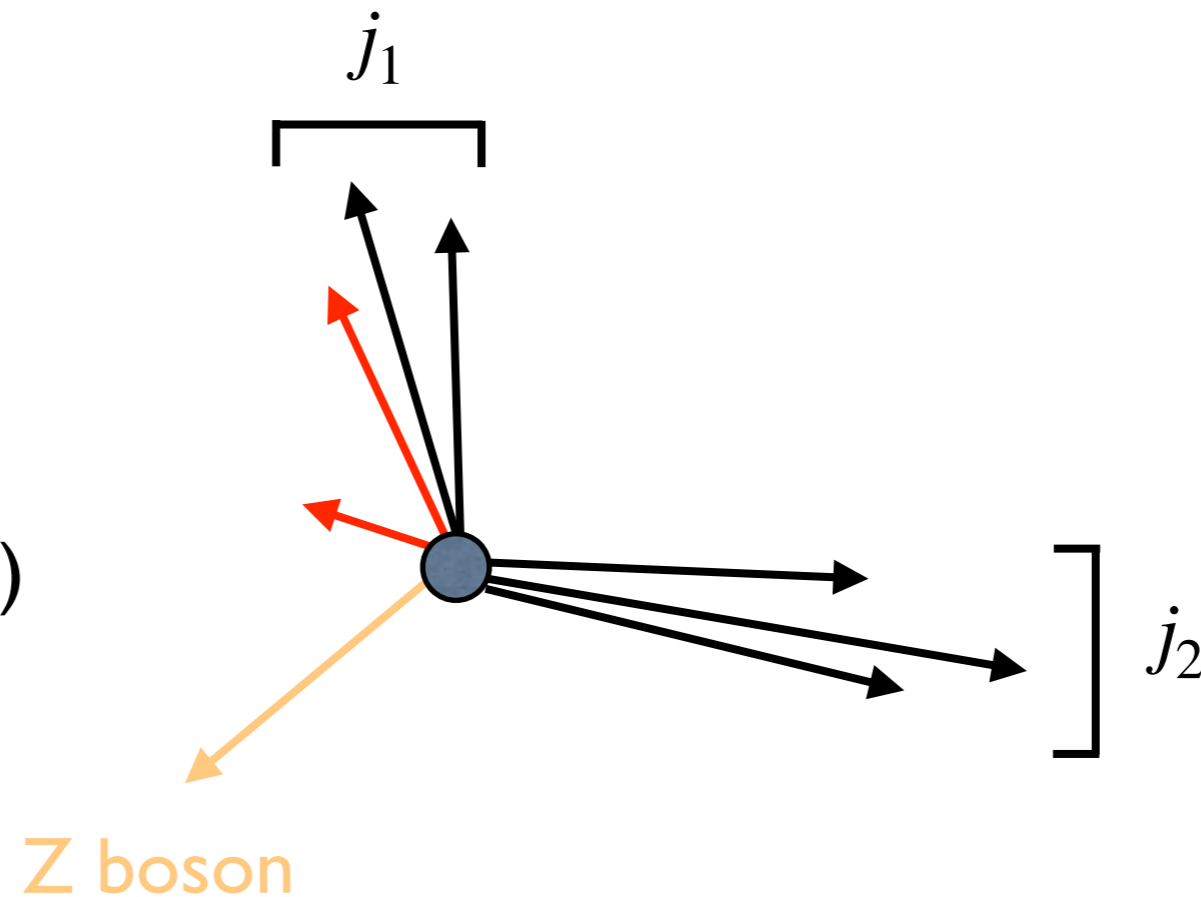
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Flavoured particles

b-quark (theory)

secondary vertex (exp.)



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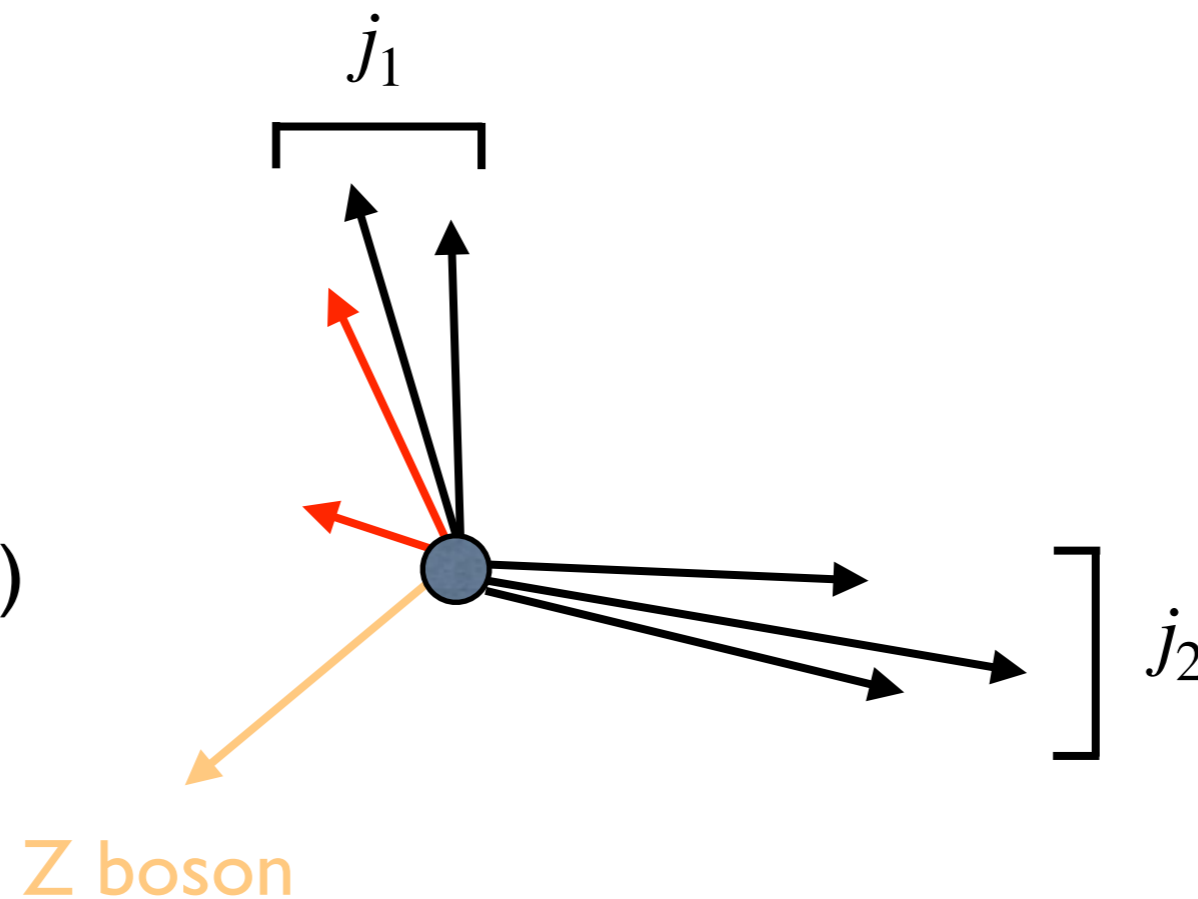
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set of anti- $k_T$  jets  $\{j_1, \dots, j_m\}$

