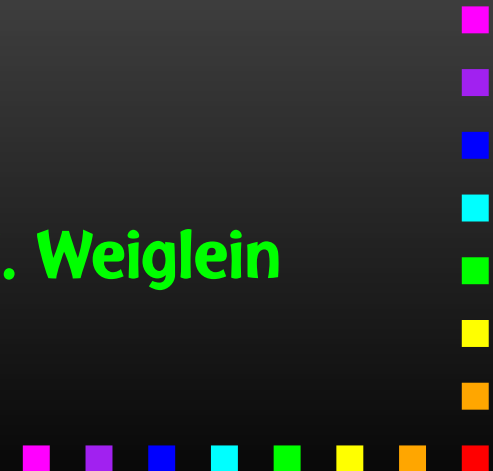


FeynHiggs - the Swiss Army Knife for Higgs Physics



T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein



The MSSM Higgs Sector

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad H_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}$$

Higgs Potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\varepsilon_{\alpha\beta} H_1^\alpha H_2^\beta + \text{h.c.}) + \frac{g_1^2 + g_2^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g_2^2}{2} |H_1 \bar{H}_2|^2$$

- **Five physical states:** h, H, A, H^+, H^- .
- **Input parameters:** $\tan \beta = v_1/v_2, M_A$ or M_{H^\pm} .
- **Unlike SM, MSSM predicts M_h (cf. Gauge Couplings).**
- **$M_h < M_Z$ at tree level, excluded by LEP searches.**



Complex Parameters

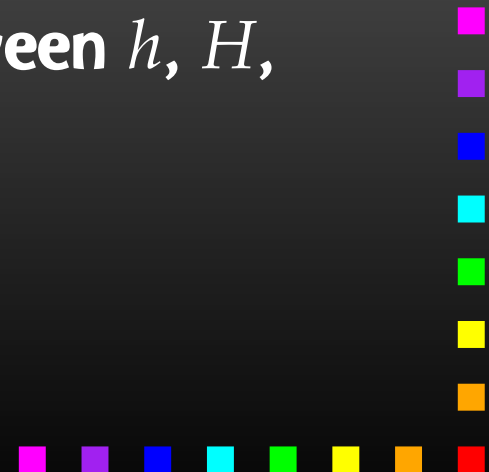
The Higgs potential contains two complex phases $\xi, \arg(m_{12}^2)$.
These can however be rotated away: **No CP at tree level.**

~~CP~~ effects are induced by **complex parameters that enter via loop corrections:**

- μ - Higgsino mass parameter,
- $A_{t,b,\tau}$ - trilinear couplings,
- $M_{1,2,3}$ - gaugino mass parameters.

They make $\hat{\Sigma}_{hA}, \hat{\Sigma}_{HA} \neq 0$ and induce mixing between $h, H,$ and A :

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} h \\ H \\ A \end{pmatrix}$$



Higgs Mass Matrix

The Higgs mass matrix has the form

$$\mathcal{M}^2 = \begin{pmatrix} q^2 - M_h^2 + \hat{\Sigma}_{hh} & \hat{\Sigma}_{hH} & \hat{\Sigma}_{hA} \\ \hat{\Sigma}_{Hh} & q^2 - M_H^2 + \hat{\Sigma}_{HH} & \hat{\Sigma}_{HA} \\ \hat{\Sigma}_{Ah} & \hat{\Sigma}_{AH} & q^2 - M_A^2 + \hat{\Sigma}_{AA} \end{pmatrix}$$

The physical Higgs states h_1, h_2, h_3 diagonalize this matrix:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = U \begin{pmatrix} h \\ H \\ A \end{pmatrix} \quad \text{where} \quad U\mathcal{M}^2U^\dagger = \begin{pmatrix} M_{h_1}^2 & 0 & 0 \\ 0 & M_{h_2}^2 & 0 \\ 0 & 0 & M_{h_3}^2 \end{pmatrix}$$

Observe: \mathcal{M}^2 is symmetric but not Hermitian.



Masses

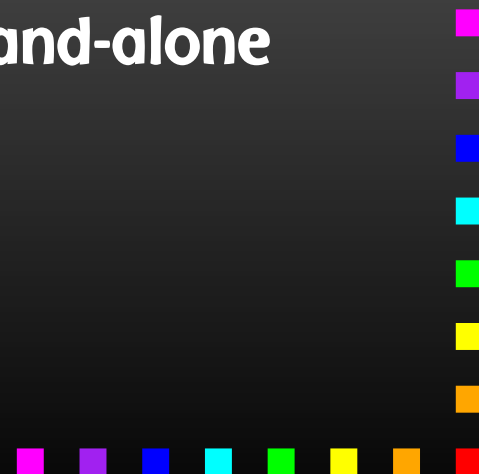
FeynHiggs performs a numerical search for the complex roots of $\det \mathcal{M}^2(q^2)$.

The Higgs masses are thus determined as the **real parts of the complex poles of the propagator**.

Complex contributions to the Higgs mass matrix ($\text{Im } \hat{\Sigma}$) are taken into account.

The diagonalization routines are available as a stand-alone package: <http://www.feynarts.de/diag>

Hahn 2006



Mixings

FeynHiggs returns two different ‘mixing’ matrices.

- **UHiggs** is a ‘true’ mixing matrix in the sense of being unitary and hence preserving probabilities. This matrix must be used **for internal Higgs bosons**.

Note: To obtain a unitary matrix, it is mathematically a necessity that \mathcal{M}^2 has no imaginary parts – making it Hermitian. This of course constrains the achievable quality of approximation.

