FIELD REDEFINITIONS, PERTURBATIVE UNITARITY, AND HIGGS INFLATION

work with Misha Shaposhnikov and Sebastian Zell

[arXiv: 2203.09534]

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Outline

- Higgs Inflation in a nutshell
- Higgs inflation is/as an EFT: Self-consistency
- Comment on different formulations of gravity
- Concluding remarks

Higgs Inflation in a nutshell
i.e. how to inflate the Universe with the SM

Starting point, the SM coupled to gravity

The relevant piece of Lagrangian is the purely scalar sector with the Higgs h in the unitary gauge

$$S_{\rm scalar} = \int d^4x\sqrt{g}\left[\frac{M_P^2+\xi h^2}{2}R - \frac{1}{2}(\partial_\mu h)^2 - V(h)\right] ,$$

where $g = \det(-g_{\mu\nu})$, $\xi \gg 1$, R is the scalar curvature, and the usual potential

$$V(h) \approx \frac{\lambda}{4}h^4 \ .$$

During inflation M_P is "small" or better say subdominant w.r.t ξh^2

Deviation from exact de Sitter is due to the presence of this "small" term

Starting point, the SM coupled to gravity

Change variables

$$g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu} \ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \ ,$$

in order to bring gravity to its conventional form (go to "Einstein frame")

$$S_{\text{scalar}} = \int d^4x \sqrt{g} \left[\frac{M_P^2}{2} R - \frac{1}{2\Omega^2} \left(1 + \frac{6\xi^2}{M_P^2} \frac{h^2}{\Omega^2} \right) (\partial_\mu h)^2 - \frac{\lambda h^4}{\Omega^4} \right] .$$

No free lunch: Canonical gravity $\mapsto$ nontrivial modification to Higgs sector...

Will come back to this point later

Starting point, the SM coupled to gravity

For inflation, $\xi h^2 \gg M_P^2$, and the action simplifies considerably

$$S_{\rm scalar} \simeq \int d^4x\sqrt{g}\left[\frac{M_P^2}{2}R - 3M_P^2\frac{(\partial_\mu h)^2}{h^2} - \frac{\lambda M_P^4}{4\xi^2}\left(1-\frac{\#}{h^2}+\dots\right)\right]$$

Highly suggestive form - canonicalize via exponential map

$$h \propto e^{\#\chi/M_P} \ ,$$

$\chi \mapsto$ pseudo Nambu-Goldstone associated with SSB of dilatations

$$S_{\rm scalar} \simeq \int d^4x\sqrt{g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial_\mu\chi)^2 - \frac{\lambda M_P^4}{4\xi^2}\left(1-2e^{-\#\chi/M_P}+\dots\right)\right] \ .$$

This is exactly what happens in Starobinsky inflation too

$$S_{\mathsf{Star}} = \int d^4x\sqrt{g}\left(\frac{M_P^2}{2}R - \alpha R^2\right) \ ,$$

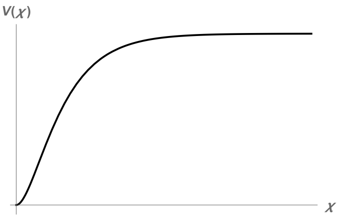
where the first term is a small explicit breaking of the theory's scale symmetry.

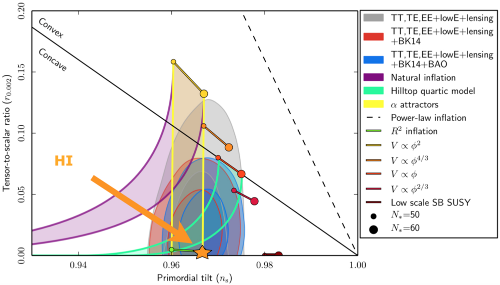
Note the following equivalent form

$$S_{\mathsf{Star}} \simeq \int d^4x\sqrt{g}\left(\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{M_P^4}{8\alpha}\left(1 - 2e^{-\#\chi/M_P} + \ldots\right)\right) \ .$$

