MAX-PLANCK-INSTITUT
FÜR PHYSIK

Investigation of unit cell configuration for the booster section of MADMAX

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Axion Electrodynamics

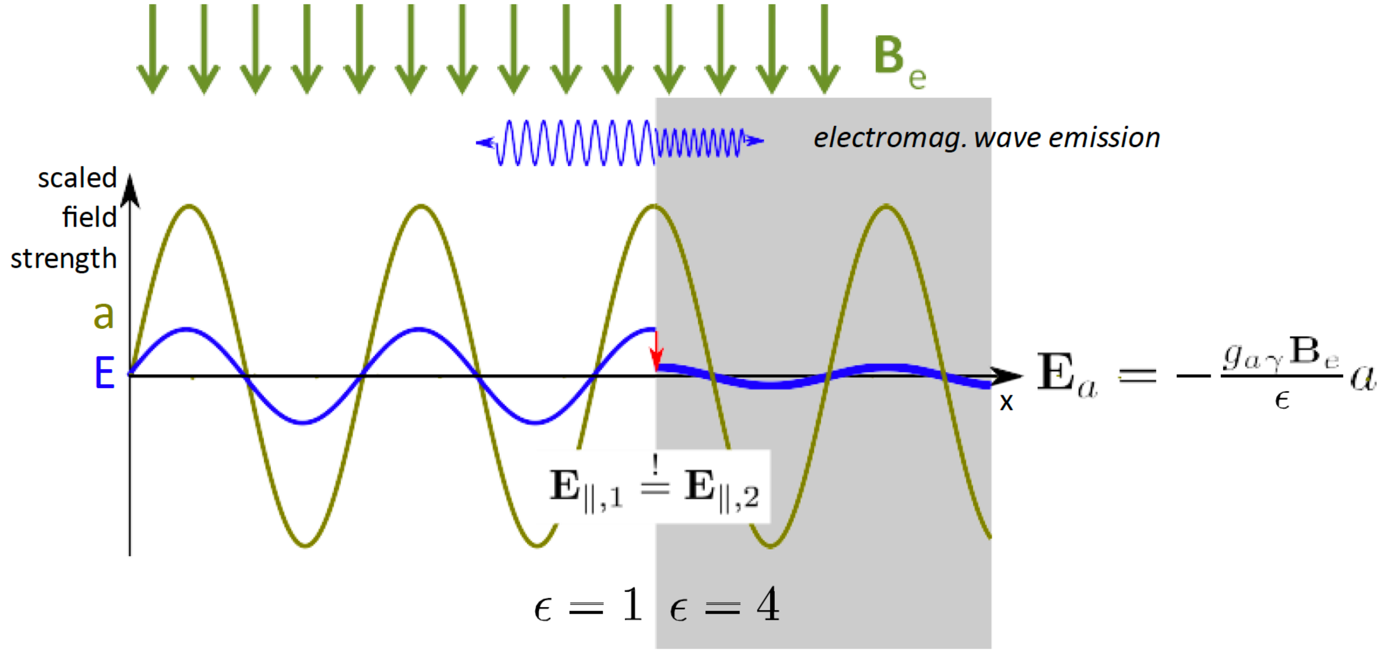
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^{\mu}A_{\mu} + \frac{1}{2}\partial_{\mu}a\,\partial^{\mu}a - \frac{1}{2}m_a{}^2a^2 \quad - \frac{g_{a\gamma}}{4}a\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Solve EOM under external homogenous magnetic field B_e :

$$\epsilon\nabla\cdot\boldsymbol{E} = 0$$
$$-\,\epsilon\,\dot{\boldsymbol{E}} = g_{a\gamma}\boldsymbol{B_e}\dot{a}$$
$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma}\,\boldsymbol{E}\cdot\boldsymbol{B_e}$$

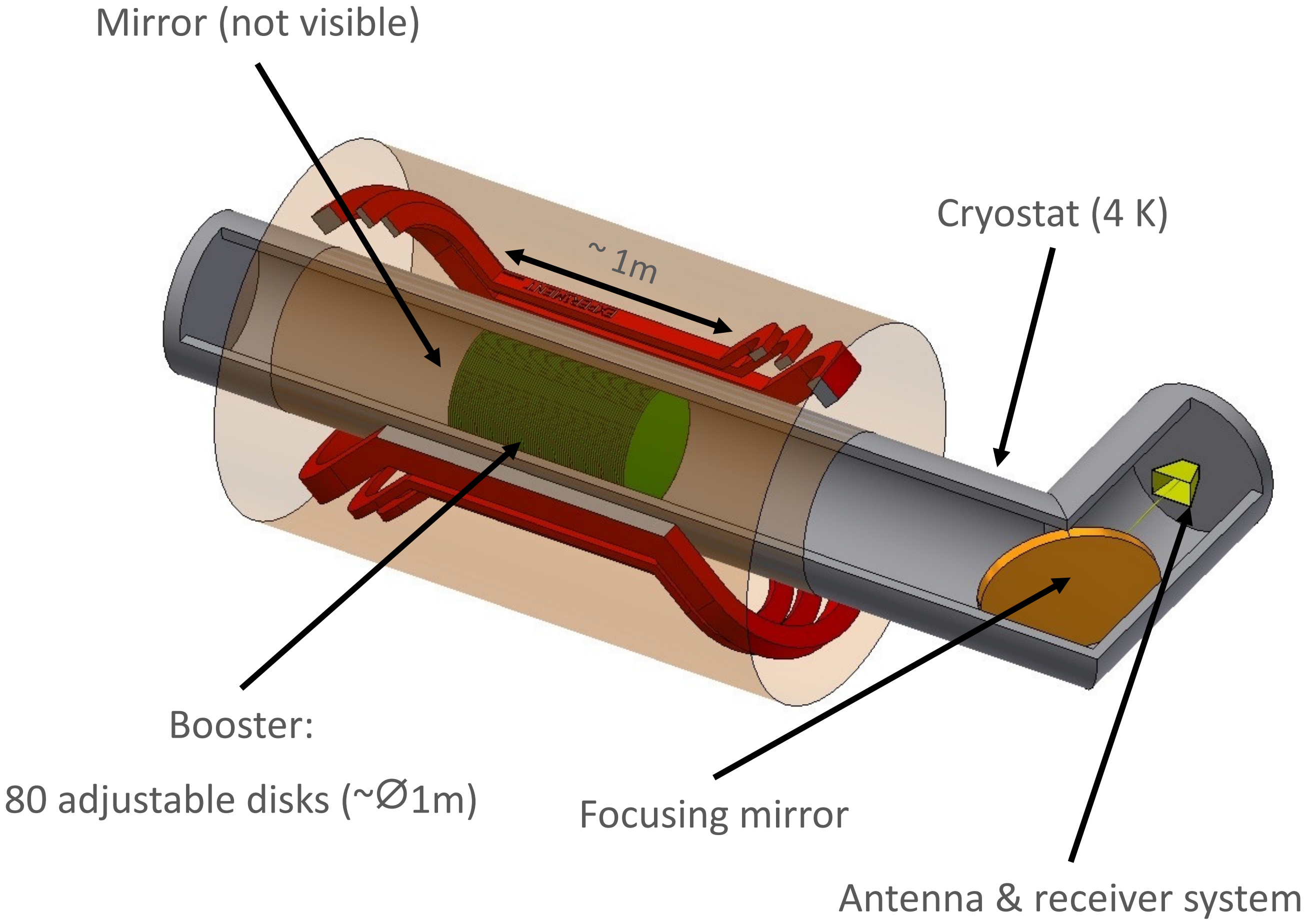
- Axion induced electric field:

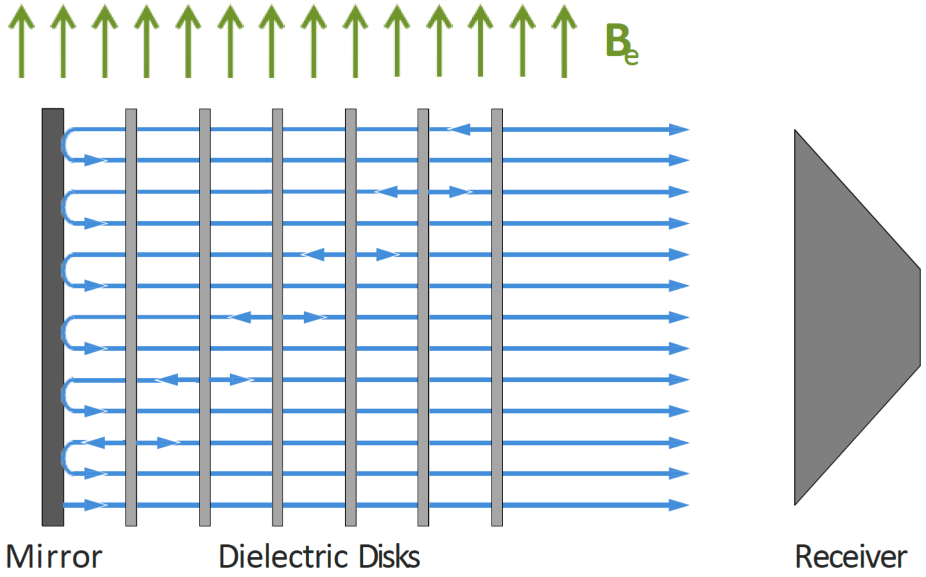
$$\mathbf{E}_a = -\frac{g_{a\gamma}\mathbf{B}_e}{\epsilon}a = 1.3\times10^{-12}\,\mathrm{V\,m^{-1}}\times\left(\frac{B_e}{10\,\mathrm{T}}\right)\frac{C_{a\gamma}f_{DM}^{1/2}}{\epsilon}$$



$$P/A = 2.2\times10^{-27}\,\mathrm{W\,m^{-2}}\left(\frac{B_e}{10\,\mathrm{T}}\right)^2 C_{a\gamma}{}^2\cdot f(\epsilon_1,\epsilon_2)$$

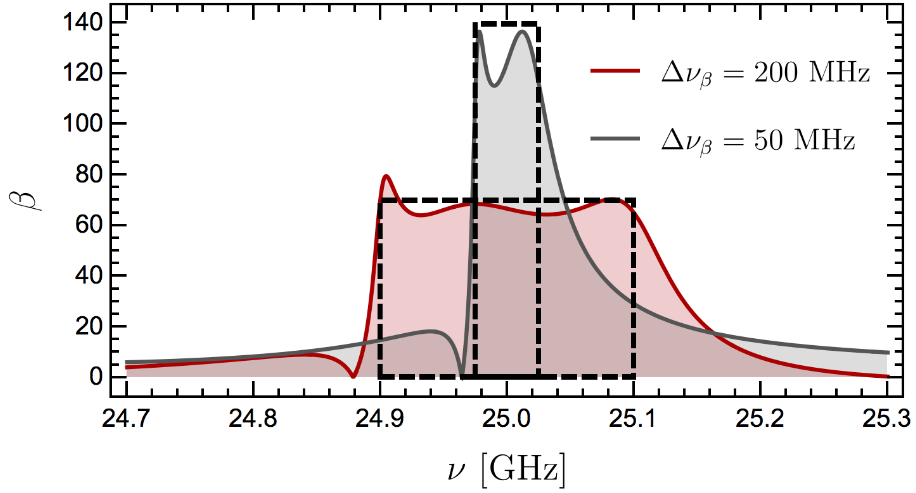
MADMAX





$$P/A = 2.2 \times 10^{-27}\,\mathrm{W\,m^{-2}} \left(\frac{B_e}{10\,\mathrm{T}}\right)^2 {C_{a\gamma}}^2 \cdot \beta^2$$

β^2: power emitted by booster / power emitted by single mirror ($\epsilon = \infty$)