set of flavoured objects  $\{\hat{f}_1, \dots, \hat{f}_n\}$

an assignment of the flavoured objects to these jets

# (collinear safe) flavoured objects

(**RG**, Huss, Stagnitto arXiv:2208.11138)

flavoured particles (quarks, hadrons) not collinear safe. Define new objects:

- i) Initialise a list of all particles
- ii) Add to the list all flavoured particles, removing any overlap
- iii) Calculate the distances  $d_{ij} = \Delta R_{ij}^2$  between all particles
- iv) If  $d_{ij}^{\min} > \Delta R_{\text{cut}}^2$  terminate the clustering.

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iv) If  $d_{ij}^{\min} > \Delta R_{\text{cut}}^2$  terminate the clustering. Otherwise:

1. (i & j flavourless) replace i & j in the list with combined object ij
2. (i & j flavoured) remove flavoured objects i & j from the list
3. (i or j flavoured) combine i and j if the criterion:

$$\frac{\min(p_{T,i}, p_{T,j})}{p_{T,i} + p_{T,j}} > z_{\text{cut}} \left( \frac{\Delta R_{ij}}{R_{\text{cut}}} \right)^\beta$$

[Soft-drop]

(Larkoski et al. arXiv:1402.2657)

Otherwise remove flavourless of i/j from list

[Repeat until list empty, or no flavoured particles left]



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ii)

This procedure alters momenta of particles (but not flavour)

$$\{f_1, \dots, f_n\} \rightarrow \{\hat{f}_1, \dots, \hat{f}_n\}$$

flavoured particles  $\rightarrow$  flavoured 'clusters' or 'objects'

Essentially dresses flavoured particles with collinear radiation!

Soft drop ensures that genuinely soft particles remain soft!

$p_{T,i} \mp p_{T,j}$

$(\Lambda_{\text{cut}})$

(Larsson et al. arXiv:1902.2657)

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[Repeat until list empty, or no flavoured particles left]

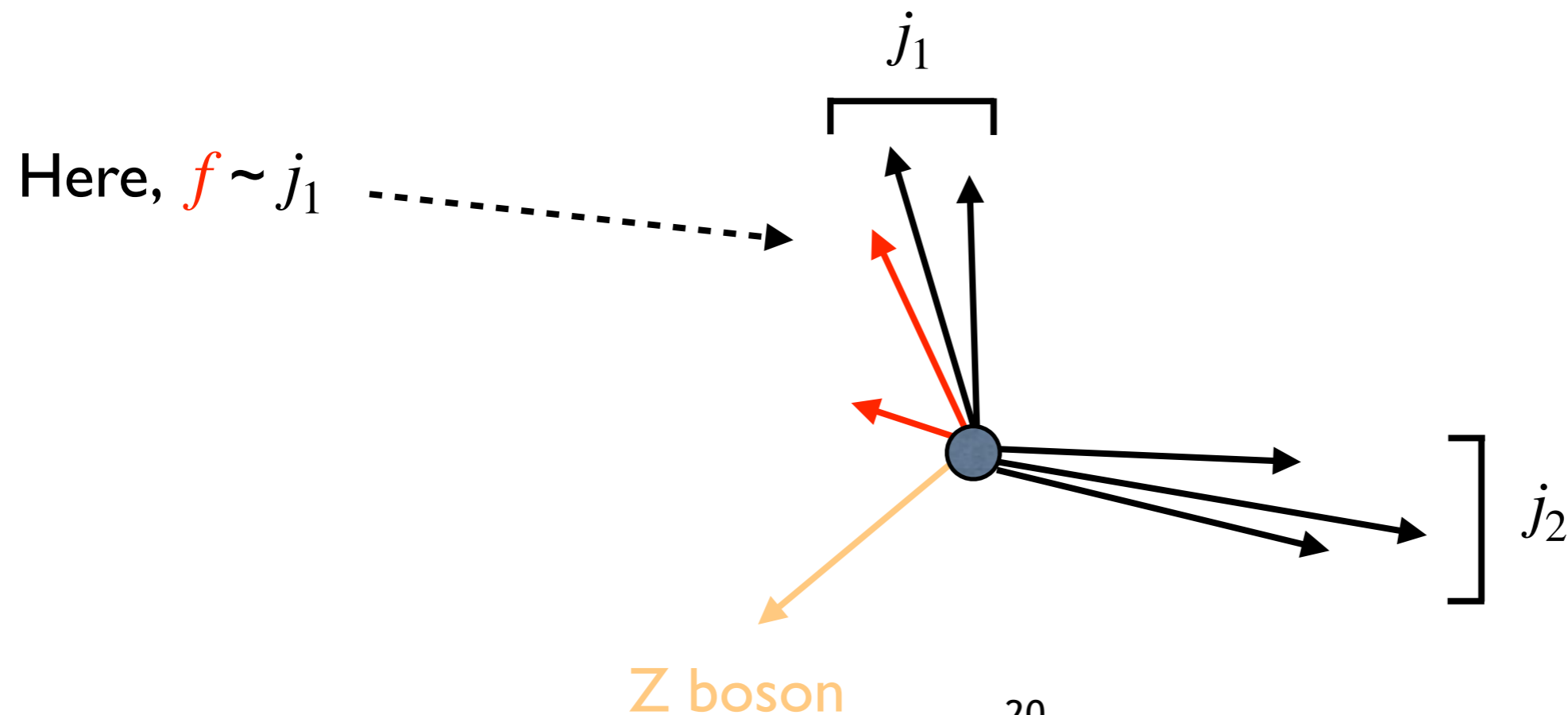
# The flavour dressing algorithm

(**RG**, Huss, Stagnitto arXiv:2208.11138)