scalaron $\mapsto$ pseudo Nambu-Goldstone associated with SSB of dilatations

Starting point, the SM coupled to gravity



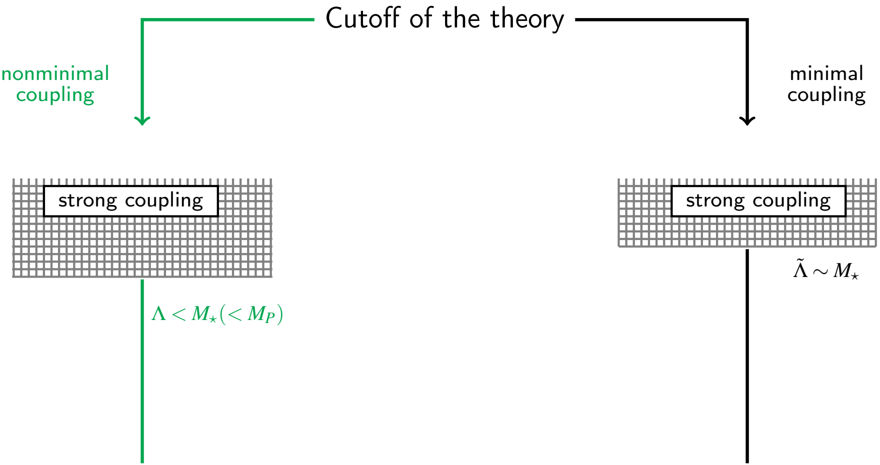


$$\text{CMB normalization} \quad \mapsto \quad \xi \simeq 10^4$$

$$n_s = 0.0966 \;, \quad r = 0.0034$$

Self-consistency

SM+gravity is nonrenormalizable EFT



$$M_\star = M_P (\#_{\text{species}})^{-1/2} < M_P \ , \quad \#_{\text{species}} \sim 10^2 \ .$$

Energies(phenomena) $\overset{?}{<} \Lambda$ decisive factor for self-consistency

The crux of it all

All information about the cutoff Λ is built-in the theory itself

The "strategy"= perform dimensional analysis[1]

Fix the Higgs background $\bar{h}$ corresponding to inflation (or whatever epoch one is interested in)

Study the behavior of perturbations on top of it

Non-renormalizability $\leftrightarrow$ higher-dimensional operators for the perturbations

Identify the scale suppressing the leading operators with (the lower bound of) the cutoff

[1] Or, compute amplitudes explicitly

The crux of it all

Take the small field limit $\xi h^2 \ll M_P^2$, then in all sectors (scalar, gauge,...) the leading higher-dimensional operators

$$\frac{\mathscr{O}_{(n)}}{(M_P/\xi)^{n-4}}$$

are suppressed by a scale $\ll M_\star$. A problem (?) if the cutoff is M_P/ξ, since during inflation

$$H_{\text{inf}} \sim \frac{M_P}{\xi}$$

The crux of it all

Take the opposite large field limit $\xi h^2 \gg M_P^2$, then in the scalar sector the leading higher-dimensional operators

$$\frac{\mathscr{O}_{(n)}}{M_P^{n-4}}$$

are suppressed by M_P. Not a problem (!) if the cutoff is M_P, since during inflation,

$$H_{\text{inf}} \sim \frac{M_P}{\xi} \ll M_P$$

Too naive: the innocent-looking nonminimal coupling to the scalar curvature is a radical modification to the heart of the SM dynamics

The crux of it all

Unitary gauge $\leftrightarrow$ Single-field $\leftrightarrow$ scalar sector borderline "trivial"

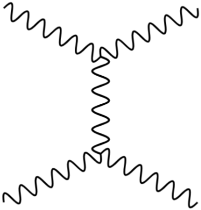
May give the (wrong) impression that issues completely disappear

Not the case: we need to pinpoint the location of the problem

The crux of it all

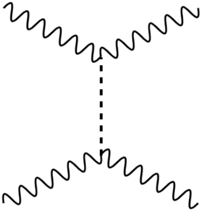
Forget for the moment about (Higgs) inflation and focus on vanilla SM
Focus on the high-energy limit of $2 \to 2$ scattering of longitudinal components of massive vectors

