Soft and Next-to-soft Resummation for processes at the LHC

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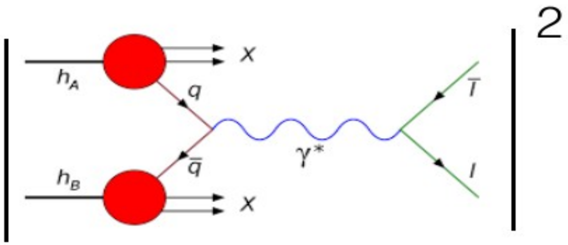
Workshop on Tools for High Precision LHC Simulations

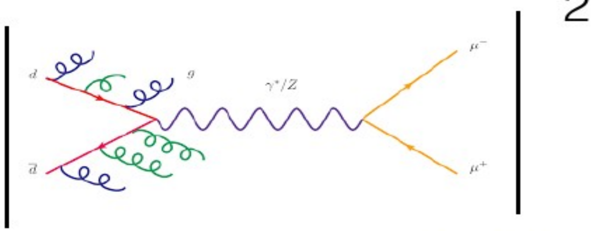
Castle Ringberg | November 3, 2022

Technical University of Munich

Incusive cross-section – QCD Improved Parton Model



$$= \sum_{ab} \Phi_{ab}(z) \otimes$$



Hadron level

Parton level $\Delta_{ab}(z)$

Drell-Yan (DY) / Higgs boson production in Hadron collisions

$$\sigma(q^2, \tau) = \sigma_0(\mu_R^2) \int \frac{dz}{z} \Phi_{ab}\left(\frac{\tau}{z}, \mu_F^2\right) \Delta_{ab}(q^2, \mu_F^2, z)$$

Partonic Coeff. function

perturbative

τ Hadronic scaling variable

q^2 Invariant mass sq

z Partonic scaling variable

μ_R^2 Renormalisation scale

μ_F^2 Factorisation scale

$\tau = \frac{q^2}{S}$ → Hadronic cm energy squared

$z = \frac{q^2}{\hat{s}}$ → Partonic cm energy squared

Partonic flux

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

Parton distribution functions (PDFs - non-perturbative)

Threshold Expansion (z->1) of the partonic CF

Soft-virtual (SV) **Regular part**

$$\Delta_{ab}(q^2,\mu_F^2,z) = \delta_{ab}\Delta_{a\bar{a}}^{SV}(q^2,\mu_F^2,z) + \Delta_{ab}^{reg}(q^2,\mu_F^2,z).$$

Power series expansion

$$\Delta_{ab}^{J}(q^2,\mu_F^2,z) = \sum_{i=0}^{\infty} a_s^i(\mu_R^2)\Delta_{ab}^{J,(i)}(q^2,\mu_R^2,\mu_F^2,z)$$

$J = SV, reg$

Plus-distribution

$$\mathscr{D}_k(z) = \left(\frac{\ln^k(1-z)}{1-z}\right)_+$$

$$\Delta_{ab}^{SV,(i)}(z) = \delta_{ab}\left(\Delta_{a\bar{a},\delta}\,\delta(1-z) + \sum_{k=0}^{2i-1}\Delta_{a\bar{a},\mathscr{D}_k}^{(i)}\mathscr{D}_k(z)\right)$$

comprises pure virtual contributions and leading threshold contributions from diagonal partonic channels with at least one emission of on-shell parton.

Plus distribution $+$ is defined by its action on test function $f(z)$

$$\int_0^1 dz\mathcal{D}_j(z)f(z) \equiv \int_0^1 dz\frac{\ln^j(1-z)}{1-z}\left[f(z)-f(1)\right]$$

- **Threshold logarithms**

$$\mathcal{D}_i(z) = \left(\frac{\ln^i(1-z)}{1-z}\right)_+$$

Linked to soft & collinear divergences

Universal process-independent form through certain IR anomalous dimensions

- **Dominate in the threshold region, namely z->1**

These large logarithms spoil the reliability of the fixed-order pertutrbative series

Resolution: Thresold resummation
[known since 1989, sterman, catani, trentedue]

Sterman-Catani-Trentedue

Sterman (`87),Catani,Trentedue `89

SUMMATION OF LARGE CORRECTIONS TO SHORT-DISTANCE HADRONIC CROSS SECTIONS

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RESUMMATION OF THE QCD PERTURBATIVE SERIES FOR HARD PROCESSES

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$$\Delta_N(Q^2) \underset{N\to\infty}{=} \exp\left(2\int_0^1 dx\, \frac{x^{N-1}-1}{1-x} \int_{Q^2}^{Q^2(1-x)} \frac{dk^2}{k^2} A\left(\alpha_s((1-x)Q^2)\right) + \frac{3}{2}\frac{C_F}{\pi}\int_0^1 dx\, \frac{x^{N-1}-1}{1-x}\, \alpha_s((1-x)Q^2)\right)$$

$$+ O\left(\alpha_s(\alpha_s \ln N)^n\right).$$

Going beyond the Soft-Virtual

Let us take a look at the regular part

$$\Delta_{ab}^{reg,(i)}(z) = \sum_{k=0}^{2i-1} \sum_{l=0}^{\infty} \Delta_{ab,l,k}^{reg,(i)} (1-z)^l \ln^k(1-z)$$

Setting l=0 in the above eqn, we obtain the Next-to-soft-virtaul (NSV) terms

The NSV terms

$$\Delta_{ab}^{NSV,(i)}(z) = \sum_{k=0}^{2i-1} \Delta_{ab,0,k}^{reg,(i)} \ln^k(1-z)$$

Perturbative structure of the CF near threshold (z->1)

$$\Delta_c^{\mathrm{SV+NSV,i}}(z,q^2) = \sum_{k=0}^{2i-1} c_{ik}^{\mathcal{D}}\ \mathcal{D}_k + c_i^{\delta}\ \delta(1-z) + \sum_{k=0}^{2i-1} c_{ik}^{L}\ \log^k(1-z)$$

$$\mathscr{D}_k(z) = \left(\frac{\ln^k(1-z)}{1-z}\right)_+$$

Plus-distribution

Soft-virtual [SV] corrections

Leading power (LP) logarithms
Most Singular when z→ 1
Corrections from diagonal Channels
Resummation to N3LL accuracy
Well-understood

Next-to SV [NSV] corrections

Next-to LP (NLP) logarithms
Next-to-dominant singular
Corrections from both diagonal & off-diagonal
Resummation to LL accuracy
Not much studied

Why Next-to-SV (NSV)?

Higgs production in the gluon fusion | N3LO

Charalampos Anastasiou[a], Claude Duhr[b], Falko Dulat[a], Elisabetta Furlan[c], Thomas Gehrmann[d], Franz Herzog[e], Bernhard Mistlberger[a]

JHEP 03 (2015) 091

$$
\begin{aligned}
\eta^{(3)}_{gg}(z)\Big|_{(1-z)^0} &= -256\log^5(1-z) && (\rightarrow \quad 115.33\%)\\
&+ 959\log^4(1-z) && (\rightarrow \quad 101.07\%)\\
&+ 1254.029198\ldots\log^3(1-z) && (\rightarrow \quad -32.15\%)\\
&- 11089.328274\ldots\log^2(1-z) && (\rightarrow \quad -89.41\%)\\
&+ 15738.441212\ldots\log(1-z) && (\rightarrow \quad -55.50\%)\\
&- 5872.588877\ldots && (\rightarrow \quad -14.31\%)
\end{aligned}
$$

The total SV contribution in z-space -> -2.25 % of the Born

The total NSV contribution in z-space -> 25 % of the Born !

The total SV contribution in Mellin N-space (conjugate space) -> 18 % of the Born

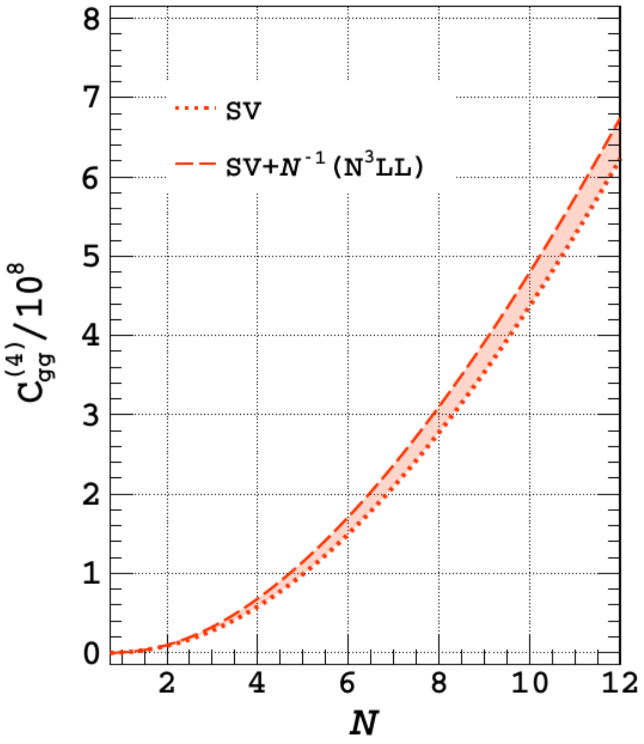
The total NSV contribution in N-space -> 11 % of the Born !

$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\; & 36 \log^6 N && (\to 0.0013\%)\\
& + 170.679\ldots \log^5 N && (\to 0.0226\%)\\
& + 744.849\ldots \log^4 N && (\to 0.2570\%)\\
& + 1405.185\ldots \log^3 N && (\to 1.0707\%)\\
& + 2676.129\ldots \log^2 N && (\to 4.0200\%)\\
& + 1897.141\ldots \log N && (\to 5.1293\%)\\
& + 1783.692\ldots && (\to 8.0336\%)\\
& + 108 \frac{\log^5 N}{N} && (\to 0.0105\%)\\
& + 615.696\ldots \frac{\log^4 N}{N} && (\to 0.1418\%)\\
& + 2036.407\ldots \frac{\log^3 N}{N} && (\to 0.9718\%)\\
& + 3305.246\ldots \frac{\log^2 N}{N} && (\to 2.9487\%)\\
& + 3459.105\ldots \frac{\log N}{N} && (\to 5.2933\%)\\
& + 703.037\ldots \frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

Approximate four-loop QCD corrections to the Higgs-boson production cross section

Phys.Lett.B 807 (2020) 135546

G. Das[*a], S. Moch[†b] and A. Vogt[‡c]



sizeable contribution from terms beyond SV (the NSV terms) due to the large coefficients

Understanding the NSV sector is important because:

- Numerically sizeable
- Provide check of higher-order corrections
- Help to reduce the scale uncertainites
- Stabilize automated fixed-order calculations

These NSV logarithms give rise to large logarithmic contributions in the threshold limit : spolis the perturbativity of the FO series

Resolution : Find a way to resum these NSV logarithms beyond LL accuracy

Previous Works

The earliest evidence that IR effects can be studied at NLP
[Low, Burnett, Kroll]

Early attempts :
[Kraemer, Laenen, Spira (98)]
[Akhoury, Sotiropoulos & Sterman (98)]

Important Results & Predictions using Physical Kernel Approach & explicit computation:
[Moch , Vogt et al. (09-20)]
[Anastasiou, Duhr, Dulat et al.(14)]

Universality of NLP effects and LL Resummation:
[Laenen, Magnea, et al. (08-19)]
[Grunberg & Ravindran (09)]
[Ball, Bonvini, Forte, Marzani, Ridolfi (13)]
[Del Duca et al. (17)]

Subleading Factorisation and LL Resummation at NLP using SCET:
[Larkoski, Nelli , Stewart et al. (14)]
[Kolodrubetz, Moult, Neill ,Stewart et al. (17)]
[Beneke et al. (19-20)]

And many other works...