Problem statement

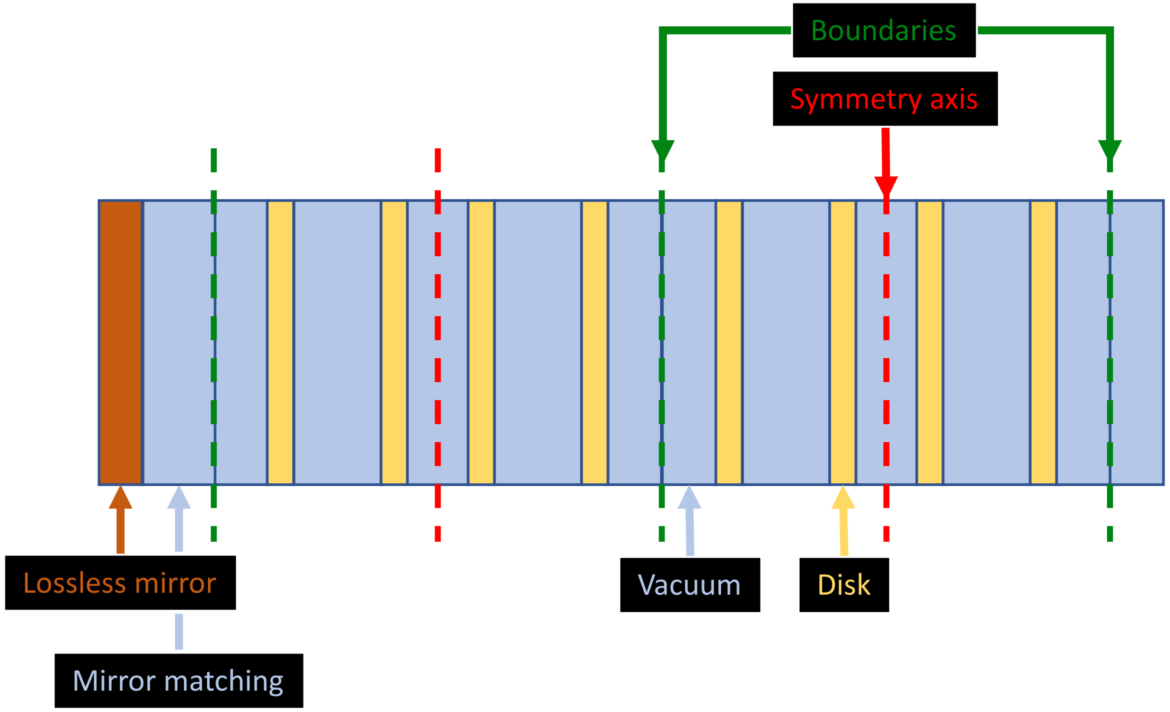
- The goal is to find approaches to the modelling of the booster that help reduce the system's complexity
  - Fewer disks
  - Fewer degrees of freedom

- Additionally, we would like to facilitate the reconstruction of high boost factors, while gaining an understanding of how the system works, in order to learn how to control the booster curve
  - Using simulations, we aim to gather the best solutions for a specific modelling of the booster, in order to assess the range of freedom the model gives
  - Based on the data gathered, specifically the disk position, we can look for correlations between neighbouring solutions, in order to identify simple "rules" to control the system

- So far, we have examined the performance of "symmetric cells"

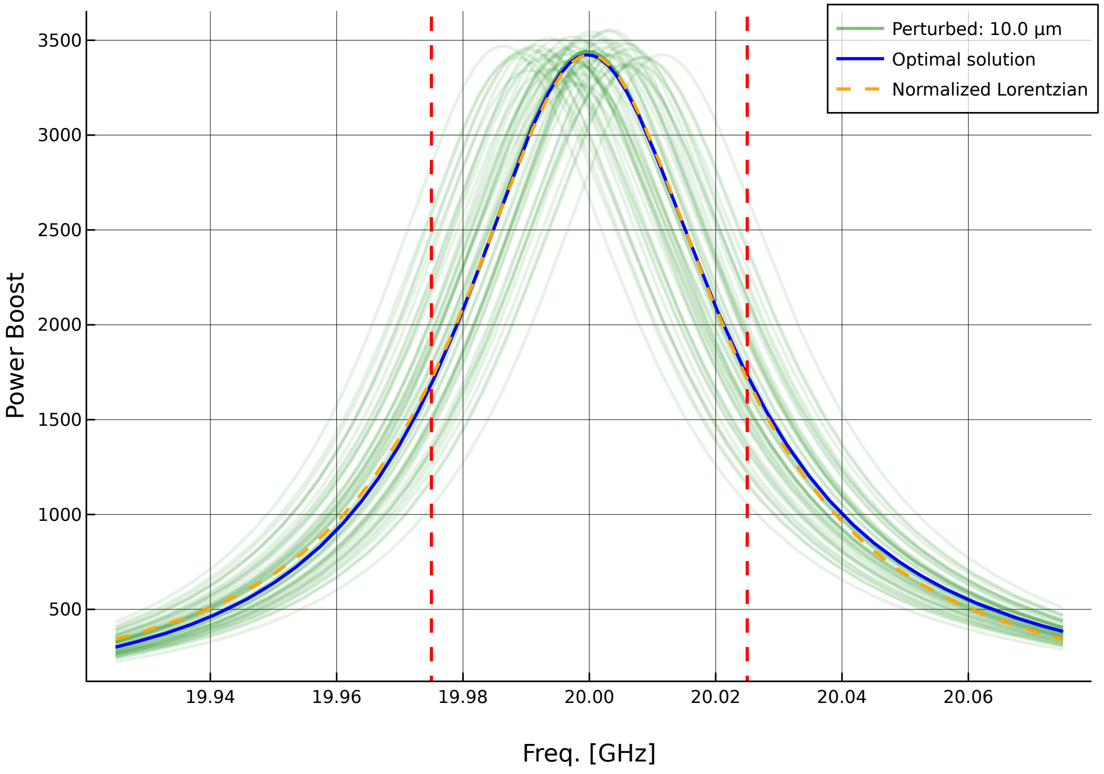
Case study: Symmetric cell approach

- The booster can be naively divided in three sections:
  - Mirror matching + Booster section + Antenna matching
- We assume that the booster section is comprised of repeating symmetric cells
  - System's degrees of freedom are the distances between disks in one cell
- Simple matching to lossless mirror obtained with one additional vacuum gap



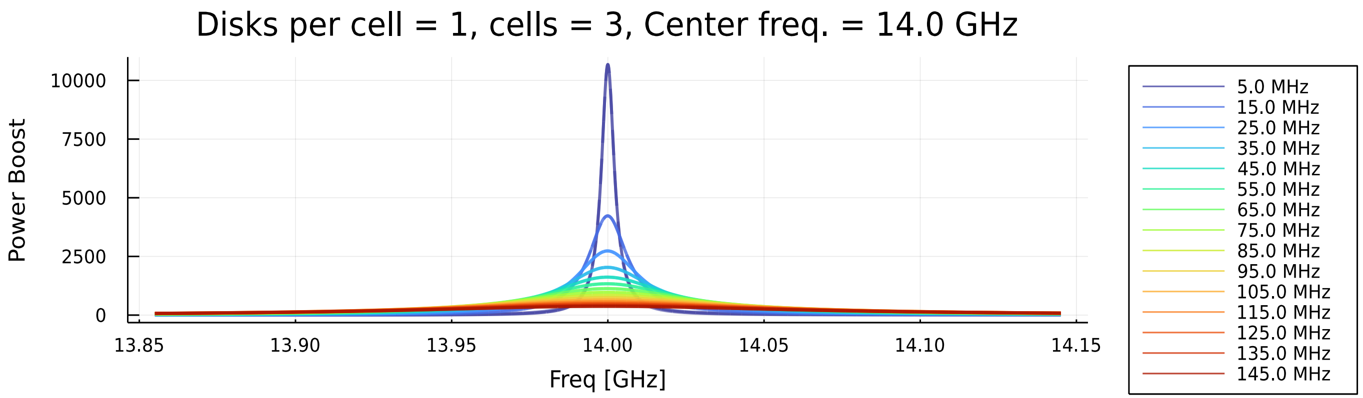
Target of optimization

- The booster acts (naively) as a transmission line or filter, so we want to understand how to control it
  - we aim to reconstruct unimodal booster curves, since they are some of the most basic filter shapes
  - Target Lorentzian distribution with predefined FWHM
  - How can we control the booster peak: shift, widen, amplitude?
  - Compare booster curve to target distribution via Least Square Sum



Data collection strategy: FWHM continuation

- Why rely only on global optimization?
  - If we trust our initial configuration we might use it to find nearby solutions
- Width continuation:
  - Target distributions with slightly different FWHM from previous solution
  - Use local optimizer (fast) and the previous solution as initial candidate



