SMEFT input schemes

Darren Scott

In collaboration with:
Anke Biekötter, Ben Pecjak, Tommy Smith

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

- (Very) basic introduction to The dim-6 Standard Model Effective Field Theory (SMEFT)
- Input schemes
  - Examples from The SM and extension to SMEFT
- Practical implementation
- Main results
  - As observed through practical decay examples (W,Z)
  - Interesting observations
- Conclusions

SMEFT in a nutshell

- Extend the SM with operators of higher mass dimension
- Gives a “model independent” way to parametrize new physics effects

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{d=5}^{\infty} \sum_{i} \frac{C_i^{(d)}}{\Lambda_{\mathrm{NP}}^{d-4}} \mathcal{O}_i^{(d)}$$

SMEFT in a nutshell

- Extend the SM with operators of higher mass dimension
- Gives a “model independent” way to parametrize new physics effects

Wilson coefficients

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{d=5}^{\infty} \sum_{i} \frac{C_i^{(d)}}{\Lambda_{\mathrm{NP}}^{d-4}} \mathcal{O}_i^{(d)}$$

“New physics” scale

Operator of mass dimension *d*
- SM fields only

$SU(3) \times SU(2) \times U(1)$

SMEFT in a nutshell

- We will restrict to baryon number conserving dimension-6 operators only
- Warsaw basis

[Grzadkowski, Iskrzynski, Misiak, Rosiek: JHEP 10 (2010) 085]

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i C_i \mathcal{O}_i \qquad\qquad C_i \sim \Lambda^{-2}$$

- 2499 such independent Wilson coefficients

SMEFT in a nutshell

$1: X^3$

Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$

Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$

$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$

Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$

Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \sigma^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \sigma^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$

Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\sigma^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\sigma^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\sigma^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$

$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_p \sigma^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \sigma^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
$Q_{Hud} + \text{h.c.}$	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

SMEFT in a nutshell

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G^A_{\mu\nu} G$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^A_{\mu\nu} G$
Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \sigma^I H\, W^I_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \sigma^I H\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G^A_{\mu\nu}$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\sigma^I \widetilde{H}\, W^I_{\mu\nu}$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G^A_{\mu\nu}$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\sigma^I H\, W^I_{\mu\nu}$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_p \sigma^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \sigma^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

+ 25 four-fermion operators

SMEFT in a nutshell

- Important to note: The presence of the dim-6 operators causes some parameters to be shifted from their SM equivalents
  - Normalization of the Higgs doublet
  - Weak mixing angle
  - Yukawas...

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} i \phi^+(x) \\ \left[1 + C_{H,\text{kin}}\right] h(x) + i \left[1 - \frac{\hat{v}_T^2}{4} C_{HD}\right] \phi^0(x) + v_T \end{pmatrix}$$

$\sim C_{HD}, C_{H\Box}$

$$\hat{c}_w^2 = \frac{M_W^2}{M_Z^2}$$

$$\bar{s}_w^2 \to \hat{s}^2 - \frac{\hat{c}_w^2 v_T^2}{2} \left(C_{HD} + \frac{\hat{s}_w}{\hat{c}_w} C_{HWB} \right)$$

$$\langle H^\dagger H \rangle = \frac{v_T^2}{2}$$

SMEFT at NLO

- Why go to NLO for SMEFT calculations?
  - Increased precision / reduced uncertainties
  - New operators may appear at loop level (which may have numerically large prefactors)
  - Check on the perturbative convergence
- Important to consider the choice of input parameters (see next slide) – <u>the focus of this talk</u>
- For the purpose of this study we have computed at NLO:

$$h \to b\bar{b} \qquad\qquad Z \to l\bar{l} \qquad\qquad W \to l\bar{\nu}$$

* I will only show a collection of results here though

Input schemes

- The SM(EFT) Lagrangian contains a number of undefined parameters
  - Gauge couplings, Yukawas, CKM elements, Higgs VEV/self coupling..
- These need to be fixed via measurement – relate to observables
  - m_H, m_t usually renormalized on-shell
  - $m_f = 0$ for all other fermions (except m_b in $h \to b\bar{b}$)
  - Approximate CKM elements $V_{ij} = \delta_{ij}$
- Still leaves us with three undetermined parameters

$$\{g_1, g_2, v_T\}$$

Input schemes

- We need three more inputs
- Some contenders...

$$M_W = 80.433(9)\,\mathrm{GeV} \qquad \alpha(M_Z) = 0.007127(2)$$
$$M_Z = 91.1876(21)\,\mathrm{GeV} \qquad G_\mu = 1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

- Subset of three will determine an input scheme
- We will consider three such schemes:

α –scheme	α_μ –scheme	LEP scheme
$\{\alpha, M_W, M_Z\}$	$\{G_\mu, M_W, M_Z\}$	$\{\alpha, G_\mu, M_Z\}$

Note: A nice discussion of scheme choices can be found in [Brivio: JHEP 04 (2021) 073]

Input schemes

- In all cases the Wilson coefficients are renormalized in the $\overline{\text{MS}}$-scheme
- Can read off the poles from the anomalous dimension calculations

[Jenkins, Manohar, Trott: JHEP 10 (2013) 087, JHEP 01 (2014) 035]
[Alonso, Jenkins, Manohar, Trott: JHEP 04 (2014) 159]

$$C_{i,0} = C_i + \delta C_i \qquad\qquad \delta C_i \equiv \frac{1}{2\epsilon} \frac{dC_i}{d\ln\mu}$$

- Operator mixing can induce large numbers of new Wilson coefficients in the anomalous dimension

The α scheme

$$\text{Inputs}:\{\alpha, M_W, M_Z\}$$

- Important to discuss the renormalization of these input parameters See eg: [Denner, Dittmaier: Phys.Rept. 864 (2020) 1-163]
- M_W, M_Z renormalized on-shell
- Tadpoles explicitly included everywhere (FJ tadpole scheme)
  [Fleischer, Jegerlehner: Phys. Rev. D 23 (1981) 2001]
- We use an "$\overline{\text{MS}}$-lite" scheme for α
  [Cullen, Pecjak, DS: JHEP 08 (2019), 173]

The α scheme

- α defined as the eeγ coupling at zero momentum transfer: $\alpha^{\mathrm{O.S.}}(0)$

