De Sitter Vacua in $\mathcal{N} = 8$ Supergravity

Gianluca Inverso

M.Sc. thesis with supervisor Dr. Gianguido Dall'Agata

IMPRS EPP Workshop, Munich

19 July, 2010

Motivation

We use a new approach to search for (stable) de Sitter vacua in $D = 4$ maximal supergravity

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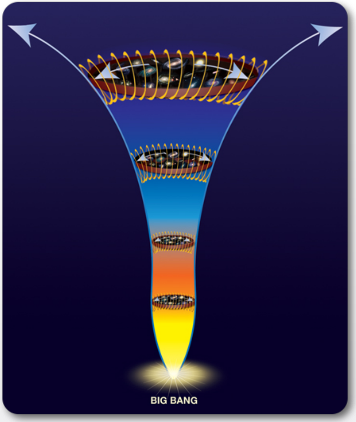

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- Dark Energy
  Accelerated expansion of the universe at the present epoch
  - candidate:
    cosmological constant
    $$\Lambda_{cosm} > 0$$
    $\hookrightarrow$ *de Sitter spacetime*

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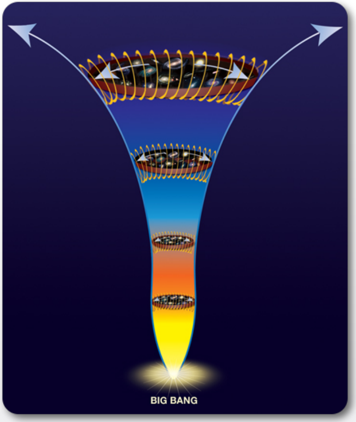

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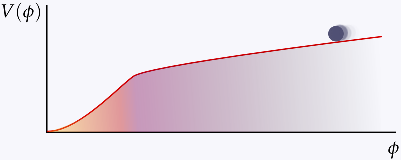
    cosmological constant
    $$\Lambda_{cosm} > 0$$
    $\hookrightarrow$ *de Sitter spacetime*
- Primordial inflation
  - *Slow roll inflation*

De Sitter spacetime

Vacuum solution of Einstein equations for $\Lambda_{cosm} > 0$

$$ds^2 = -dt^2 + e^{Ht} dl^2, \quad H^2 = \Lambda_{cosm}/3.$$

Slow roll inflation



$$\left| V'(\phi)/V(\phi) \right| \ll 1$$

$$\left| V''(\phi)/V(\phi) \right| \ll 1$$

Supergravity (SUGRA)

Supersimmetry (SUSY) + gravity (GR)
$\Rightarrow$ every particle has *superpartners*:

$$\text{bosons} \overset{\text{SUSY}}{\longleftrightarrow} \text{fermions}$$

- SUGRA models descend from String Theory
  - low energy limit
  - compactifications of the extra dimensions
- de Sitter vacua in SUGRA $\longrightarrow$ test for ST models
- better theoretical framework for Inflation and Dark Energy

$\Lambda_{cosm} > 0$ in Supergravity

SUGRA $\supset$ RG $\Rightarrow$ *de Sitter vacua in supergravity*

$\Lambda_{cosm} > 0$ in supergravity *only if* there is a scalar potential $V(\phi)$

stationary points $\longrightarrow$ vacuum solutions

$$V(\phi) = V(\phi_0) + \tfrac{1}{2} V''|_{\phi_0} (\phi - \phi_0)^2 + \ldots$$

$$V'|_{\phi_0} = 0, \quad V(\phi_0) \equiv \Lambda_{cosm}$$

difficulty: stable de Sitter vacua in extended supergravity

$\mathcal{N} = 8$ Supergravity

We look for de Sitter vacua of maximal SUGRA in $D = 4$ ($\mathcal{N} = 8$ supercharges Q^i).

- origin in terms of String Theory is well understood (at least for some models[1])
- matter content is fixed, although not realistic
  - only freedom: choice of G_{gauge}
  - 70 real scalar fields ϕ^{ijkl}
- SUSY $\Rightarrow$ potential $V(\phi)$ only depends on G_{gauge}
- good testing ground for our method

[1] D'Auria, Ferrara, Trigiante, JHEP, 01:081, 2006.