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- Width continuation:
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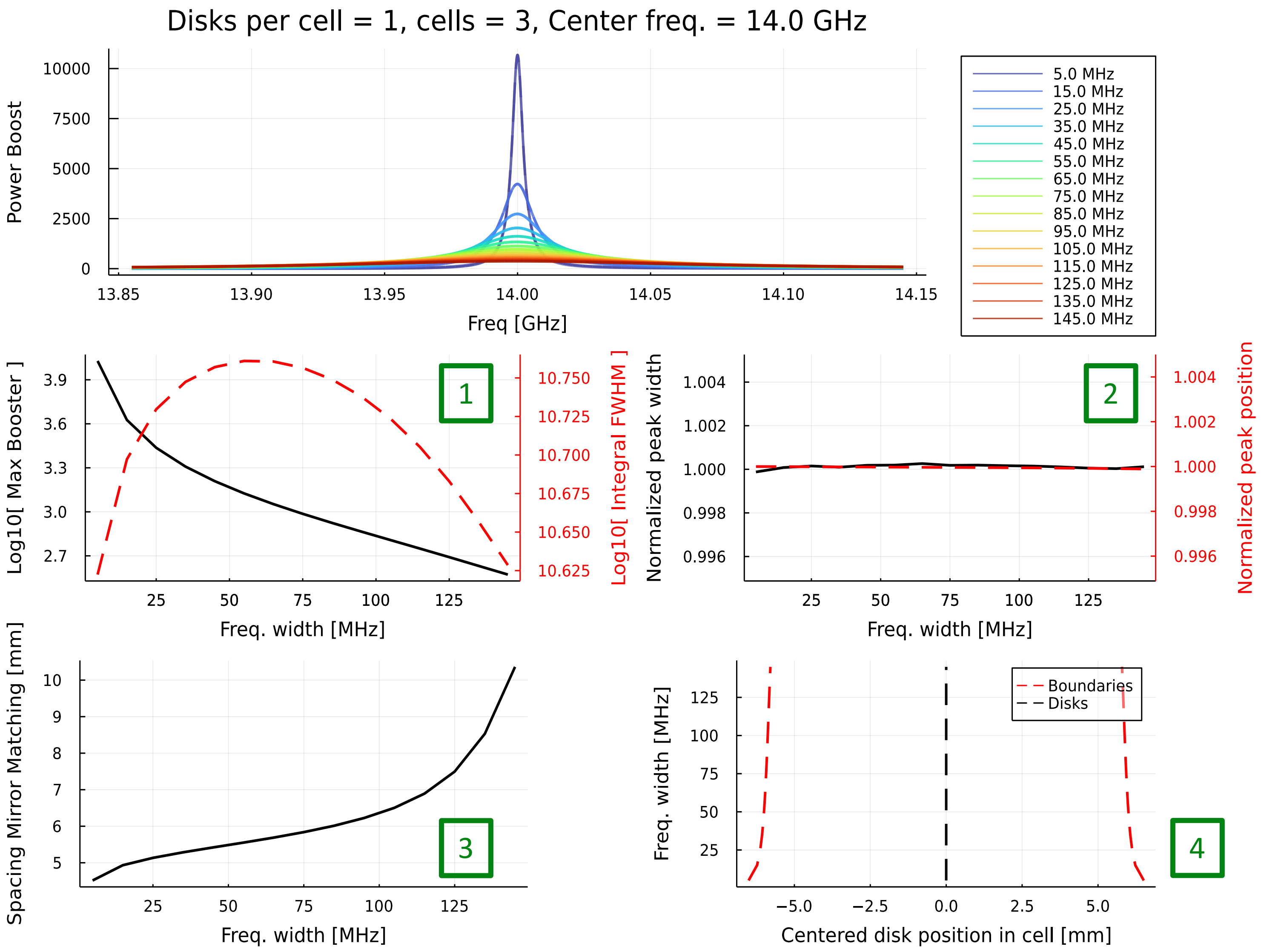
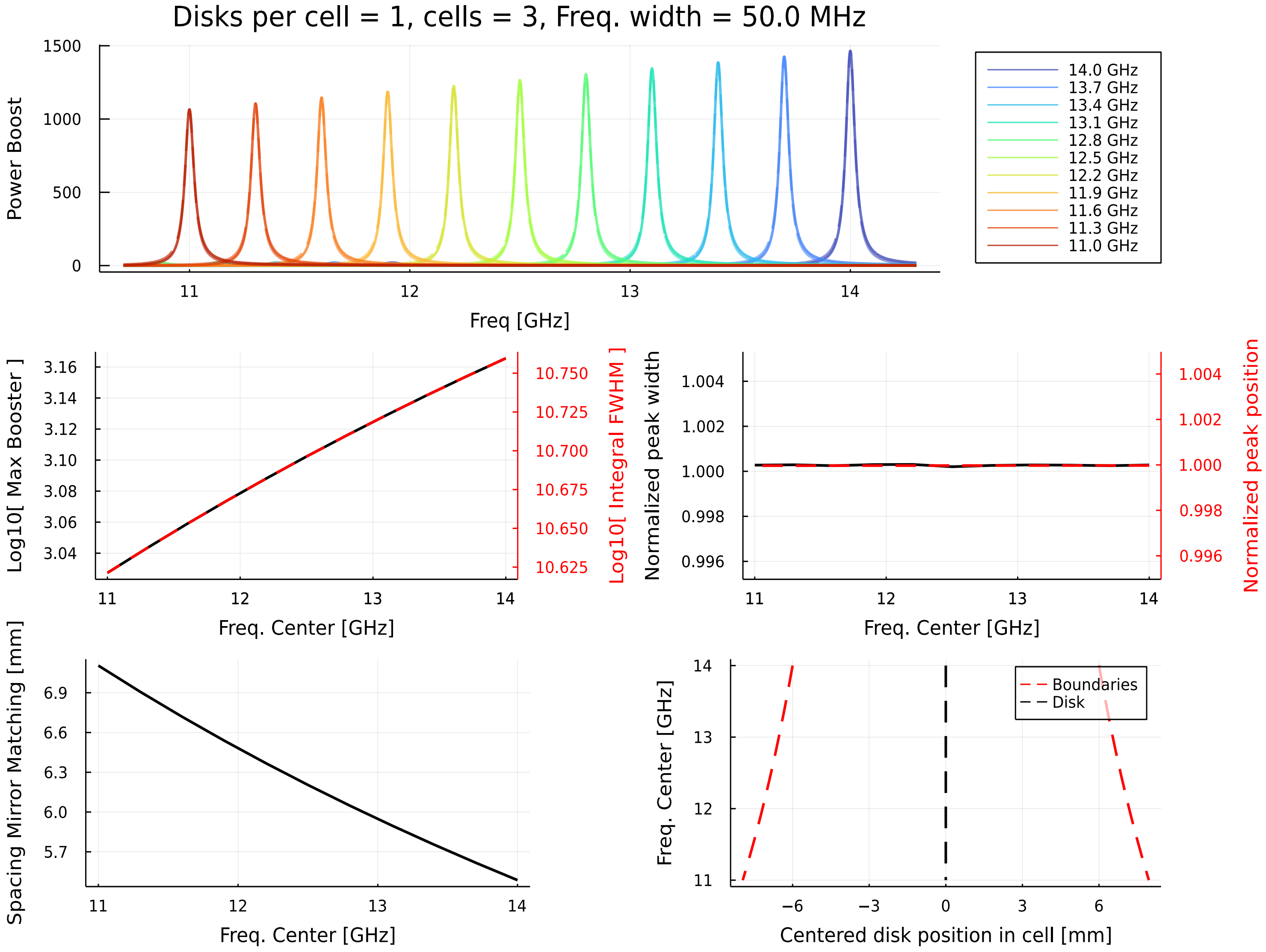
- Check goodness of continuation:
  - Performance of booster → 1
  - Agreement with target → 2
  - Smoothness of solution → 3 + 4



Data collection strategy: shift continuation

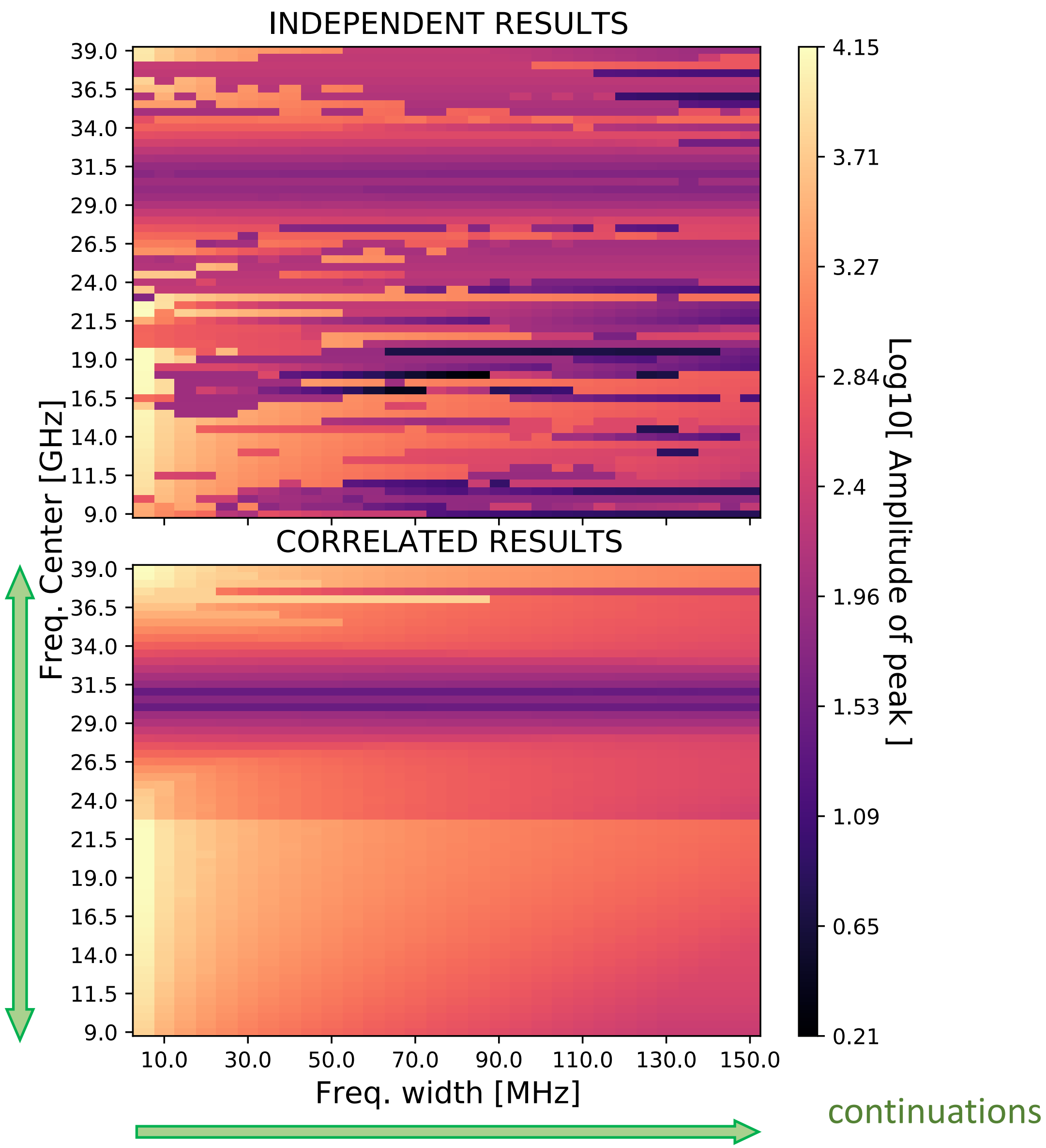
- We notice that width and position continuation seem to be quite stable and yield smooth variations of disk positions

    - Using a short booster we can test the validity of this claim



FWHM + Shift continuation

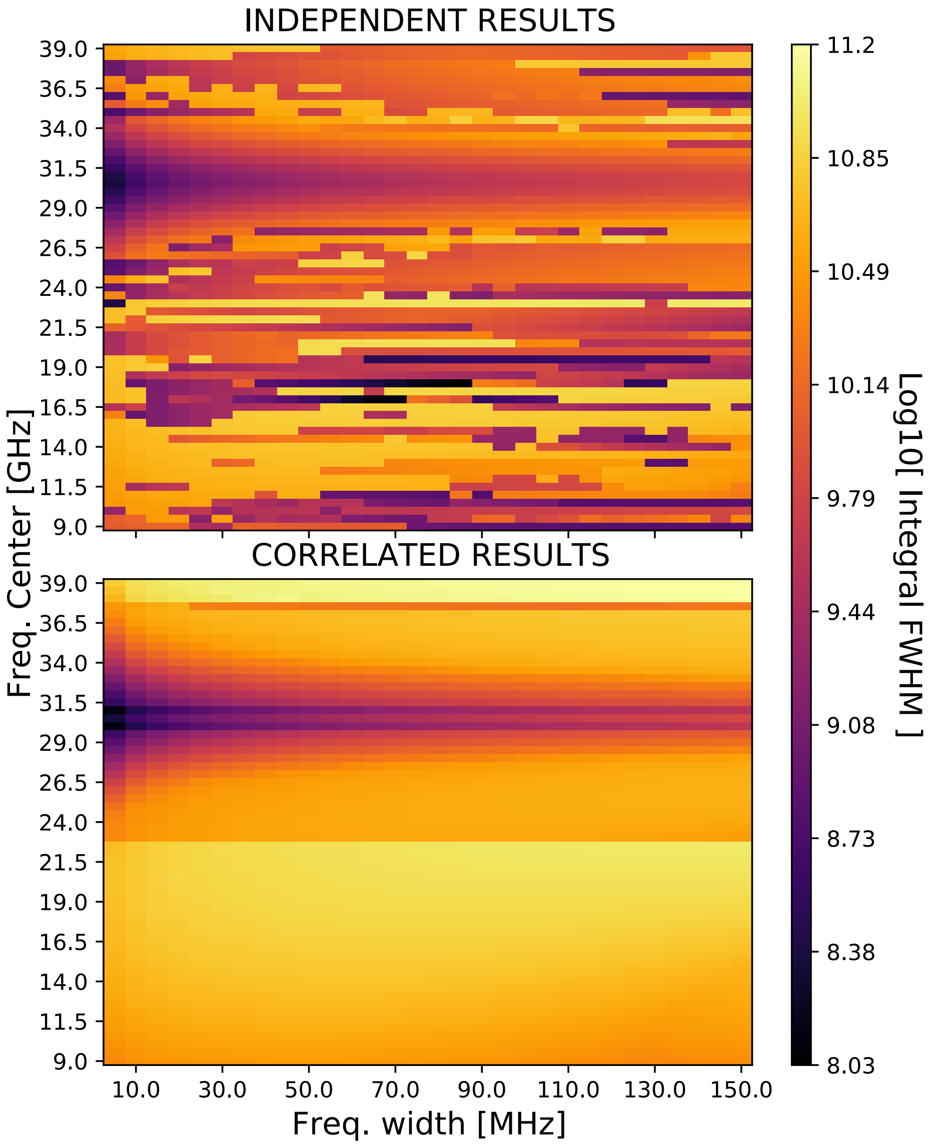
Disks per cell: 1 # Cells: 3



- Comparing solution between global optimization (INDEPENDENT RESULTS) and local continuation (CORRELATED RESULTS)
- Compare booster amplitude