  $e_0 = e + \delta e$

- Often "run" to an effective value: $\alpha^{O.S.}(M_Z) = \dfrac{\alpha^{O.S.}(0)}{1 - \Delta\alpha(M_Z)}$
- For our $\overline{\text{MS}}$-lite scheme: $\bar{\alpha}^{(\ell)}(\mu)$
  - All particles heavier than the b-quark decoupled (defined in a five-flavour QEDxQCD scheme)
- Can relate to the on-shell value:

$$\bar{\alpha}^{(\ell)}(M_Z) = \frac{\alpha^{O.S.}(0)}{1 - \Delta\bar{\alpha}^{(\ell)}(M_Z)} \longrightarrow \bar{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z)\left[1 + \frac{\alpha(M_Z)}{\pi}\frac{100}{27}\right]$$

The α scheme

- For the purpose of later comparison it is helpful to use

$$v_\alpha^2 = \frac{M_W^2 s_w^2}{\pi\alpha}$$

- Renormalization of α then appears in the relation between the bare and renormalized vev

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\alpha^2}\left[1 - v_\alpha^2 \Delta v_\alpha^{(6,0,\alpha)} - \frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)} - \Delta v_\alpha^{(6,1,\alpha)}\right]$$

With the tree level result: $\Delta v_\alpha^{(6,0,\alpha)} = -2\frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{4s_w} C_{HD}\right]$

The α_μ scheme

$$\text{Inputs}: \{G_\mu, M_W, M_Z\}$$

- We can include G_μ by relating it to the bare and renormalized VEV
- Define:

$$v_\mu \equiv \left(\sqrt{2}G_\mu\right)^{-\frac{1}{2}}$$

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\mu^2}\left[1 - v_\mu^2 \Delta v^{(6,0,\mu)} - \frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)} - \Delta v_\mu^{(6,1,\mu)}\right]$$

- Renormalization condition provided through muon decay:

  SMEFT amplitude = SM tree level amplitude

At tree level: $\Delta v_\mu^{(6,0)} = C_{\substack{Hl\\11}}^{(3)} + C_{\substack{Hl\\22}}^{(3)} - C_{\substack{ll\\1221}}$

The “LEP” scheme

$$\text{Inputs} : \{G_\mu, \alpha, M_Z\}$$

- G_μ and α defined as before
- Now M_W is a derived parameter

$$M_{W,0} = \hat{M}_W \left(1 + v_\mu^2 \Delta\hat{M}_W^{(6,0,\mu)}(\hat{M}_W) + \frac{1}{v_\mu^2}\Delta\hat{M}_W^{(4,1,\mu)}(\hat{M}_W) + \Delta\hat{M}_W^{(6,1,\mu)}(\hat{M}_W)\right)$$

Where: $$\hat{M}_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha v_\mu^2}{M_Z^2}}\right)$$

- Relate this definition to the on-shell mass:

$$M_W = \hat{M}_W \left[1 + v_\mu^2 \hat{\Delta}_W^{(6,0,\mu)} + \frac{1}{v_\mu^2}\hat{\Delta}_W^{(4,1,\mu)} + \hat{\Delta}_W^{(6,1,\mu)}\right]$$

The “LEP” scheme

- The relation between M_W and $\hat{M}_W$ also requires the tree level expression for the W mass in the SMEFT in this scheme

Can be derived using the expressions in for example: [Alonso, Jenkins, Manohar, Trott: JHEP 1404 (2014) 159]

$$M_W = \hat{M}_{W,T}\left[1 + v_T^2 \frac{\hat{c}_{w,T}\hat{s}_{w,T}}{1-2\hat{c}^2_{w,T}}\left(C_{HWB} + \frac{\hat{c}_{w,T}}{4\hat{s}_{w,T}}C_{HD}\right)\right]$$

Subscript T implies full bare VEV used $v_{T,0}$

- Interpret as a relation between bare parameters and expand in counterterms $\rightarrow$ gives $\Delta\hat{M}_W$ from previous slide
- Combining with counterterms from OS W mass, we can derive the $\hat{\Delta}_W$

E.g. $$\Delta\hat{M}_W^{(6,0,\mu)} = \frac{1}{1-2\hat{c}_w^2}\left[\hat{c}_w\hat{s}_w\left(C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w}C_{HD}\right) + \frac{\hat{s}_w^2}{2}\hat{\Delta}v_\mu^{(6,0,\mu)}\right]$$

From vev relation in the for G_μ from muon decay

Questions

- Key questions:
  - How does the perturbative convergence differ between different schemes?
  - How does the number of Wilson coefficients differ when calculating processes in different schemes?
  - How large are the numerical prefactors of different Wilson coefficients in different schemes?

- Before jumping to results for the decays we calculated, we can already examine some features of the schemes themselves...

Scheme features

- SM

$$\alpha_\mu : \qquad \frac{1}{v_\mu^2}\left(1 - \frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)}\right) = \frac{1}{v_\mu^2}\left(1 - 0.0003 - 0.051\,[\mathrm{top},\mathrm{tadpole}]\right)$$

$$\alpha : \qquad \frac{1}{v_\alpha^2}\left(1 - \frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)}\right) = \frac{1}{v_\alpha^2}\left(1 - 0.044 - 0.052\,[\mathrm{top},\mathrm{tadpole}]\right)$$

$$= \frac{1}{v_\mu^2}\left(1 - 0.008 - 0.054\,[\mathrm{top},\mathrm{tadpole}]\right)$$

Scheme features

- SM

$$\alpha_\mu : \quad \frac{1}{v_\mu^2}\left(1 - \frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)}\right) = \frac{1}{v_\mu^2}\left(1 - 0.0003 - \boxed{0.051\,[\text{top, tadpole}])}\right.$$

$$\alpha : \quad \frac{1}{v_\alpha^2}\left(1 - \frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)}\right) = \frac{1}{v_\alpha^2}\left(1 - 0.044 - \boxed{0.052\,[\text{top, tadpole}])}\right.$$

$$= \frac{1}{v_\mu^2}\left(1 - 0.008 - \boxed{0.054\,[\text{top, tadpole}])}\right.$$

Tadpoles from tops – should cancel out in the end

Scheme features

- SM

$$\alpha_\mu : \qquad \frac{1}{v_\mu^2}\left(1 - \frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)}\right) = \frac{1}{v_\mu^2}\left(1 - 0.0003 - 0.051\,[\text{top}, \text{tadpole}]\right)$$

$$\alpha : \qquad \frac{1}{v_\alpha^2}\left(1 - \frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)}\right) = \frac{1}{v_\alpha^2}\left(1 - 0.044 - 0.052\,[\text{top}, \text{tadpole}]\right)$$

$$= \frac{1}{v_\mu^2}\left(1 - 0.008 - 0.054\,[\text{top}, \text{tadpole}]\right)$$