We now have  $\{j_1, \dots, j_m\}$  and  $\{\hat{f}_1, \dots, \hat{f}_n\}$

We introduce an **Association criterion** for  $\hat{f}_a$  and  $j_b$  (some possibilities):

- the flavoured particle  $f_a$  is a constituent of jet  $j_b$  (applicable to unstable  $f_a$ )
- or  $\Delta R(\hat{f}_a, j_b) < R_{\text{tag}}$
- or Ghost association of  $\hat{f}_a$  (include direction of  $\hat{f}_a$  in anti- $k_T$  clustering)



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Introduce a **Counting** or **Accumulation** for flavour:

- with charge info. ( $q$  vs  $\bar{q}$ ), then  $q = +1$  and  $\bar{q} = -1$  (net flavour is sum)
- if one cannot (e.g. experiment),  $q = \bar{q} = 1$  (net flavour is sum modulo 2)  
[i.e. jets with even number of  $q_i + \bar{q}_j$  are NOT flavoured]

# The flavour dressing algorithm

(**RG**, Huss, Stagnitto arXiv:2208.11138)

We now have  $\{j_1, \dots, j_m\}$ ,  $\{\hat{f}_1, \dots, \hat{f}_n\}$ , association, and counting rules

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We now have  $\{j_1, \dots, j_m\}$ ,  $\{\hat{f}_1, \dots, \hat{f}_n\}$ , association, and counting rules

Dressing algorithm:

- Calculate a set of distances between the flavoured objects, jets and beam:
  - ▶ [ff]  $d_{ab}$  between all all flavoured objects  $\hat{f}_a$  and  $\hat{f}_b$
  - ▶ [fj]  $d_{ab}$  between  $\hat{f}_a$  and  $j_b$  ONLY if there is an association
  - ▶ [fB]  $d_{aB}$  for all  $\hat{f}_a$  without a jet association



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  - [fB]  $d_{aB}$  for all  $\hat{f}_a$  without a jet association
- Find the minimum distance of all entries in the list
  - if it is an [fj] assign  $\hat{f}_a$  to  $j_b$  (removing entries involving  $\hat{f}_a$  from list)
  - otherwise just remove  $\hat{f}_a$  [fB] or  $\hat{f}_a$  and  $\hat{f}_b$  [ff] from the list

[repeat until list empty]

- The flavour of each jet is then just the accumulation of its flavour

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Dressing algorithm:

Here we use the distance measures proposed in flavour- $k_T$

(Banfi, Salam, Zanderighi hep-ph/0601139)

$$d_{ab} = \Delta R_{ab}^2 \max \left( p_{T,a}^\alpha, p_{T,b}^\alpha \right) \min \left( p_{T,a}^{2-\alpha}, p_{T,b}^{2-\alpha} \right)$$

$$d_{aB_\pm} = \max \left( p_{T,a}^\alpha, p_{T,B_\pm}^\alpha(y_{\hat{f}_a}) \right) \min \left( p_{T,a}^{2-\alpha}, p_{T,B_\pm}^{2-\alpha}(y_{\hat{f}_a}) \right)$$

Another viable option is Jade:  $d_{ab} = 2p_a \cdot p_b$

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# tests of the algorithm ( $e^+e^-$ )

(**RG**, Huss, Stagnitto arXiv:2208.11138)

Consider the process  $e^+e^- \rightarrow 2$  jets at fixed-order using  $k_T$  algorithm

Look at ‘bad’ events (i.e. where we do not find 2 flavoured jets)

The ‘bad’ cross-section should vanish in the  $y_3 \rightarrow 0$  limit  
(that is the limit of extremely soft and/or collinear emissions)

These tests originally proposed/shown in the original flavour- $k_T$  study

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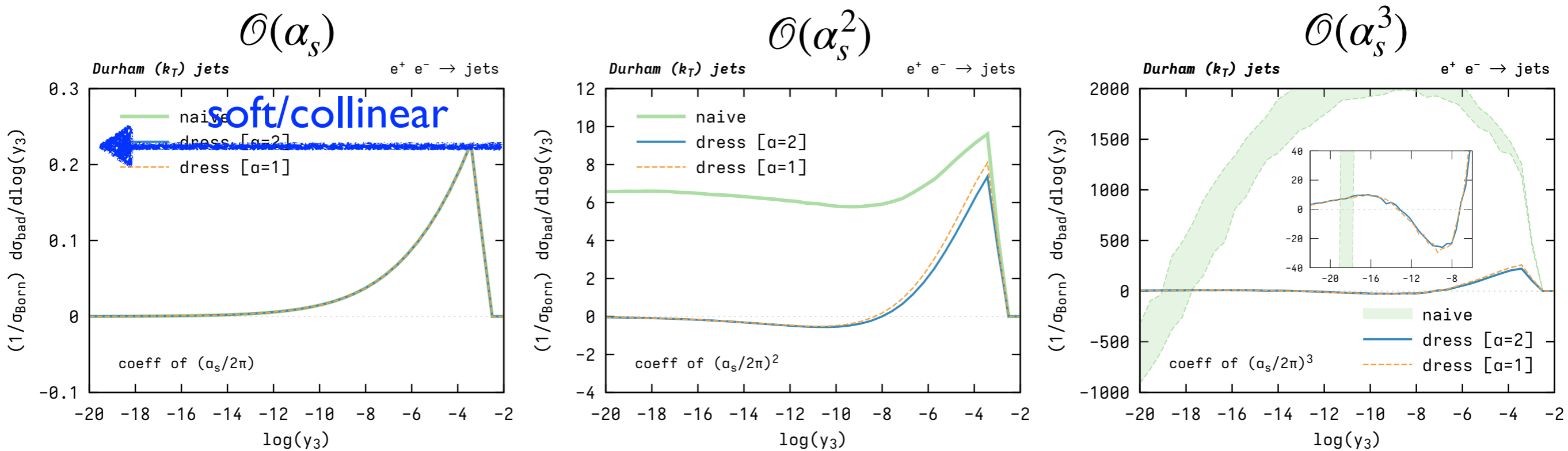
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# tests of the algorithm (pp)