Our method

We considered *gauged* maximal supergravity in 4 dimensions:
Einstein–Hilbert + Yang–Mills

- gauge covariant derivatives: $\partial_\mu \to D_\mu = \partial_\mu + gA_\mu$
- add mass-like terms in δ_{SUSY} and in $\mathcal{L}$, $\mathcal{O}(g)$
- SUSY $\Rightarrow V(\phi)$, $\mathcal{O}(g^2)$

Explicit calculations for $G_{gauge} = (\mathrm{C})\mathrm{SO}(p,q,r)$, $\quad (p+q+r=8)$

$$U \in G_{gauge} \;\Leftrightarrow\; U\theta U^T = \theta, \quad \theta = \begin{pmatrix} \mathbb{1}_{p\times p} & & \\ & -\mathbb{1}_{q\times q} & \\ & & \mathbb{O}_{r\times r} \end{pmatrix}$$

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The potential is completely parametrized by θ: $V(\phi, \theta)$

Scalar fields ϕ are coordinates on the manifold $\mathcal{M}_\phi = \mathrm{E}_7/\mathrm{SU}(8)$
Isometry transformations on $\mathcal{M}_\phi$ can also act on θ:

$$\widetilde{\phi} = U\phi \quad \longleftrightarrow \quad \widetilde{\theta} = U^{-1}\theta \qquad U \in ISO(\mathcal{M}_\phi)$$

The potential is invariant!

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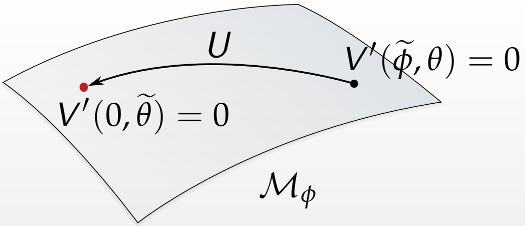
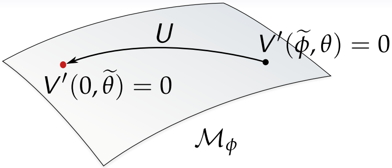


$\Rightarrow$ we look for stationary points at $\phi = 0$ and modify θ



$$V(0,\widetilde{\theta}) \;=\; V(\widetilde{\phi},\theta)$$

We look for stationary points at $\phi = 0$ and modify θ

Can we find vacua $\forall\,\phi$?

YES: $\mathcal{M}_\phi$ is a homogeneous space*

Advantages:

- calculate the derivatives of $V(\phi)$ only once
- obtain all 70 mass^2 eigenstates and eigenvalues
  $\hookrightarrow$ vacuum stability
- no numerical approximations

*however: assumption on gauge connection not containing dual fields $\rightarrow\ \phi \in \mathbb{R}$

First derivative

$$V(\phi) = g^2 \left(\frac{1}{24} A_{2\,i}{}^{jkl} A_2{}^i{}_{jkl} - \frac{3}{4} A_1^{ij} A_{1\,ij} \right)$$

Shifts in δ_{SUSY}

$$V'(\phi) \propto \Omega^{ijkl}|_{selfdual}$$

$$\Omega^{ijmn} \equiv \frac{3}{4} A_{2\,l}{}^{u[ij} A_{2\,u}{}^{mn]l} - A_1{}^{l[i} A_{2\,l}{}^{jmn]}$$

After some algebra:

$$\Omega^{ijkl}|_{\phi=0} = \frac{1}{16} \mathrm{Tr}(\overbrace{\Gamma^{ijkl}}^{Cliff(8)} \theta^2) - \frac{1}{32} \mathrm{Tr}(\theta)\, \mathrm{Tr}(\Gamma^{ijkl}\theta)$$

$\Rightarrow$ vacuum condition: $\Omega^{ijkl}|_{\phi=0} = 0$

Second derivative

$$V''(\phi_0) \ \rightarrow \ \text{effective masses for} \ \ \Delta\phi^{ijkl} = \phi^{ijkl} - \phi_0^{ijkl}$$

$$V''|_{\phi=0} \propto M_{ijkl}{}^{mnpq} = \left(\frac{1}{8}\,\mathrm{Tr}(\theta^2) + \frac{1}{16}(\mathrm{Tr}\,\theta)^2\right)\delta^{mnpq}_{i\,j\,k\,l}$$
$$- \frac{2}{3}A_2{}^{[m}{}_{[ijk}A_{2\,l]}{}^{npq]} + \frac{3}{16}\delta_{[ij}{}^{[mn}\,\mathrm{Tr}\left(\Gamma^{pq]}\theta\Gamma_{kl]}\theta\right)$$
$$- \frac{3}{4}\delta_{[ij}{}^{[mn}\,\mathrm{Tr}\left(\Gamma_k{}^{p}\theta\Gamma^{q]}{}_{l]}\theta\right)$$