Correction much larger in the α scheme.
We can look at the individual counterterms which make up this correction

Scheme features

$$-\frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)} = -2\Delta s_w^{(4,1,\alpha)} + 0.006 - 0.052\,[\text{top, tadpole}]$$

Where: $$\Delta s_w^{(i,1,\alpha)} = -\frac{c_w^2}{s_w^2}\left(\Delta M_W^{(i,1,\alpha)} - \Delta M_Z^{(i,1,\alpha)}\right)$$

Related to mass counterterms

Can examine the large m_t limit:

$$-2\Delta s_w^{(4,1,\alpha)} = 2\frac{c_w^2}{s_w^2}\left(-\frac{3}{32\pi^2}\frac{m_t^2}{v_\alpha^2} - 0.002\right) = \boxed{2\frac{c_w^2}{s_w^2}}(-0.005 - 0.002) = -0.050$$

Factor of 7

Compare to α_μ scheme:

$$-\frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)} = \frac{3}{16\pi^2}\frac{m_t^2}{v_\mu^2}\left(1 + 2\ln\frac{\mu^2}{m_t^2}\right) + 0.002 - 0.050\,[\text{top, tadpole}]$$

$$= -0.003 + 0.002 - 0.050\,[\text{top, tadpole}]$$

Scheme features

What about SMEFT? Examining the large m_t limit

$$\alpha: \quad \Delta v_{\alpha,t}^{(6,1,\alpha)} = \frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{2s_w}C_{HD}\left(1 + 2s_w^2 \ln\frac{\mu^2}{m_t^2}\right) + 4c_w s_w\left(C_{\underset{33}{Hq}}^{(1)} - C_{\underset{33}{Hu}}\right)\ln\frac{\mu^2}{m_t^2}\right]\frac{3m_t^2}{16\pi^2 s_w^2}$$

$$- \Delta v_{\alpha,t}^{(4,1,\alpha)}\left(\Delta v_\alpha^{(6,0,\alpha)} - 2C_{\underset{33}{Hq}}^{(3)} + 2\sqrt{2}\frac{M_W}{m_t}C_{\underset{33}{uW}}\right)$$

$$- \frac{M_W m_t}{2\sqrt{2}\pi^2 s_w}\left(3c_w C_{\underset{33}{uB}} + 8s_w C_{\underset{33}{uW}}\right)\ln\frac{\mu^2}{m_t^2} + \ldots$$

$$= v_\alpha^2\left[0.22C_{HWB} + 0.12C_{HD}\right.$$

$$\left.+ 10^{-2}\left(7.3C_{\underset{33}{Hq}}^{(3)} + 8.8C_{\underset{33}{Hu}} - 8.8C_{\underset{33}{Hq}}^{(1)} + 3.1C_{\underset{33}{uB}} - 0.4C_{\underset{33}{uW}}\right)\right] + \ldots$$

Where: $$\Delta v_{\alpha,t}^{(4,1,\alpha)} = \frac{3m_t^2}{16\pi^2 s_w^2}\left(1 - 2s_w^2 - 2s_w^2\ln\frac{\mu^2}{m_t^2}\right) + \ldots.$$

Scheme features

What about SMEFT? Examining the large m_t limit

$$\alpha: \quad \Delta v_{\alpha,t}^{(6,1,\alpha)} = \frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{2s_w}C_{HD}\left(1 + 2s_w^2 \ln\frac{\mu^2}{m_t^2}\right) + 4c_w s_w\left(C_{\substack{Hq\\33}}^{(1)} - C_{\substack{Hu\\33}}\right)\ln\frac{\mu^2}{m_t^2}\right]\boxed{\frac{3m_t^2}{16\pi^2 s_w^2}}$$

$$\boxed{-\Delta v_{\alpha,t}^{(4,1,\alpha)}}\left(\Delta v_{\alpha}^{(6,0,\alpha)} - 2C_{\substack{Hq\\33}}^{(3)} + 2\sqrt{2}\frac{M_W}{m}C_{\substack{uW\\33}}\right)$$

$$-\frac{M_W m_t}{2\sqrt{2}\pi^2 s_w}\left(3c_w C_{\substack{uB\\33}} + 8s_w C_{\substack{uW\\33}}\right)\ln$$

$$= v_\alpha^2\Big[0.22C_{HWB} + 0.12C_{HD}$$

$$+10^{-2}\left(7.3C_{\substack{Hq\\33}}^{(3)} + 8.8C_{\substack{Hu\\33}} - 8.8C_{\substack{Hq\\33}}^{(1)}\right.$$

Largest corrections (>5%) are of the same origin as in the SM.
Enhanced by

$$m_t^2 \qquad 1/s_w^2$$

Where: $$\Delta v_{\alpha,t}^{(4,1,\alpha)} = \boxed{\frac{3m_t^2}{16\pi^2 s_w^2}}\left(1 - 2s_w^2 - 2s_w^2\ln\frac{\mu^2}{m_t^2}\right) + \dots .$$

Scheme features

What about SMEFT?
Again in the large m_t limit

$$\alpha_\mu : \quad \Delta v_{\mu,t}^{(6,1,\mu)} = -\left(C_{\substack{Hl \\ 11}}^{(3)} + C_{\substack{Hl \\ 33}}^{(3)} - 2C_{\substack{ll \\ 1221}} - 2C_{\substack{Hq \\ 33}}^{(3)} + C_{\substack{lq \\ 1133}}^{(3)} + C_{\substack{lq \\ 2233}}^{(3)} \right) \Delta v_{\mu,t}^{(4,1,\mu)}$$

Where: $$\Delta v_{\mu,t}^{(4,1,\mu)} = -\frac{3m_t^2}{16\pi^2}\left(1 + 2\ln\frac{\mu^2}{m_t^2}\right)$$

The m_t^2 corrections are still present, but missing the additional enhancement by factors of $1/s_w^2$

The corrections here are typically larger in the α scheme than in the α_μ scheme

Results

- How do the scheme choices affect practical examples?
- Study decays of the W, Z, and H. Will present only W and Z here
- Results normalized to LO SM:

$$\Delta^{\rm sch}_{X f_1 f_2, {\rm LO}} = \frac{\Gamma^{(4,0,{\rm sch})}_{X f_1 f_2} + \Gamma^{(6,0,{\rm sch})}_{X f_1 f_2}}{\Gamma^{(4,0,{\rm sch})}_{X f_1 f_2}}$$