Some of the recent works

Factorisation and RG invariance approach to study NSV resummation effects
[Ajjath, Pooja, Ravindran , Phys. Rev. D 105, 094035 , Phys. Rev. D 105, L091503, (2020)]

On next to soft threshold corrections to DIS and SIA processes
[Ajjath, Pooja, Ravindran, A.S, S.Tiwari, JHEP 04 (2021) 131]

Next-to SV resummed Drell-Yan cross section beyond Leading-logarithm
[Ajjath, Pooja, Ravindran, A.S, S.Tiwari, Eur. Phys. J. C 82, 234, (2021)]

Resummed Higgs boson cross section at next-to SV to NNLO + $\overline{\text{NNLL}}$
[Ajjath, Pooja, Ravindran, A.S, S.Tiwari, Eur. Phys. J. C 82, 774, (2021)]

Next-to-soft corrections for Drell-Yan and Higgs boson rapidity distributions beyond N3LO
[Ajjath, Pooja, Ravindran, A.S, S.Tiwari, Phys.Rev.D 103 (2021) L111502, (2021)]

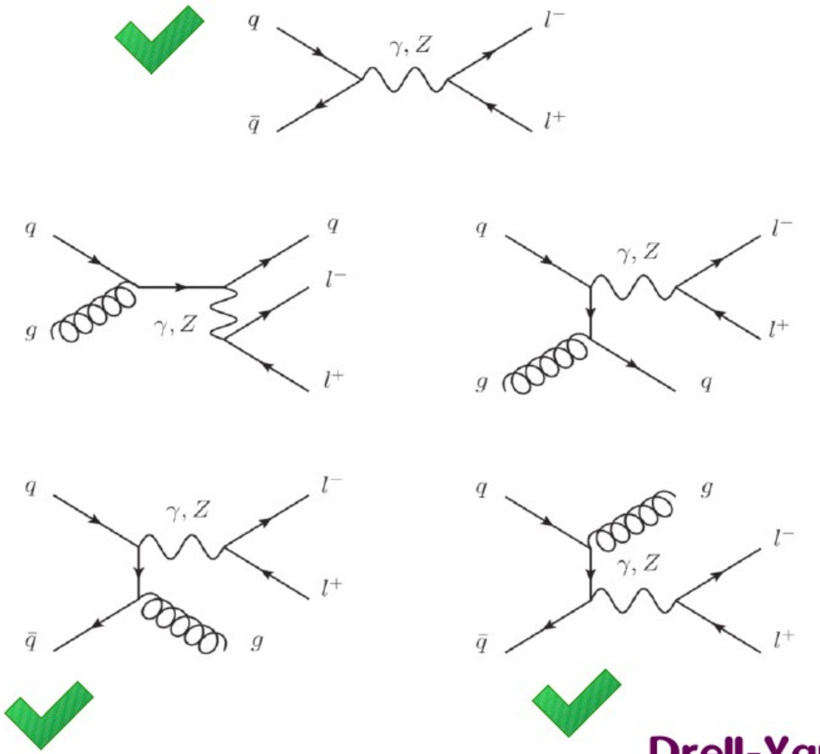
Next-to SV resummed rapidity distribution for Drell-Yan to NNLO + $\overline{\text{NNLL}}$
[Ajjath, Pooja, Ravindran, A.S, S.Tiwari, Phys. Rev. D 106, 034005, (2022)]

Resummed next-to-soft corrections to rapidity distribution of Higgs Boson to NNLO + $\overline{\text{NNLL}}$
[Ravindran, A.S, S.Tiwari, arXiv:2205.11560 [hep-ph], (2022)]

The Approach

Works for 2-> ‘n’ colourless Final states

Considered only diagonal channels :



Drell-Yan

Keypoints

- Collinear Factorisation
- Renormalisation Group (RG) Invariance
- Logarithmic structure of higher order perturbative results

The Formalism

Factoring out the pure virtual contributions

$$\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \left(Z_{c,UV}\right)^2 |\hat{F}_c(\epsilon)|^2 S_c(z,\epsilon)$$

Partonic cross-section

UV Renormalisation constant

Unrenormalised Form Factor (FF) (pure virtual corrections)

Soft-collinear function

Mass Factorisation

$$\frac{1}{z}\hat{\sigma}_{ab}(z,\epsilon) = \sigma_0 \sum_{a'b'} \Gamma_{aa'}(\mu_F^2, z, \epsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2, z, \epsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2, z, \epsilon)$$

Altarelli-Parisi (AP) kernel

Partonic cross-section containing only Initial state collinear singularities

Collinear Finite

Collinear Singular

Factorisation – Diagonal channel

For Drell-Yan process:

Contributes to beyond NSV

Diagonal Channel: $$\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \frac{\Delta_{q\bar{q}}}{z} \otimes \Gamma_{\bar{q}\bar{q}} + \Gamma^T_{qq} \otimes \frac{\Delta_{qg}}{z} \otimes \Gamma_{g\bar{q}} + \cdots$$

In the threshold limit z -> 1 , keeping only

$$\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+ \qquad \delta(1-z_i) \qquad \text{SV}$$

$$\log^k(1-z_i), \quad k = 0, \cdots \infty \qquad \text{next to SV}$$

dropping $(1-z_i)^k, \quad k = 1, \cdots \infty$

$$\frac{\hat{\sigma}^{\text{sv+nsv}}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\text{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.$$

Remarkably Simple form !

Coefficient function – Diagonal channel

Finite mass-factorised SV+NSV partonic coefficient function for the diagonal channels:

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Altarelli-Parisi splitiing kernel

UV Renormalisation constant

Form Factor

Soft-collinear function

The Building blocks

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Set of governing differential eqns

[Sen, Sterman,Magnea]

$$Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

poles

finite

Sudakov K+G diff eqn

$$\mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \quad \gamma_{i-1}^c$$

UV anomalous dimension

RGE

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z, \mu_F^2, \epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}(z, a_s(\mu_F^2)) \otimes \Gamma_{a'b}(z, \mu_F^2, \epsilon)$$

Splitting function

AP evolution eqn

[Moch,Vogt,Vermaseren]

Altarelli-Parisi kernels

Required to remove the initial state collinear singularities

AP kernels which satisfy renormalisation group equations

[Moch,Vogt,Vermaseren]

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z,\mu_F^2,\epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}\left(z, a_s(\mu_F^2)\right) \otimes \Gamma_{a'b}(z,\mu_F^2,\epsilon)\,, \qquad a,b = q,\bar{q},g$$

AP Splitting function

Expansion around z=1

Collinear ano dim (contributes to NSV)

$$P_{cc}(z,\mu_F^2) = 2\left[\frac{A^c}{(1-z)_+} + B^c\,\delta(1-z) + C^c\,\log(1-z) + D^c + \mathcal{O}(1-z)\right]$$

SV NSV beyond NSV

We consider only diagonal parts of splitting functions

Building blocks- Summary

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Building blocks		
$Z_{c,UV}$	Renormalisation const	✓
$\hat{F}_c$	Form Factor (FF)	✓
Γ_c	AP Kernels	✓
S_c	Soft-collinear function	?

How to obtain the soft-collinear function S_c ?

Guiding factors to obtain the soft-collinear function S_c

- Finiteness of the partonic coefficient function Δ_c
- Sudakov differential eqn of FFs (K+G eqn)
- RGE and AP evolution eqn of the splitting kernels

Soft-collinear function also satisfies a K+G type differential eqn

$$q^2 \frac{d}{dq^2} \mathcal{S}_c = \frac{1}{2} \left[\overline{K}_c \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) + \overline{G}_c \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) \right] \otimes \mathcal{S}_c$$

IR singular   IR finite

Soft-collinear contributions exhibits exponential behaviour:

$$\mathcal{S}_c = \mathcal{C} \exp \left(2\Phi_c \right)$$

$$\mathcal{C} \exp \left(2\Phi_c(z) \right) = \frac{\hat{\sigma}_{c\bar{c}}(z)}{Z_{c,UV}^2 \left| \hat{F}_c \right|^2}, \qquad c = q, b, g$$

No pure virtual , Only Real-Virtual (RV), Real-Real (RR) etc

$$\mathcal{C} e^{f(z)} = \delta(1-z) + \frac{1}{1!} f(z) + \frac{1}{2!} (f \otimes f)(z) + \cdots .$$

Solution for the exponent Φ_c

Alreday known : [Ravindran (`05,`06), j.nuclphysb.2006.06.025]

$$\Phi_c(\hat{a}_s, q^2, \mu^2, \epsilon, z) = \sum_i \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \left(\frac{i\epsilon}{1-z}\right)\left[\hat{\phi}_{\mathrm{SV}}^{c,(i)}(\epsilon) + (1-z)\,\hat{\phi}_{\mathrm{NSV}}^{c,(i)}(z,\epsilon)\right]$$

Phase-space factor

From matrix elements

Inspired from explicit results
Solution verified up to 3rd order

$A^c, f^c, \overline{\mathcal{G}}^c$

singularities get canceled against those in FF entirely and AP kernels partially

$C^c, D^c, \varphi_c(z)$

Process dependent

Singularities get canceled against the residual div in AP kernels

- Expanding the ansatz:

$$\frac{1}{(1-z)}\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \frac{\delta(1-z)}{i\epsilon} + \sum_{k=0}^{\infty} [i\epsilon]^k \frac{\mathcal{D}_k}{k!}$$

Contributes to SV

$$\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \sum_{n=0}^{\infty} \frac{\left[i\epsilon\ \log(1-z)\right]^n}{n!}$$

Contributes to NSV

SV+NSV in Nutshell

Restricting to diagonal channels and around z=1

↓

Mass Factorization Formula

↓

All-order factorisation formula for $\Delta_{c\bar{c}}$

↓

Sudakov type differential Equation & RG Eq.

↓

$$\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = \mathcal{C} \exp\Big(\Psi^c\big(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\big)\Big)\Big|_{\varepsilon=0}$$

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!} f(z) + \frac{1}{2!} f(z) \otimes f(z) + \cdots .$$

The All-order Formula | The Master Formula

$$\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = C \exp\left(\Psi^c\left(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\right) \right)\Big|_{\varepsilon=0}$$

$$\Psi_c = \left(\ln\left(Z_{c,UV}\left(\hat{a}_s, \mu^2, \mu_R^2, \epsilon\right) \right)^2 + \ln\left|\hat{F}_c\left(\hat{a}_s, \mu^2, q^2, \epsilon\right)\right|^2 \right) \delta(1-z)$$
$$+ 2\Phi_c\left(\hat{a}_s, \mu^2, q^2, z, \epsilon\right) - 2\mathcal{C} \ln \Gamma_{cc}\left(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon\right)$$

The Predictions to all orders (z-space)

GIVEN				PREDICTIONS		
$\Psi_c^{(1)}$	$\Psi_c^{(2)}$	$\Psi_c^{(3)}$	$\Psi_c^{(n)}$	$\Delta_c^{(2)}$	$\Delta_c^{(3)}$	$\Delta_c^{(i)}$
$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^3	$\mathcal{D}_5, \mathcal{D}_4$ L_z^5	$\mathcal{D}_{(2i-1)}, \mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$
	$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^2, L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^4	$\mathcal{D}_{(2i-3)}, \mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$
		$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^3, \cdots, L_z^0$				$\mathcal{D}_{(2i-5)}, \mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$
			$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^n, \cdots, L_z^0$			$\mathcal{D}_{(2i-(2n-1))}, \mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$

$$\mathcal{D}_j \equiv \left(\frac{\ln^j(1-z)}{1-z}\right)_+$$

$$L_z^i = \ln^i(1-z)$$

$$\delta = \delta(1-z)$$

NSV predictions Checked up to 4th order
For DIS, DY, ggH, bBH

[Moch, Vogt et.al], [De Florian et al.], [Das et al.]

- But there are certain logarithms which we cannot predict completely from previous order information.
- For instance : $\log^3(1-z)$ coefficient at the 3rd order.
- Even though 2-loop cannot predict completely, but we find predictions for many color factors agree with known exact results using 2-loop.

	$gg \to H$			Drell-Yan (DY)	
C_A^3	$\frac{-111008}{27} + 3584\zeta_2$	$\frac{-110656}{27} + 3584\zeta_2 + \chi_1$	C_F^3	$2272 + 3072\zeta_2$	$2272 + 3072\zeta_2$
$C_A^2 n_f$	$\frac{6560}{9}$	$\frac{19616}{27} + \chi_2$	$C_F^2 n_f$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$
$C_A n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$C_A C_F^2$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$
			$C_F n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$
			$C_A C_F n_f$	$\frac{2816}{27}$	$\frac{2816}{27}$
			$C_A^2 C_F$	$\frac{-7744}{27}$	$\frac{-7744}{27}$

In general, using $a_s^{(n-1)}$ info:

$$\log^k(1-z),\quad n+1 \le k \le 2n-1$$

at order a_s^n

[Anastasiou et al.] [Duhr et al.]

The left column stands for the exact result and the right for the predictions using two loop.