(**RG**, Huss, Stagnitto arXiv:2208.11138)

Can also perform all-order ‘sensitivity’ tests using Parton Shower framework

In this case study, also use resolution variable to probe IRC sensitive regions  
(here we study the behaviour, rather than the bad cross-section vanishing)

Here consider dijet events (exclusive  $k_T$  algorithm) with  $E_T \geq 1$  TeV

We use the resolution variable:  $y_3^{k_T} = d_3^{k_T} / (E_{T,1} + E_{T,2})$  [Luca R’s talk]

(Buonocore et al. arXiv:2201.11519)

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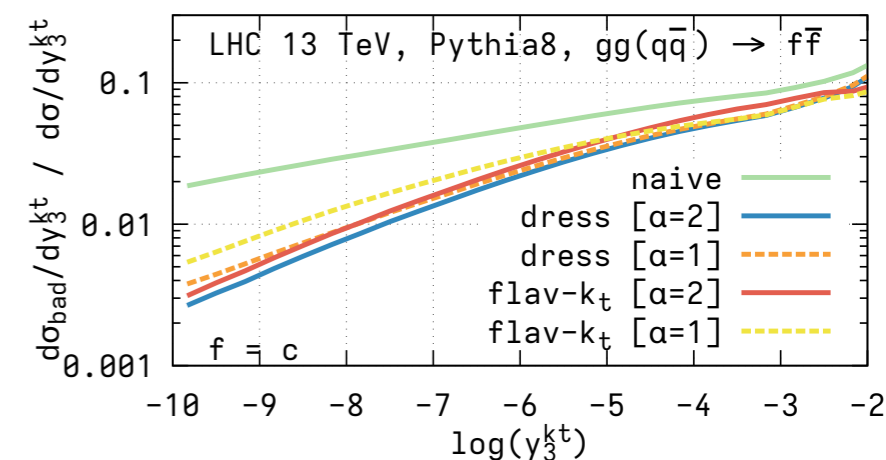
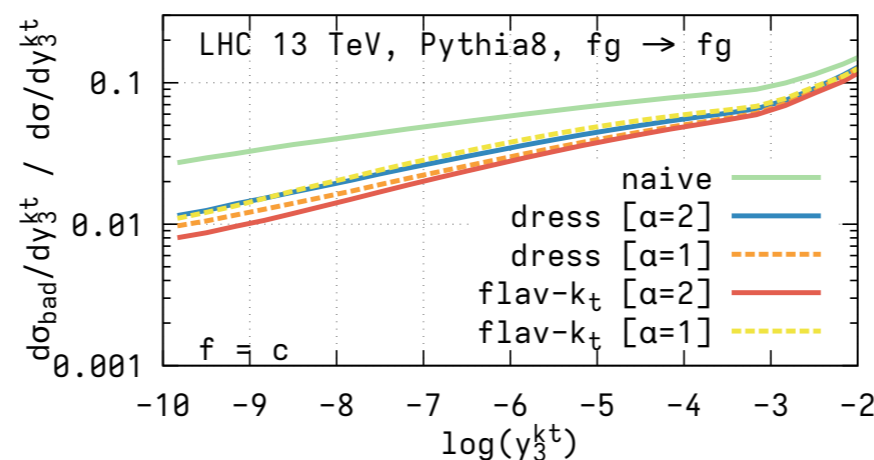
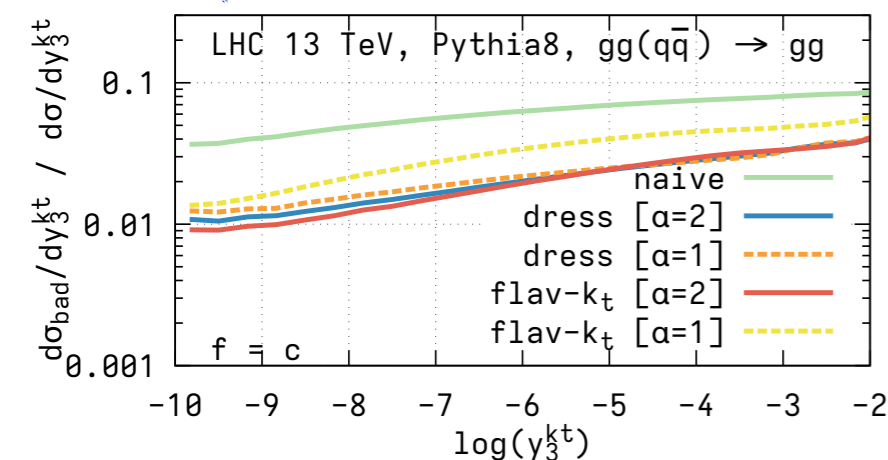
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soft/collinear



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# application of the algorithm (pp)

(**RG**, Huss, Stagnitto arXiv:2208.11138)

Now consider the process  $pp \rightarrow Z + b - \text{jet}$  in Fiducial region (13 TeV, CMS-like)