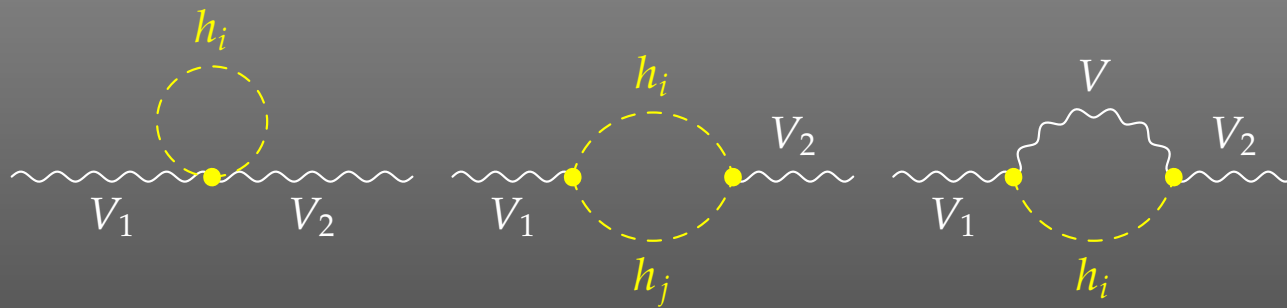
- **ZHiggs** is a matrix of Z-factors. It guarantees on-shell properties **for external Higgs bosons**.

It is important to understand that ZHiggs and UHiggs are two objects with physically and mathematically distinct properties. Neither is universally ‘better’ than the other.

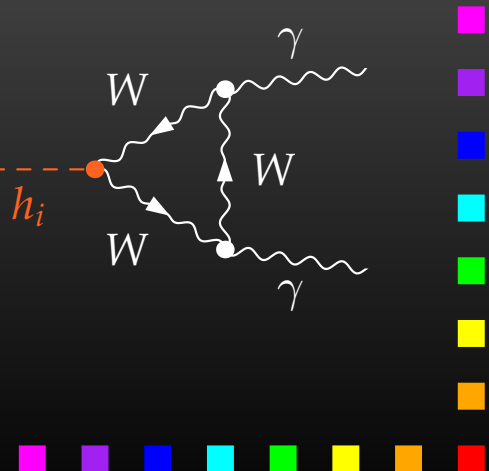
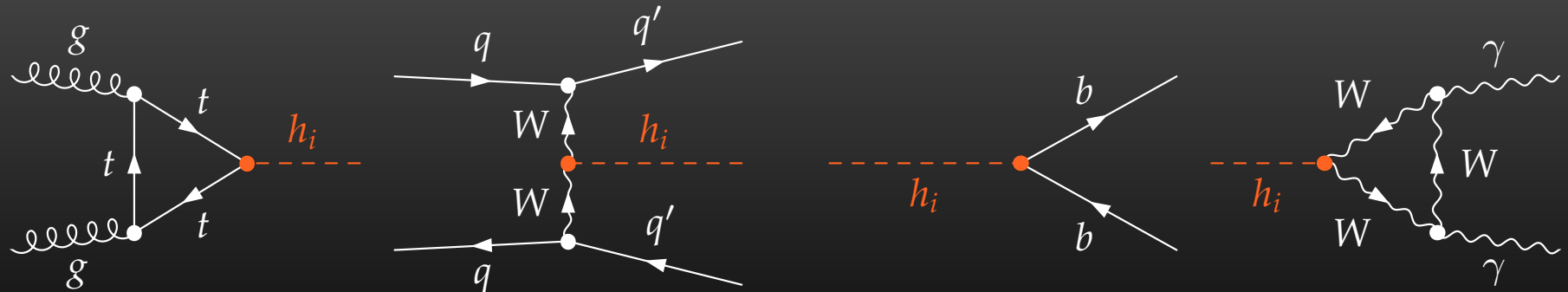


Examples of Internal and External Higgs Bosons

Internal Higgs bosons:



External Higgs bosons (production and decay):



UHiggs

FeynHiggs offers two approximations for UHiggs:

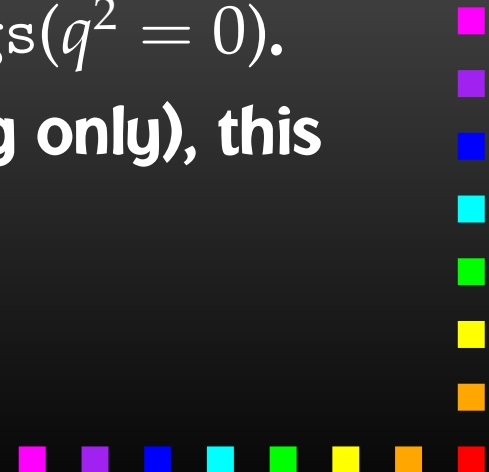
- q^2 on-shell

meaning $\hat{\Sigma}_{ii}(q^2 = m_i^2)$,
 $\hat{\Sigma}_{ij}(q^2 = \frac{1}{2}(m_i^2 + m_j^2))$.

- $q^2 = 0$

In this limit, UHiggs corresponds to the effective potential approach and coincides with ZHiggs($q^2 = 0$).

In the absence of ~~CP~~ effects (i.e. 2×2 mixing only), this is identical to the α_{eff} description.



