The Strong CP Problem and the Axion Solution

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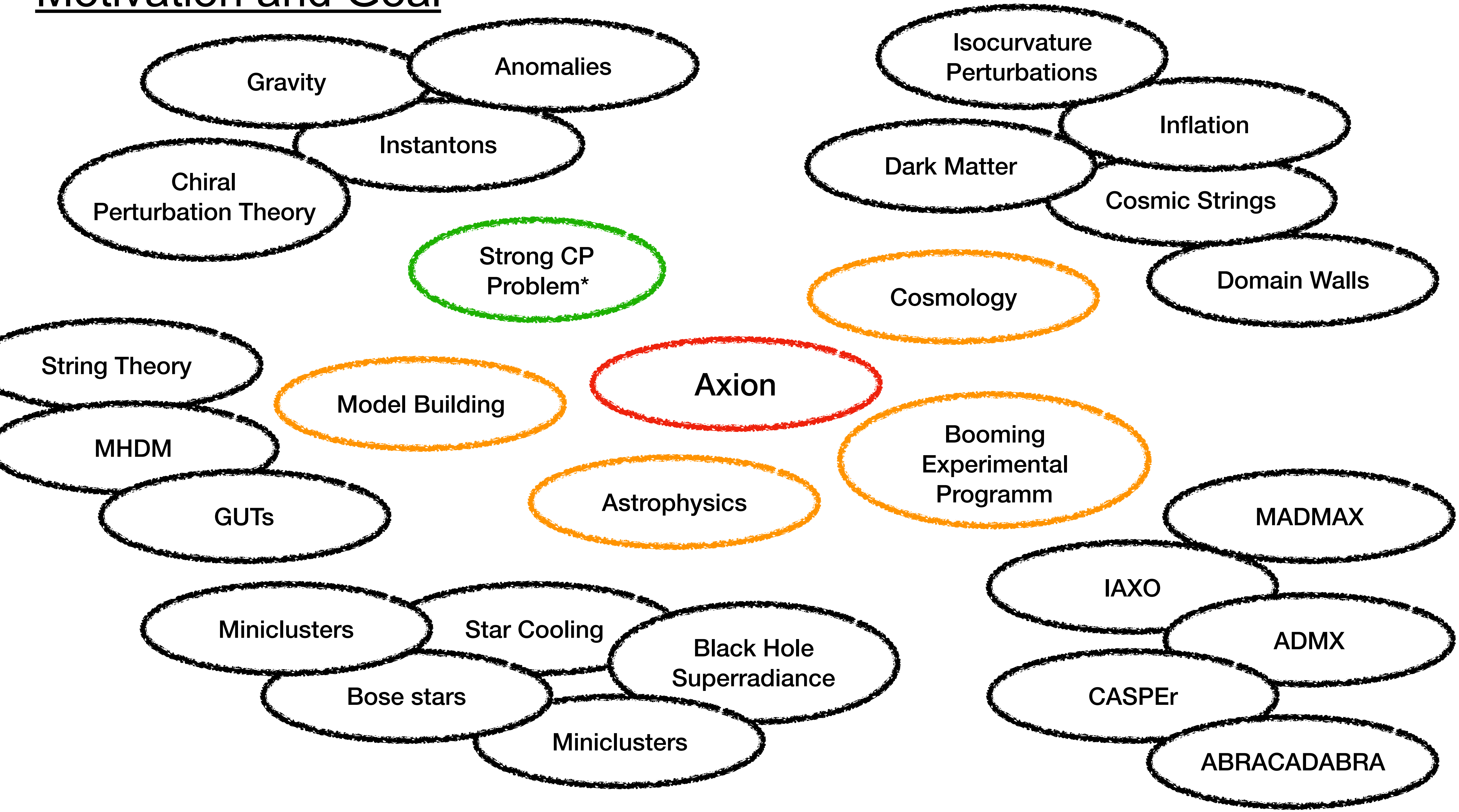
IMPRS Colloquium
20th October 2022