FWHM + Shift continuation

Disks per cell: 1   # Cells: 3



- Comparing solution between global optimization (INDEPENDENT RESULTS) and local continuation (CORRELATED RESULTS)
- Compare booster amplitude
- Compare FWHM integral

FWHM + Shift continuation

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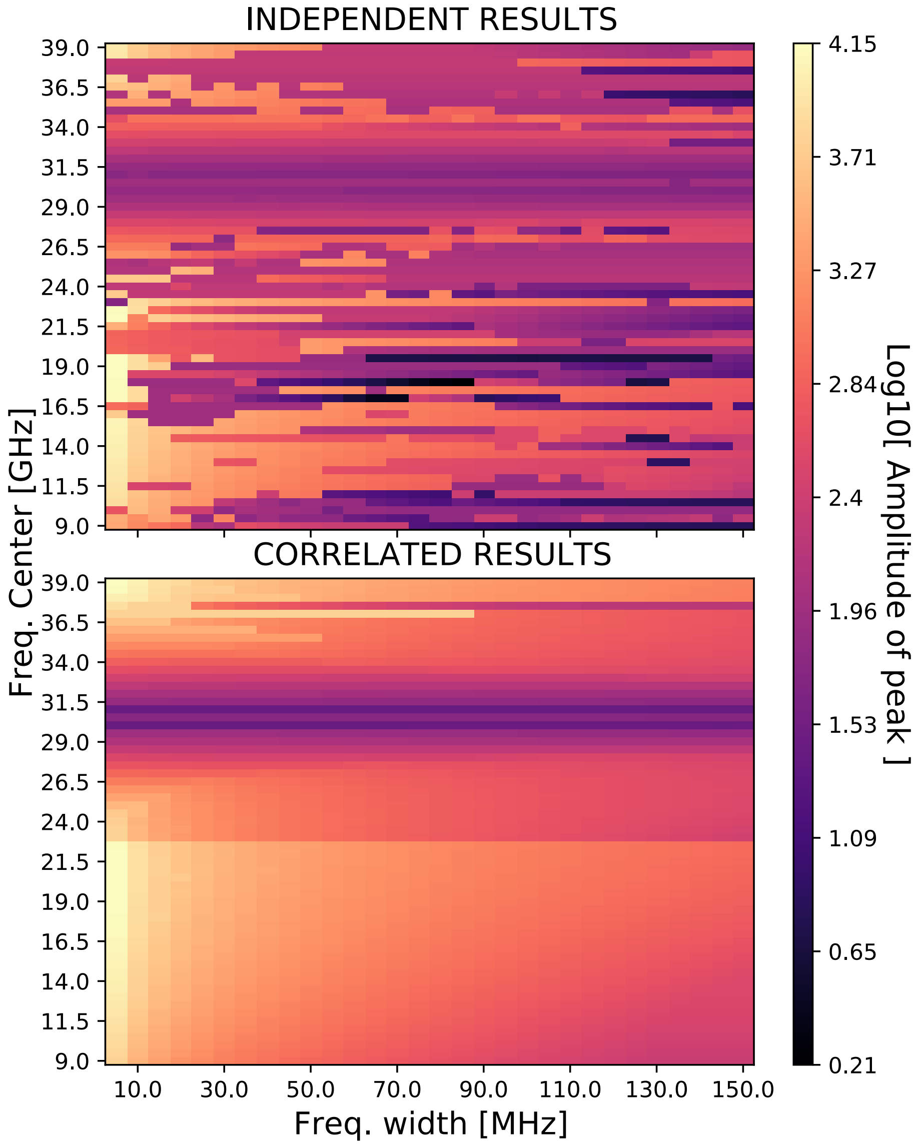
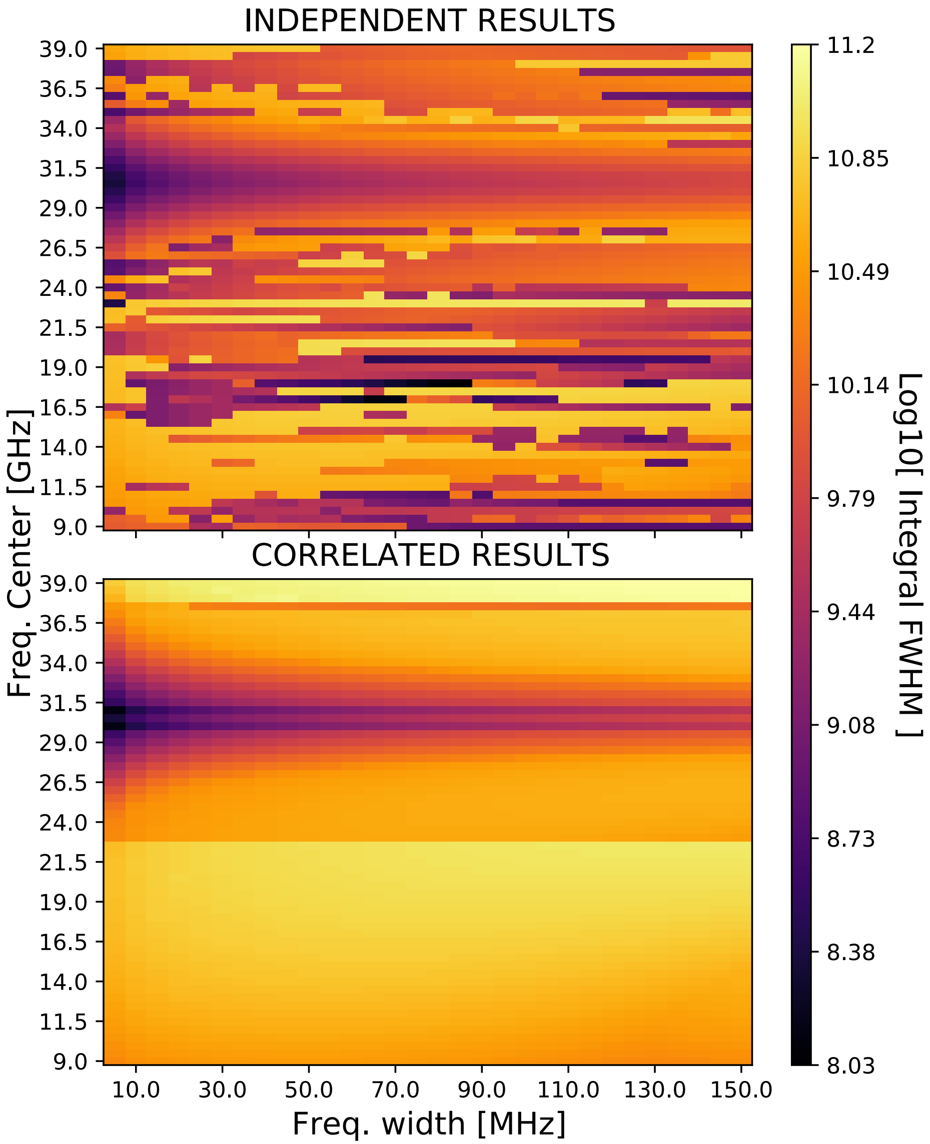
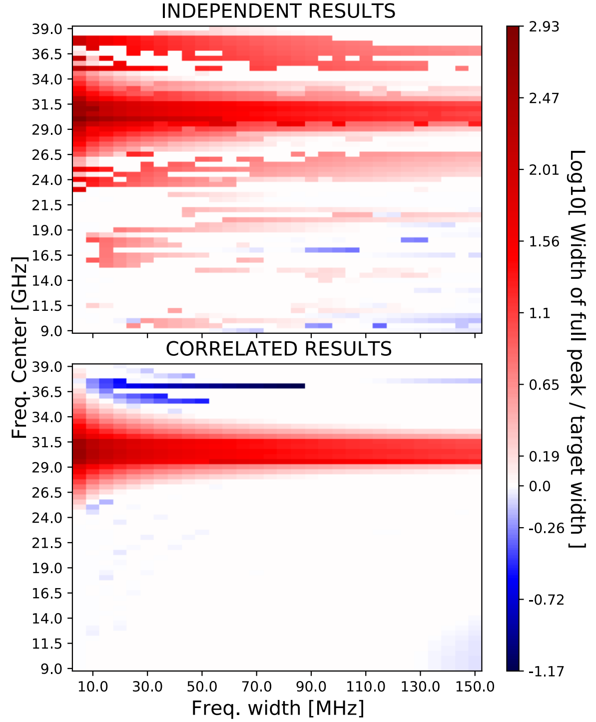