Integral representation of CF in z-space

Knowing the functional form of each building blocks one can derive the integral form as:

captures the delta contribution from FF and S_c

$$\Delta_c(q^2, z) = C_0^c(q^2) \ \mathcal{C} \exp\left(2\Psi_{\mathcal{D}}^c(q^2, z)\right),$$

Exponent:

$$\Psi_{\mathcal{D}}^c(q^2, z) = \frac{1}{2}\int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + \mathcal{Q}^c(a_s(q^2(1-z)^2), z)$$

Finite contributions from cancellation between Γ_{cc} & S_c

$$P'_{cc} = 2\Big[A^c \mathcal{D}_0(z) + C^c \ln(1-z) + D^c\Big]$$

$$\mathcal{Q}^c(a_s(q^2(1-z)^2), z) = \left(\frac{1}{1-z}\overline{G}_{SV}^c(a_s(q^2(1-z)^2))\right)_+ + \varphi_{f,c}(a_s(q^2(1-z)^2), z).$$

Finite contribution coming from S_c

In the Mellin N space

- Mellin moment of CFs

$$\Delta_N^c = \int_0^1 dz\ z^{N-1} \Delta_c(z)$$

- Threshold limit $z \to 1$ in z-Space translates to $N \to \infty$ in N-Space

- $N \to \infty$ Taking into account SV and NSV terms

$$\left(\frac{\log(1-z)}{1-z}\right)_+ = \frac{\log^2 N}{2} - \frac{\log N}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\log^k(1-z) = \frac{\log^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Tower of NSV logarithms

Structure of Next to SV terms

$$\Delta_N^c = 1 + a_s\Big[c_1^2 \log^2 N + c_1^1 \log N + c_1^0 + d_1^1 \frac{\log N}{N} + \mathcal{O}(1/N)\Big]$$

$$+a_s^2\Big[c_2^4 \log^4 N + \cdots + c_2^0 + d_2^3 \frac{\log^3 N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$$+\cdots$$

$$+a_s^n\Big[c_n^{2n} \log^{2n} N + \cdots + d_n^{2n-1} \frac{\log^{2n-1} N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

SV+NSV Resummation Formula

Inclusion of the NSV logarithms modifies the existing resummed expression as :

$$\omega = 2\beta_0 a_s(\mu_R^2) \log N$$

$$\Delta_{c,N}(q^2,\mu_R^2,\mu_F^2) = C_0^c(q^2,\mu_R^2,\mu_F^2) \exp\left(\Psi^c_{\mathrm{SV,N}}(q^2,\mu_F^2) + \Psi^c_{\mathrm{NSV,N}}(q^2,\mu_F^2)\right)$$

N-independent coefficient

$$\Psi^c_{\mathrm{SV,N}} = \log(g_0^c(a_s(\mu_R^2))) + g_1^c(\omega)\log N + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g_{i+2}^c(\omega)$$

Known since 1989 [Sterman et.al] [Catani et.al]

New Result

$$\Psi^c_{\mathrm{NSV},N} = \frac{1}{N}\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\Big(\bar{g}^c_{i+1}(\omega) + h_i^c(\omega,N)\Big)$$

$$h_i^c(\omega,N) = \sum_{k=0}^{i} h_{ik}^c(\omega)\ \log^k N.$$

NSV Resummation exponents

Resummed exponents & Logarithmic accuracy

Logarithmic Accuracy	Resummed Exponents
$\overline{\text{LL}}$	$\tilde{g}^q_{0,0}, g^q_1, \overline{g}^q_1, h^q_0$
$\overline{\text{NLL}}$	$\tilde{g}^q_{0,1}, g^q_2, \overline{g}^q_2, h^q_1$
$\overline{\text{NNLL}}$	$\tilde{g}^q_{0,2}, g^q_3, \overline{g}^q_3, h^q_2$

Nomenclature :
$\overline{\text{N}^n\text{LL}}$ -> NSV included resummed results at $\overline{\text{N}^n\text{LL}}$ accuracy

$$a_s \frac{1}{N} \log N, \quad a_s^2 \frac{1}{N} \log^3 N, \quad a_s^3 \frac{1}{N} \log^5 N, \quad \ldots, \quad a_s^i \frac{1}{N} \log^{2i-1} N$$

Only 1-loop info

$$a_s^2 \frac{1}{N} \log^2 N, \quad a_s^3 \frac{1}{N} \log^4 N, \quad a_s^4 \frac{1}{N} \log^6 N, \quad \ldots, \quad a_s^i \frac{1}{N} \log^{2i-2} N$$

Only 2-loop info

. . .

$$a_s^n \frac{1}{N} \log^n N, \quad \ldots, \quad a_s^i \frac{1}{N} \log^{2i-n} N$$

Only n-loop info

Tower of NSV logarithms that we sum over

Checked till $\overline{\text{LL}}$ accuracy
[Beneke et.al]
[Laenen et.al]

The Matched Result - F.O + Res

Now we perform Mellin Inversion of the resummed result and add it to the F.O results to study the numerical impact.

$$\sigma_N^{\mathrm{N^nLO}+\overline{\mathrm{N^nLL}}} = \sigma_N^{\mathrm{N^nLO}} + \sigma^{(0)} \sum_{ab\in\{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} \delta_{a\bar{b}} f_{a,N}(\mu_F^2) f_{b,N}(\mu_F^2) \times \left(\Delta_{q,N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{q,N}\Big|_{tr\ \mathrm{N^nLO}} \right).$$

The resummed results are matched to the fixed order result in order to avoid any double counting of threshold logarithms

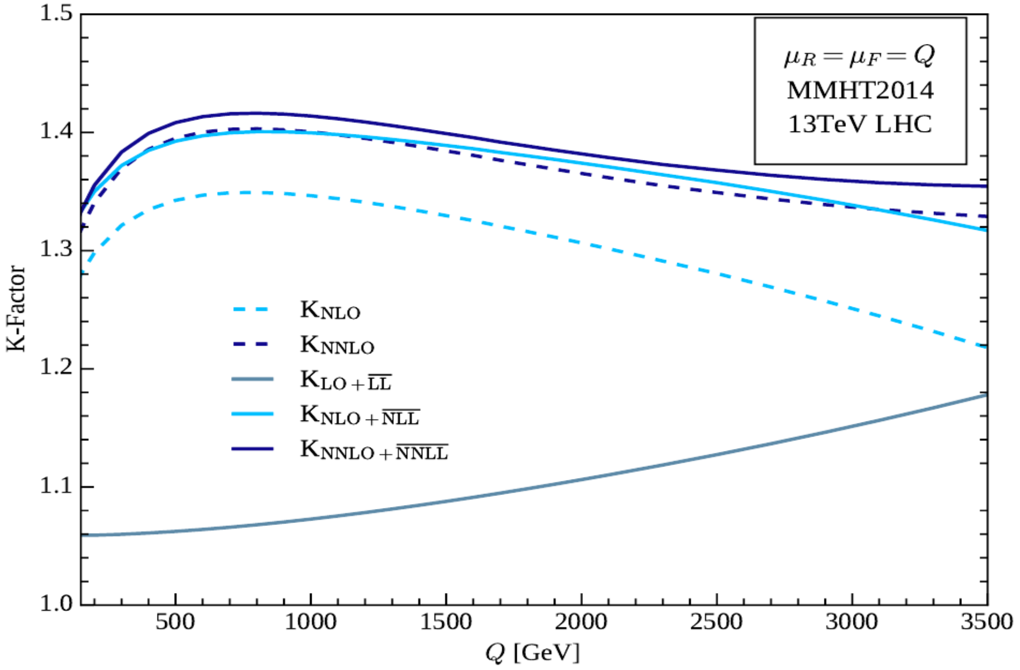
The contour c in the Mellin inversion is chosen according to Minimal prescription

Used for phenomenological studies

Drell-Yan Inclusive cross-section

K-Factor Analysis

Eur. Phys. J. C 82, 234, (2021)



resummed curves lie above their corresponding fixed order ones - enhancement due to the resummed corrections

[For Q= 500 GeV, LO->LO+$\overline{LL}$: 6.2 %
NLO->NLO+$\overline{NLL}$: 3.7 % , NNLO->NNLO+$\overline{NNLL}$: 0.94 %]

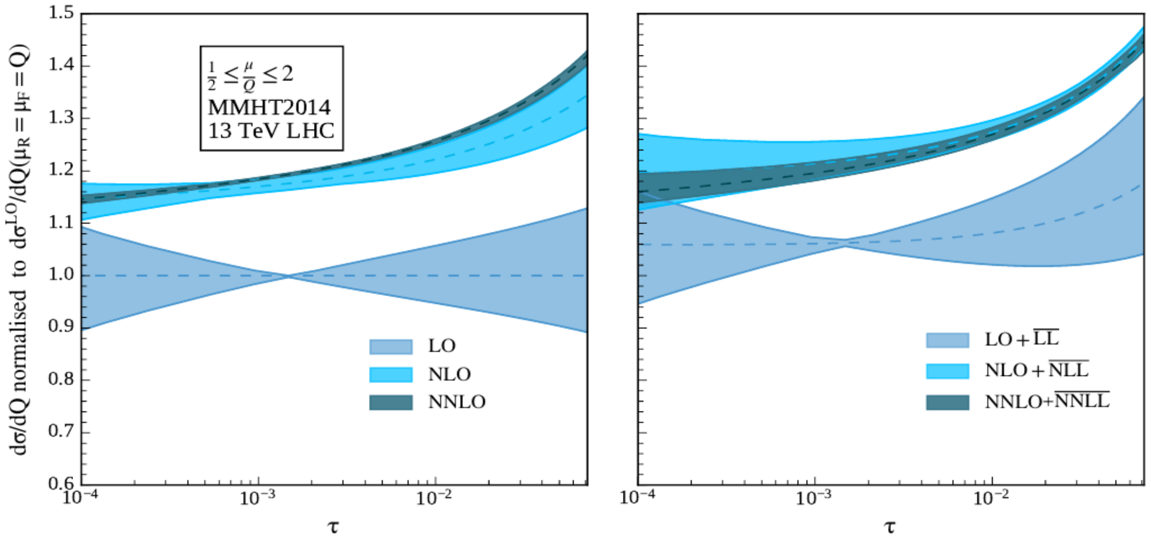
resummed curves are closer
resummed correction decreases as we go for higher order resummed contributions

Perturbative convergence in the RES predictions
reliability of RES predictions

$\mu_R = \mu_F = Q$(GeV)	$LO + \overline{LL}$	NLO	$NLO + \overline{NLL}$	NNLO	$NNLO + \overline{NNLL}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

$$K(Q) = \frac{\dfrac{d\sigma}{dQ}(\mu_R = \mu_F = Q)}{\dfrac{d\sigma^{LO}}{dQ}(\mu_R = \mu_F = Q)}$$

7-point scale uncertainities of the resummed results



resummed result shows a systematic reduction of the uncertainties with the inclusion of each logarithmic corrections

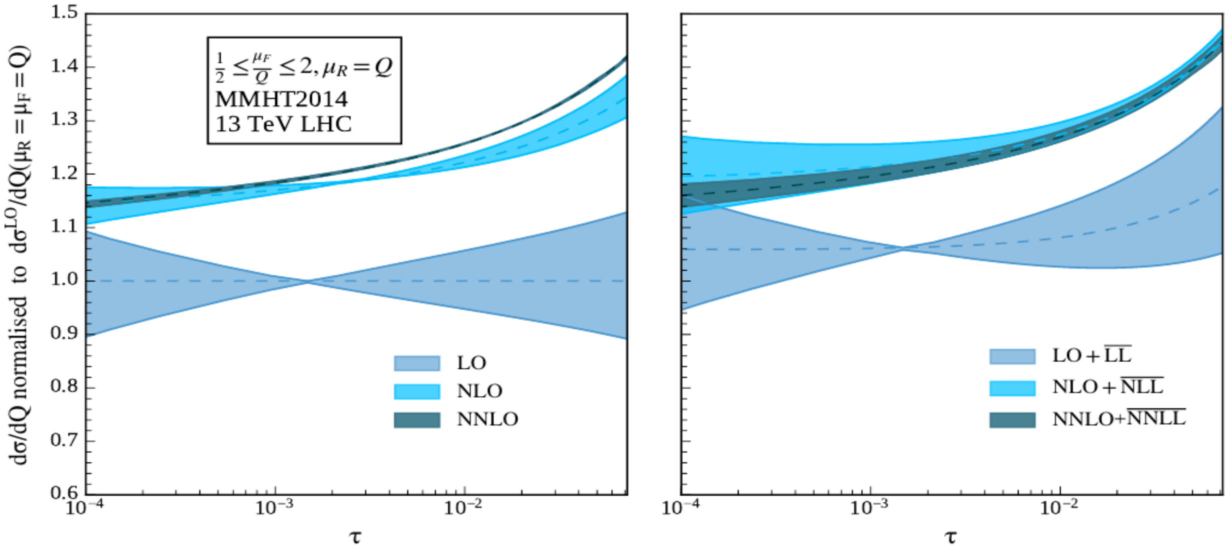
improvement at the $NLO+\overline{NLL}$ than at the $NNLO+\overline{NNLL}$ in comparison to their respective F.O predictions

Q	LO	$LO+\overline{LL}$	NLO	$NLO+\overline{NLL}$	NNLO	$NNLO+\overline{NNLL}$
1000	$2.3476^{+4.10\%}_{-3.92\%}$	$2.5184^{+4.49\%}_{-4.25\%}$	$3.1609^{+1.79\%}_{-1.69\%}$	$3.2857^{+2.08\%}_{-1.18\%}$	$3.2876^{+0.20\%}_{-0.31\%}$	$3.3191^{+1.13\%}_{-0.86\%}$
2000	$0.0501^{+8.50\%}_{-7.46\%}$	$0.0554^{+9.10\%}_{-7.91\%}$	$0.0654^{+2.83\%}_{-2.98\%}$	$0.0688^{+1.43\%}_{-1.23\%}$	$0.0684^{+0.37\%}_{-0.62\%}$	$0.0692^{+0.89\%}_{-0.78\%}$

cross section in 10^−5 pb/GeV

7-point var: μ = {μF , μR} is varied in the range [1/2Q, 2Q] keeping the ratio μR /μF not larger than 2 and smaller than 1/2.