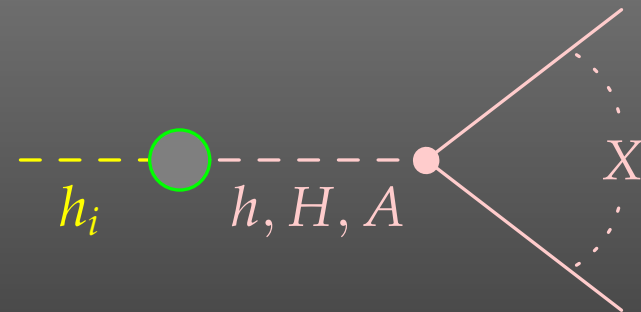
ZHiggs

ZHiggs is engineered to deliver the correct on-shell properties of an external Higgs boson, but is not necessarily unitary.

$$\Gamma_{h_1} = \sqrt{Z_h} (\Gamma_h + Z_{hH} \Gamma_H + Z_{hA} \Gamma_A)$$

$$\Gamma_{h_2} = \sqrt{Z_H} (Z_{Hh} \Gamma_h + \Gamma_H + Z_{HA} \Gamma_A)$$

$$\Gamma_{h_3} = \sqrt{Z_A} (Z_{Ah} \Gamma_h + Z_{AH} \Gamma_H + \Gamma_A)$$



- $\Gamma_{h,H,A}$ - amplitude for $h, H, A \rightarrow X$,
- $\sqrt{Z_h}$ - sets residuum of the external Higgs boson to 1,
- Z_{hH}, Z_{hA} - describe the transition $h \rightarrow H, A$.



ZHiggs

For convenience, the Z factors can be arranged in matrix form:

$$\text{ZHiggs} = \begin{pmatrix} \sqrt{Z_h} & \sqrt{Z_h} Z_{hH} & \sqrt{Z_h} Z_{hA} \\ \sqrt{Z_H} Z_{Hh} & \sqrt{Z_H} & \sqrt{Z_H} Z_{HA} \\ \sqrt{Z_A} Z_{Ah} & \sqrt{Z_A} Z_{AH} & \sqrt{Z_A} \end{pmatrix}$$

In this guise, ZHiggs can be used very much like UHiggs, even though its theoretical origin is quite different.

Reassuringly, ZHiggs and UHiggs coincide in the limit $q^2 = 0$.

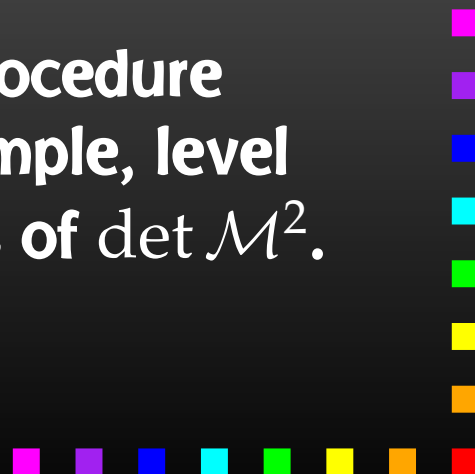
ZHiggs

The transition factors Z_{ij} involve both the **tree-level mass** m_j and the **loop-corrected mass** M_i of each Higgs boson:

$$Z_{ij} = \frac{\hat{\Sigma}_{ik}(M_i^2) \hat{\Sigma}_{jk}(M_i^2) - \hat{\Sigma}_{ij}(M_i^2) [M_i^2 - m_j^2 + \hat{\Sigma}_j(M_i^2)]}{[M_i^2 - m_j^2 + \hat{\Sigma}_j(M_i^2)] [M_i^2 - m_k^2 + \hat{\Sigma}_k(M_i^2)] - \hat{\Sigma}_{jk}^2(M_i^2)}$$

To compute Z_{ij} we thus have to make the connection between the **'loop'** (h_1, h_2, h_3) and the **'tree'** (h, H, A) states.

Neither the zero-search nor the diagonalization procedure allow to do this in an unambiguous way. For example, level crossings may occur when searching for the zeros of $\det \mathcal{M}^2$.



ZHiggs

FeynHiggs computes ZHiggs and the associated masses \tilde{M}_i for **all permutations π of Higgs states** involved in the mixing and chooses the one which minimizes

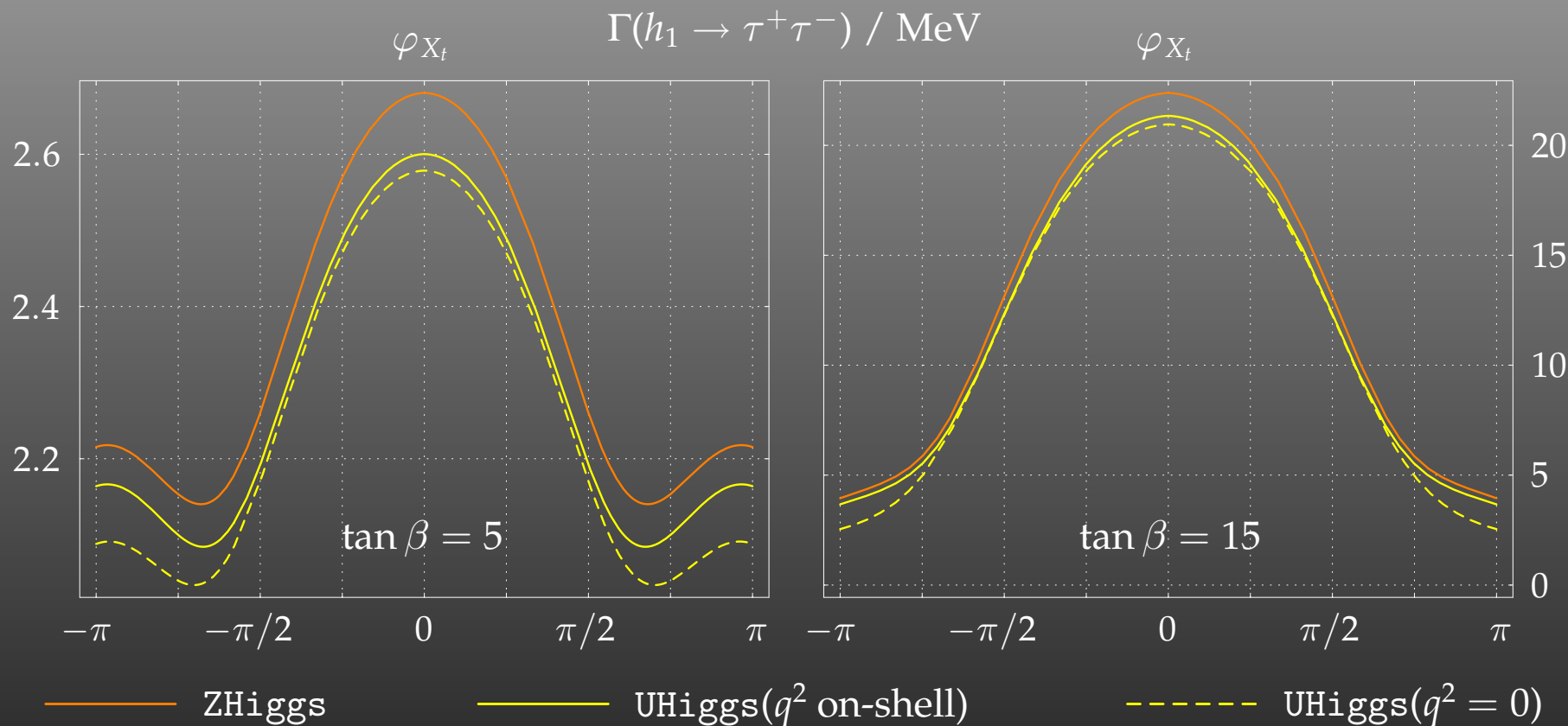
$$\sum_i |M_i - \tilde{M}_{\pi(i)}| + \sum_{i,j} |C_{ij} - \text{ZHiggs}_{\pi(i)j}|$$