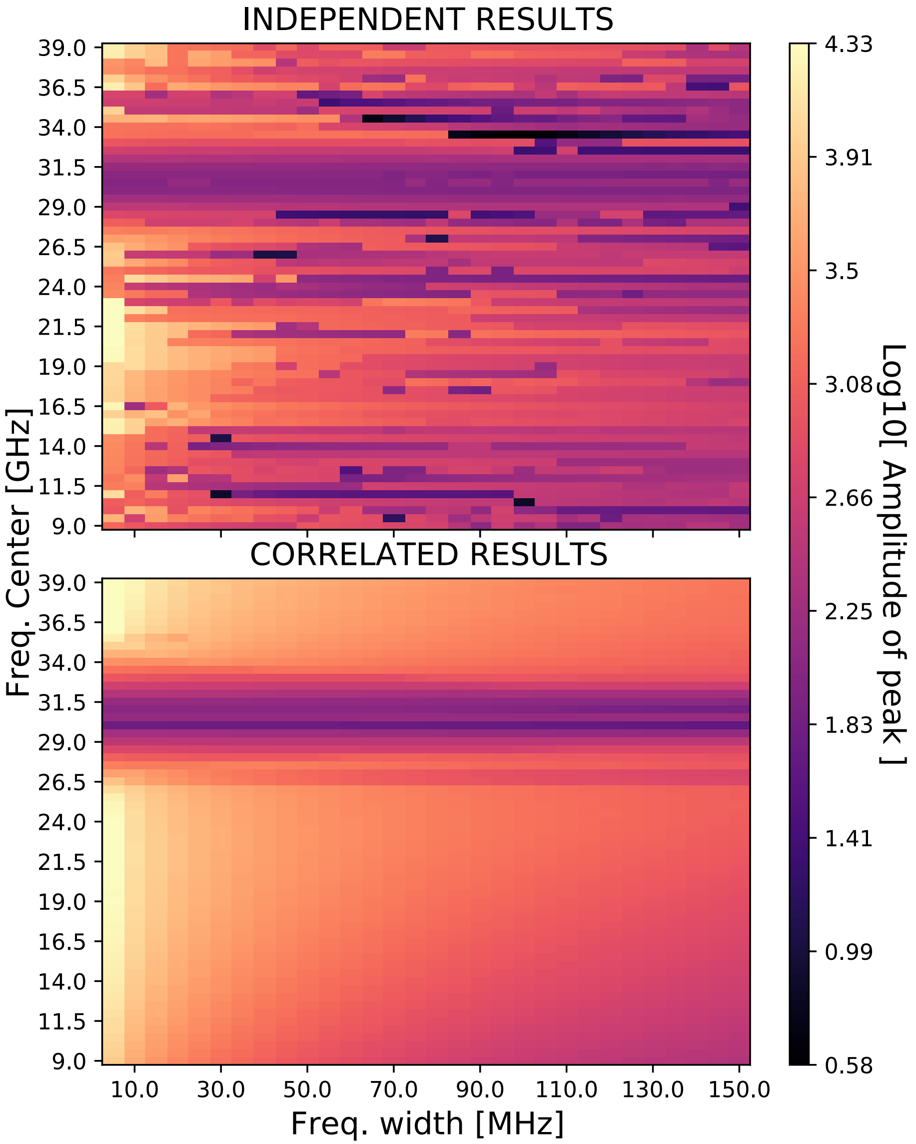


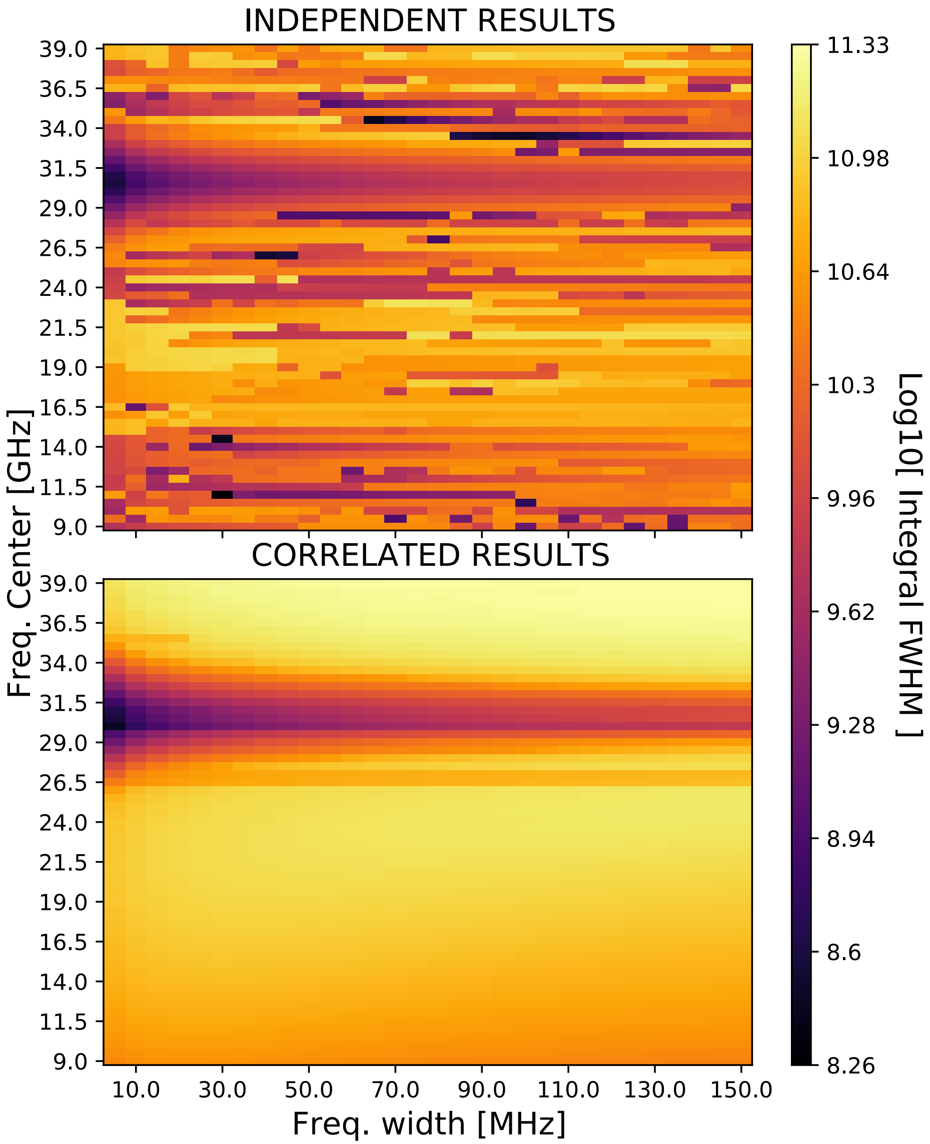


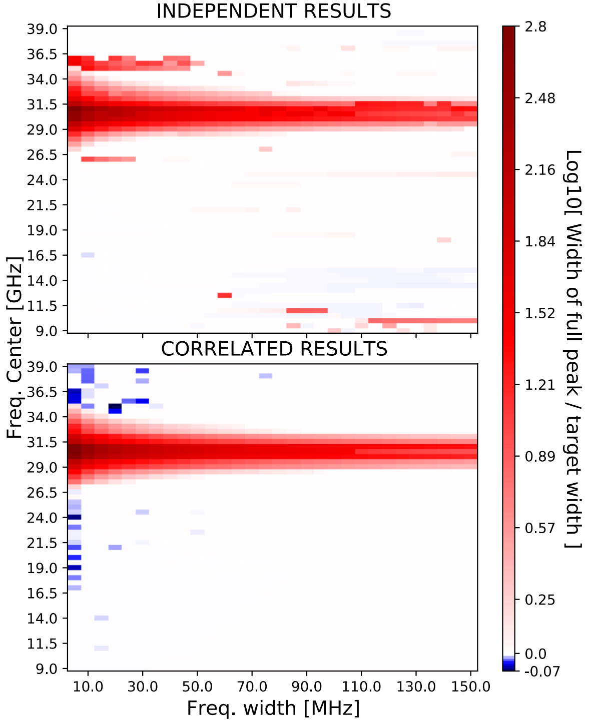
- Compare booster FWHM

FWHM + Shift continuation

Disks per cell: 2   # Cells: 2



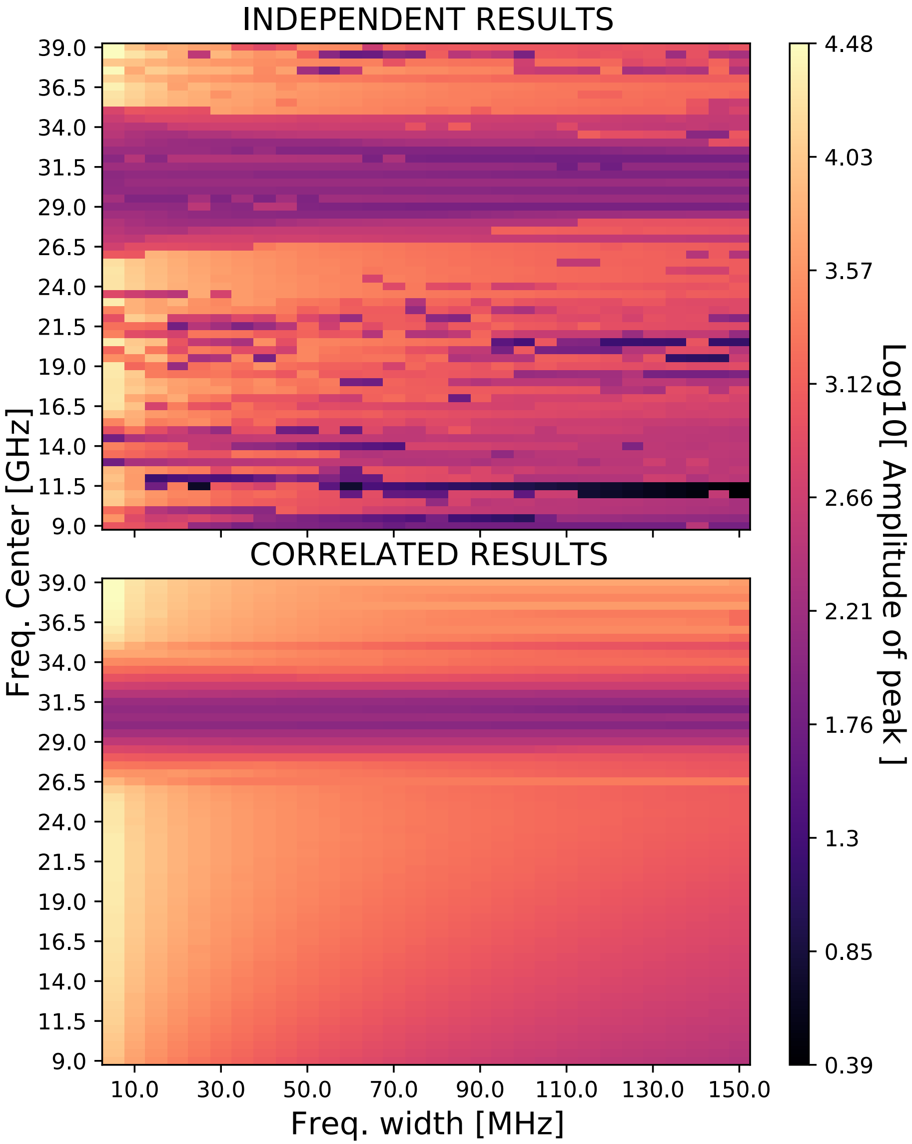


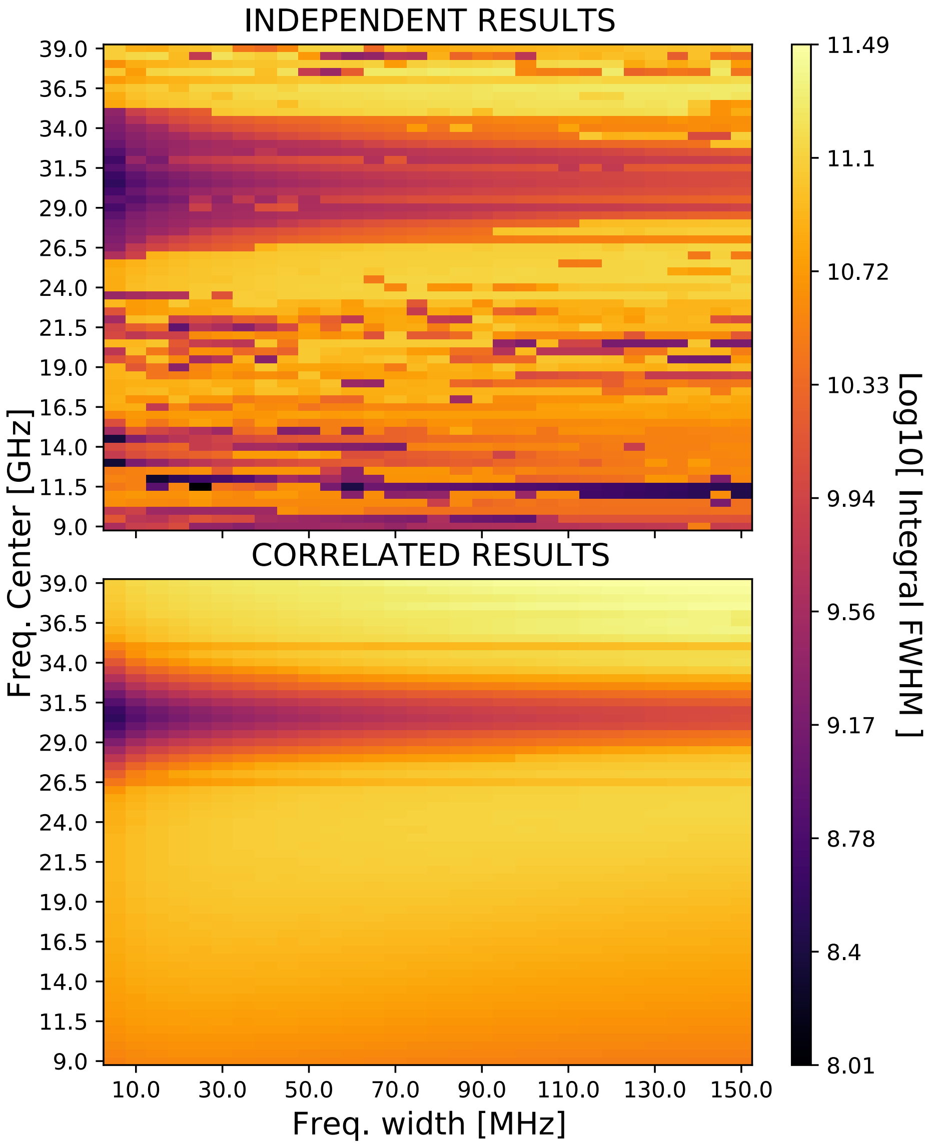


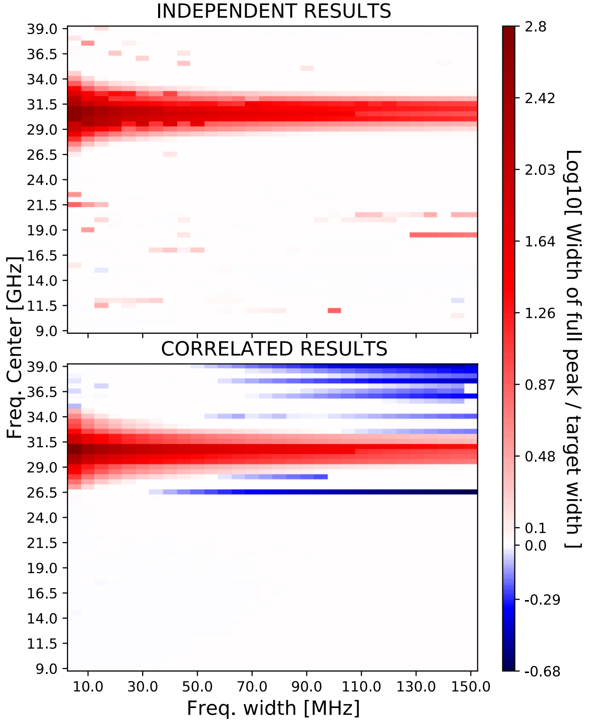
- Initial solutions appear to be quite robust across the phase-space
- continuations offer a smoother transition between neighbouring solution