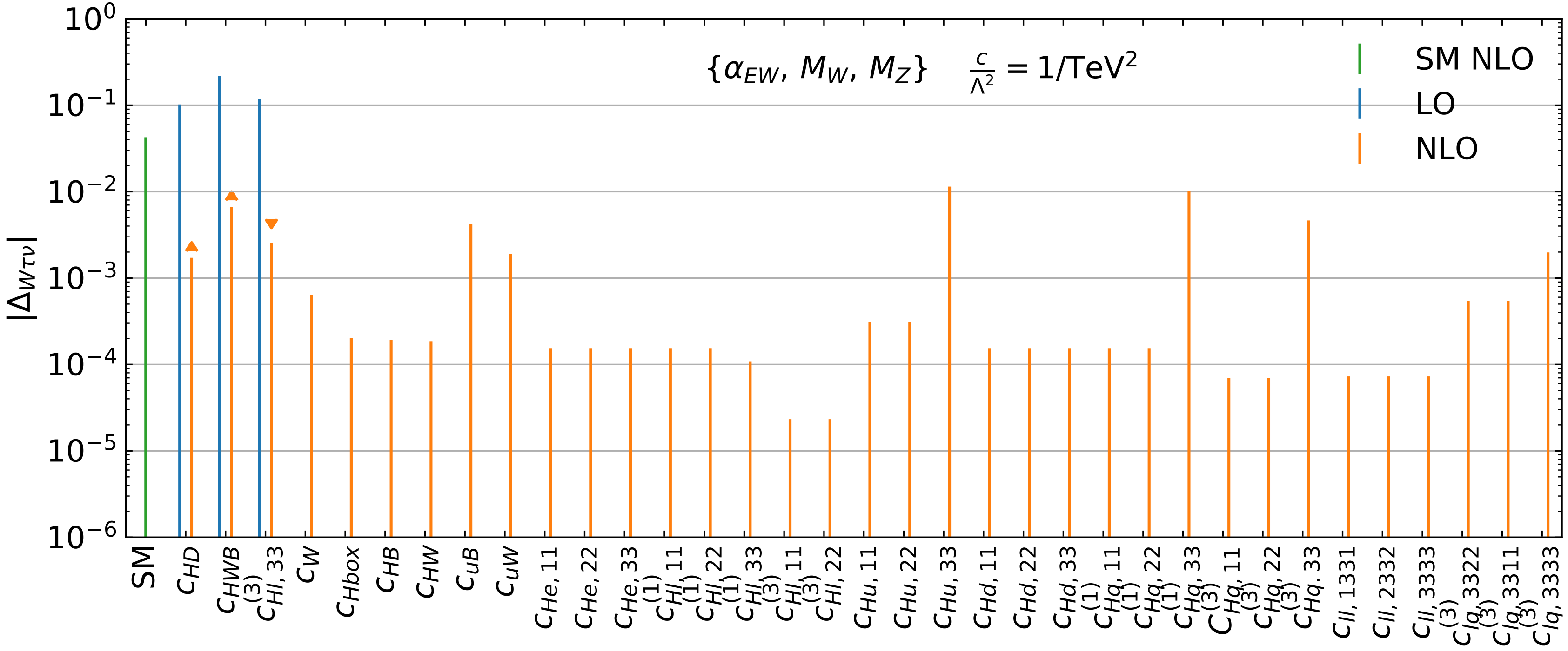
$$\Delta^{\rm sch}_{X f_1 f_2, {\rm NLO}} = \frac{\Gamma^{(4,1,{\rm sch})}_{X f_1 f_2} + \Gamma^{(6,1,{\rm sch})}_{X f_1 f_2}}{\Gamma^{(4,0,{\rm sch})}_{X f_1 f_2}}$$