where C is the mixing matrix that comes out of the diagonalization of \mathcal{M}^2 , i.e. a by-product of the zero-search.

This is an **empirical recipe**, so don't be confused by the different dimensions of M and Z . The permutation is decided in 99+% of all cases by the mass pattern. The $|C - Z|$ term becomes relevant only for (almost) degenerate masses where it can tell e.g. the symmetric from the antisymmetric state.



Phenomenological Effects



$[M_{\text{SUSY}} = M_3 = M_2 = 500 \text{ GeV}, \mu = 1000 \text{ GeV}, M_{H^\pm} = 150 \text{ GeV}, X_t = 700 e^{i\varphi_{X_t}} \text{ GeV}]$

UHiggs(q^2 on-shell) gives results closer to the full result than UHiggs($q^2 = 0$) with deviations at the few-percent level.

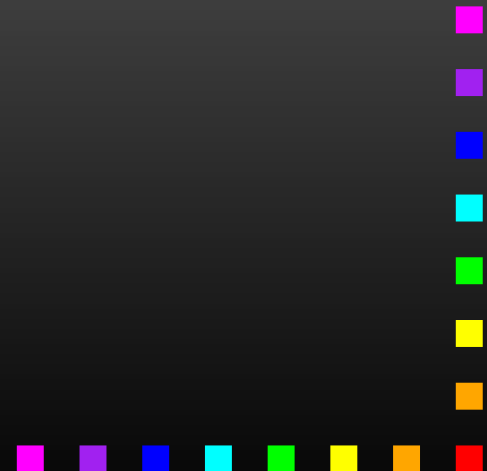


Mixing Matrix Overview

- **Internal** Higgs boson: use **UHiggs**.
Two approximations:
 - q^2 on-shell,
 - $q^2 = 0$ = effective potential approximation.
- **External** Higgs boson: use **ZHiggs**.

There exists a version of the MSSM Model File for FeynArts (HMix.mod) with

- $S[0, \{h\}] = \sum_{i=1}^3 \text{UHiggs}[h, i] S[i],$
- $S[10, \{h\}] = \sum_{i=1}^3 \text{ZHiggs}[h, i] S[i],$
inserted only on external lines.



Benchmark Scenarios

FeynHiggs has long included **Benchmark Scenarios** which are useful in the search for the MSSM Higgs bosons:

- Vary only M_A and $\tan \beta$,
- Keep all other SUSY parameters fixed.

m_h^{\max} scenario

Yields conservative $\tan \beta$ exclusion bounds ($X_t = 2 M_{\text{SUSY}}$).

gluophobic Higgs scenario

Looks at a small hgg coupling, such that a main LHC production mode vanishes.

no-mixing scenario

No mixing in the scalar top sector ($X_t = 0$).

small α_{eff} scenario

Explores $\alpha_{\text{eff}} \rightarrow 0$ where the $hb\bar{b}$ coupling $\sim \sin \alpha_{\text{eff}} / \cos \beta$ and thus a main decay mode and important search channel vanishes.

Carena, Heinemeyer, Wagner, Weiglein 2002

But: constraints such as CDM so far ignored.

Wanted: M_A - $\tan \beta$ planes in agreement with CDM.



Parameter Planes

Candidate models:

- **CMSSM (or mSUGRA)**

characterized by $m_0, m_{1/2}, A_0, \tan \beta, \text{sign } \mu$.

But: Too restricted.

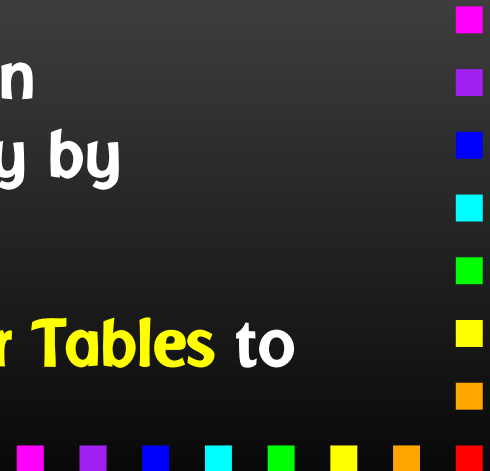
- **NUHM (Non-universal Higgs mass model)**

Assumption: no unification of scalar fermion and scalar Higgs parameters at the GUT scale.

additional parameters: M_A, μ .