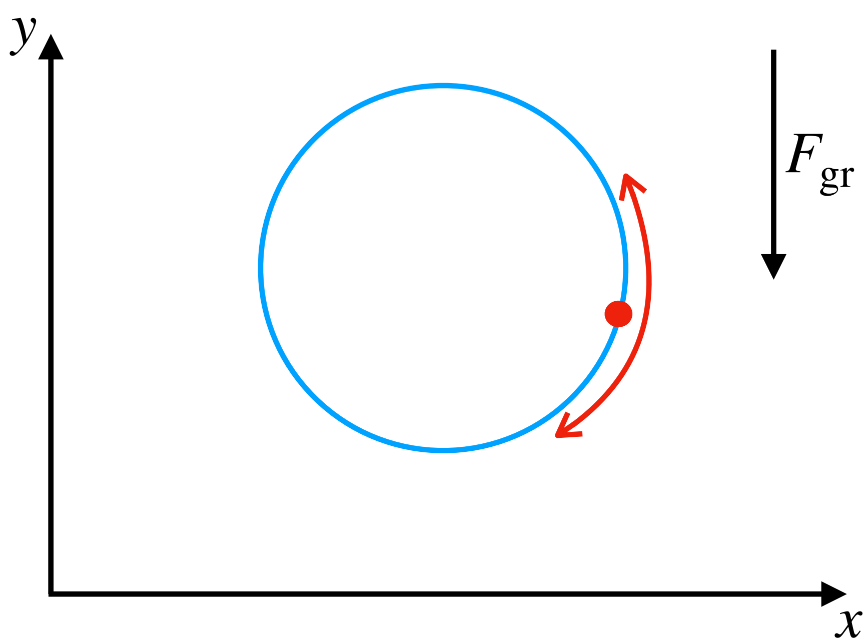
Motivation and Goal



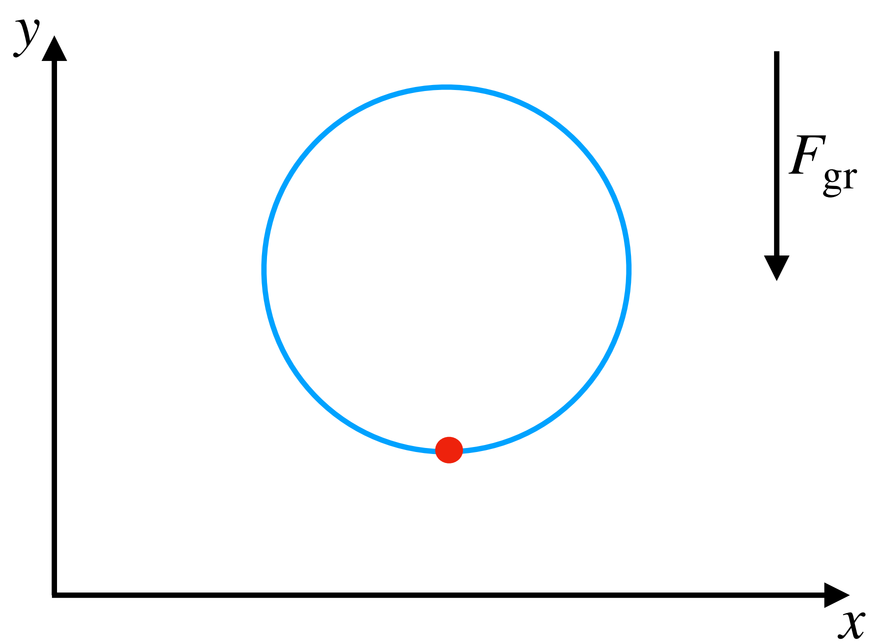
*One can give a whole course on strong CP and the axion solution, so goal is to give a non-technical overview

Theta Vacua - Introductory Example

System:



Classical Vacuum:



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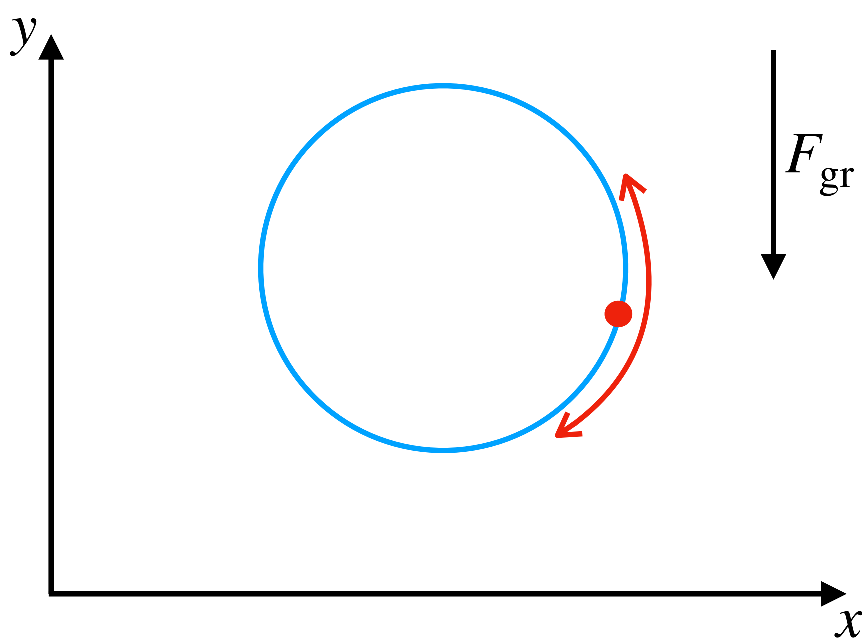

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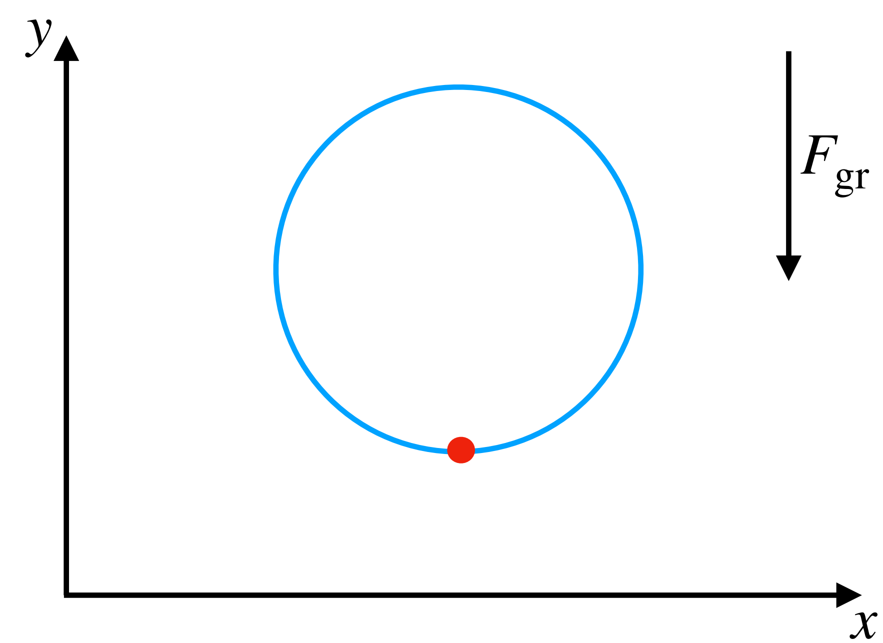
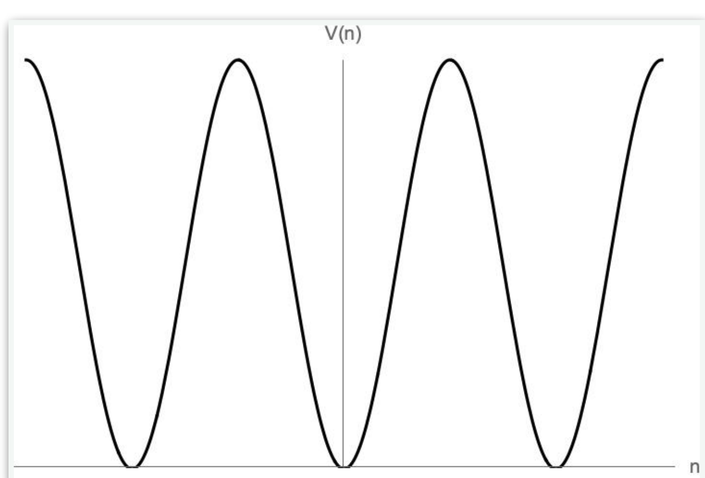