Uncertainities w.r.t μ_F scale variation



resummed bands look similar to that of 7-point bands --width of the 7-point bands mainly comes from the μF uncertainties

NLO band gets improved with the inclusion of $\overline{\mathrm{NLL}}$, but NNLO band increses with the inclusion of $\overline{\mathrm{NNLL}}$

Missing qg-channel resummed contribution leads to more uncertainty at NNLO+$\overline{\mathrm{NNLL}}$

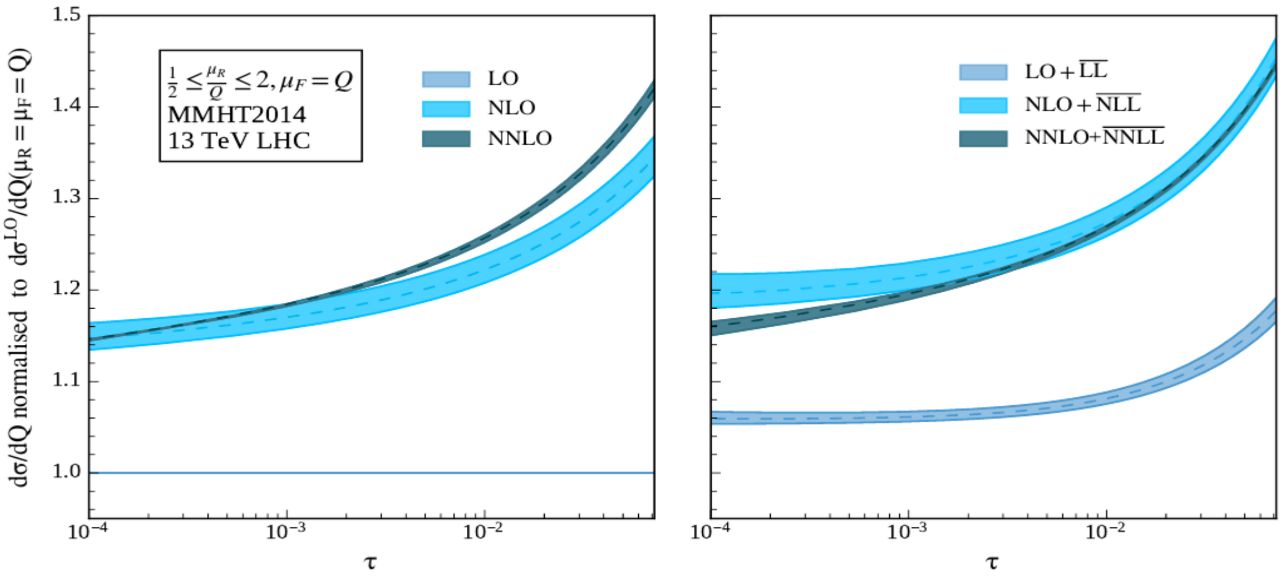
NLO : $q\bar{q}$ -> 22 %
qg -> -5 %

NNLO : $q\bar{q}$ -> 4.9 %
qg -> -2.8 %

$q\bar{q}$ dominating

Bigger cancellation b/w $q\bar{q}$ & qg

Uncertainities w.r.t μ_R scale variation

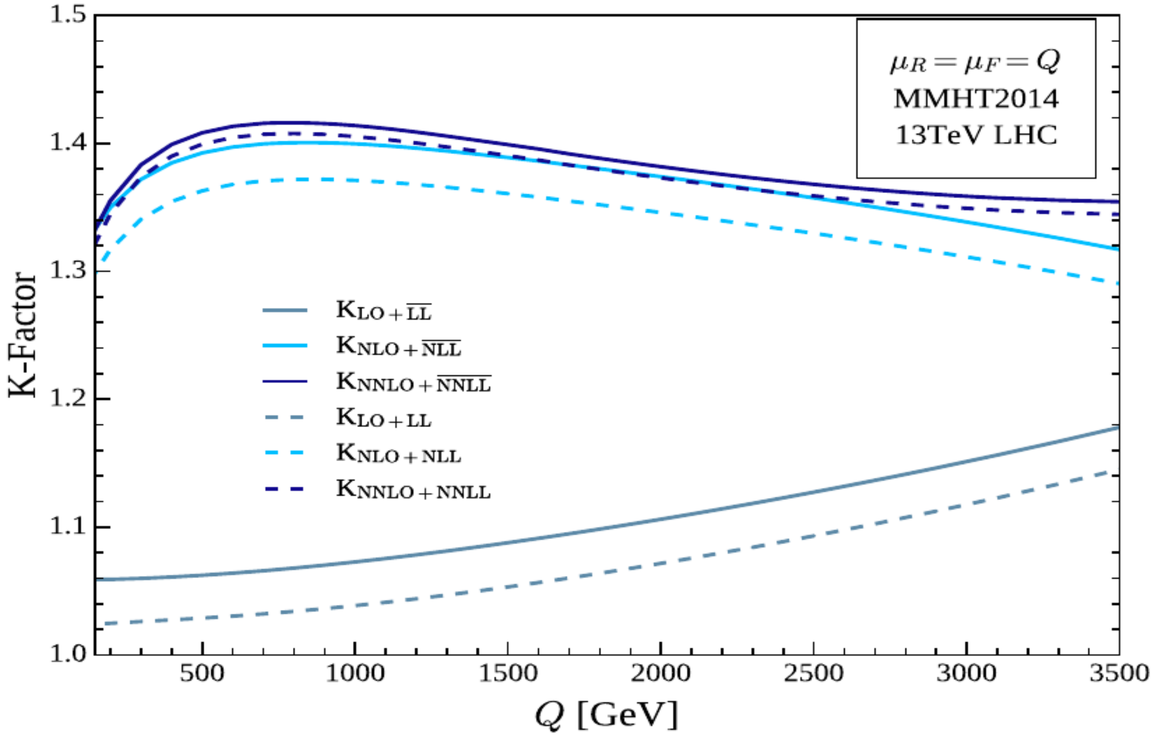


NNLO+$\overline{\text{NNLL}}$ the error band becomes substantially thinner

each partonic channel is invariant under μ_R variation and hence inclusion of more corrections within a channel is expected to reduce the uncertainity

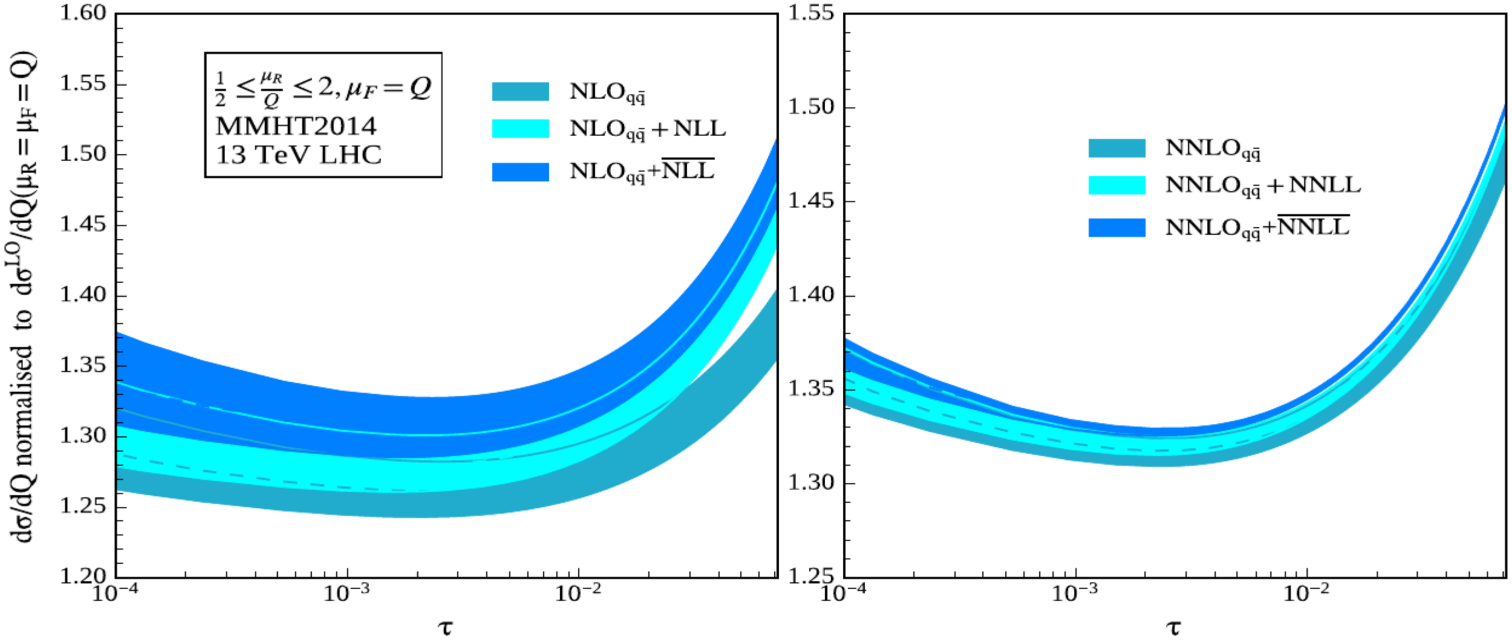
Inclusion of resummed result reduces the μ_R uncertainty remarkably as compared to the fixed order ones

SV versus SV+NSV Resummation: K- Factor Analysis



Perturbative convergence when the NSV effects are included

Uncertainities w.r.t μ_R scale variation in the $q\bar{q}$ channel

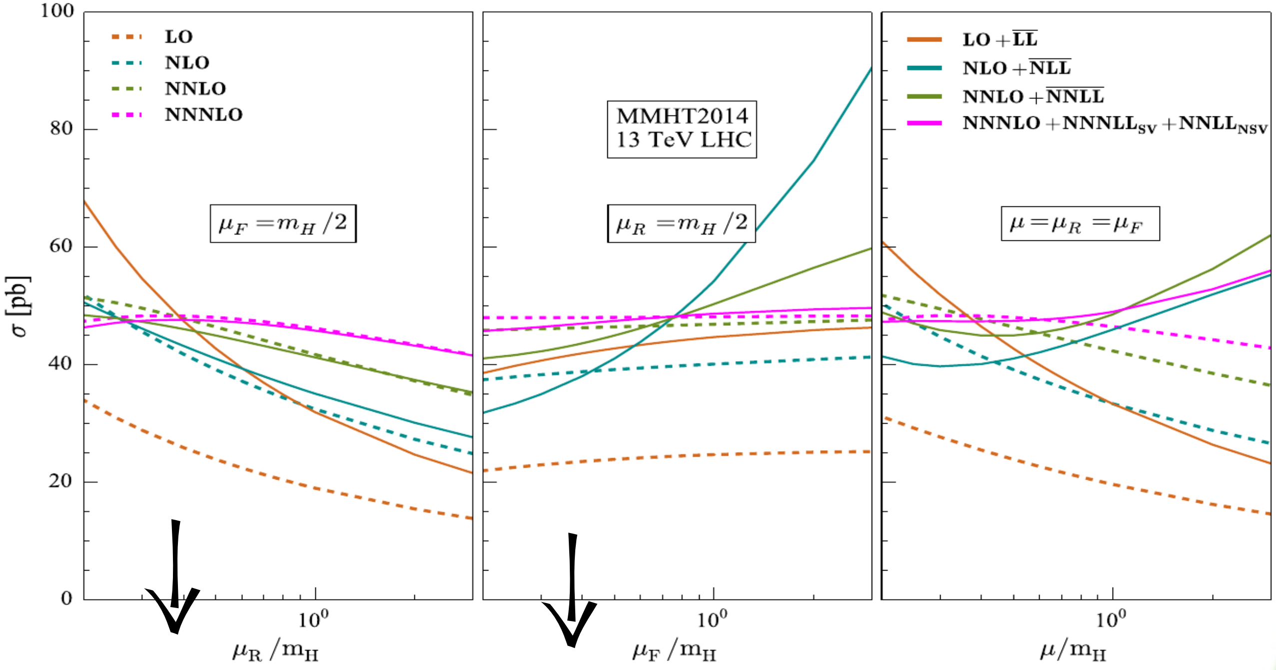


$Q = \mu_R = \mu_F$	$NNLO_{q\bar{q}}$	$NNLO_{q\bar{q}} + NNLL$	$NNLO_{q\bar{q}} + \overline{NNLL}$
1000	$3.5260^{+0.49\%}_{-0.58\%}$	$3.5376^{+0.25\%}_{-0.39\%}$	$3.5576^{+0.006\%}_{-0.20\%}$
2000	$0.0717^{+0.54\%}_{-0.62\%}$	$0.0721^{+0.19\%}_{-0.33\%}$	$0.0725^{+0.0\%}_{-0.15\%}$

cross section in 10^−5 pb/GeV for $q\bar{q}$-channel

Interestingly, the behaviour of NNLO $q\bar{q}$ + $\overline{NNLL}$ is significantly improved from the corresponding SV results, NNLO$q\bar{q}$ + NNLL, for a wide range of Q.