$\mathrm{tr}(F^2) \supset$



$$+\ldots \;\sim -g^2\left(\frac{E}{m_V}\right)^2 + \mathscr{O}(E^0)\,, \quad m_V = g\langle \mathrm{Higgs}\rangle$$

$g\, m_V\, h\, \mathrm{tr}(V^2) \supset$



$$+\ldots \;\sim +g^2\left(\frac{E}{m_V}\right)^2 + \mathscr{O}(E^0)$$

Delicate cancellation possible iff $\mathscr{L}_{\mathrm{cubic}} \propto m_V\, h\, \mathrm{tr}(V^2)$

The crux of it all
Back to Higgs inflation

Coupling the field nonminimally to gravity has a twofold effect:

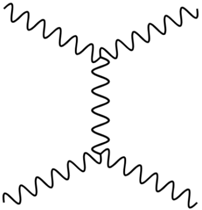
1. Gauge bosons V acquire effective masses $m_V \propto M_P/\sqrt{\xi}$
2. Higgs interaction with the vectors not proportional to m_V anymore, but exponentially suppressed $\mapsto$ practically decoupled

At the same time, vector kinetic & self-interaction terms as in usual SM

This partial modification of the gauge dynamics is the problem

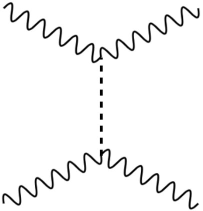
The crux of it all
Back to Higgs inflation

$$\mathrm{tr}(F^2) \supset$$



$$+\dots \sim -g^2\left(\frac{E}{m_V}\right)^2 + \mathcal{O}(E^0)\,, \quad m_V = g\,\frac{M_P}{\sqrt{\xi}}$$

$$g\,\mathrm{e}^{-\#}\,h\,\mathrm{tr}(V^2) \supset$$



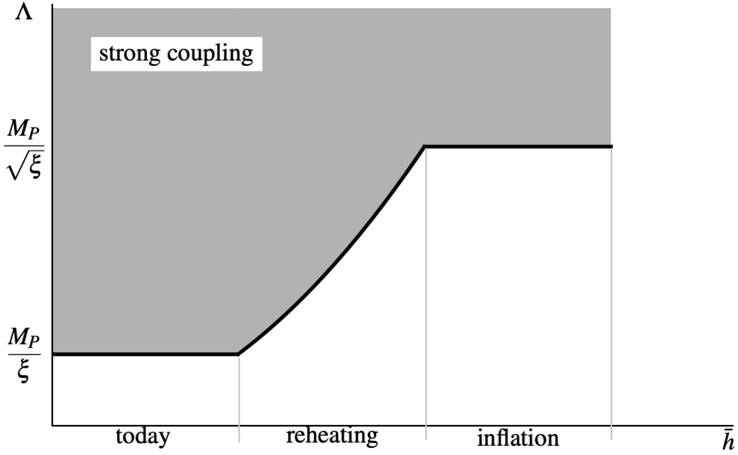
$$+\dots \sim \mathrm{e}^{-\#}\left(g^2\left(\frac{E}{m_V}\right)^2 + \mathcal{O}(E^0)\right)$$

Delicate cancellation not possible, so

$$\Lambda_{\text{inflation}} \sim m_V \sim \frac{M_P}{\sqrt{\xi}}$$

The big picture

The cutoff is background-dependent



What if we did not use the unitary gauge?

Presence of would-be Nambu-Goldstone (NG) modes $\leftrightarrow$ scalar sector "not trivial"

Can something really change? Remember: NG's are the longitudinal components of the vectors!

Nevertheless, a poor choice of variables results into false appearances...