Note this is only the NLO correction

Results: W decay

α −scheme : $W \to \tau\nu$
$\mu = M_W$

\# Wilson coefficients at LO: 3
\# Wilson coefficients at NLO: 36

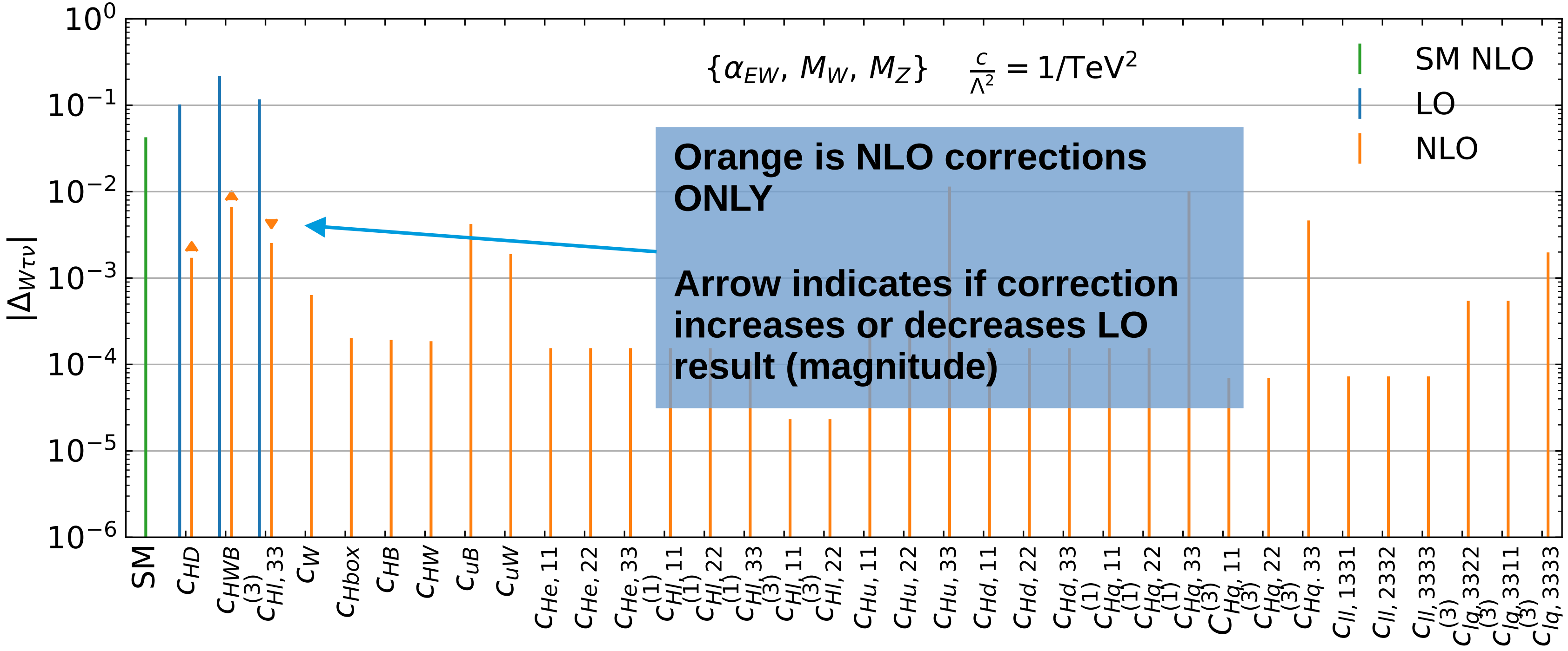


Results: W decay

$\alpha\ \text{–scheme}: \ W \to \tau\nu$

$\mu = M_W$

\# Wilson coefficients at LO: 3
\# Wilson coefficients at NLO: 36



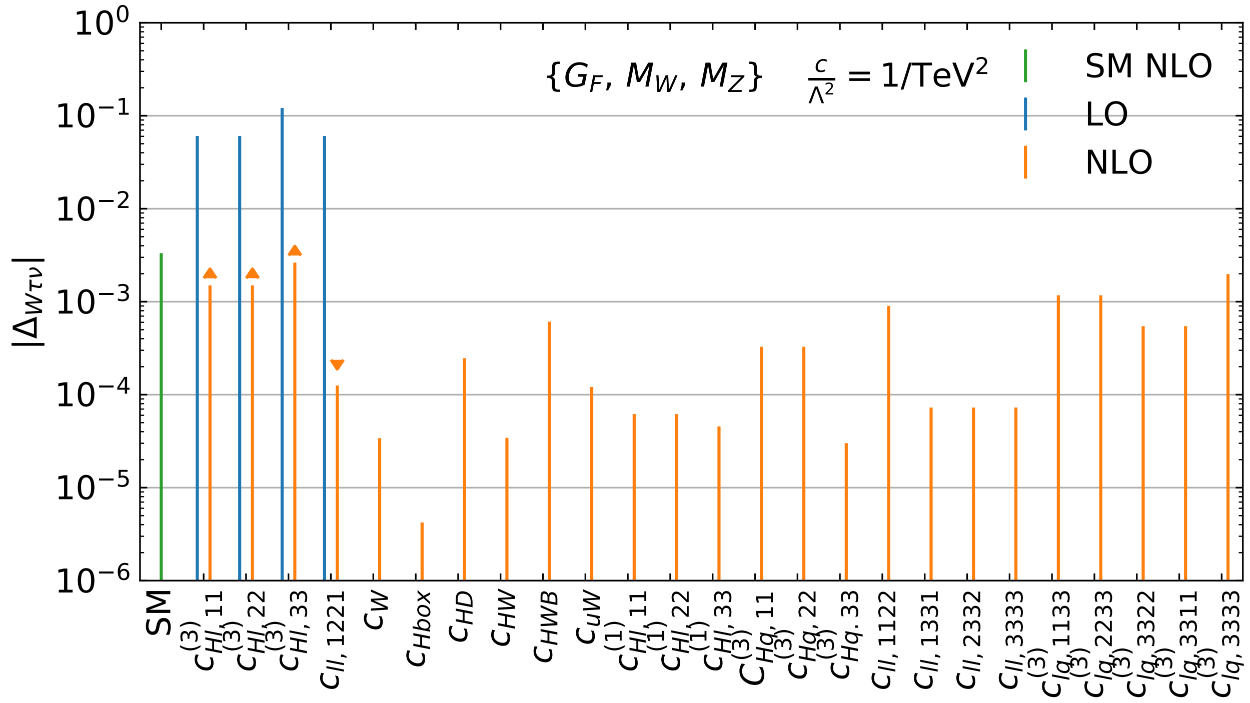
Orange is NLO corrections ONLY

Arrow indicates if correction increases or decreases LO result (magnitude)

Results: W decay

$$\alpha_\mu - \text{scheme}: \; W \to \tau\nu$$
$$\mu = M_W$$

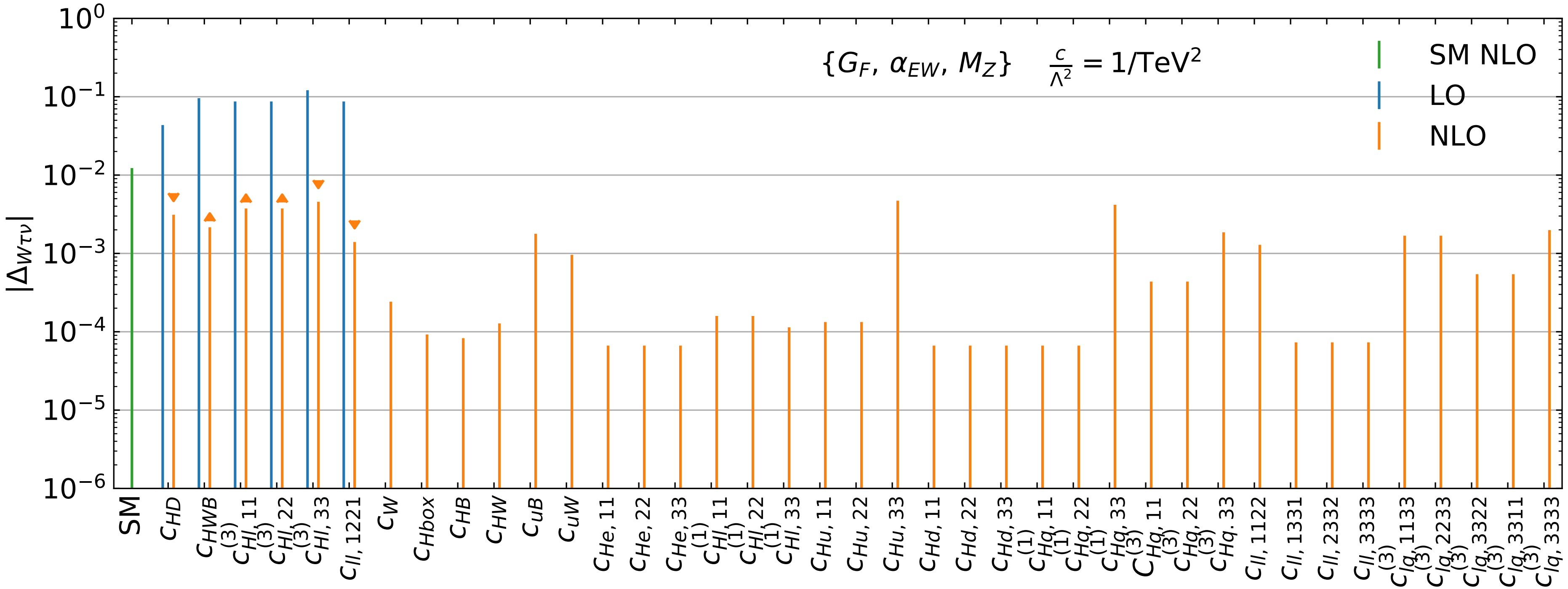
\# Wilson coefficients at LO: 4
\# Wilson coefficients at NLO: 25