Results

Vacuum condition $V'|_{\phi=0} = 0 \longrightarrow$ equation for θ:

$$2\,\theta^2\big|_{traceless} = \mathrm{Tr}\,\theta\;\theta\big|_{traceless}$$

Value of the potential at the vacuum:

$$V(\phi = 0) = \frac{1}{4}\,\mathrm{Tr}\,\theta^2 - \frac{1}{8}(\mathrm{Tr}\,\theta)^2$$

Invariant under GL(8): $\theta \to G\theta G^{-1}$, $G \in GL(8,\mathbb{R})$
$\Rightarrow$ condition on the eigenvalues of θ

$$V'|_{\phi=0} = 0 \quad \Leftrightarrow \quad 2\,\theta^2|_{traceless} = \mathrm{Tr}\,\theta\;\theta|_{traceless}$$

5 diagonal solutions (up to an overall factor)

G_{gauge}	eigenvalues(θ)	$\Lambda_{cosm} = V(0)$	type of vacuum
SO(8)	(1, 1, 1, 1, 1, 1, 1, 1)	< 0	anti de Sitter
	(5, 1, 1, 1, 1, 1, 1, 1)	< 0	anti de Sitter
SO(3,5)	(-3, -3, -3, 1, 1, 1, 1, 1)	> 0	de Sitter
SO(4,4)	(-1, -1, -1, -1, 1, 1, 1, 1)	> 0	de Sitter
CSO(2,0,6)	(1, 1, 0, 0, 0, 0, 0, 0)	$= 0$	Minkowski

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2 de Sitter vacua + 1 Minkowski

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θ in a non-standard form $\Rightarrow$ corresponds to $\phi \neq 0$

these vacua were already known for $\phi \neq 0$ $\Rightarrow$ our method works!

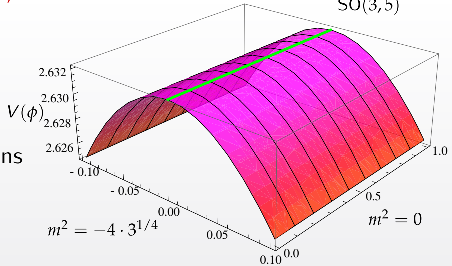
C. M. Hull and N. P. Warner, Nucl. Phys., B253:675, 1985
C. M. Hull, Class. Quant. Grav., 2:343, 1985

Vacuum properties

G_{gauge}	Λ_{cosm}	m^2 [multiplicity]
$SO(3,5)$	$2 \times 3^{1/4}$	0 [15], $8 \cdot 3^{1/4}$ [5], $4 \cdot 3^{1/4}$ [30], $8 \cdot 3^{-3/4}$ [14], $-4 \cdot 3^{-3/4}$ [5], $-4 \cdot 3^{1/4}$ [1]
$SO(4,4)$	2	0 [16], 2 [16], 4[36], -4 [2]
$CSO(2,0,6)$	0	0 [48], $1/2$ [20], 2 [2]

$SO(3,5)$ and $SO(4,4)$ (de Sitter)

- *unstable*
- $|V''|/V \approx 1$
  $\Rightarrow$ ~~slow roll inflation~~
- flat directions $\to$ Goldstone bosons
  $SO(p,q) \to SO(p) \times SO(q)$



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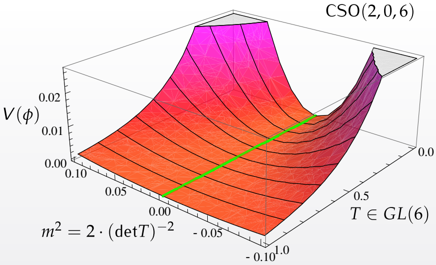
$CSO(2,0,6)$ (Minkowski)

- *stable* ($\mathcal{N} = 2$)
- 12 Goldstone bosons
  $CSO(2,0,6) \to SO(2) \times SO(6)$
- other 36 flat directions
  $T \in GL(6)$
  $\theta \to (\det T)^{-1} \theta$



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Conclusions

- We reproduced known vacua of gauged $\mathcal{N}=8$ supergravity $G_{gauge} = (\mathrm{C})\mathrm{SO}(p,q,r)$.
- We provided the whole mass spectra for the first time
  - de Sitter vacua are unstable, no slow roll
  - the Minkowski vacuum is stable
- We can reproduce vacuum solutions for $\phi \neq 0$ just by modifying the gauge group embedding
  - $\theta \to \widetilde{\theta} = U^{-1}\theta, \quad U \in ISO(\mathcal{M}_\phi)$
- The general method holds $\forall\ G_{gauge},\ \mathcal{N} \neq 8$
  - instead of θ: ${\Theta_M}^\alpha$, *embedding tensor*
  - generic vacuum conditions
  - classification of the vacua