The Higgs production in gluon fusion



Eur. Phys. J. C 82, 774, (2021)

The inclusion of $\overline{\text{NNLL}}$ to NNLO decreases the cross section by 3.15%

N3LO | 48.05 pb
N3LO + N3LL_SV + N2LL_NSV | 47.45 pb
1.25% reduction

μ_R uncertainty decreases for the RES results

N3LO -> (-3.87%, 0.54%)

N3LO + N3LL_SV + N2LL_NSV
-> (-3.46% , 0.38%)

neligible μ_F dependency for F.O results
significant μ_F uncertainty for RES results

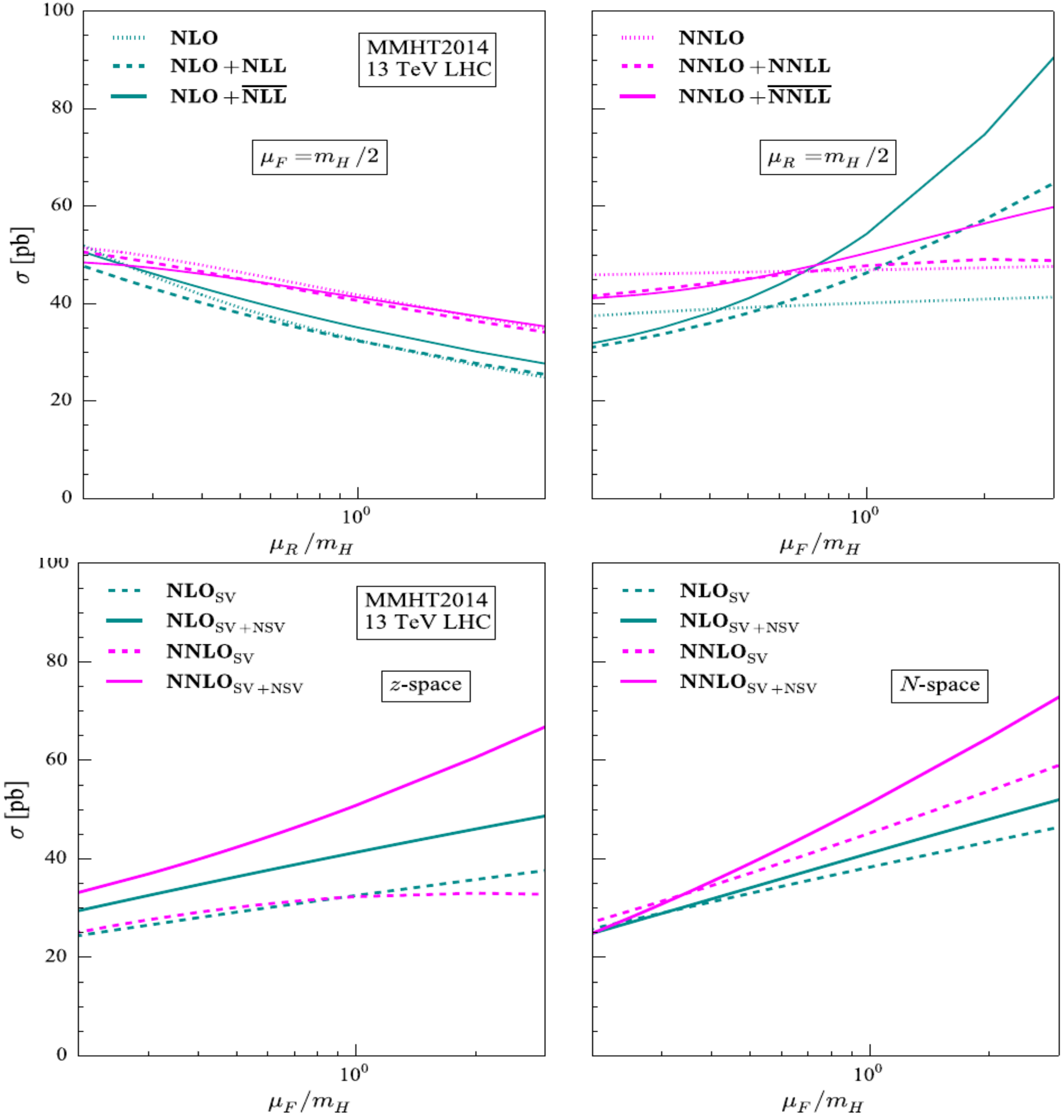
N3LO -> (-0.1%, 0.2%)

N3LO + N3LL_SV + N2LL_NSV
-> (-2.86% , 2.59%)

qg contribution is very minuscule !

NLO -- gg : 48.35 % , qg : - 0.79 %

NNLO -- gg : 19.7 % , qg : -2 %



NLO+$\overline{\text{NLL}}$ (SV+NSV) > NLO+NLL (SV) >>NLO

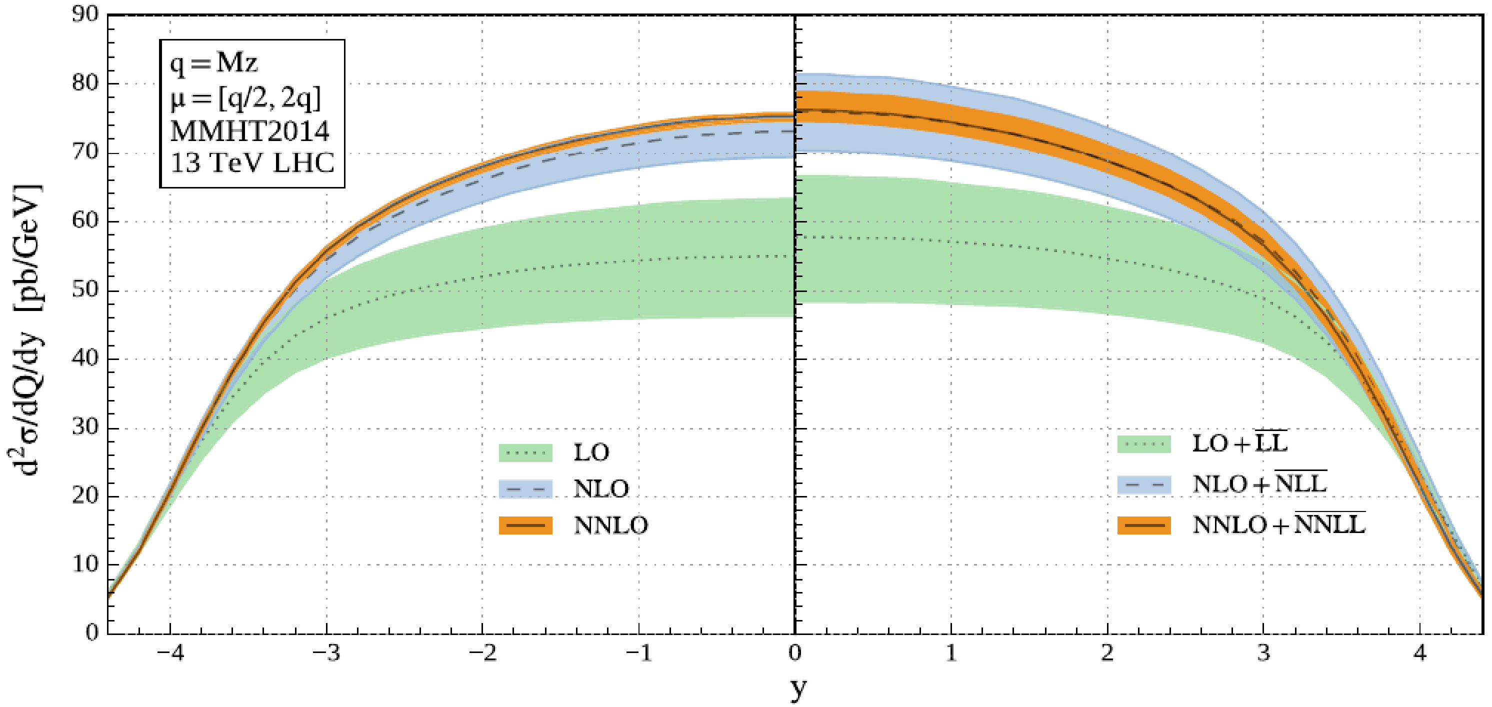
NNLO+$\overline{\text{NNLL}}$ (SV+NSV) > NNLO + NNLL (SV)>NNLO

Adding the NSV terms in the threshold expansion increases the μ_F uncertainity

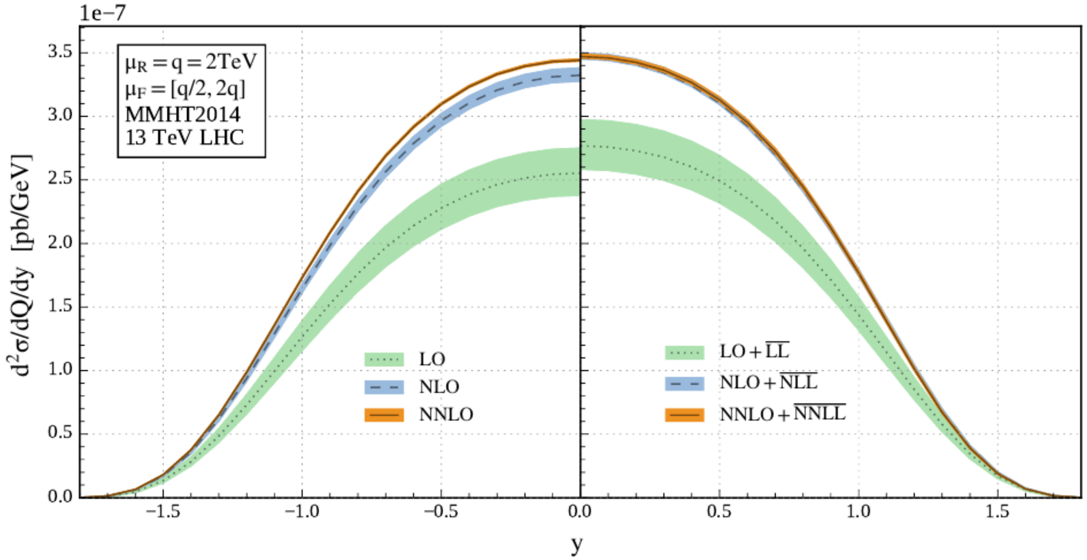
-> beyond NSV is needed to regulate this