An interlude: toy model

Take two scalars φ_1 & φ_2 in flat 4-dimensional spacetime

$$S = -\frac{1}{2}\int d^4x \left[(\partial_\mu\varphi_1)^2 + (\partial_\mu\varphi_2)^2 + \frac{2\varphi_2}{\tilde{\Lambda}}\partial_\mu\varphi_1\partial^\mu\varphi_2 + \frac{c}{\tilde{\Lambda}^2}\varphi_2^2\partial_\mu\varphi_2\partial^\mu\varphi_2\right] ,$$

with $[\tilde{\Lambda}] = M$, $c = \text{const.}$

At first sight:

- two fields interacting nontrivially via derivative mixings
- $\tilde{\Lambda}$ is the cutoff

At second thought, appearances can be deceiving:

- two massless fields completely decoupled
- $\tilde{\Lambda}$ is spurious

Trivial to complete the square and rewrite the action as

$$S = -\frac{1}{2}\int d^4x \left[\left(\partial_\mu \varphi_1 + \frac{\varphi_2}{\tilde{\Lambda}} \partial_\mu \varphi_2 \right)^2 + \left(1 + (c-1)\frac{\varphi_2^2}{\tilde{\Lambda}^2} \right) (\partial_\mu \varphi_2)^2 \right] \ .$$

Introduce χ_1 and χ_2,

$$\chi_1 = \varphi_1 + \frac{\varphi_2^2}{2\tilde{\Lambda}} \ , \quad \chi_2 = \int^{\varphi_2} d\varphi \sqrt{1 + (c-1)\frac{\varphi^2}{\tilde{\Lambda}^2}}$$

such that the true nature of the toy model is revealed

$$S = -\frac{1}{2}\int d^4x \left[(\partial_\mu \chi_1)^2 + (\partial_\mu \chi_2)^2 \right] \ .$$

Cartesian parametrization

The starting point

Focus on the purely kinetic sector for the Higgs doublet H - in the Einstein frame this reads

$$S = -\int d^4x \frac{1}{\Omega^2}\left[\partial_\mu H^\dagger \partial^\mu H + \frac{3\xi^2}{M_P^2\Omega^2}\partial_\mu (H^\dagger H)\partial^\mu (H^\dagger H)\right] ,$$

with

$$\Omega^2 = 1 + \frac{2\xi H^\dagger H}{M_P^2} .$$

As I said before, coupling nonminimally to gravity constitutes a radical modification of the dynamics, even if the flat limit is taken...

The methodology

- Write the Higgs doublet in its Cartesian form

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_1 + i\pi_2 \\ \bar{h} + h + i\pi_3 \end{pmatrix} ,$$

  where $\bar{h} = \text{background}$, $h = \text{physical Higgs},$ $\pi_a =$ would-be NG modes

- Plug H into the action and expand in h and π's.
- Keep terms at most quartic in the perturbations
- Normalize canonically the kinetic terms by trivial rescalings

$$h = f(\bar{h})\chi \ , \quad \pi_a = g(\bar{h})\sigma_a$$

- Take the limit $\bar{h} \gg M_P/\sqrt{\xi}$, relevant for inflation

A confusion (?)

Find the following (after some trivial massaging)

$$S \simeq -\frac{1}{2}\int d^4x \Bigg[(\partial_\mu\chi)^2 + (\partial_\mu\sigma_a)^2 + \frac{\xi}{M_P}\sqrt{6}\partial_\mu\chi\partial^\mu\sigma_a^2 + \frac{\xi^2}{M_P^2}\frac{3}{2}(\partial_\mu\sigma_a^2)^2 - \frac{\xi}{M_P^2}\left(2(\partial_\mu\chi)^2\sigma_a^2 + \frac{3}{2}\partial_\mu\chi^2\partial^\mu\sigma_a^2 + \sigma_a^2(\partial_\mu\sigma_b)^2\right) + \mathscr{O}\left(\frac{1}{M_P}\right)\Bigg] .$$

Notice that M_P/ξ appears explicitly in the action... Since that's the smallest scale, dimensional analysis dictates that this should be the cutoff...

What is going on?!

No confusion, after all...