The NUHM introduces non-trivial relations between parameters, which thus cannot be scanned naively by independent loops.

FeynHiggs 2.6 offers the new format of **Parameter Tables** to deal with such cases.



Parameter Tables

Input parameters can either be given in an input file (as before) or interpolated from a table, in almost any mixture.

The table format is pretty straightforward:

MT	MSusy	MA0	TB	At	MUE ...
171.4	500	200	5	1000	761
171.4	500	210	5	1000	753
...					
171.4	500	200	6	1000	742
171.4	500	210	6	1000	735

For **two given inputs** (typically M_A and $\tan \beta$) the four **neighbouring grid points** are searched in the table and the other parameters are **interpolated** from those points. An error is returned if the inputs fall outside of the table boundaries (i.e. no extrapolation).



Tables and Records

Four predefined **NUHM M_A -tan β planes** can be downloaded from **www.feynhiggs.de**.

Definition of new planes by the user is possible.

The Table concept is actually embedded into the new **FeynHiggs Record**. This is a data type which captures the entire content of a FeynHiggs parameter file.

Using a Record, the programmer can process FeynHiggs parameter files independently of the frontend.



Corrections included in FeynHiggs 2.6

$$\left(\begin{array}{ccc} q^2 - M_h^2 + \hat{\Sigma}_{hh}^{\bullet\bullet\bullet} & \hat{\Sigma}_{hH}^{\bullet\bullet\bullet} & \hat{\Sigma}_{hA}^{\bullet\bullet} \\ \hat{\Sigma}_{Hh}^{\bullet\bullet\bullet} & q^2 - M_H^2 + \hat{\Sigma}_{HH}^{\bullet\bullet\bullet} & \hat{\Sigma}_{HA}^{\bullet\bullet} \\ \hat{\Sigma}_{Ah}^{\bullet\bullet} & \hat{\Sigma}_{AH}^{\bullet\bullet} & q^2 - M_A^2 + \hat{\Sigma}_{AA}^{\bullet\bullet} \end{array} \right), \hat{\Sigma}_{H^+H^-}^{\bullet\bullet}$$

- **Leading $\mathcal{O}(\alpha_s\alpha_t)$ two-loop corrections in the cMSSM.**

Heinemeyer, Hollik, Rzehak, Weiglein 2007

- **Leading $\mathcal{O}(\alpha_t^2)$ + subleading $\mathcal{O}(\alpha_s\alpha_b, \alpha_t\alpha_b, \alpha_b^2)$ two-loop corrections in the rMSSM (phases only partially included).**

Degrassi, Slavich, Zwirner 2001 – Brignole, Degrassi, Slavich, Zwirner 2001, 02
Dedes, Degrassi, Slavich 2003

- **Full one-loop evaluation (all phases, q^2 dependence).**

Frank, Heinemeyer, Hollik, Weiglein 2002

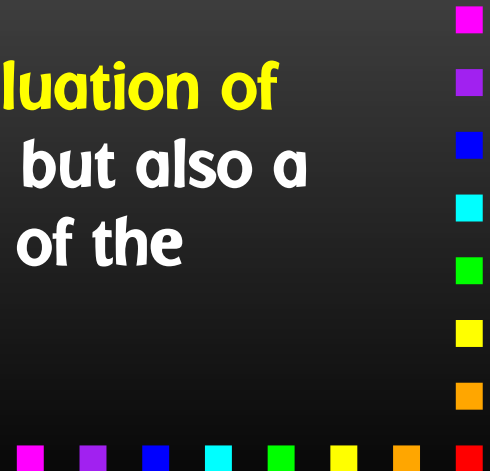


Treatment of Phases

A **new flag** controls the treatment of phases in the part of the two-loop corrections known only in the rMSSM so far:

- all corrections ($\alpha_s\alpha_t, \alpha_s\alpha_b, \alpha_t\alpha_t, \alpha_t\alpha_b$) in the rMSSM,
- only the cMSSM $\alpha_s\alpha_t$ corrections,
- the cMSSM $\alpha_s\alpha_t$ corrections combined with the remaining corrections in the rMSSM, truncated in the phases,
- the cMSSM $\alpha_s\alpha_t$ corrections combined with the remaining corrections in the rMSSM, interpolated in the phases [default].

FeynHiggs thus not only has the **most precise evaluation of the Higgs masses in the cMSSM** available to date, but also a method to obtain a reasonably objective estimate of the **uncertainties due to the rMSSM-only parts.**