(N)NLO at fixed-order w/ NNLOJET, **RG** et al. arXiv:2005.03016

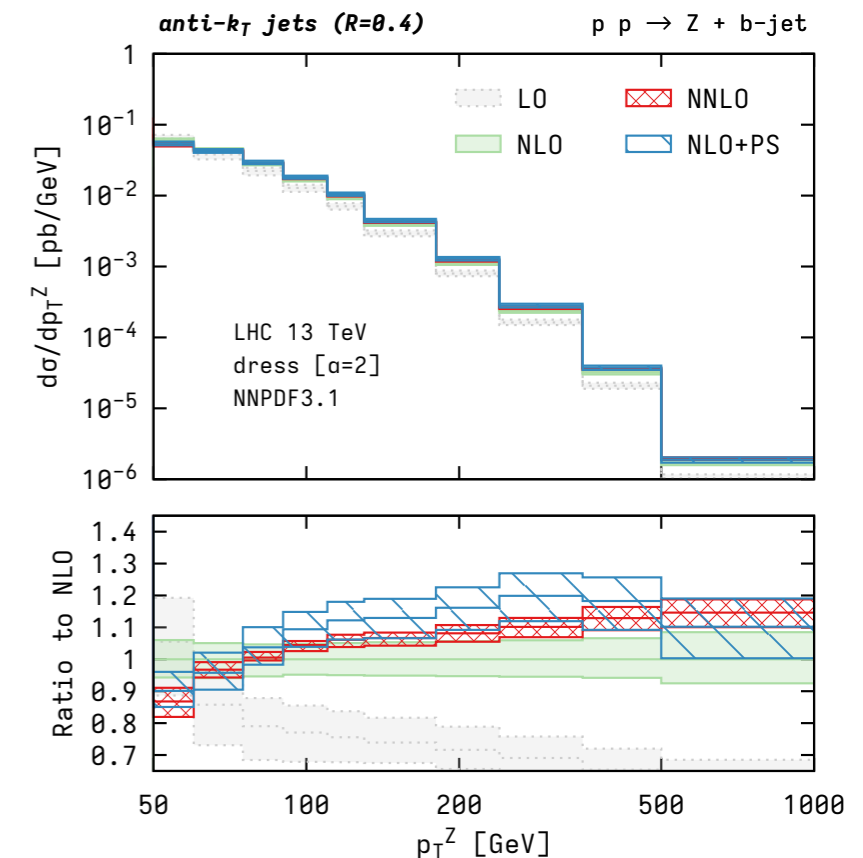
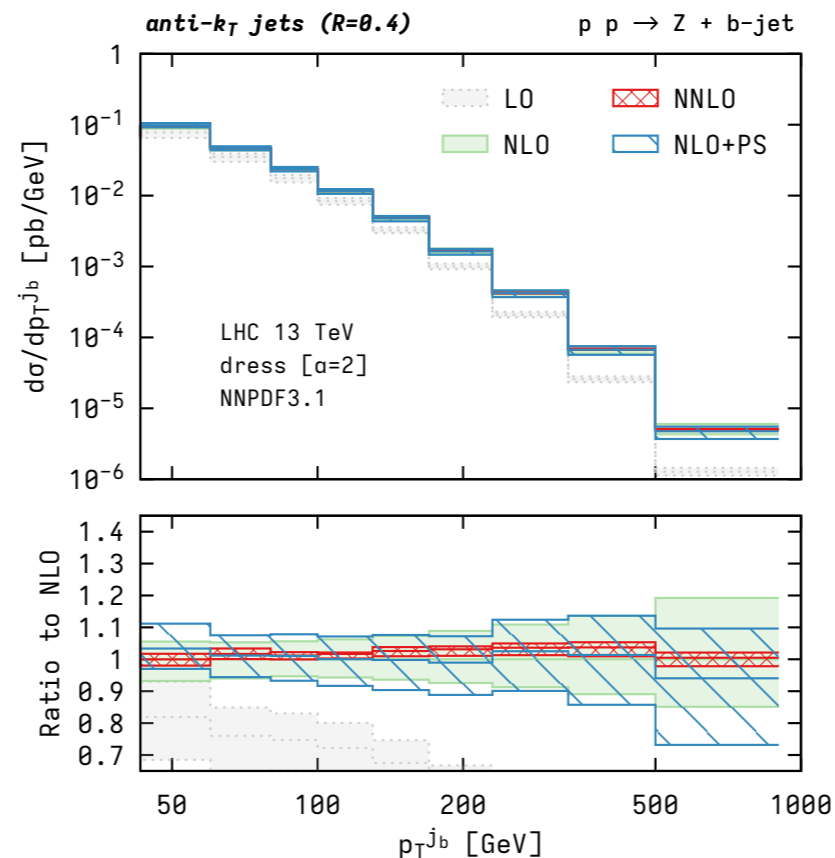
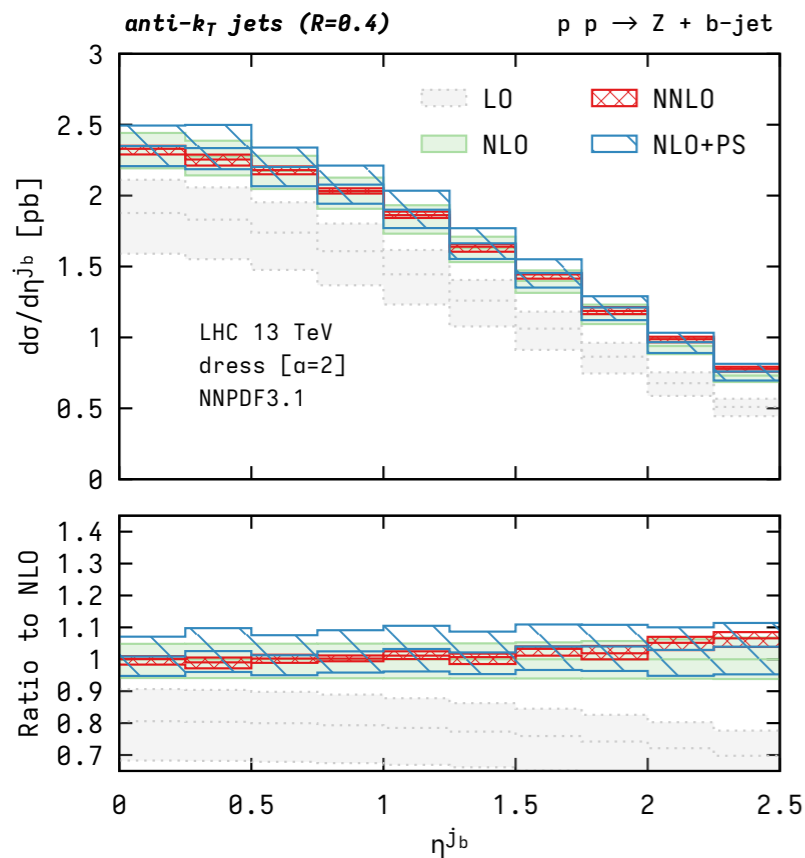
NLO+PS Hadron-level with aMC@NLO interfaced to Pythia8

Tests sensitivity to: all-order effects, hadronisation (also FO IRC safety in pp)

$\eta_{b\text{-jet}}$

$p_{T,b\text{-jet}}$

$p_{T,Z}$



We have proposed an algorithm for assigning flavour to jets:

- ▶ The approach is IRC safe (at least until N4LO, maybe more)
- ▶ It can be applied to any set of jets (provided they are IRC safe)
- ▶ It does **not** require flavoured particles to be part of the initial jet reco. (i.e. it can be applied to heavy flavour tagging at an experiment!)
- ▶ As only few evaluations of distance measures is required for the flavoured objects, an experimental realisation seems feasible (LHCb is very optimistic about this)

Finally (my opinion), IRC safety should be taken seriously for all processes.