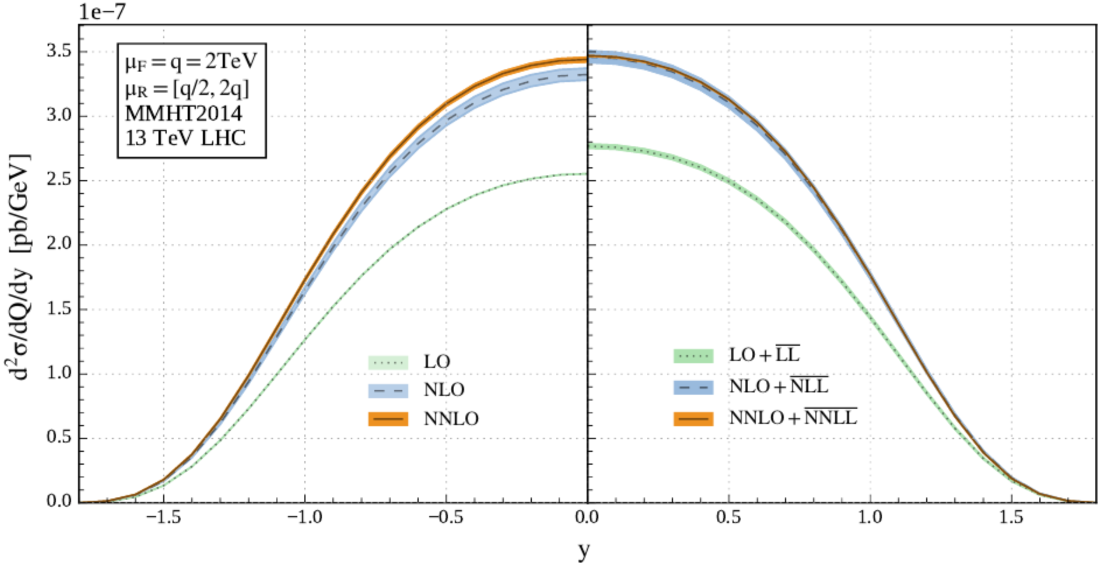
In the SV+NSV resummed results, Spurious beyond NSV terms due to "inexact" Mellin inversion give rise to huge μ_F unceratinity

Rapidity distribution
Drell-Yan



Phys. Rev. D 106, 034005, (2022)

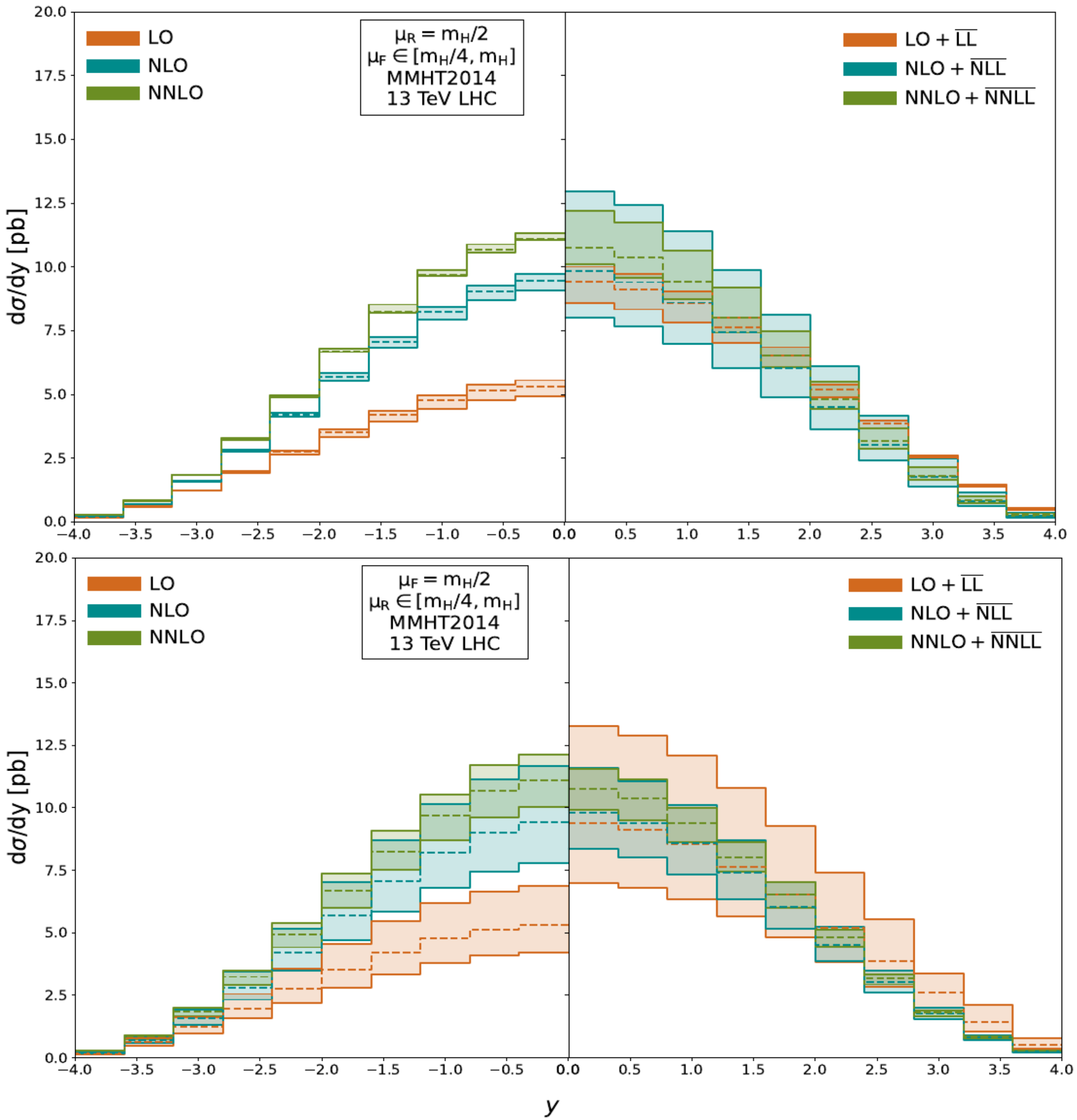




the uncertainty at NNLO + $\overline{\text{NNLL}}$ is reduced from (−2.18%; +3.3%) to (−0.31%; +0.53%) As we go from Mz to 2 Tev around the central rapidity

Resummed contribution at $\overline{\text{NNLL}}$ brings in 0.86% correction to NNLO

Rapidity distribution Higgs production in gluon fusion



arXiv:2205.11560 [hep-ph], (2022)

the inclusion of $\overline{\text{NNLL}}$ result decreases the rapidity distribution at NNLO level by 3 % at the central rapidity region

higher order uncertainty bands are completely included within the lower order uncertainty bands for the resummed predictions.

Conclusions and Outlook

- We set up a formalism to resum the next-to-soft-virtual terms using factorisation & RG Invariance.
- The resummation, taking into account the NSV terms, appreciably increases the cross section while decreasing the sensitivity to renormalisation scale.
- The inclusion of resummed NSV terms improves perturbative convergence.
- The absence of quark gluon initiated contributions to NSV part in the resummed terms leaves large factorisation scale dependence indicating their importance at NSV level for DY.
- The sensitivity to factorisation scale increases in the presence of resummed NSV terms implying the importance of beyond NSV terms for ggH

What more to do ?

- All-order structure and resummation for off-diagonal NSV
- Extending the current formalism to the case of mixed gauge theroy
  Eg: QCD-QED

THANK YOU

Additional Slides ...

Factorisation – off-diagonal channel

Off-diagonal Channel:
$$\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots$$

In the threshold limit z -> 1 , keeping only $\log^k(1 - z_i), \quad k = 0, \cdots \infty$ next to SV

$$\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\mathrm{nsv}}_{qg} \otimes \Gamma_{gg} \,.$$

dropping $(1 - z_i)^k, \quad k = 1, \cdots \infty$ NNSV terms

Getting complicated due to Mixing of channels

Form Factor – The Sudakov differential Eqn

IR singularities factorise

[Sen,sterman,Magnea]
[Moch,Vogt,Vermasern]

$$\hat{F}^c(Q^2,\mu^2,\epsilon) = Z_{IR}(Q^2,\mu^2,\mu_R^2,\epsilon)\hat{F}_c^{fin}(Q^2,\mu^2,\mu_R^2,\epsilon)$$

universal IR counter term contains poles

Finite part

Differentiating both sides with respect to Q^2, we obtain **K+G equation** for the FFs

$$Q^2\frac{d}{dQ^2}\log\hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s,\frac{\mu_R^2}{\mu^2},\varepsilon\right) + G^c\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\varepsilon\right)\right]$$

Poles

No Poles

RG Invariance

$$\mu_R^2\frac{d}{d\mu_R^2}K^c(a_s(\mu_R^2)) = -\mu_R^2\frac{d}{d\mu_R^2}G^c(a_s(\mu_R^2)) = -\overline{A}^c(a_s(\mu_R^2))$$

$$A_q = \frac{C_F}{C_A}A_g$$

Maximally non-abelian, verified up to 4 loops

Form Factor – Perturbative structure

Solution in d =4+ ϵ

$$\log \hat{F}^c(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \hat{\mathcal{L}}_F^{c,(i)}(\varepsilon)$$

$$\hat{\mathcal{L}}_F^{I,(1)} = \frac{1}{\varepsilon^2}\Big(-2A_1^I\Big) + \frac{1}{\varepsilon}\Big(G_1^I(\varepsilon)\Big)$$

$$\hat{\mathcal{L}}_F^{I,(2)} = \frac{1}{\varepsilon^3}\Big(\beta_0 A_1^I\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_2^I - \beta_0 G_1^I(\varepsilon)\Big) + \frac{1}{2\varepsilon}G_2^I(\varepsilon)$$

$$\hat{\mathcal{L}}_F^{I,(3)} = \frac{1}{\varepsilon^4}\Big(-\frac{8}{9}\beta_0^2 A_1^I\Big) + \frac{1}{\varepsilon^3}\Big(\frac{2}{9}\beta_1 A_1^I + \frac{8}{9}\beta_0 A_2^I + \frac{4}{3}\beta_0^2 G_1^I(\varepsilon)\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{2}{9}A_3^I - \frac{1}{3}\beta_1 G_1^I(\varepsilon) - \frac{4}{3}\beta_0 G_2^I(\varepsilon)\Big) + \frac{1}{\varepsilon}\Big(\frac{1}{3}G_3^I(\varepsilon)\Big)$$

$$G_1^I(\varepsilon) = 2(B_1^I - \delta_{I,g}\beta_0) + f_1^I + \sum_{k=1}^{\infty}\varepsilon^k g_1^{I,k}$$

$$G_2^I(\varepsilon) = 2(B_2^I - 2\delta_{I,g}\beta_1) + f_2^I - 2\beta_0 g_1^{I,1} + \sum_{k=1}^{\infty}\varepsilon^k g_2^{I,k}$$

Function of

$$\{A^c, B^c, f^c, \gamma^c, g^c\}$$

Cusp ano dim

Collinear ano dim

Soft ano dim

UV ano dim

Process dependent

[Ravindran, j.nuclphysb.2006.04.008]

All the Anomalous Dimensions have power series expansion in terms of a_s

For Drell-Yan :
SV and NSV contributions

Table 3 % contribution of SV distributions and NSV logarithms to the Born cross section at NNLO for $Q = 200$ GeV

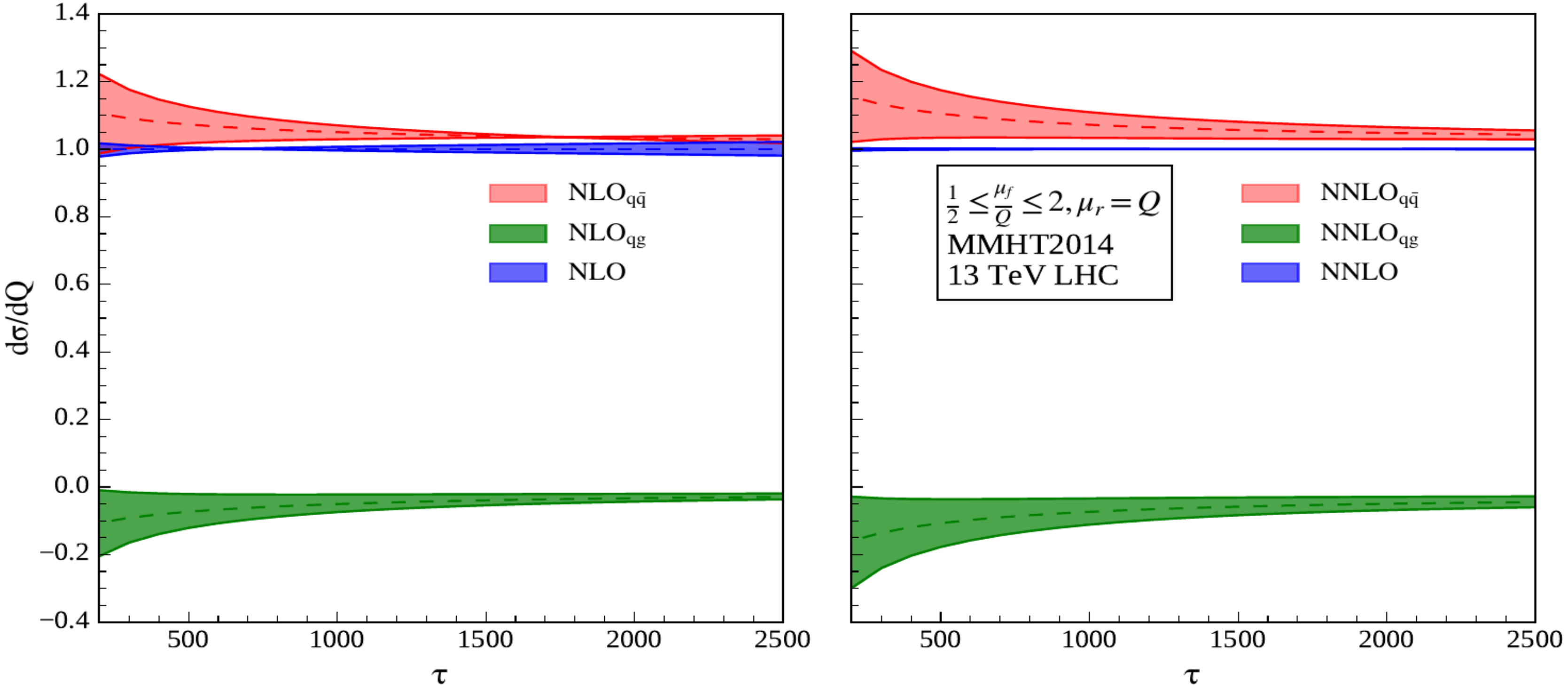
$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\ln^4 N$	0.0144%	$\frac{\ln^4 N}{N}$	0%
	$\ln^3 N$	0.125%	$\frac{\ln^3 N}{N}$	0.05%
	$\ln^2 N$	2.70%	$\frac{\ln^2 N}{N}$	0.392%
	$\ln N$	6.07%	$\frac{\ln N}{N}$	4.08%
	$\ln^0 N$	17.7%	$\frac{1}{N}$	3.35%
Total		34.5%		7.87%