Size matters

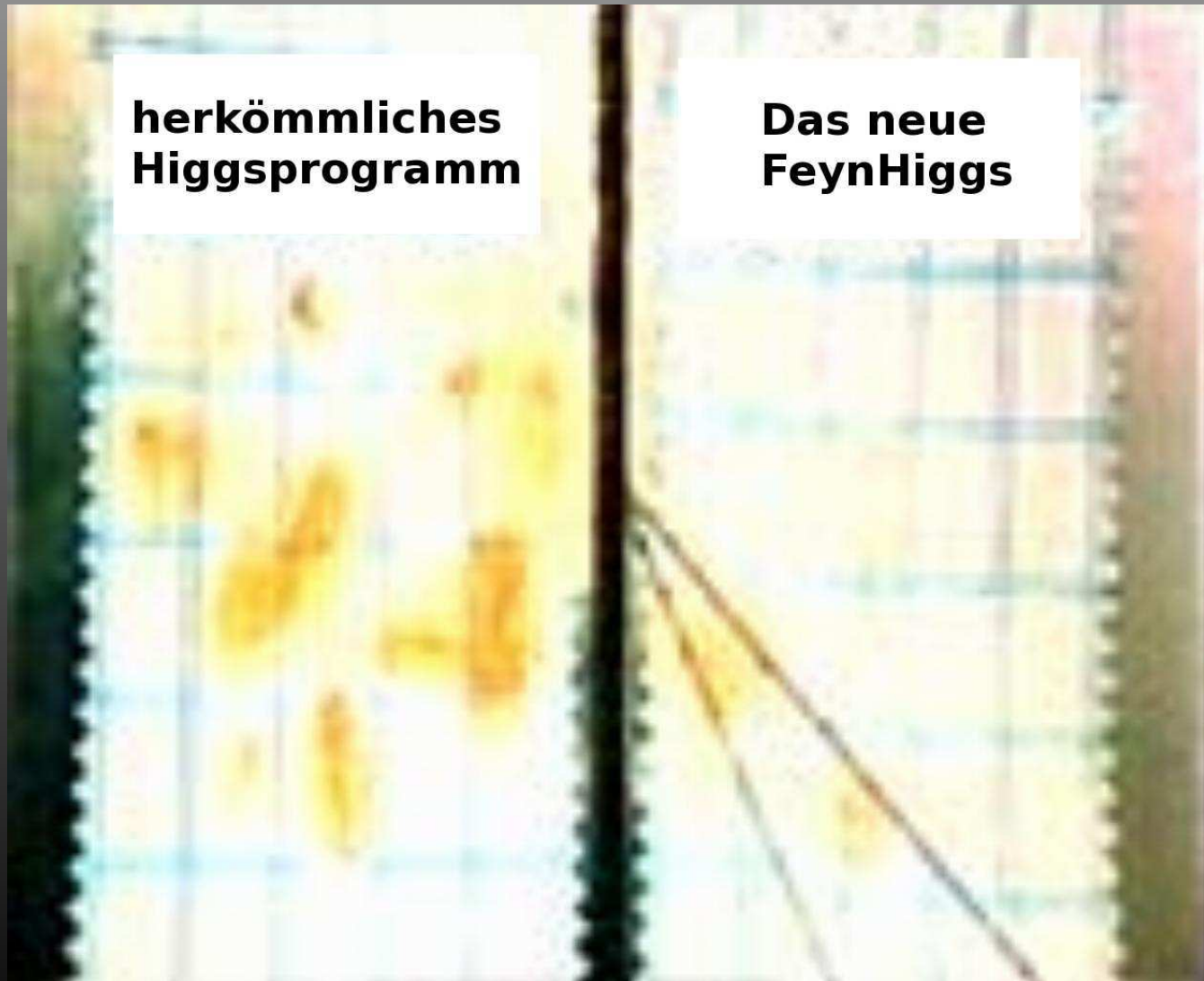
Implementing the $\alpha_s\alpha_t$ cMSSM corrections in FeynHiggs was a major piece of work. The amplitudes could be shrunk from 38 MB to less than 1.5 MB, mainly by abbreviation techniques and exploiting the unitarity of the sfermion mixing matrices.

- **Compile time is about 3 min**
(up from 45 sec in FeynHiggs 2.5).
- **Run time is 28 msec per parameter point**
(up from 27 msec in FeynHiggs 2.5).

These figures show that the full cMSSM evaluation is actually usable in everyday life.



FeynHiggs



Output of FeynHiggs 2.6

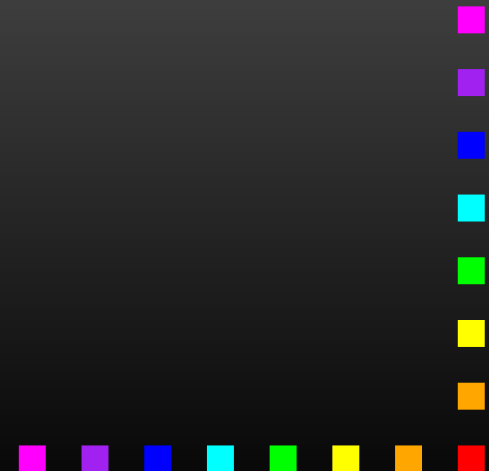
- FHHiggsCorr - **All Higgs-boson masses and mixings:**
 $M_{h_1}, M_{h_2}, M_{h_3}, M_{H^\pm}, \alpha_{\text{eff}}, \text{UHiggs}, \text{ZHiggs}, \dots$
- FHUncertainties - **Uncertainties of masses and mixings.**
- FHCouplings

- **Couplings and Branching Ratios for the channels**

$h_{1,2,3} \rightarrow f\bar{f}, \gamma\gamma, ZZ^*, WW^*, gg$ $h_i Z^*, h_i h_j, H^+ H^-$ $\tilde{f}_i \tilde{f}_j,$ $\tilde{\chi}_i^\pm \tilde{\chi}_j^\pm, \tilde{\chi}_i^0 \tilde{\chi}_j^0$	$H^\pm \rightarrow f\bar{f}'$ $h_i W^{\pm*}$ $\tilde{f}_i \tilde{f}'_j,$ $\tilde{\chi}_i^0 \tilde{\chi}_j^\pm$	$t \rightarrow W^+ b$ $H^+ b$
---	---	----------------------------------