Sometimes a signal process is less sensitive to such effects,  $Z(h \rightarrow b\bar{b})$

But background processes are often highly sensitive to them,  $Zb\bar{b}$



# Whiteboard

# Massive calculation (IRC unsafe def.)

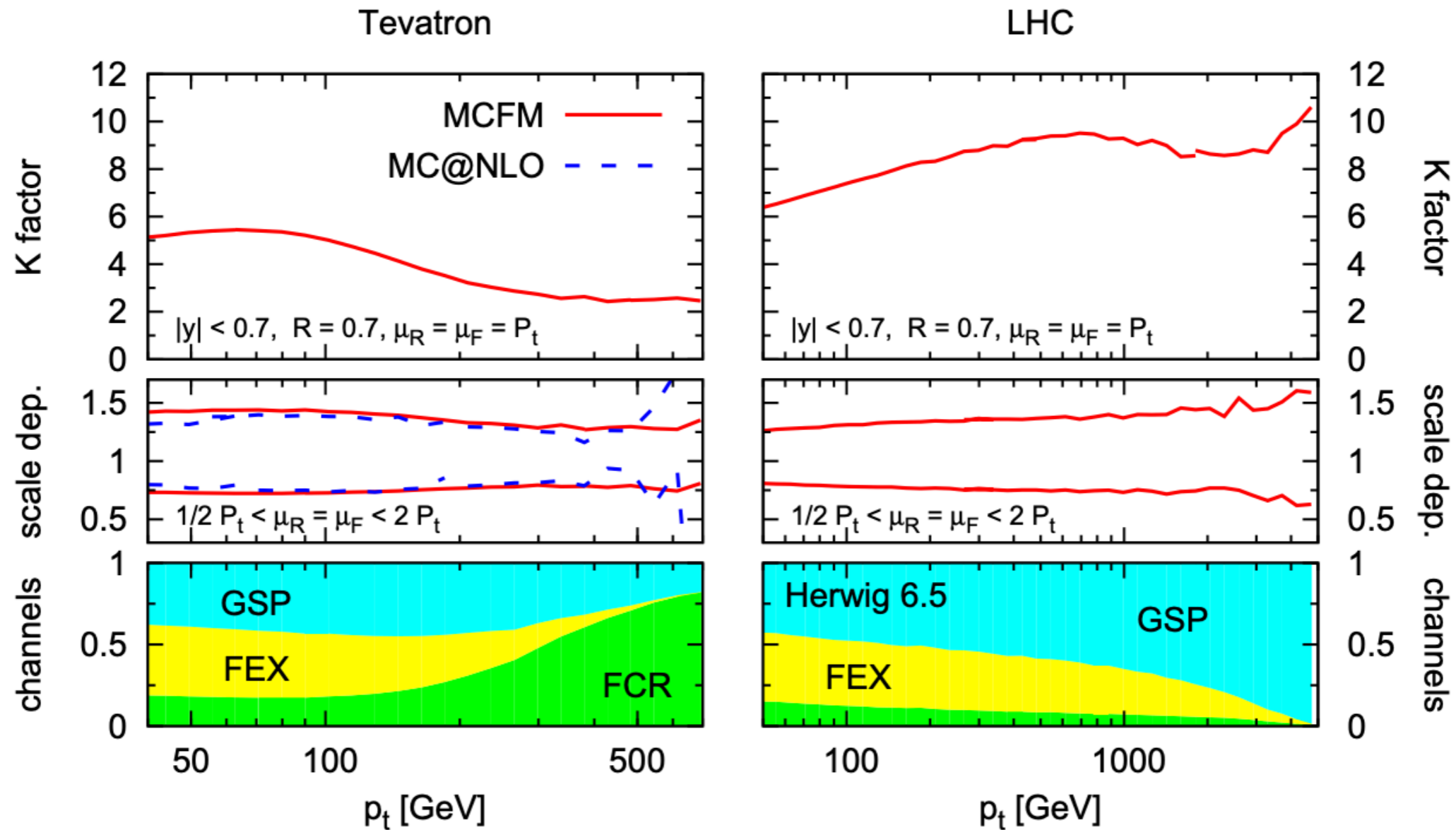


Figure 2: Top:  $K$ -factor for inclusive  $b$ -jet spectrum as computed with MCFM [10], clustering particles into jets using the  $k_t$  jet-algorithm [9] with  $R=0.7$ , and selecting jets in the central rapidity region ( $|y| < 0.7$ ). Middle: scale dependence obtained by simultaneously varying the renormalisation and factorisation scales by a factor two around  $p_t$ , the transverse momentum of the hardest jet in the event. Bottom: breakdown of the Herwig [11] inclusive  $b$ -jet spectrum into the three major hard underlying channels cross sections (for simplicity the small  $bb \rightarrow bb$  is not shown).