FWHM + Shift continuation

Disks per cell: 4   # Cells: 1

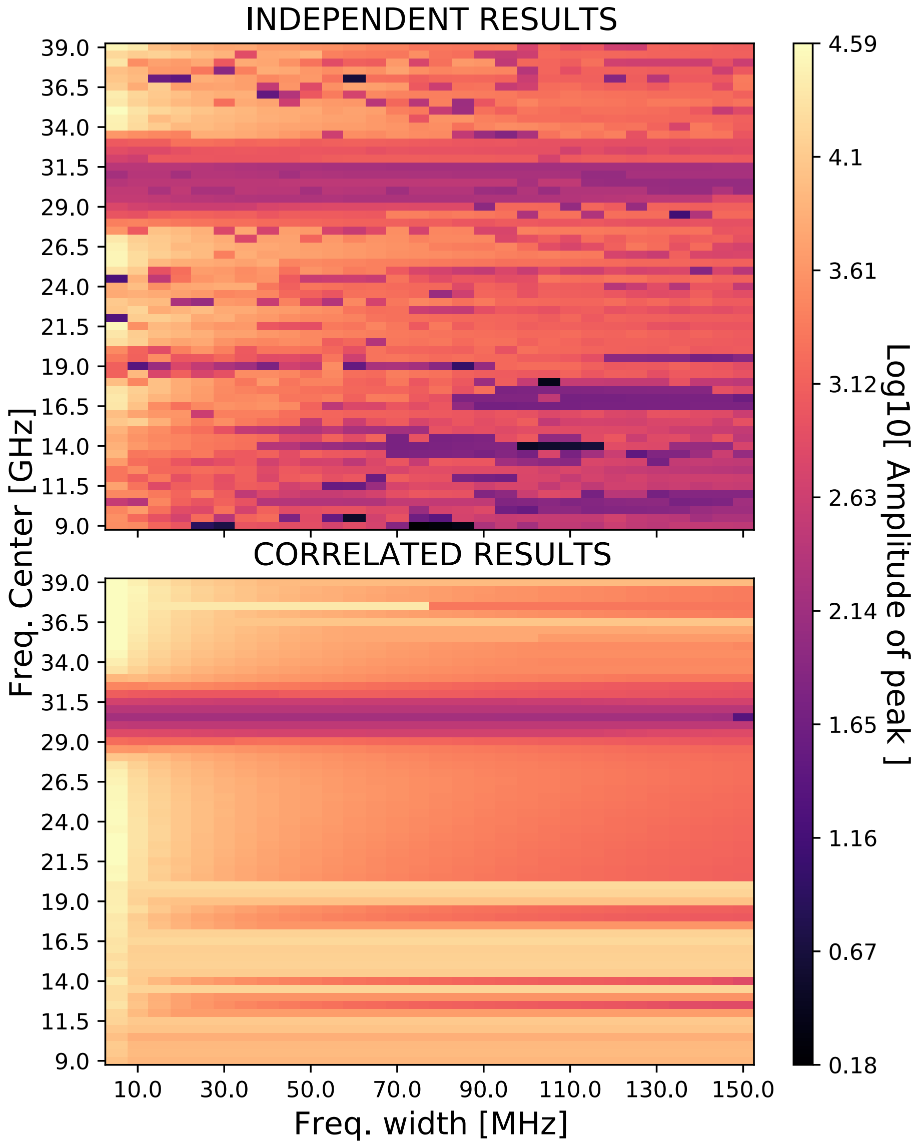


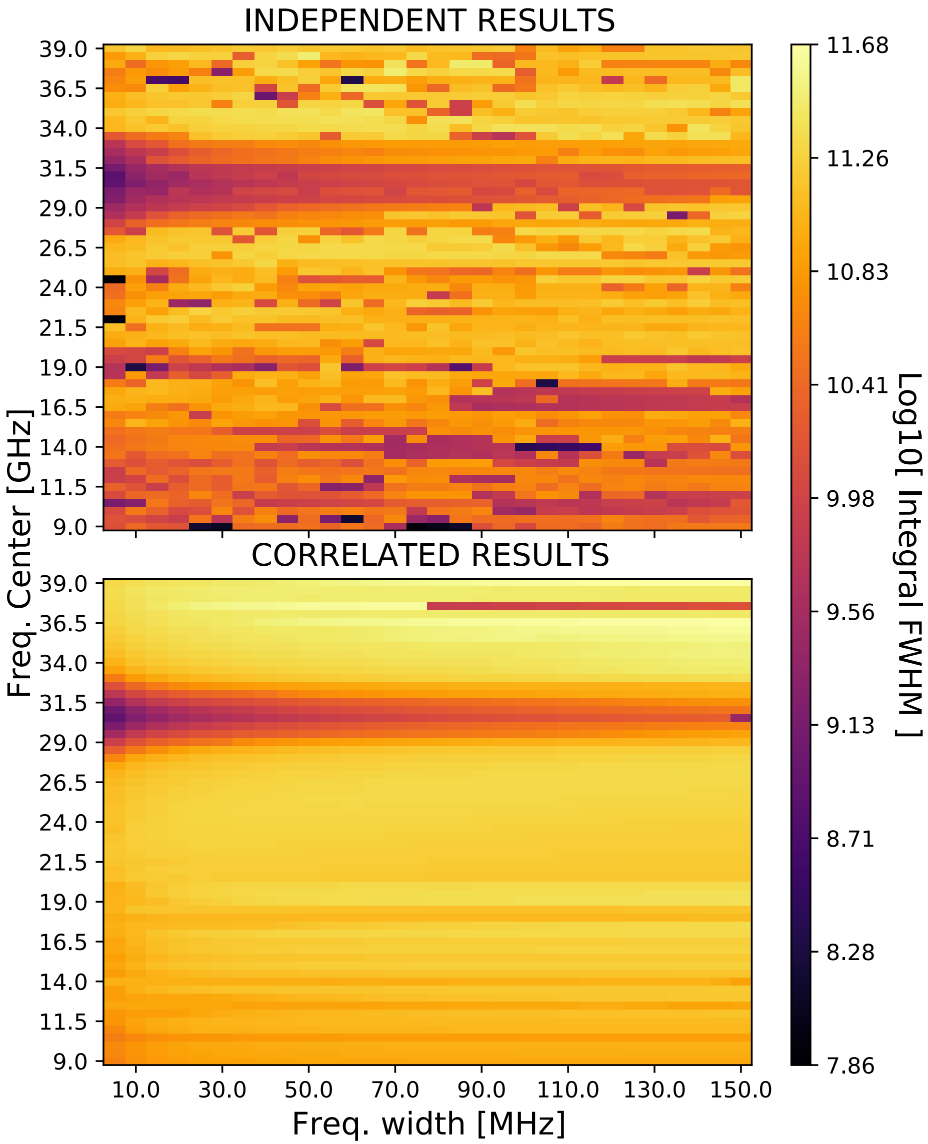


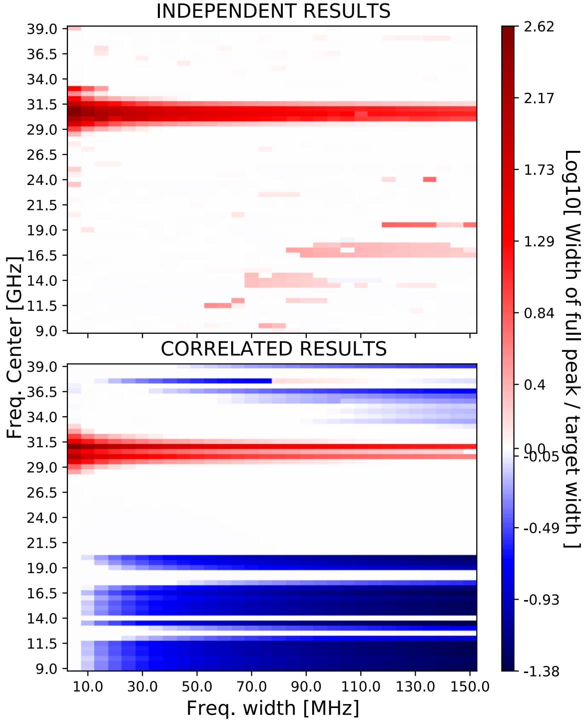


FWHM + Shift continuation

\# Disks per cell: 6   # Cells: 1



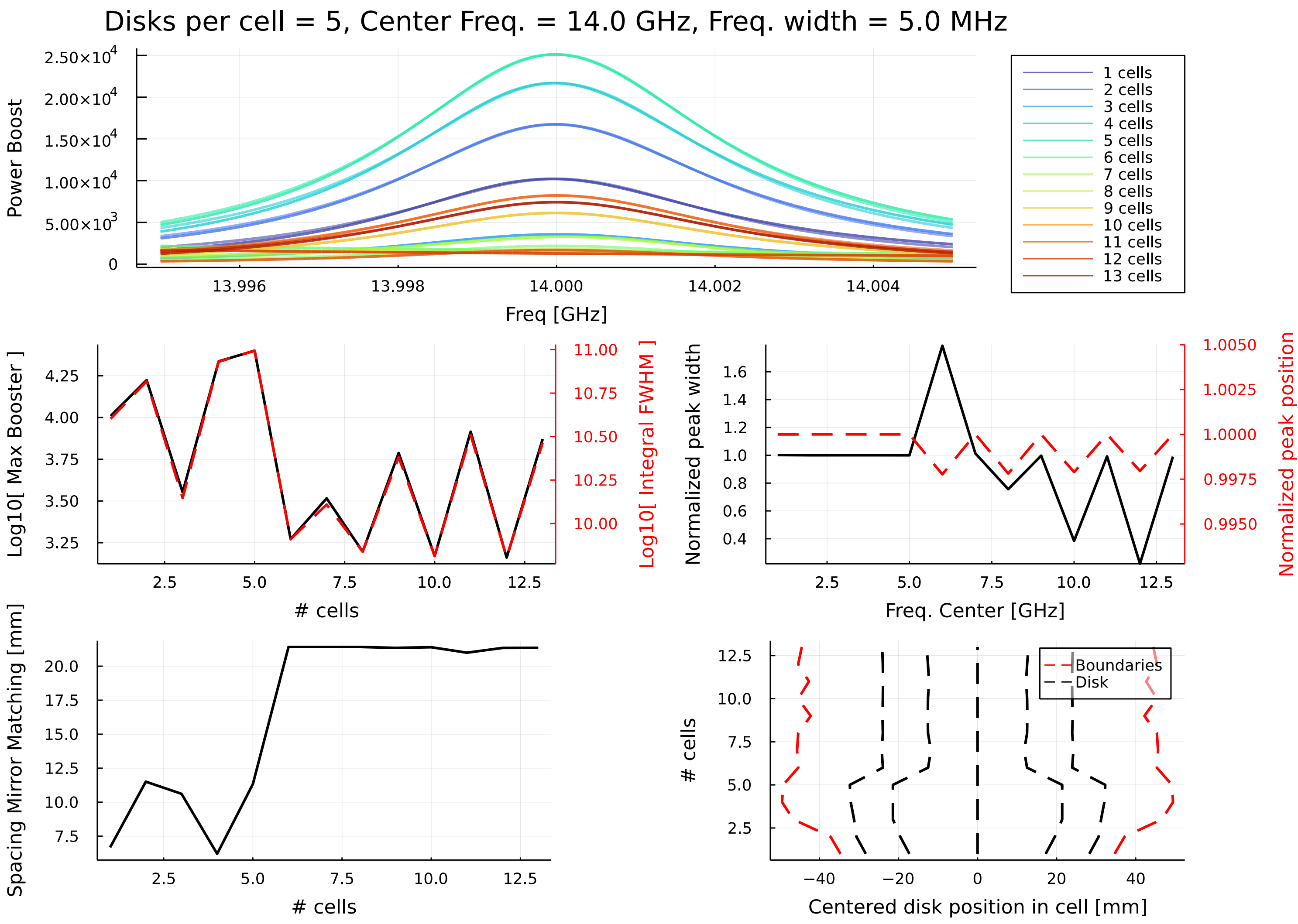




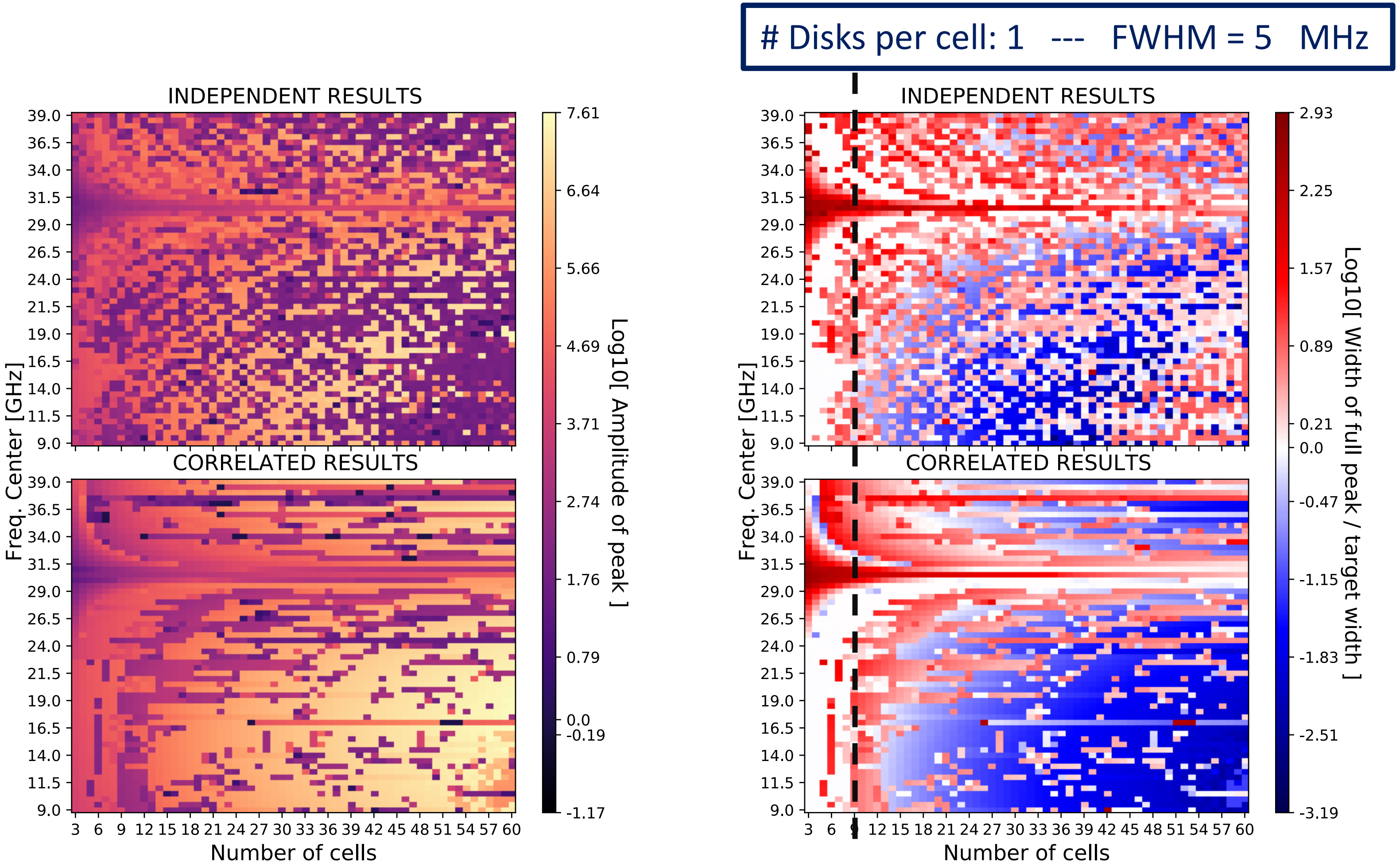
- as the complexity of the cell grows, the number of local maxima increases and we need more seeds for the continuations

Increasing number of cells

- We try to increase the number of cells in the booster
  - The system is more unstable with respect to these changes
  - We do not notice a smooth transition between booster curves anymore