Table 4 % contribution of SV distributions and NSV logarithms to the Born cross section at N^3LO for $Q = 200$ GeV

$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\ln^6 N$	− 0.0025%	$\frac{\ln^6 N}{N}$	0%
	$\ln^5 N$	− 0.001%	$\frac{\ln^5 N}{N}$	0.0004%
	$\ln^4 N$	0.0244%	$\frac{\ln^4 N}{N}$	0.006%
	$\ln^3 N$	0.171%	$\frac{\ln^3 N}{N}$	0.1%
	$\ln^2 N$	2.85%	$\frac{\ln^2 N}{N}$	0.56%
	$\ln N$	6.23%	$\frac{\ln N}{N}$	4.31%
	$\ln^0 N$	18.3%	$\frac{1}{N}$	3.30%
Total		27.6%		8.28%

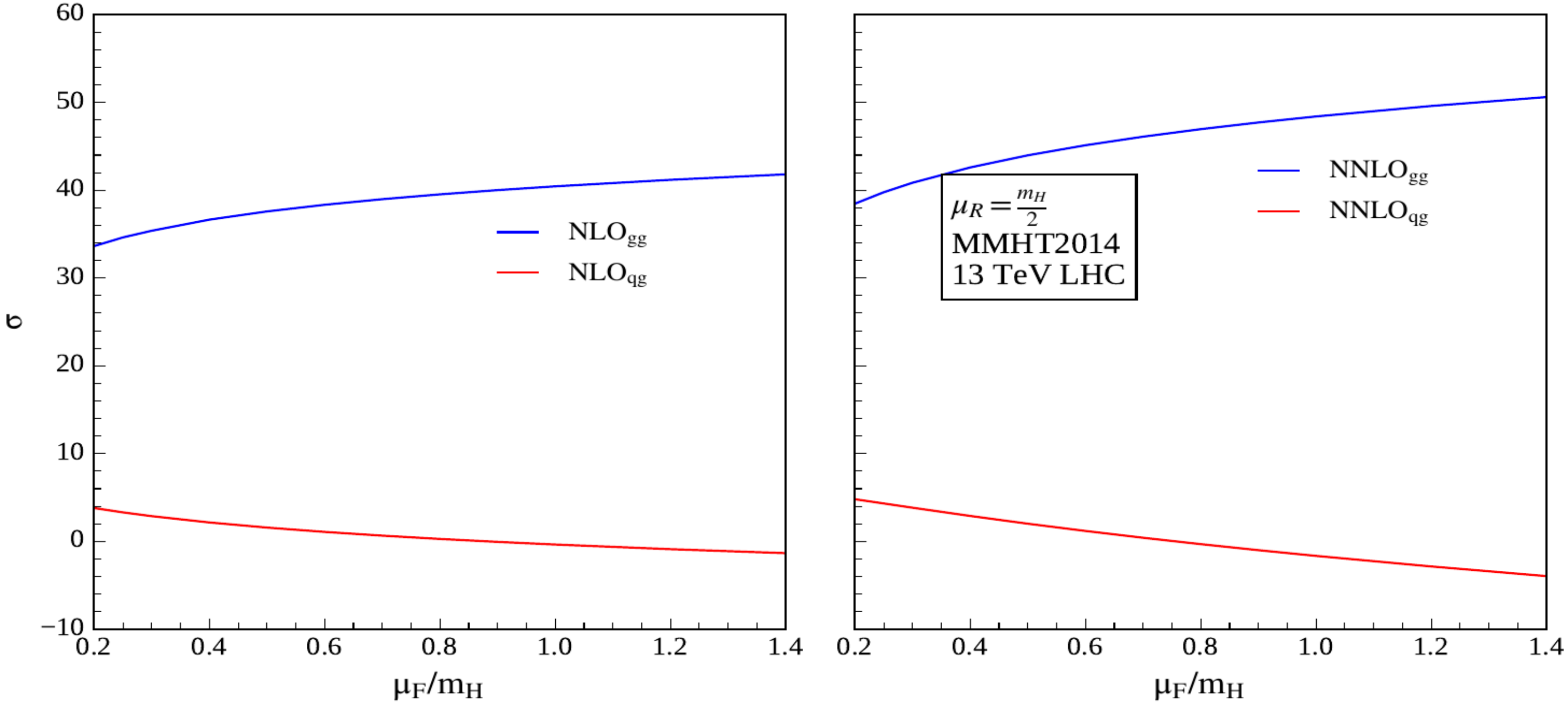
Phenomenology – Drell-Yan Process

$q\bar{q}$ & qg contributions under μ_F variation keeping μ_R fixed



Phenomenology – Higgs production in gg fusion

gg & qg contributions under μ_F variation keeping μ_R fixed



$$\Phi_B^c(\hat{a}_s, \mu^2, q^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \varphi_c^{(i)}(z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \; \hat{\Phi}_{NSV,c}^{(i)}(z, \epsilon)\,,$$

$$\begin{aligned}\hat{\Phi}_{NSV,c}^{(1)}(z,\epsilon) = {}& \frac{1}{\epsilon}\Big[\hat{\phi}_0^{c,(1,-1)}\Big] + \Big[\hat{\phi}_0^{c,(1,0)} + \hat{\phi}_1^{c,(1,0)}\log(1-z)\Big] + \epsilon\Big[\hat{\phi}_0^{c,(1,1)} + \hat{\phi}_1^{c,(1,1)}\log(1-z) \\ & + \hat{\phi}_2^{c,(1,1)}\log^2(1-z)\Big] + \epsilon^2\Big[\hat{\phi}_0^{c,(1,2)} + \hat{\phi}_1^{c,(1,2)}\log(1-z) + \hat{\phi}_2^{c,(1,1)}\log^2(1-z) \\ & + \hat{\phi}_3^{c,(1,3)}\log^3(1-z)\Big] + \mathcal{O}(\epsilon^3)\end{aligned}$$

$$\begin{aligned}\hat{\Phi}_{NSV,c}^{(2)}(z,\epsilon) = {}& \frac{1}{\epsilon^2}\Big[\hat{\phi}_0^{c,(2,-2)}\Big] + \frac{1}{\epsilon}\Big[\hat{\phi}_0^{c,(2,-1)} + \hat{\phi}_1^{c,(2,-1)}\log(1-z)\Big] + \Big[\hat{\phi}_0^{c,(2,0)} + \hat{\phi}_1^{c,(2,0)}\log(1-z) \\ & + \hat{\phi}_2^{c,(2,0)}\log^2(1-z)\Big] + \epsilon\Big[\hat{\phi}_0^{c,(2,1)} + \hat{\phi}_1^{c,(2,1)}\log(1-z) + \hat{\phi}_2^{c,(2,1)}\log^2(1-z) \\ & + \hat{\phi}_3^{c,(2,1)}\log^3(1-z)\Big] + \mathcal{O}(\epsilon^2)\end{aligned}$$

$$\begin{aligned}\hat{\Phi}_{NSV,c}^{(3)}(z,\epsilon) = {}& \frac{1}{\epsilon^3}\Big[\hat{\phi}_0^{c,(3,-3)}\Big] + \frac{1}{\epsilon^2}\Big[\hat{\phi}_0^{c,(3,-2)} + \hat{\phi}_1^{c,(3,-2)}\log(1-z)\Big] + \frac{1}{\epsilon}\Big[\hat{\phi}_0^{c,(3,-1)} + \hat{\phi}_1^{c,(3,-1)}\log(1-z) \\ & + \hat{\phi}_2^{c,(3,-1)}\log^2(1-z)\Big] + \Big[\hat{\phi}_0^{c,(3,0)} + \hat{\phi}_1^{c,(3,0)}\log(1-z) + \hat{\phi}_2^{c,(3,0)}\log^2(1-z) \\ & + \hat{\phi}_3^{c,(3,0)}\log^3(1-z)\Big] + \mathcal{O}(\epsilon)\end{aligned}$$

$$\Phi_B^c \;=\; \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=-i}^{\infty} \sum_{k=0}^{i+j} \hat{\Phi}_k^{c,(i,j)} \epsilon^j \log^k(1-z).$$

$$\frac{d\sigma^c}{dy} = \sigma_{\mathrm{B}}^c(\tau, q^2) \sum_{a,b=q,\bar{q},g} \int_{x_1^0}^1 \frac{dz_1}{z_1} \int_{x_2^0}^1 \frac{dz_2}{z_2} f_a\left(\frac{x_1^0}{z_1}, \mu_F^2\right) \times f_b\left(\frac{x_2^0}{z_2}, \mu_F^2\right) \Delta_{d,ab}^c(z_1, z_2, q^2, \mu_F^2, \mu_R^2),$$

$$\Delta_{d,c}^{\mathrm{SV+NSV}} = \mathcal{C} \exp(\Psi_d^c(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon))|_{\epsilon=0},$$

$$\Phi_d^c = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2 \bar{z}_1 \bar{z}_2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \left[\frac{(i\epsilon)^2}{4\bar{z}_1\bar{z}_2} \hat{\phi}_d^{c,(i)}(\epsilon) + \frac{i\epsilon}{4\bar{z}_1} \varphi_{d,c}^{(i)}(\bar{z}_2, \epsilon) + \frac{i\epsilon}{4\bar{z}_2} \varphi_{d,c}^{(i)}(\bar{z}_1, \epsilon)\right],$$

$$\Psi_d^c = \frac{\delta(\bar{z}_1)}{2}\left(\int_{\mu_F^2}^{q^2\bar{z}_2}\frac{d\lambda^2}{\lambda^2}\mathcal{P}^c(a_s(\lambda^2),\bar{z}_2) + \mathcal{Q}_d^c(a_s(q_2^2),\bar{z}_2)\right)_+$$
$$+\frac{1}{4}\left(\frac{1}{\bar{z}_1}\left\{\mathcal{P}^c(a_s(q_{12}^2),\bar{z}_2) + 2L^c(a_s(q_{12}^2),\bar{z}_2)\right.\right.$$
$$\left.\left.+\,q^2\frac{d}{dq^2}\left(\mathcal{Q}_d^c(a_s(q_{12}^2),\bar{z}_2) + 2\varphi_{d,c}^f(a_s(q_2^2),\bar{z}_2)\right)\right\}\right)_+$$
$$+\frac{1}{2}\delta(\bar{z}_1)\delta(\bar{z}_2)\ln\left(g_{d,0}^c(a_s(\mu_F^2))\right) + \bar{z}_1 \leftrightarrow \bar{z}_2, \qquad (6)$$