Just like in the toy model, it's trivial to complete the square

$$S \simeq -\frac{1}{2}\int d^4x \Bigg[\left(\partial_\mu \chi + \sqrt{\frac{3}{2}}\frac{\xi}{M_P}\partial_\mu \sigma_a^2\right)^2 + (\partial_\mu \sigma_a)^2 - \frac{\xi}{M_P^2}\left(2(\partial_\mu \chi)^2\sigma_a^2 + \frac{3}{2}\partial_\mu \chi^2 \partial^\mu \sigma_a^2 + \sigma_a^2(\partial_\mu \sigma_b)^2\right) + \mathscr{O}(M_P^{-1}) \Bigg] .$$

Introduce

$$\rho = \chi + \sqrt{\frac{3}{2}}\frac{\xi}{M_P}\sigma_a^2 ,$$

in terms of which the action becomes

$$S \simeq -\frac{1}{2}\int d^4x \left[(\partial_\mu \rho)^2 + (\partial_\mu \sigma_a)^2 + \frac{\xi}{M_P^2}\left(\sigma_a^2 \rho \Box \rho + \frac{1}{4}(\partial_\mu \sigma_a^2)^2\right) + \mathscr{O}(M_P^{-1}) \right] .$$

The fictitious (significantly lower) scale M_P/ξ disappears, as it should!

No confusion, after all...

It's important to make sure to eliminate redundancies

One may of course use the redundant form of the action and compute scattering amplitudes for processes involving the χ and π_a's.

At order M_P/ξ these must vanish

At order M_P/ξ these indeed vanish, as explicitly showed in a number of works

We can do a better choice of variables

Angular (exponential) parametrization [2]

[2] Other parametrizations for the Higgs doublet are of course possible

The starting point

Take again

$$S = -\int d^4x \frac{1}{\Omega^2}\left[\partial_\mu H^\dagger \partial^\mu H + \frac{3\xi^2}{M_P^2\Omega^2}\partial_\mu (H^\dagger H)\partial^\mu (H^\dagger H)\right]$$

with

$$\Omega^2 = 1 + \frac{2\xi H^\dagger H}{M_P^2}$$

- Now write the Higgs doublet in its angular (exponential) form

$$H = \frac{1}{\sqrt{2}}(\bar{h}+h)\mathrm{e}^{i\frac{\pi_a\tau_a}{\bar{h}}}\begin{pmatrix}0\\1\end{pmatrix}$$

$\bar{h} = \text{background}$, $h = \text{physical Higgs}$, $\pi_a =$ would-be NG modes , $\tau_a =$ Pauli matrices

The methodology
(same as in Cartesian analysis)

- Plug H into the action and expand in h and π's.

- Keep terms at most quartic in the perturbations

- Normalize canonically the kinetic terms by trivial rescalings (same as in the Cartesian case)

$$h = f(\bar{h})\chi \ , \quad \pi_a = g(\bar{h})\sigma_a$$

- Take the limit $\bar{h} \gg M_P/\sqrt{\xi}$, relevant for inflation

No confusion, to start with...

End up with

$$S \simeq -\frac{1}{2}\int d^4x \Bigg[(\partial_\mu \chi)^2 + (\partial_\mu \sigma_a)^2 \\ - \frac{\xi}{M_P^2}\left(\frac{1}{3}\sigma_a^2(\partial_\mu \sigma_b)^2 - \frac{1}{12}(\partial_\mu \sigma_a^2)^2\right) + \mathscr{O}(M_P^{-1})\Bigg] \,.$$

Notice the presence of the true cutoff scale $M_P/\sqrt{\xi}$, without the need for field redefinitions...

