Aspects of flux compactifications in string theory

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String theory

String theory is a quantum theory of gravity.

Quantisation of a vibrating string.

Features:

- Infinite set of different particle states
- In particular: graviton states
- Extra dimensions
- Supersymmetry

Flux compactifications

Compactification is compulsory.

- Compactifications without flux:
  - $\mathcal{M}_{1,9} = \mathcal{M}_{1,3} \times \mathcal{M}_6$
  - Typically: $\mathcal{M}_6$ is a Calabi-Yau
  - But: too much supersymmetry and a set of massless fields (moduli)
- Flux compactifications:
  - Warped geometries
  - Moduli stabilisation
  - Supersymmetry breaking

Supergravity

Investigations of flux compactifications are conveniently pursued in the context of supergravity.

- Theory of supersymmetric general relativity
- But also: low energy effective theory of string theory
- Graviton, Dilaton, B-field, R-R fields
- Maximal supersymmetry in $d = 10$: types IIA and IIB

Supersymmetric supergravity backgrounds

The main task is to solve the supersymmetry equations.

- Supergravity background = solution to the field equations
- Investigate $\mathcal{M}_{1,9} = \mathbb{R}^{1,5} \times_\omega \mathcal{M}_4$:
  - Complete solutions available
  - Discover general features by contrasting with usual $4+6$ split
- Strategy:
  - Supersymmetry equations + Bianchi identities $\Rightarrow$ equations of motion
  - Fermionic fields $= 0$
  - $\delta(\text{gravitino}) = 0$ and $\delta(\text{dilatino}) = 0$

Supersymmetry equations

A beauty in its own

$$0 = \delta_\varepsilon \Psi_\mu = \left(\nabla_\mu + \frac{1}{8} \mathcal{P} \not{H}_\mu \right) \varepsilon + \frac{1}{16} e^\Phi \sum_n \frac{1}{(2n)!} \not{F}^{(2n)} \Gamma_\mu \mathcal{P}_n \varepsilon$$

$$0 = \delta_\varepsilon \lambda = \left(\not{\partial} \Phi + \frac{1}{12} \not{H} \mathcal{P} \right) \varepsilon + \frac{1}{8} e^\Phi \sum_n (-1)^{2n} \frac{5 - 2n}{(2n)!} \not{F}^{(2n)} \mathcal{P}_n \varepsilon,$$

Solving

Everything has to be decomposed to the tiniest bits and pieces.

- Spinor ansatz with $\mathcal{N} = (1, 0)$ supersymmetry

$$\varepsilon_1 = a\zeta \otimes \eta + a\zeta_c \otimes \eta_c$$
$$\varepsilon_2 = b\zeta \otimes \eta + c\zeta \otimes \eta_c + b\zeta_c \otimes \eta_c - c^*\zeta_c \otimes \eta$$

- Fluxes decomposed as

$$e^\Phi \not{F} = 2f^{(1)}_m \Gamma^m + \frac{1}{3} f^{(3)}_q \varepsilon_{mnpq} \Gamma^{mnp}$$
$$\not{H} = \frac{1}{3!} \varepsilon_{mnpq} \Gamma^{mnp} h_q$$

in the type IIB case

Solutions

Few degrees of freedom remain.

- Geometry determined by the fluxes
  - Torsion classes
  - Warp factor
- Constraints on the fluxes

Generalised geometry

Differential geometry on $T\mathcal{M} \oplus T^*\mathcal{M}$

- Powerful mathematical description:

$$\text{Spinors of } T\mathcal{M} \oplus T^*\mathcal{M}$$

$$\updownarrow$$

$$\text{Polyform of } T^*\mathcal{M}$$

$$\updownarrow$$

$$\text{Bispinor of } S(\mathcal{M})$$

- Providing convenient formulations of
  - Supersymmetry equations
  - Effective actions including branes

Pure spinor equations

A completely equivalent formulation

- Pure spinor equations

$$\mathrm{d}_H\left(e^{4A-\Phi}\Psi_1\right)=0$$
$$\mathrm{d}_H\left(e^{6A-\Phi}\mathrm{Re}\Psi_2\right)=-\tfrac{1}{4}*\sigma(F)e^{6A}$$
$$\mathrm{d}_H\left(e^{4A-\Phi}\mathrm{Im}\Psi_2\right)=0$$

- Additional constraint

$$2\mathrm{d}A\wedge\mathrm{Re}\Psi_2=-\tfrac{1}{8}e^{\Phi}*\sigma(F)$$

Outlook

Learning from the toy model

- Find manifolds $\mathcal{M}_4$ in the set of group manifolds (e.g. solve- and nilmanifolds)
- Investigate also AdS solutions or even dS solutions including orientifolds
- Find a low energy effective action for the six-dimensional field theory
- Investigate the landscape (i.e. the effective potential)