QM Vacuum:



$|\cdots\rangle \quad |1\rangle \quad |0\rangle \quad |1\rangle \quad |\cdots\rangle$

Periodic Bloch boundary condition

$$|n+1\rangle = \mathrm{e}^{i\theta}|n\rangle$$

$\xrightarrow{\text{superposition}}$

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} \mathrm{e}^{in\theta}|n\rangle \,, \qquad \theta = \text{const}\,.$$

Theta Vacua - QCD

QCD w/out quarks: $\mathscr{L} = -\frac{1}{4}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}$

Situation is similar to previous example! But compact direction “hidden” in gluon field space

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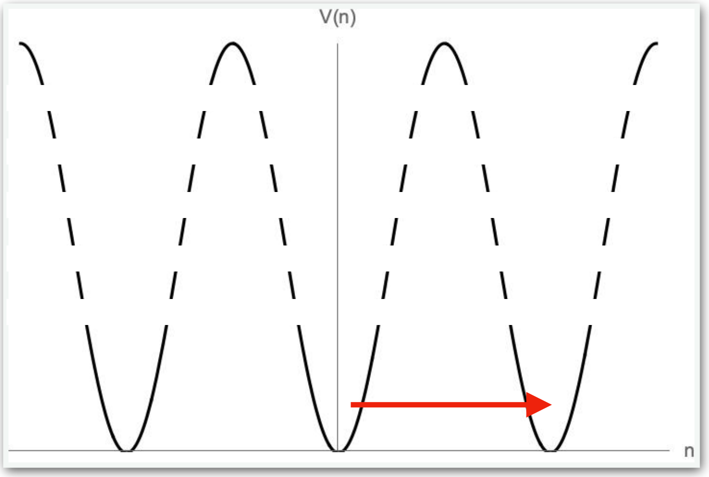

$|n_1\rangle$ $|n_2\rangle$ $|n_3\rangle$

instantons
= leading order field configuration
of tunneling process

$\xrightarrow{\text{superposition}}$

$$\boxed{|\theta\rangle = \sum_{n=-\infty}^{\infty} \mathrm{e}^{in\theta}|n\rangle}$$

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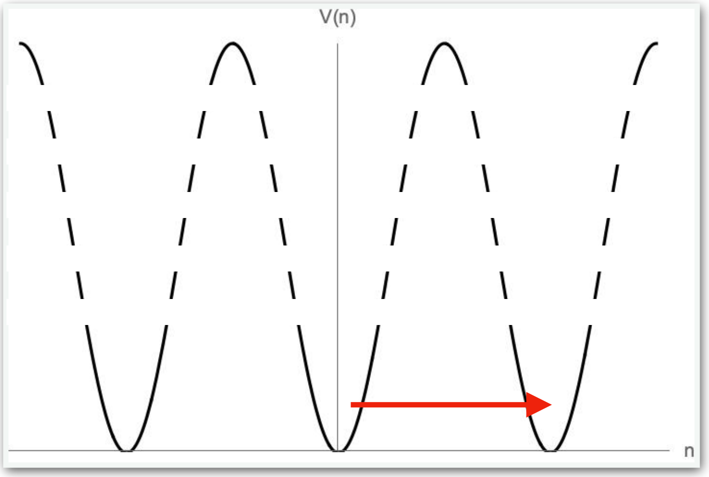