Results: W decay

$$\text{LEP}-\text{scheme}: \ W \to \tau\nu$$
$$\mu = M_W$$

\# Wilson coefficients at LO: 6
\# Wilson coefficients at NLO: 39



Results: W decay

$W \to \tau\nu$
$\mu = M_W$

$W \to \tau\nu_\tau$	SM	C_{HD}	C_{HWB}	$C^{(3)}_{\substack{Hl\\11}}$	$C^{(3)}_{\substack{Hl\\22}}$	$C^{(3)}_{\substack{Hl\\33}}$	$C_{\substack{ll\\1221}}$
α	-4.2%	-1.7%	-3.0%	—	—	2.2%	—
α_μ	-0.3%	—	—	2.5%	2.5%	2.2%	-0.2%
LEP	1.2%	7.1%	2.0%	4.2%	4.2%	3.8%	1.5%

Size of NLO corrections to Wilson coefficients which appear at LO

- Far fewer Wilson coefficients at NLO in the α_μ scheme (25 vs 36 or 39)
- Slight differences between LO coefficients in different schemes
- Corrections to LO coefficients at the 1% level
- Largest Wilson coefficients first appearing at NLO arise from top loops

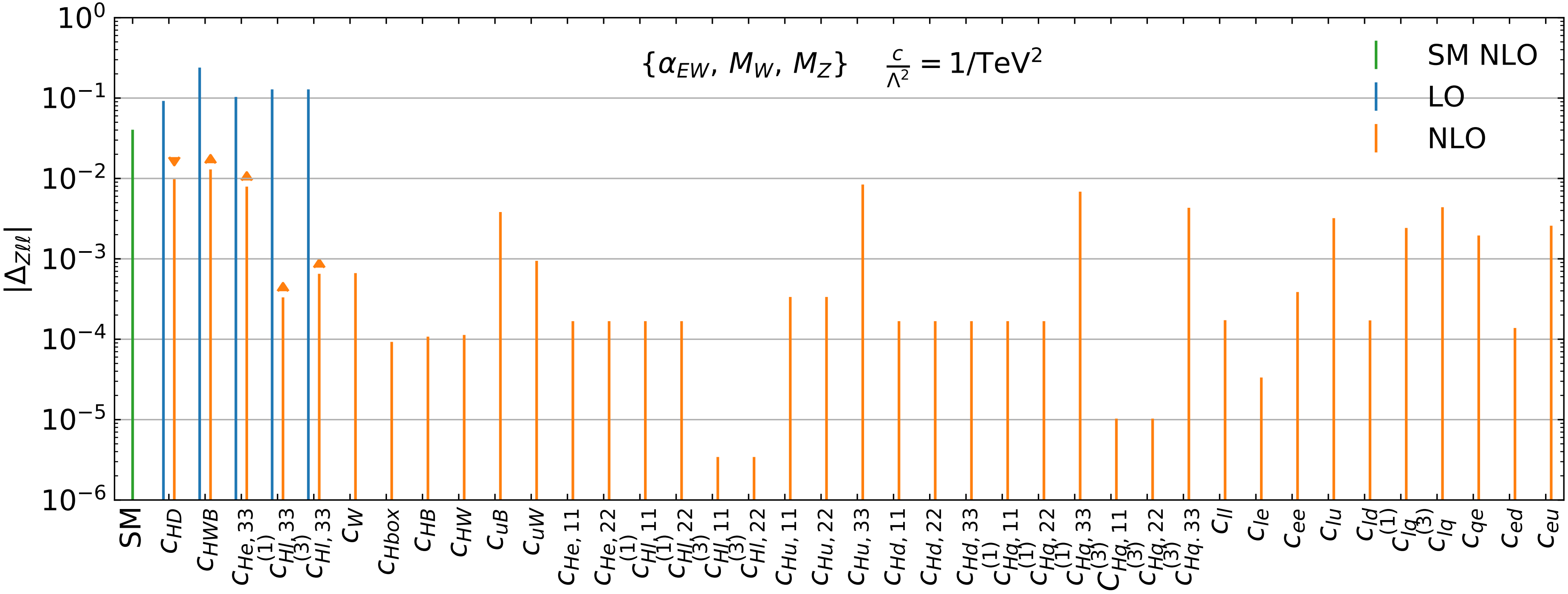
E.g: $C_{\substack{Hu\\33}}, C^{(1)}_{\substack{Hq\\33}}, C^{(3)}_{\substack{Hq\\33}}, C^{(3)}_{\substack{lq\\xx33}}$