  - Using local optimization is not the optimal strategy
  - Continuation of number of cells seems to not work well



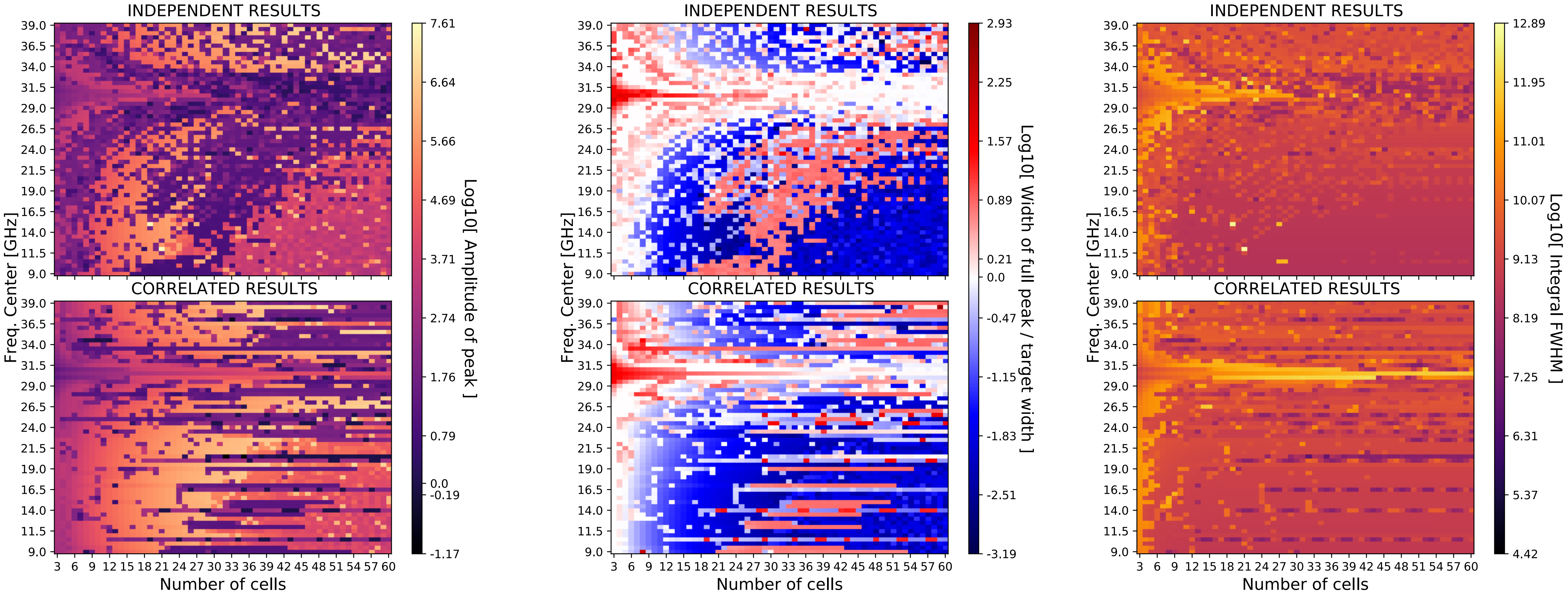
Number of cell continuation



- Increasing the number of cells seems to work up to a total of 7/10 disks
- Beyond this value the target FWHM is not reconstructed correctly
- Results consistent between independent and correlated samples
- Increasing the number of cells renders the system very resonant

Number of cell continuation

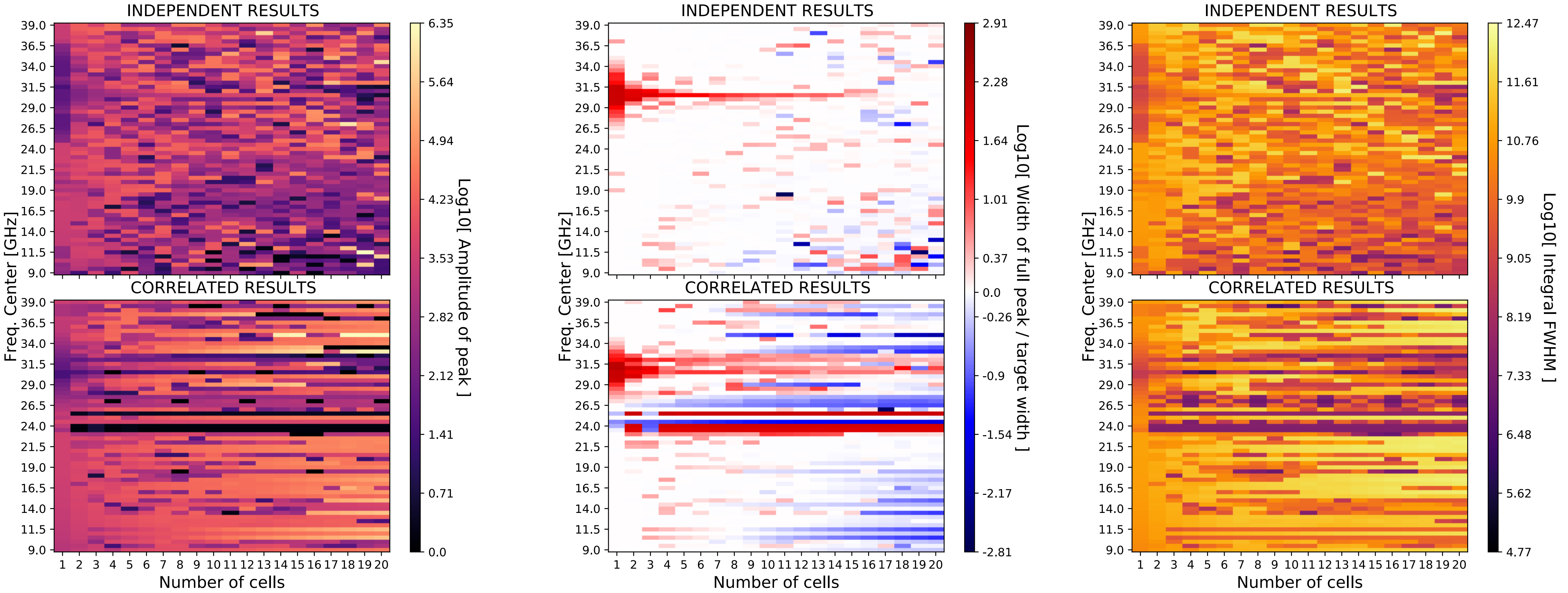
Disks per cell: 1 --- FWHM = 50 MHz



- The equidistant disk configuration appears to be highly resonant, thus only very narrow boosters have noticeable amplitudes