# Unfolding for Z+b-jet

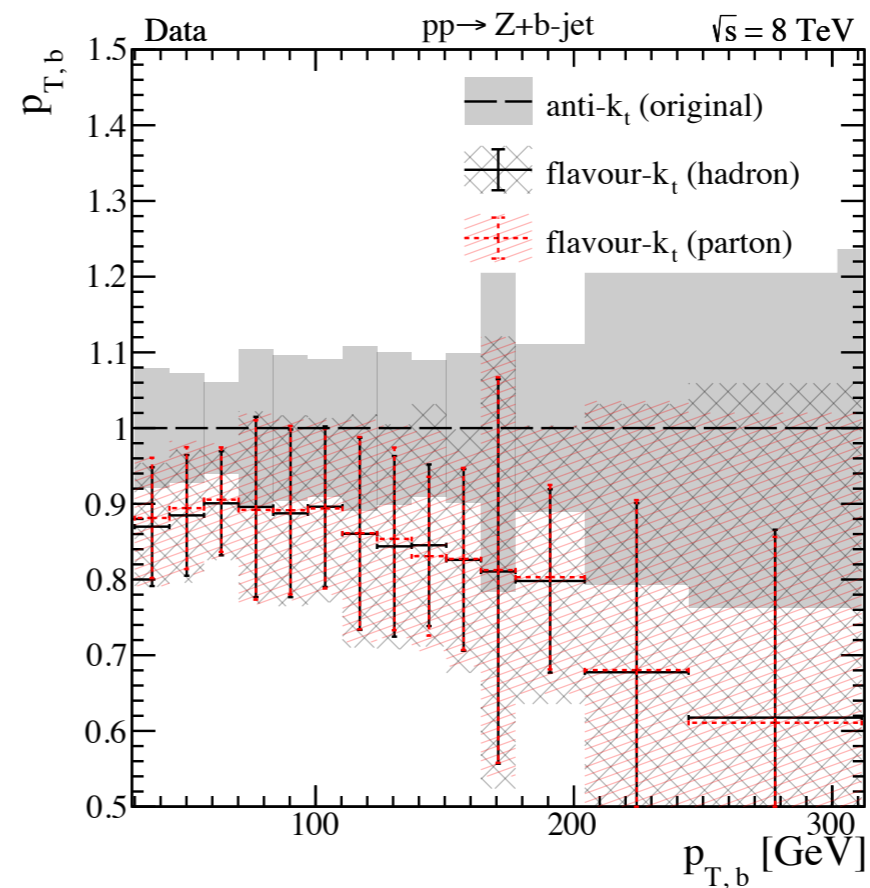
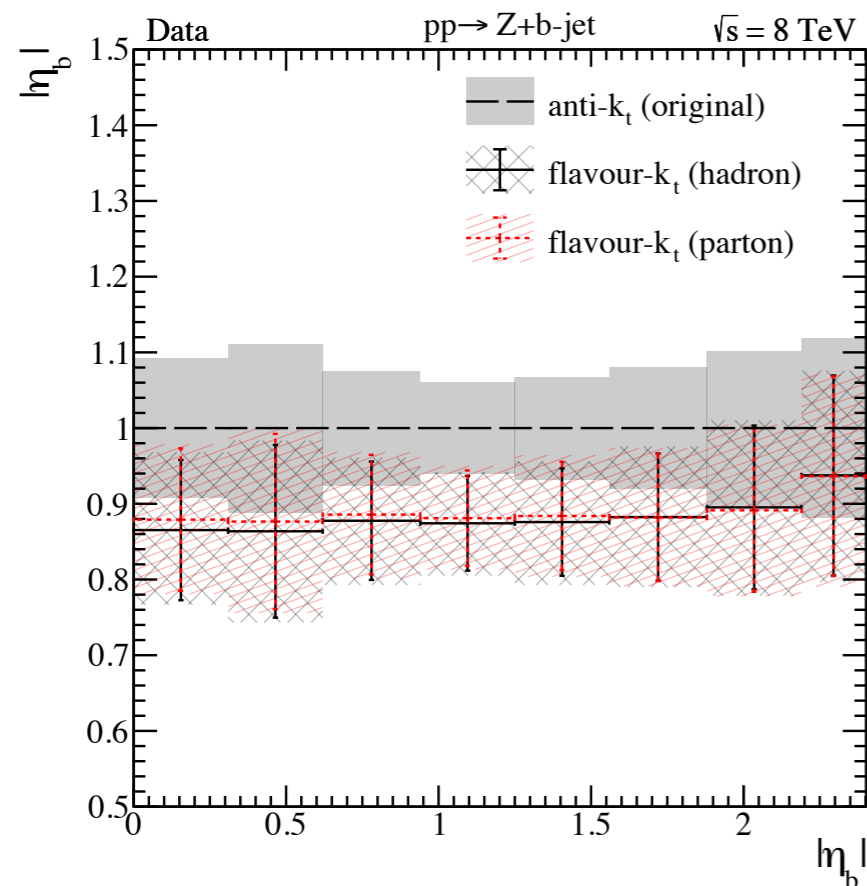
How to account for theory-experiment mismatch?

Use an NLO + Parton Shower prediction (which can evaluate both)

1) Prediction at parton-level, flavour- $k_T$  algorithm (**Theory**)

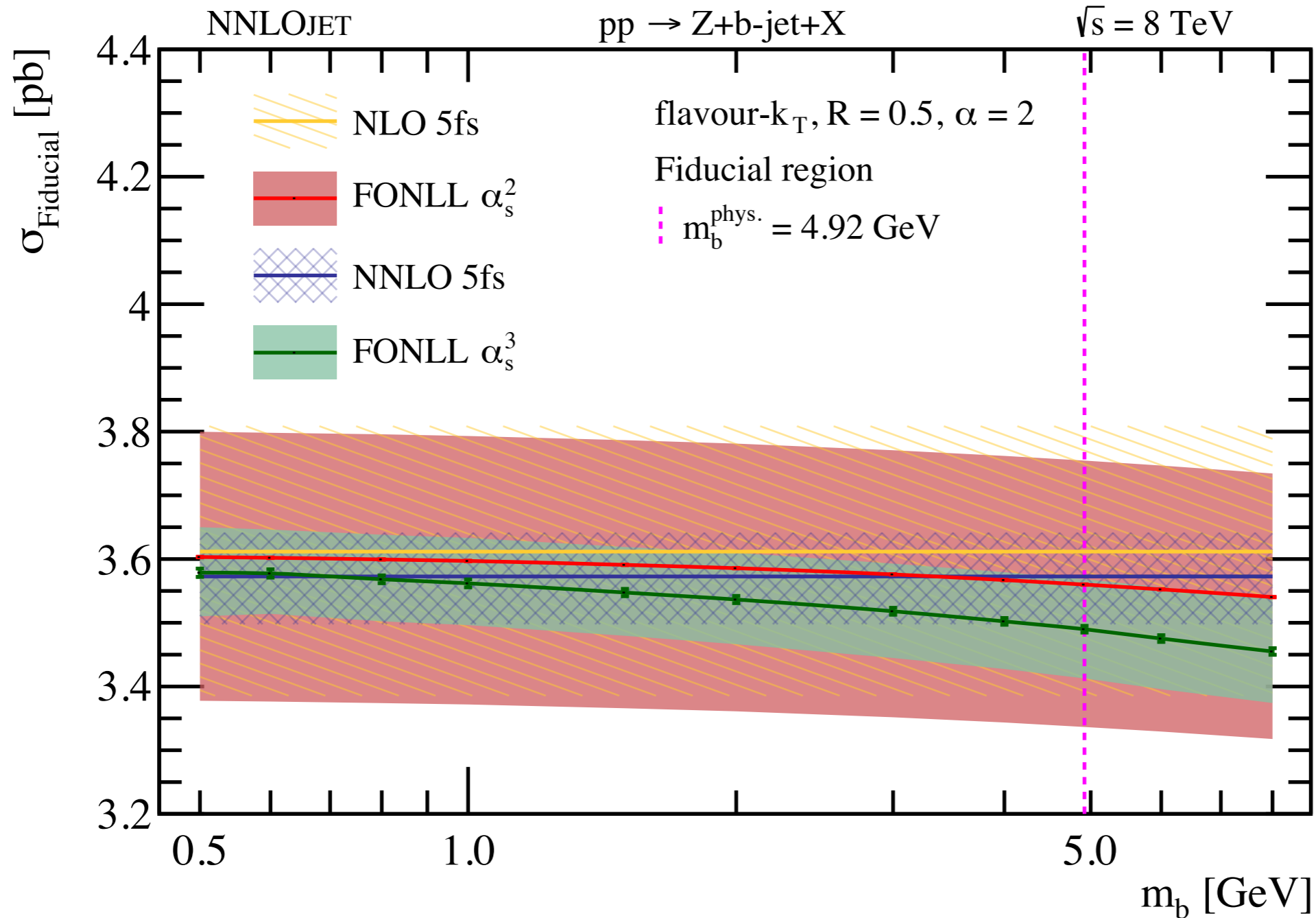
2) Prediction at hadron-level, anti- $k_T$  algorithm (**Experiment**)