MATCHING WITH THE INCLUSIVE

$$\int_0^1 dx_1^0 \int_0^1 dx_2^0 (x_1^0 x_2^0)^{N-1} \frac{d\sigma^c}{dy} = \int_0^1 d\tau \tau^{N-1} \sigma^c,$$

$$\sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{\frac{i\epsilon}{2}} S_\epsilon^i \Bigg[t_1^i(\epsilon) \hat{\phi}_d^{c,(i)}(\epsilon) - t_2^i(\epsilon) \hat{\phi}^{c,(i)}(\epsilon) + \sum_{k=0}^{\infty} \left(t_3^{(i,k)}(\epsilon) \varphi_{d,c}^{(i,k)}(\epsilon) - t_4^{(i,k)}(\epsilon) \varphi_c^{(i,k)}(\epsilon) \right) \Bigg] = 0.$$

$$
\begin{aligned}
t_1^i &= \frac{i\epsilon(2-i\epsilon)}{4N^{i\epsilon}}\Gamma^2\left(1+i\frac{\epsilon}{2}\right), \qquad t_2^i = \frac{i\epsilon(1-i\epsilon)}{2N^{i\epsilon}}\Gamma(1+i\epsilon), \\
t_3^{(i,k)} &= \Gamma\left(1+i\frac{\epsilon}{2}\right)\frac{\partial^k}{\partial\alpha^k}\left(\frac{\Gamma(1+\alpha)}{N^{\alpha+i\epsilon/2}}\right)_{\alpha=i\frac{\epsilon}{2}}, \\
t_4^{(i,k)} &= \frac{\partial^k}{\partial\hat{\alpha}^k}\left(\frac{\Gamma(1+\hat{\alpha})}{N^{\hat{\alpha}}}\right)_{\hat{\alpha}=i\epsilon}.
\end{aligned} \tag{11}
$$

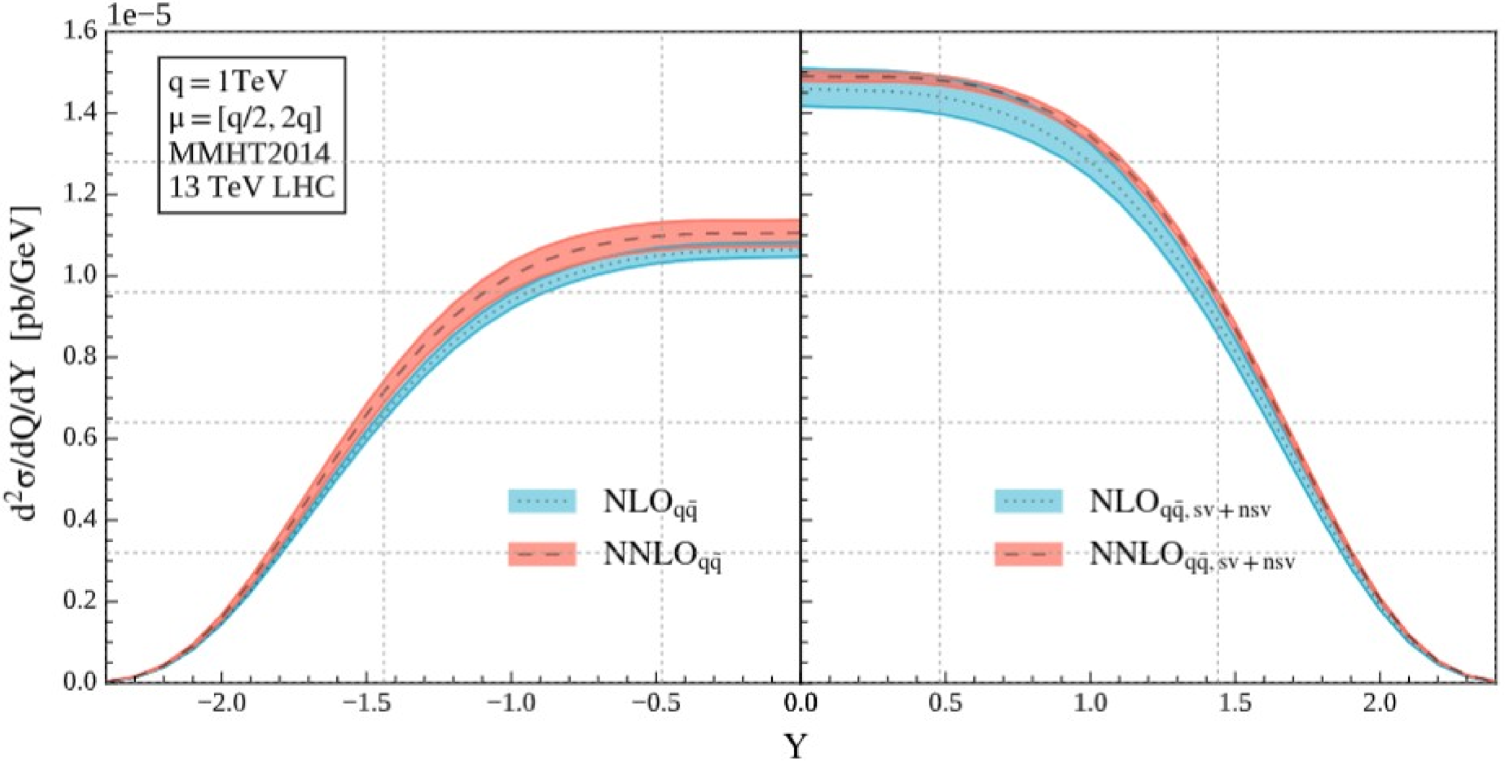
$$\Psi^c_{d,\vec{N}} = \left(g^c_{d,1}(\omega) + \frac{1}{N_1} \bar{g}^c_{d,1}(\omega) \right) \ln N_1$$
$$+ \sum_{i=0}^{\infty} a_s^i \left(\frac{1}{2} g^c_{d,i+2}(\omega) + \frac{1}{N_1} \bar{g}^c_{d,i+2}(\omega) \right)$$
$$+ \frac{1}{N_1} \sum_{i=0}^{\infty} a_s^i h^c_{d,i}(\omega, N_1) + (N_1 \leftrightarrow N_2),$$

where

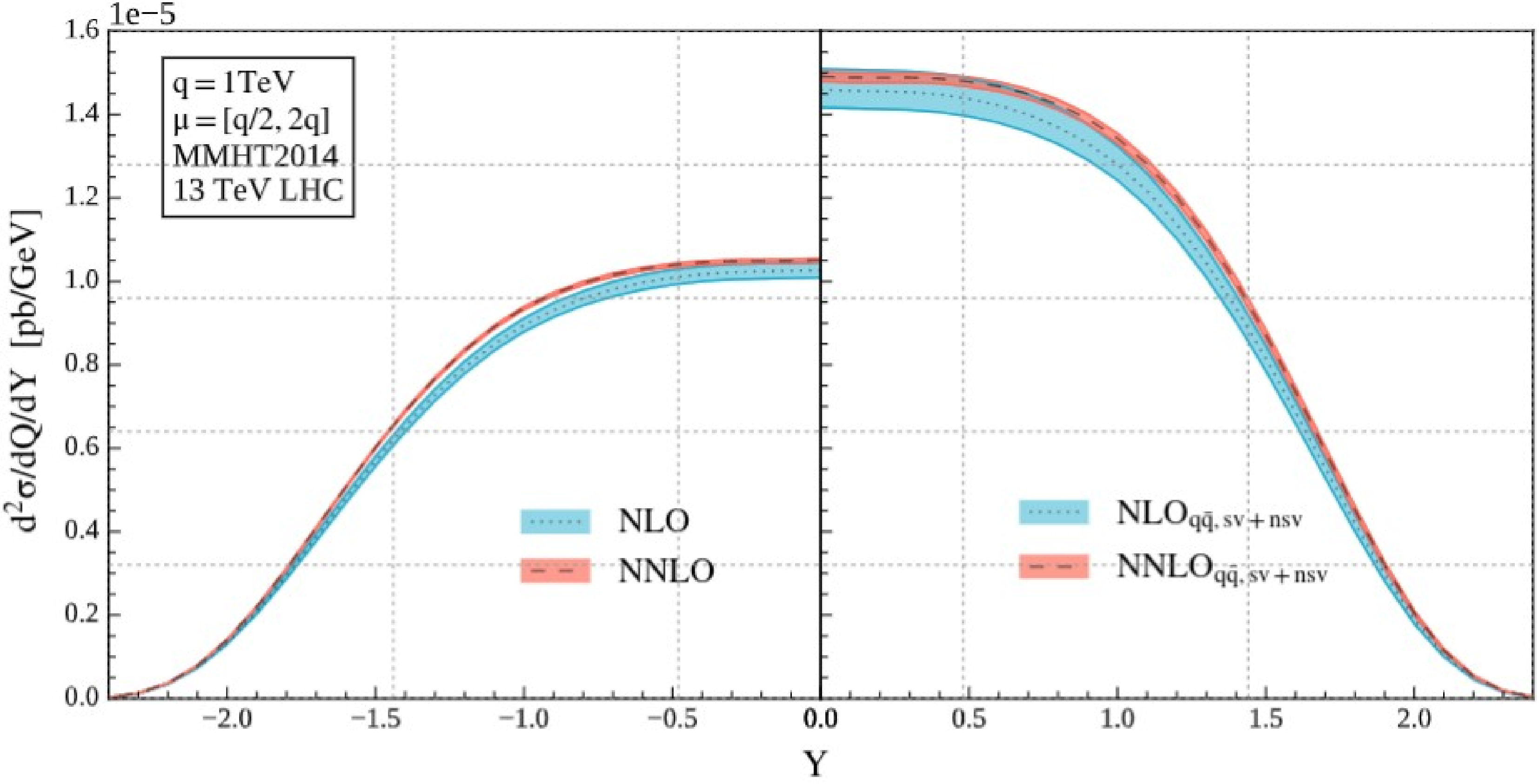
$$h^c_{d,0}(\omega, N_l) = h^c_{d,00}(\omega) + h^c_{d,01}(\omega) \ln N_l,$$
$$h^c_{d,i}(\omega, N_l) = \sum_{k=0}^{i} h^c_{d,ik}(\omega) \ln^k N_l,$$

GIVEN Resummed exponents	PREDICTIONS: SV logarithms $\Delta^{q,(2)}_{d,N_1,N_2}$	$\Delta^{q,(3)}_{d,N_1,N_2}$	$\Delta^{q,(4)}_{d,N_1,N_2}$	$\cdots$	$\Delta^{q,(n)}_{d,N_1,N_2}$	Logarithmic accuracy
$\tilde{g}^q_{d,0,0}, g^q_{d,1}$	$\{L_1^i L_2^j\}\vert_{i+j=4}$	$\{L_1^i L_2^j\}\vert_{i+j=6}$	$\{L_1^i L_2^j\}\vert_{i+j=8}$	$\cdots$	$\{L_1^i L_2^j\}\vert_{i+j=2n}$	LL
$\tilde{g}^q_{d,0,1}, g^q_{d,2}$		$\{L_1^i L_2^j\}\vert_{i+j=5,4}$	$\{L_1^i L_2^j\}\vert_{i+j=7,6}$	$\cdots$	$\{L_1^i L_2^j\}\vert_{i+j=2n-1,2n-2}$	NLL
$\tilde{g}^q_{d,0,2}, g^q_{d,3}$			$\{L_1^i L_2^j\}\vert_{i+j=5,4}$	$\cdots$	$\{L_1^i L_2^j\}\vert_{i+j=2n-3,2n-4}$	NNLL

GIVEN Resummed exponents	PREDICTIONS: NSV logarithms $\Delta^{q,(2)}_{d,N_1,N_2}$	$\Delta^{q,(3)}_{d,N_1,N_2}$	$\Delta^{q,(4)}_{d,N_1,N_2}$	$\cdots$	$\Delta^{q,(n)}_{d,N_1,N_2}$	Logarithmic accuracy
$\tilde{g}^q_{d,0,0}, g^q_{d,1}, \bar{g}^q_{d,1}, h^q_{d,0}$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=3}$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=5}$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=7}$	$\cdots$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=2n-1}$	$\overline{\text{LL}}$
$\tilde{g}^q_{d,0,1}, g^q_{d,2}, \bar{g}^q_{d,2}, h^q_{d,1}$		$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=4}$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=6}$	$\cdots$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=2n-2}$	$\overline{\text{NLL}}$
$\tilde{g}^q_{d,0,2}, g^q_{d,3}, \bar{g}^q_{d,3}, h^q_{d,2}$			$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=5}$	$\cdots$	$\{L^{i,j}_{N_1,2}, L^{i,j}_{N_2,1}\}\vert_{i+j=2n-3}$	$\overline{\text{NNLL}}$



1e−5
q = 1TeV
μ = [q/2, 2q]
MMHT2014
13 TeV LHC
d²σ/dQ/dY [pb/GeV]
NLO$_{q\bar{q}}$
NNLO$_{q\bar{q}}$
NLO$_{q\bar{q},\,sv+nsv}$
NNLO$_{q\bar{q},\,sv+nsv}$
Y



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Y