Number of cell continuation

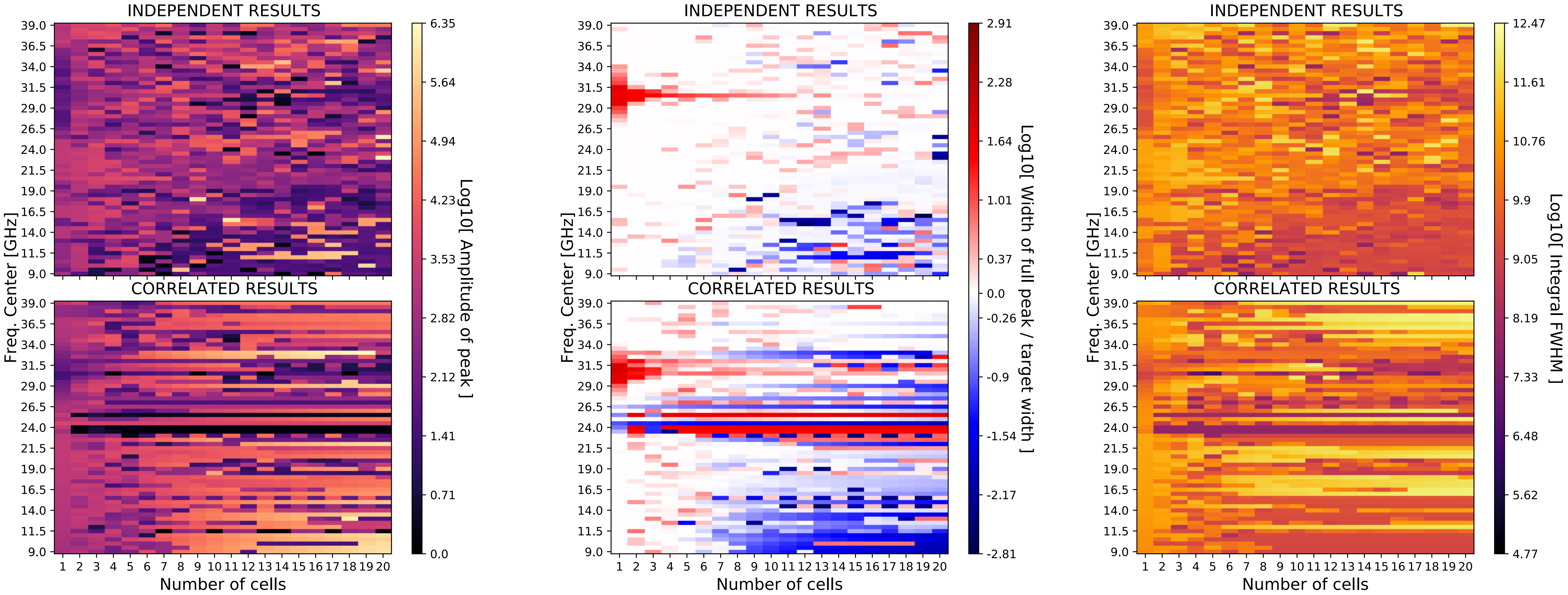
Disks per cell: 3 --- FWHM = 25 MHz



- Including more disks in the unit cell allows for larger booster curves

Number of cell continuation

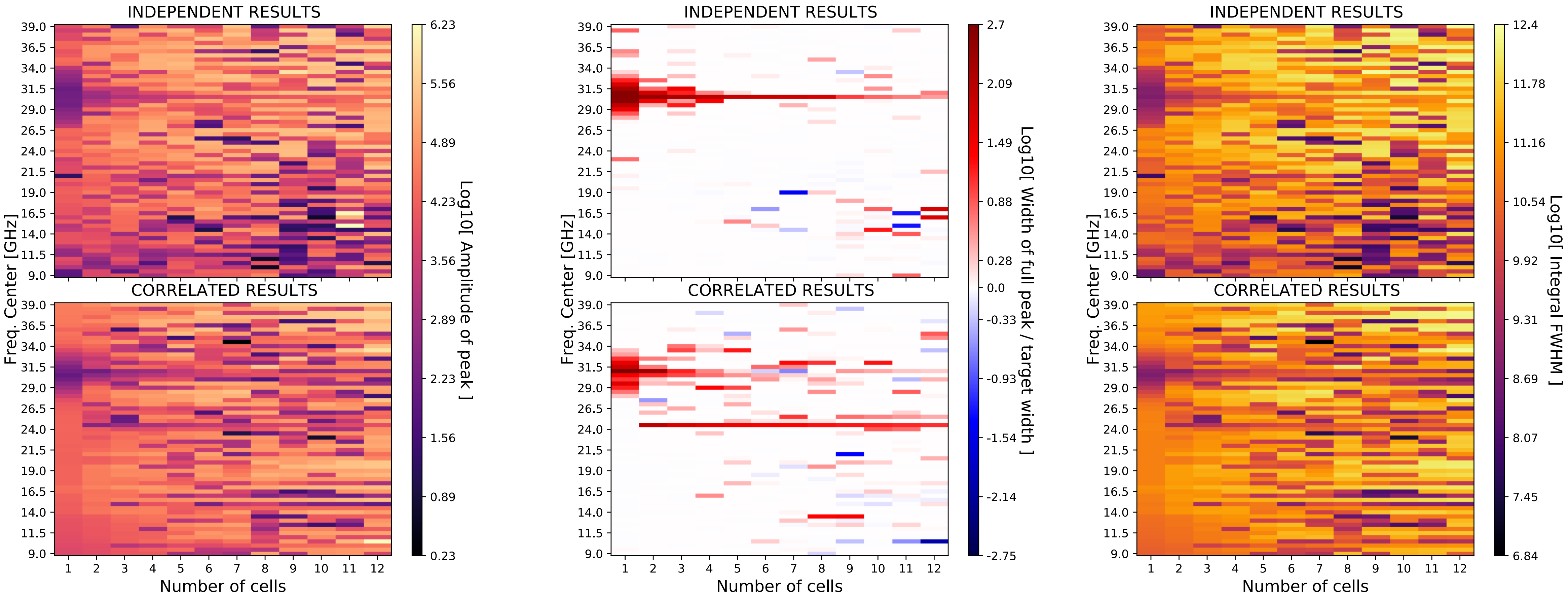
Disks per cell: 3 --- FWHM = 50 MHz



- Still, only relatively narrow peaks are reconstructed correctly when the the number of cells increases, while wide ones are not supported by the system

Number of cell continuation

Disks per cell: 5 --- FWHM = 5 MHz



- Good reconstruction of narrow peaks

Number of cell continuation

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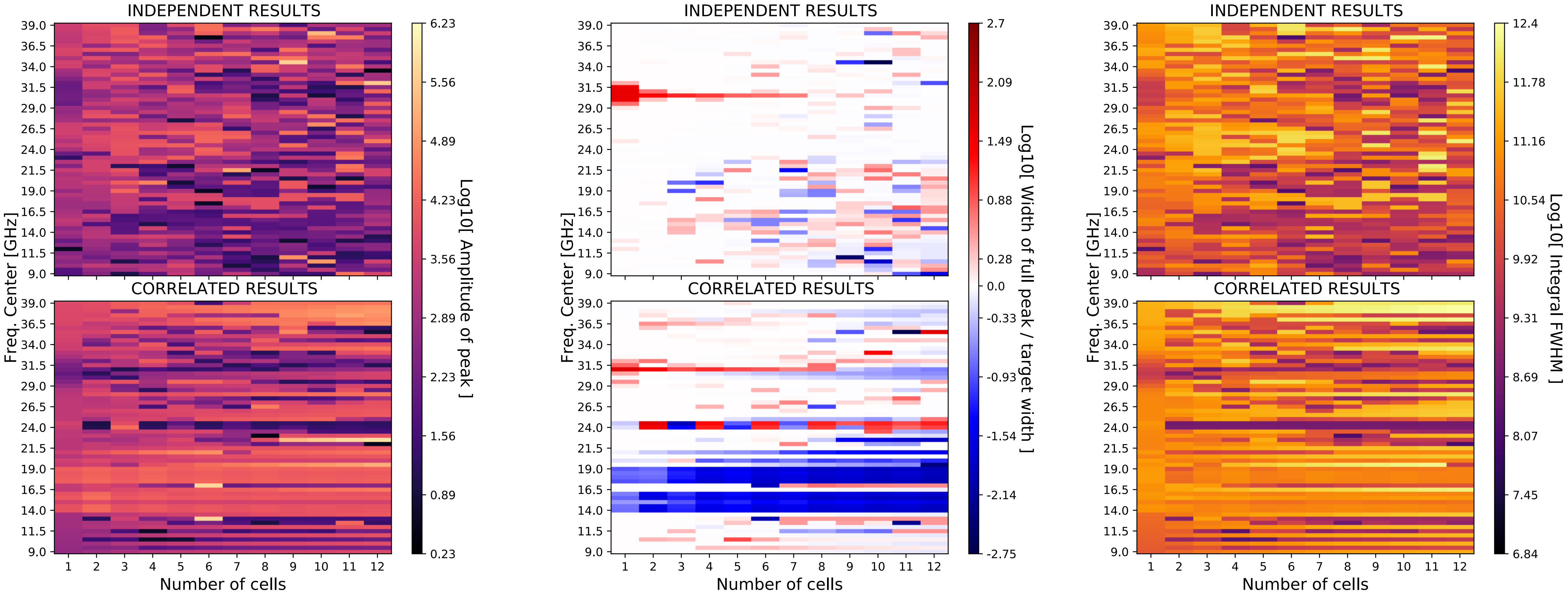
 # Disks per cell: 5 --- FWHM = 50 MHz



- Symmetric unit cells seems a good candidate for short boosters since they become more and more resonant in long boosters

Conclusions

- The symmetries we include in the system seem to work fine when moving the booster curve and in a slightly more limited capacity in widening the booster curve

- Increasing the number of cells seems to be much more difficult
  - The system becomes more resonant as we increase the number of cells → This is a contradiction to the request for a "wide" boost factor curve
  - Mainly narrow boosters with high amplitude or faint wide boosters

- This is not a good solution for long boosters, since it renders the system unstable and does not allow a meaningful increase in power boost [FWHM booster integral]
  - Reasonable candidate for short booster or resonant booster (narrow peaks)

New ideas and future studies

- Instead of symmetries we plan to study the variation of the “local Q-factor” across the booster:
  - Impedance transformation should keep the local Q-factor constant,
  - unit cells repeat exactly, so the same impedance transformation occurs at every cell, regardless of the position in the booster → This leads to very different local conditions along the booster

- An idea would be to optimize trying to keep local Q-factor roughly constant (might work well for medium boosters)
  - After gaining understanding of how the system works, relax this requirement to aid constructive interference and gain from long boosters

Thank you for your attention !