Calculate an “Unfolding” correction from 2) Experiment  $\rightarrow$  1) Theory



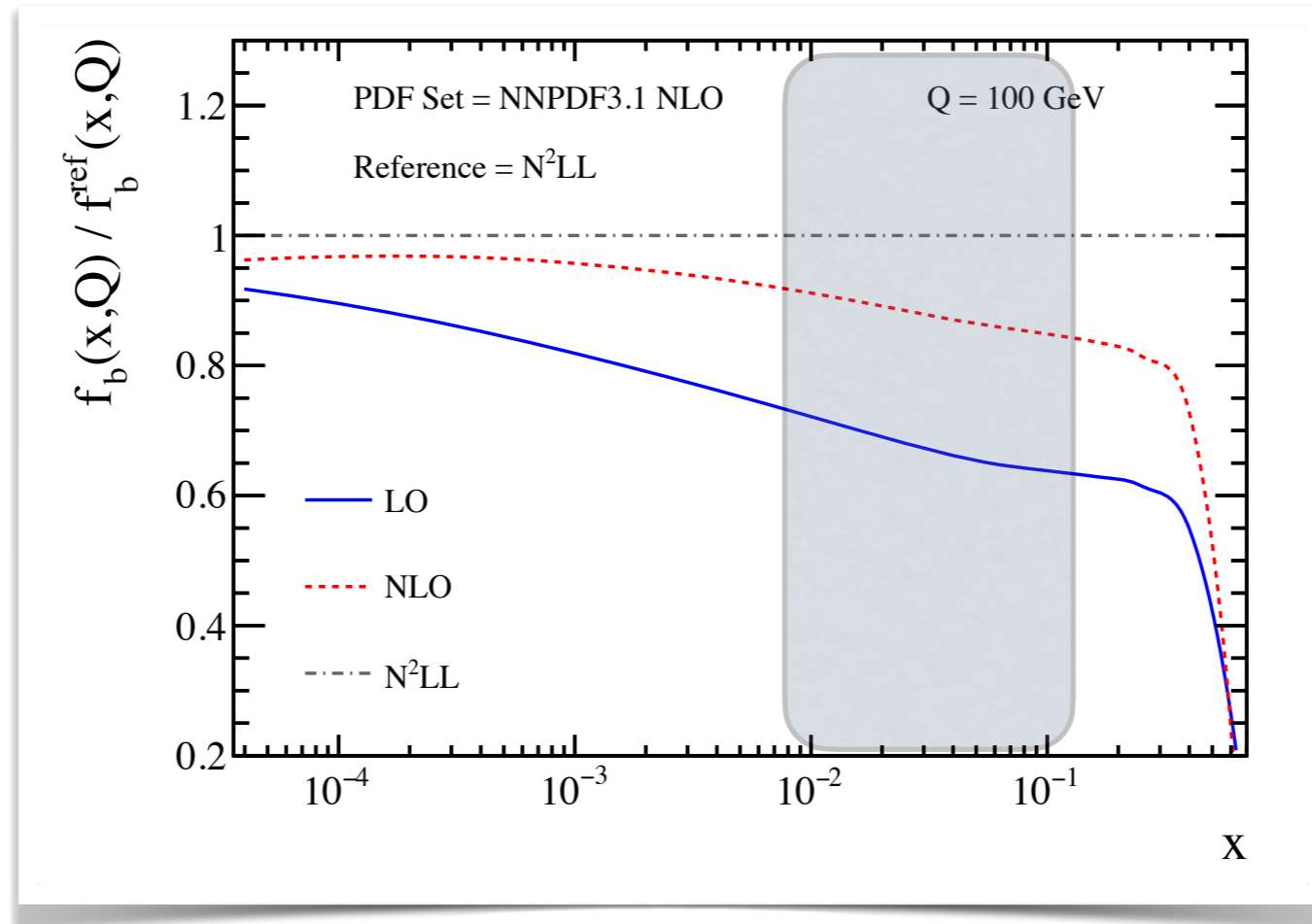
We use RooUnfold (following the procedure used in the exp. analyses)

# Validation of M-VFNS (Z+b-jet)



For IRC safe observables one can always construct a MVFNS scheme

# The b-quark PDF



I am showing fixed-order pdf versus a resummed one (PDF evolution)

$$\alpha_s^m \ln^n[\mu_F^2/m_b^2], \quad m \geq n$$

Note!  $\alpha_s \ln[m_Z^2/m_b^2] \approx 0.7$