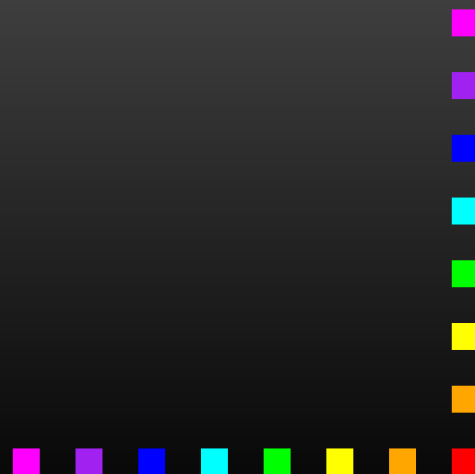
- **Branching Ratios of an SM Higgs with mass M_{h_i} :**

$$h_{1,2,3}^{\text{SM}} \rightarrow f\bar{f}, \gamma\gamma, ZZ^*, WW^*, gg$$



Output of FeynHiggs 2.6

- FHHiggsProd - Higgs production-channel cross-sections:
(SM: most up-to-date, MSSM: effective coupling approximation)
 - $gg \rightarrow h_i$ - gluon fusion.
 - $WW \rightarrow h_i, ZZ \rightarrow h_i$ - gauge-boson fusion.
 - $W \rightarrow Wh_i, Z \rightarrow Zh_i$ - Higgs-strahlung.
 - $b\bar{b} \rightarrow b\bar{b}h_i$ - Yukawa process.
 - $b\bar{b} \rightarrow b\bar{b}h_i, h_i \rightarrow b\bar{b}$, **one b tagged.**
 - $t\bar{t} \rightarrow t\bar{t}h_i$ - Yukawa process.



Output of FeynHiggs 2.6

- FHConstraints - Electroweak precision observables:

- $\Delta\rho$
at $\mathcal{O}(\alpha, \alpha\alpha_s)$ including NMFV effects.
- M_W, s_w^{eff}
via SM formula + $\Delta\rho$.
- $\text{BR}(b \rightarrow s\gamma)$
including NMFV effects.
Hahn, Hollik, Illana, Peñaranda 2006
- $(g_\mu - 2)_{\text{SUSY}}$
full one-, leading/subleading two-loop SUSY corrections.
Heinemeyer, Stöckinger, Weiglein 2004
- EDMs of electron (Th), neutron, Hg.



Download and Build

- Get the FeynHiggs tar file from www.feynhiggs.de.
- Unpack and configure:

```
tar xfz FeynHiggs-2.6.1.tar.gz
cd FeynHiggs-2.6.1
./configure
```
- Type **make** to build the Fortran/C++ part only.
Type **make all** to build also the Mathematica part.
Takes about 3 min to build on a Pentium IV.
- Type **make install** to install the package.
- Type **make clean** to remove unnecessary files.

Build tested on Linux, Tru64 Unix, Mac OS, Windows (Cygwin).

Usage








Four operation modes:

- **Library Mode:** Invoke the FeynHiggs routines from a Fortran or C/C++ program linked with `libFH.a`.
- **Command-line Mode:** Process parameter files in FeynHiggs or SLHA format at the shell prompt or in scripts with the standalone executable `FeynHiggs`.
- **Web Mode:** Interactively choose the parameters at the FeynHiggs User Control Center (FHUCC) and obtain the results on-line.
- **Mathematica Mode:** Access the FeynHiggs routines in Mathematica via MathLink with `MFeynHiggs`.

All programs and subroutines are documented in man pages.



Library Mode

- Static Fortran 77 library `libFH.a`.
- All **global symbols prefixed** to prevent symbol collision.
- Uses **only subroutines** (no functions):
No include files needed (except for couplings).
C/C++ users include `CFeynHiggs.h` for prototypes.
- Detailed **debugging output** can be turned on at run time.
- **Main routines:**
 - `FHSetFlags` - set the flags of the calculation, 
 - `FHSetPara` - set the MSSM input parameters, 
 - `FHHiggsCorr` - compute Higgs masses and mixings, 
 - `FHUncertainties` - estimate their uncertainties, 
 - `FHCouplings` - compute the Higgs couplings and BRs, 
 - `FHHiggsProd` - estimate Higgs production cross-sections, 
 - `FHConstraints` - evaluate additional constraints. 



Command-line Mode

Input File

```
MT      178
MB      4.7
MW     80.450
MZ     91.1875
MSusy   975
MAO     200
Abs(M_2) 332
Abs(MUE) 980
TB      50
Abs(At) -300
Abs(Ab) 1500
Abs(M_3) 975
```

Command

`FeynHiggs file [flags]`

Screen Output

```
----- HIGGS MASSES -----
| Mh0    = 116.022817
| MHH    = 199.943497
| MA0    = 200.000000
| MHp    = 216.973920
| SAeff  = -0.02685112
| UHiggs = 0.99999346 -0.00361740 0.00000000 \
|         0.00361740 0.99999346 0.00000000 \
|         0.00000000 0.00000000 1.00000000
----- ESTIMATED UNCERTAINTIES -----
| DeltaMh0 = 1.591957
| DeltaMHH = 0.004428
| DeltaMA0 = 0.000000
| DeltaMHP = 0.152519
| ...
```

- Mask off details with
`FeynHiggs file [flags] | grep -v %`
- table utility converts to machine-readable format, e.g.
`FeynHiggs file [flags] | table TB Mh0 > outfile`

Access to Tables

Input File “normal”

```

MT      170.9
MB      4.7
MW      80.392
MZ      91.1875
MSusy   975
MAO     200
Abs(M_2) 332
Abs(MUE) 980
TB      50
Abs(At) -300
Abs(Ab) 1500
Abs(M_3) 975
    
```

“table”

```

MT      170.9
MB      4.7
MW      80.392
MZ      91.1875
MAO     200
TB      50
table file.dat MAO TB
    
```

“inline table”

```

MT      170.9
MB      4.7
MW      80.392
MZ      91.1875
MAO     200
TB      50
table - MAO TB
MAO TB At MUE ...
200 5 1000 761
210 5 1000 753
...
    
```

Loops over parameter values possible (parameter scans).

- MAO 200 400 50 - linear: 200, 250, 300, 350, 400,
- TB 5 40 *2 - logarithmic: 5, 10, 20, 40,
- TB 5 50 /6 - # of steps: 5, 14, 23, 32, 41, 50.