Results: Z decay

α −scheme : $Z \to \tau\tau$

$\mu = M_Z$

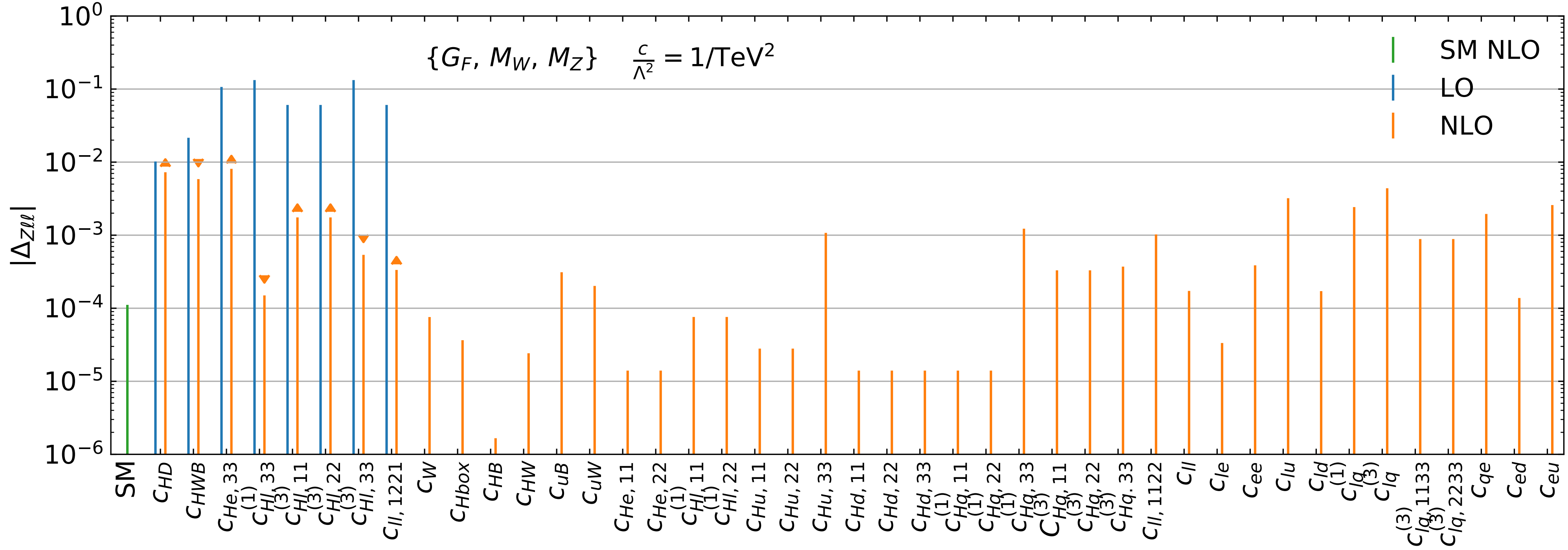
Wilson coefficients at LO: 5
Wilson coefficients at NLO: 63



Results: Z decay

$$\alpha_\mu\,\text{-scheme}:\ Z \to \tau\tau$$
$$\mu = M_Z$$

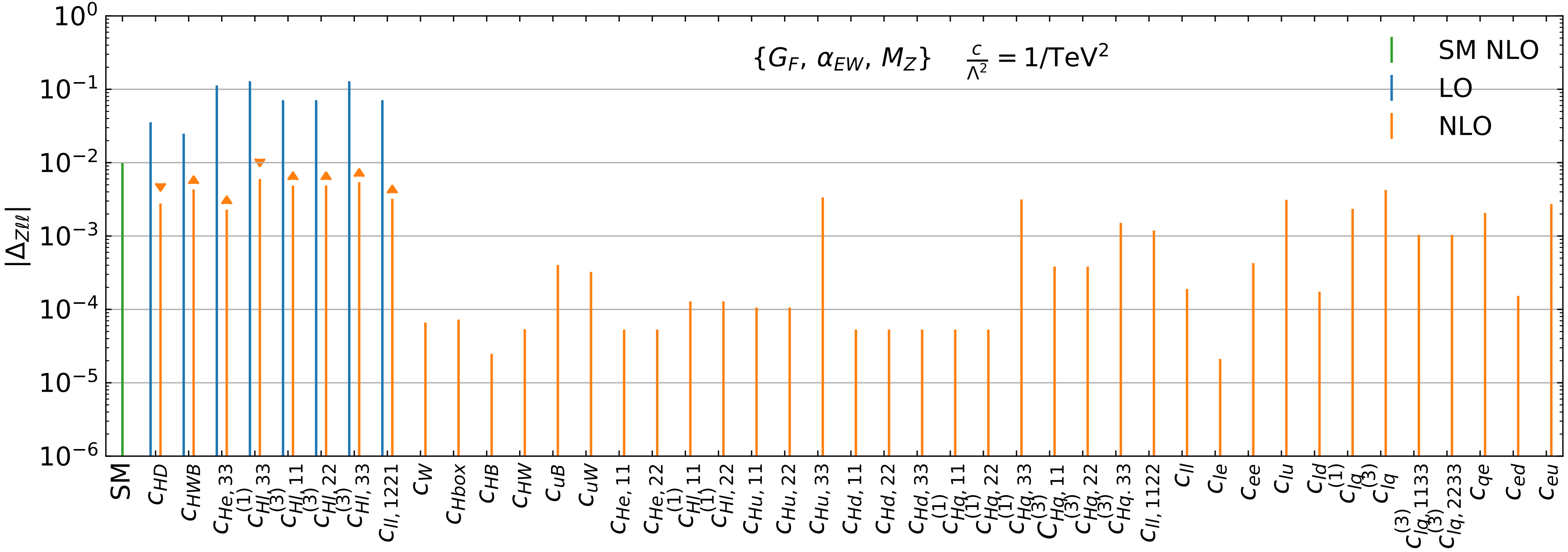
\# Wilson coefficients at LO: 8
\# Wilson coefficients at NLO: 67



Results: Z decay

$$\mathrm{LEP-scheme}: \; Z \to \tau\tau$$
$$\mu = M_Z$$

\# Wilson coefficients at LO: 8
\# Wilson coefficients at NLO: 67



Results: Z decay

$Z \to \tau\tau$
$\mu = M_Z$

$Z \to \tau^+\tau^-$	SM	C_{HD}	C_{HWB}	$C_{\underset{33}{He}}$	$C^{(1)}_{\underset{33}{Hl}}$	$C^{(3)}_{\underset{11}{Hl}}$	$C^{(3)}_{\underset{22}{Hl}}$	$C^{(3)}_{\underset{33}{Hl}}$	$C_{\underset{1221}{ll}}$
α	-4.0%	-10.6%	-5.4%	7.7%	0.3%	—	—	-0.5%	—
α_μ	$< 0.1\%$	71.1%	-27.2%	7.6%	0.1%	2.9%	2.9%	-0.4%	0.5%
LEP	0.1%	7.8%	17.4%	2.0%	0.3%	6.9%	6.9%	4.1%	4.5%

Size of NLO corrections to Wilson coefficients which appear at LO

- Appearance of $C^{(3)}_{\underset{11}{Hl}}, C^{(3)}_{\underset{22}{Hl}}$, and $C_{\underset{1221}{ll}}$ at tree level only in α_μ and LEP schemes
- Dramatic difference for the coefficients C_{HWB} and C_{HD}. Very large corrections in the α_μ scheme. Corrections actually of a larger magnitude in the α scheme (their LO values are radically different - more on next slides)
- Again, largest corrections arise via top loops (can be seen in large m_t limit)

Results: Z decay

Subset of the LO results

$$\Delta^{\alpha_\mu}_{Z,\mathrm{LO}} = v_\mu^2 \left\{ \ldots \boxed{+\, 0.355^{+0.012}_{-0.012} C_{HWB}} \boxed{-\, 0.169^{+0.011}_{-0.011} C_{HD}} + \ldots \right\}$$

$$\Delta^{\alpha}_{Z,\mathrm{LO}} = v_\alpha^2 \left\{ \ldots \boxed{+\, 4.088^{+0.145}_{-0.143} C_{HWB}} + \boxed{1.573^{+0.109}_{-0.108} C_{HD}} + \ldots \right\}$$

$$\Delta^{\mathrm{LEP}}_{Z,\mathrm{LO}} = v_\mu^2 \left\{ \ldots \boxed{-\, 0.410^{+0.049}_{-0.043} C_{HWB}} - \boxed{0.587^{+0.027}_{-0.025} C_{HD}} + \ldots \right\}$$

Particularly stark contrast in the relative contributions of C_{HD} and C_{HWB} .