What if we “change” gravity?

e.g. Palatini formulation

Same logic, only now the dimension-six operator

$$\frac{3\xi^2}{M_P^2\Omega^2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H)\ ,$$

is absent—in the Palatini formulation the scalar curvature is Weyl-inert

Scalar sector much simpler (as compared to the metric case)

$$S = -\int d^4x \frac{1}{\Omega^2}\partial_\mu H^\dagger \partial^\mu H$$

Can this affect self-consistency as far as inflation is concerned?

e.g. Palatini formulation

Answer to this question is well (?) known and negative

Gauge bosons still get effective masses $\propto M_P/\sqrt{\xi}$

Higgs excitations still suppressed heavily, although not so much as in the metric formulation

(Higgs) mechanism underlying the cancellations of the diverging parts of amplitudes again nullified

Conclusions/Take-home messages

- Higgs inflation is a self-consistent EFT
- Be careful to understand what you're doing if nontrivially modifying the SM
- Depending on the choice of gauge, one needs to look at different sector(s)
- Be careful to choose suitable variables to study the problem
- Be careful to rid of artifacts/redundancies when choosing not so suitable variables

Thank you!

Backup

Weyl transformation

Rescale the metric as (Weyl transformation)

$$g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu} \ .$$

Since $g = \det(-g_{\mu\nu})$, then

$$\sqrt{g} \to \Omega^{-4} \sqrt{g} \ .$$

At the same time,

$$R \to \Omega^2 \left(R + 6\Omega^{-1} \Box \Omega - 12 \Omega^{-2} (\partial_\mu \Omega)^2 \right) \ .$$

de Sitter in disguise

Take the following action

$$S = \int d^4x\sqrt{g}\left[\frac{\zeta\phi^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\kappa}{4}\phi^4\right] ,$$

Weyl transform with conformal factor

$$\Omega^2 = \frac{\zeta\phi^2}{M_P^2} .$$

Obtain

$$S = \int d^4x\sqrt{g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\frac{M_P^2(1+6\zeta)}{\zeta}\frac{(\partial_\mu\phi)^2}{\phi^2} - \frac{\kappa M_P^4}{4\zeta^2}\right] ,$$

i.e. a massless scalar minimally coupled to gravity
$\zeta = -1/6 \mapsto$ Weyl invariant action $\mapsto$ ϕ artifact.

The contributions from the potential

Start from the potential that in the inflationary regime reads

$$U(H) \simeq \frac{\lambda (H^\dagger H)^2}{\Omega^4} \ .$$

Take the Higgs in the unitary gauge

$$H = \frac{h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ ,$$

plug into the above, and end up with

$$U(H) \simeq \frac{\lambda M_P^4}{4\xi^2} \left[1 + \frac{M_P^2}{\xi \bar{h}^2} \sum_{n=1} c_n \left(\frac{h}{\bar{h}} \right)^n \right] \simeq \frac{\lambda M_P^4}{4\xi^2} \left[1 + \frac{M_P^2}{\xi \bar{h}^2} \sum_{n=1} \tilde{c}_n \left(\frac{\chi}{M_P} \right)^n \right] \ ,$$

with c_n , $\tilde{c}_n$ numerical factors $\mathcal{O}(1)$.
Exponentially suppressed contributions, aftermath of the exponential map between h and the canonically normalized χ.

Kinetic sector in a sigma-model form

In case it's not so trivial to "complete the square":

1. Write the action as a sigma-model

$$S = -\frac{1}{2}\int d^4x\, G_{IJ}(\varphi)\partial_\mu\Phi_I\partial^\mu\Phi_J \ ,$$

   where $\Phi_I = (\varphi_1, \varphi_2, \ldots, \varphi_N)$, $I, J = 1, \ldots, N$.

2. Compute the Riemann tensor of the target manifold

$$\begin{aligned} \kappa^I{}_{JKL} &= \partial_K\gamma^I_{LJ} - \partial_L\gamma^I_{KJ} + \gamma^I_{KM}\gamma^M_{LJ} - \gamma^I_{LM}\gamma^M_{KJ} \ , \\ \gamma^I_{MN} &= \tfrac{1}{2}G^{IJ}\left(\partial_M G_{JN} + \partial_N G_{MJ} - \partial_J G_{MN}\right) \ . \end{aligned}$$

3. If $\kappa^I{}_{JKL} = 0$, then $\exists\ \chi_I(\varphi)$ such that

$$S = -\frac{1}{2}\int d^4x\ \partial_\mu X_I\partial^\mu X_I \ .$$