SUSY Les Houches Accord Format

Input File

```
BLOCK MODSEL
  1      1
BLOCK MINPAR
  1  0.100000000E+03 # m0
  2  0.250000000E+03 # m12
  3  0.100000000E+02 # tanb
  4  0.100000000E+01 # Sign(mu)
  5 -0.100000000E+03 # A
BLOCK SMINPUTS
  4  0.911870000E+02 # MZ
  5  0.425000000E+01 # mb(mb)
  6  0.175000000E+03 # t
...
```

Command

FeynHiggs *file* [*flags*]

file.fh

```
BLOCK MASS
  25  1.12697840E+02 # Mh0
  35  4.00145460E+02 # MHH
  36  3.99769788E+02 # MA0
  37  4.08050556E+02 # MHp
  ...
BLOCK ALPHA
      -1.10658125E-01 # Alpha
  ...
```

- Uses the SLHA 2 and the SLHA Library.

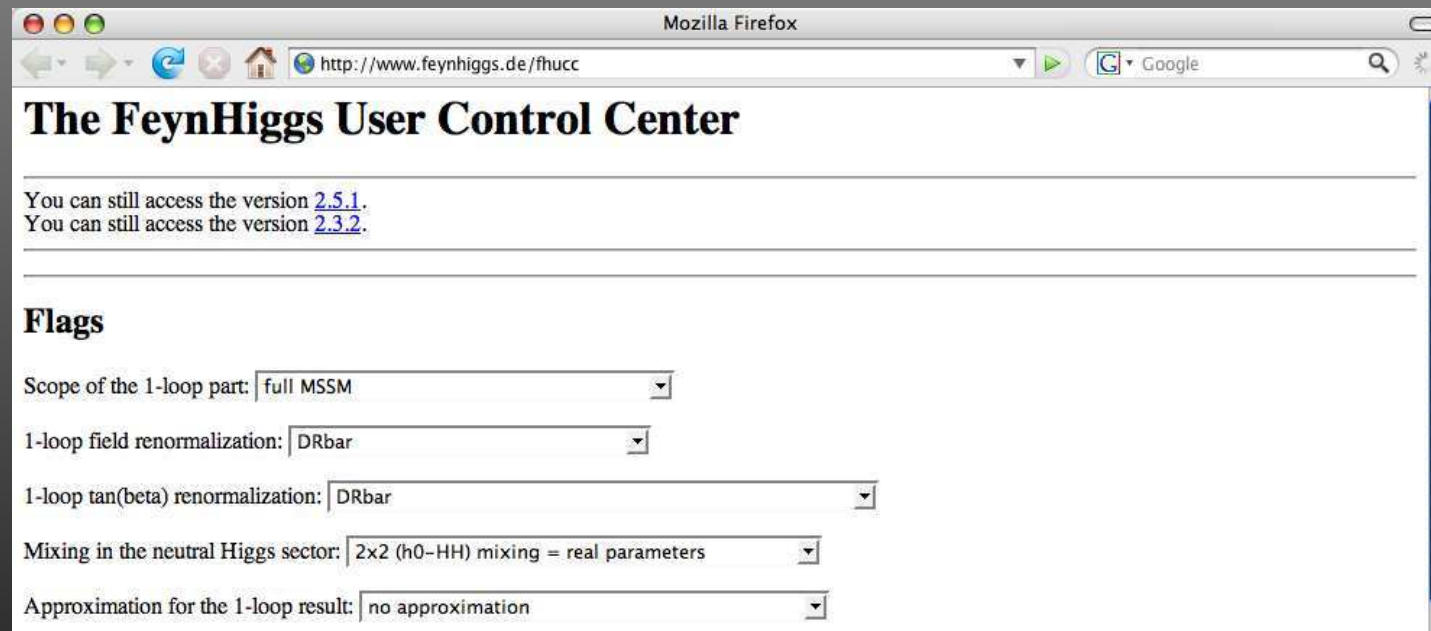
Hahn 2004, 06

- SLHA can also be used in Library Mode with FHSetSLHA.
- FeynHiggs tries to read each file in SLHA format first.
If that fails, fallback to native format.



Web Mode

The **FeynHiggs User Control Center (FHUCC)** is on-line at
<http://www.feynhiggs.de/fhucc>



**FHUCC is a Web interface for the Command-line Frontend.
The user gets the results together with the input file for the
Command-line Frontend.**

Mathematica Mode

Provides the FeynHiggs functions in Mathematica, e.g.

```
In[1]:= Install["MFeynHiggs"];
```

```
In[2]:= FHSetFlags[...];
```

```
In[3]:= FHSetPara[...];
```

```
In[4]:= FHHiggsCorr[]
```

```
Out[4]= {MHiggs -> {117.184, 194.268, 200., 212.67},  
>      SAeff -> -0.37575,  
>      UHiggs -> {{0.994782, 0.102021, 0},  
>                {-0.102021, 0.994782, 0},  
>                {0, 0, 1.}}}
```

- Can use all Mathematica functions on the results (e.g. ContourPlot, FindMinimum).
- Convenient interactive mode for FeynHiggs.



Summary: Main New Features

- Higgs masses are the **real part of the complex pole**.
- **Two kinds of ‘mixing’ matrices** (UHiggs, ZHiggs).
Choice of mixing matrices in all Higgs production and decay channels (default: ZHiggs).
- Inclusion of the **full cMSSM two-loop $\alpha_s\alpha_t$ corrections** in highly optimized form.
- Inclusion of **full one-loop NMFV effects**.
- Possibility to **interpolate parameters from data tables**.
Availability of **M_A - $\tan\beta$ planes** in agreement with CDM constraints.
- Estimates of Higgs **production cross-sections**.
- **EDMs** of electron (Th), neutron, Hg.