Results: Z decay

How do the amplitudes change? α and α_μ schemes

SM:

$$\mathcal{M}^{\alpha}_{Z\ell\ell} \propto \frac{\sqrt{4\pi\alpha}}{\sqrt{s_w^2(1-s_w^2)}}\left[P_L\left(-\frac{1}{2}+s_w^2\right)+P_R s_w^2\right]$$

$$\mathcal{M}^{\alpha_\mu}_{Z\ell\ell} \propto \frac{2M_W}{v_\mu^2\sqrt{(1-s_w^2)}}\left[P_L\left(-\frac{1}{2}+s_w^2\right)+P_R s_w^2\right]$$

LH and RH couplings the same in both schemes, but with different signs

$-1/2+s_w^2 \approx -0.27$ $\qquad$ $s_w^2 \approx 0.22$

C_{HD} contributes to these decay rates, only through the shifts in s_w

The two amplitudes differ in this quantity only by $\sqrt{s_w^2}$

$s_w^2 \to s_w^2 + \delta s_w^2$ ← Contains C_{HD}

Results: Z decay

How do the amplitudes change? α and α_μ schemes

SM:

$$\mathcal{M}^{\alpha}_{Z\ell\ell} \propto \frac{\sqrt{4\pi\alpha}}{\sqrt{s_w^2(1-s_w^2)}}\left[P_L\left(-\frac{1}{2}+s_w^2\right)+P_R s_w^2\right]$$

$$\mathcal{M}^{\alpha_\mu}_{Z\ell\ell} \propto \frac{2M_W}{v_\mu^2\sqrt{(1-s_w^2)}}\left[P_L\left(-\frac{1}{2}+s_w^2\right)+P_R s_w^2\right]$$

$$\mathcal{M}^{(6,0,\alpha)}_{Z\ell\ell} \propto -i\delta s_w^2 \frac{\sqrt{4\pi\alpha}}{4c_w^3 s_w^3}\left[P_L + P_R 2s_w^2\right]$$

$$\mathcal{M}^{(6,0,\alpha_\mu)}_{Z\ell\ell} \propto -i\delta s_w^2 \frac{2M_W s_w}{v_\mu^2}\frac{1}{4c_w^3 s_w^3}\left[P_L s_w^2\left(-3+2s_w^2\right)+P_R 2s_w^2\left(-2+2s_w^2\right)\right]$$

The C_{HD} terms in each scheme have same sign couplings to both LH and RH fermions → large cancellation on squaring with the SM

Results: Z decay

The shift from s_w is the same in both schemes.
Straightforward to calculate the ratio of the relative contribution to C_{HD} in both schemes.

$$\frac{\Gamma^{\alpha}_{Z\to\ell\ell}}{\Gamma^{\alpha_\mu}_{Z\to\ell\ell}} = \frac{\left(-\frac{1}{2}+s_w^2\right)+2s_w^4}{\left(-\frac{1}{2}+s_w^2\right)s_w^2(-3+2s_w^2)+2s_w^4(-2+s_w^2)} \approx -9.3$$

- Gives some reasoning to the large LO discrepancies between the α and α_μ scheme
- In the LEP scheme there are two contributions to δs_w^2

$$s_w^2 \to s_w^2 + \delta s_w^2 \qquad M_W = \hat{M}_W\left[1+v_\mu^2\hat{\Delta}_W^{(6,0,\mu)}\right]$$

Results: Z decay

Total contribution: $$(\delta s_w^2)^{\text{LEP}}(C_{HD}) = \frac{s_w^2}{1-2c_w^2}(\delta s_w^2)^{\alpha}(C_{HD}) \approx -0.4(\delta s_w^2)^{\alpha}(C_{HD})$$

$$\Gamma^{LEP}_{Z\to\ell\ell}/\Gamma^{\alpha_\mu}_{Z\to\ell\ell} \approx 3.7$$

- Similar cancellations occur for C_{HWB}
- Cancellation more complicated due to other sources of C_{HWB}

Conclusions / Summary 1

- Choice of input scheme can be important for the precision of high order SM calculations
- Precision not (yet) as important for SMEFT calculations, but important from the perspective of performing fits:
  - Fits must be done in a consistent scheme
  - Choice of scheme impacts:
    - Relative contribution of different Wilson coefficients
    - Perturbative convergence
    - Number of Wilson coefficients which enter an observable
- Begin investigation using decays of the Higgs, W, and Z as testing grounds

Conclusions / Summary 2

- Begun investigations in to three schemes

α −scheme	α_μ −scheme	LEP scheme
$\{\alpha, M_W, M_Z\}$	$\{G_\mu, M_W, M_Z\}$	$\{\alpha, G_\mu, M_Z\}$

- Identified set of corrections $\sim 1/s_w^2$ and m_t^2 which can lead to large contributions in the SMEFT (and SM) due to renormalization conditions
- Calculated NLO decays of W, Z to leptons and Higgs to $b\bar{b}$ in each of the schemes
- Identified some already large discrepancies at LO – some accounted for through chirality (But still seeing large corrections at NLO in some cases)
- Work in progress

Conclusions / Summary 2

- Begun investigations in to three schemes

α –scheme | α_μ –scheme | LEP scheme

$\{\alpha, M_W, M_Z\}$ | $\{G_\mu, M_W, M_Z\}$ | $\{\alpha, G_\mu, M_Z\}$

- Identified set of corrections $\sim 1/s_w^2$ and m_t^2 which can lead to large contributions in the SMEFT (and SM) due to renormalization conditions
- Calculated NLO decays of W, Z to leptons and Higgs to b$\bar{\text{b}}$ in each of the schemes
- Identified some already large discrepancies at LO – some accounted for through chirality (But still seeing large corrections at NLO in some cases)
- Work in progress

Thank you for your attention