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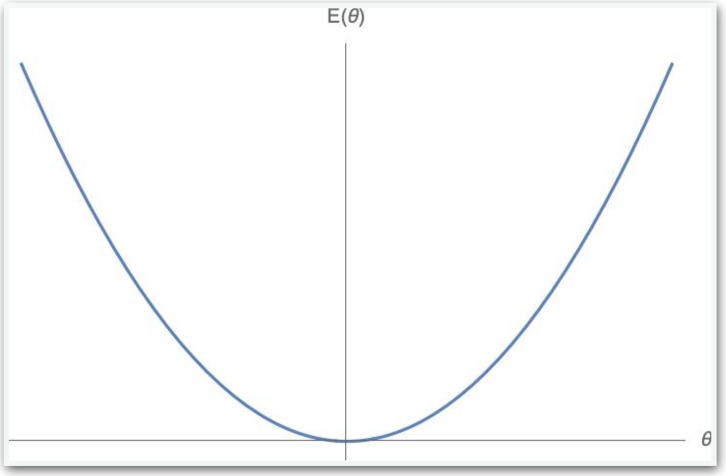
$$\boxed{|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta}|n\rangle}$$

1) different super selection sectors: $\langle\theta|\mathscr{O}|\theta'\rangle = 0$ if $\theta \neq \theta'$

2) Vacuum energy density associated with θ- vacua: $E(\theta) \sim \theta^2$



Strong CP Puzzle

Account for these tunneling field configurations in path integral

$$Z = \sum_{\Delta n} f(\Delta n) \int [DA] e^{iS[A]} \quad \xrightarrow{\text{S-matrix locality}} \quad \delta\mathscr{L}_\theta = \theta \frac{g^2}{32\pi^2} \underbrace{\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}}_{\text{CP violating}} \quad \text{with } \tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$$

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Is θ observable? Yes

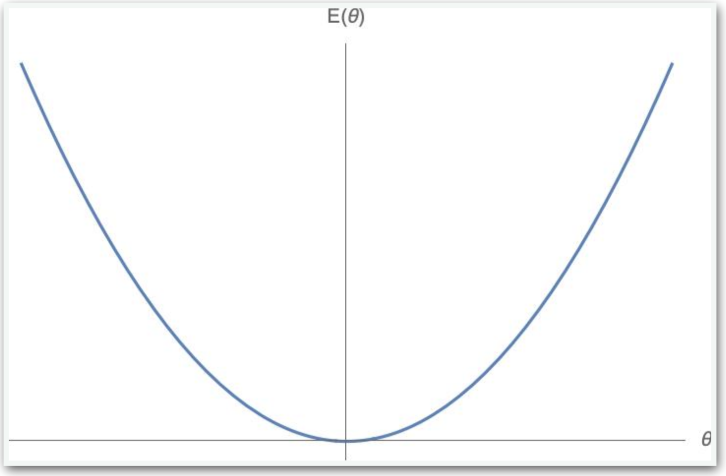
Contributes to electric dipole moment of e.g. neutron, but we do not measure CP violation!

$$\Longrightarrow \theta < 10^{-10} \text{ or so}$$

$\boxed{\text{Strong CP puzzle: Why is } \theta \text{ so small?}}$

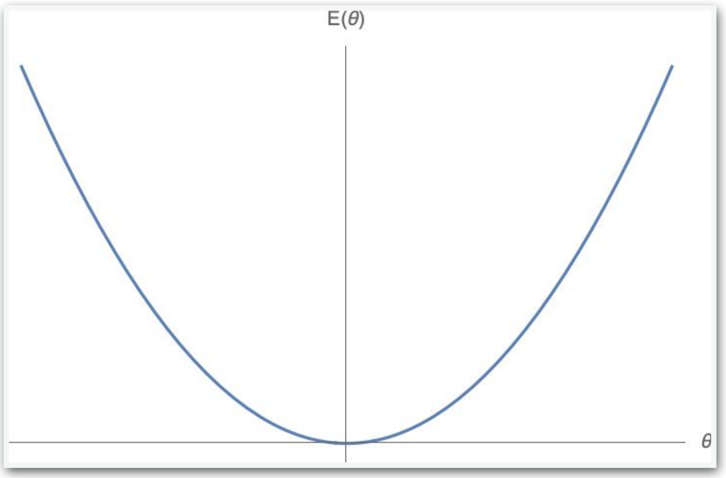
Not really a problem, quantum corrections to θ within SM are tiny (unlike e.g. Higgs mass)

Strong CP Problem



$\theta \neq 0 \quad \Longrightarrow$ positive vacuum energy
(positive cosmological constant; de Sitter)

Strong CP Problem



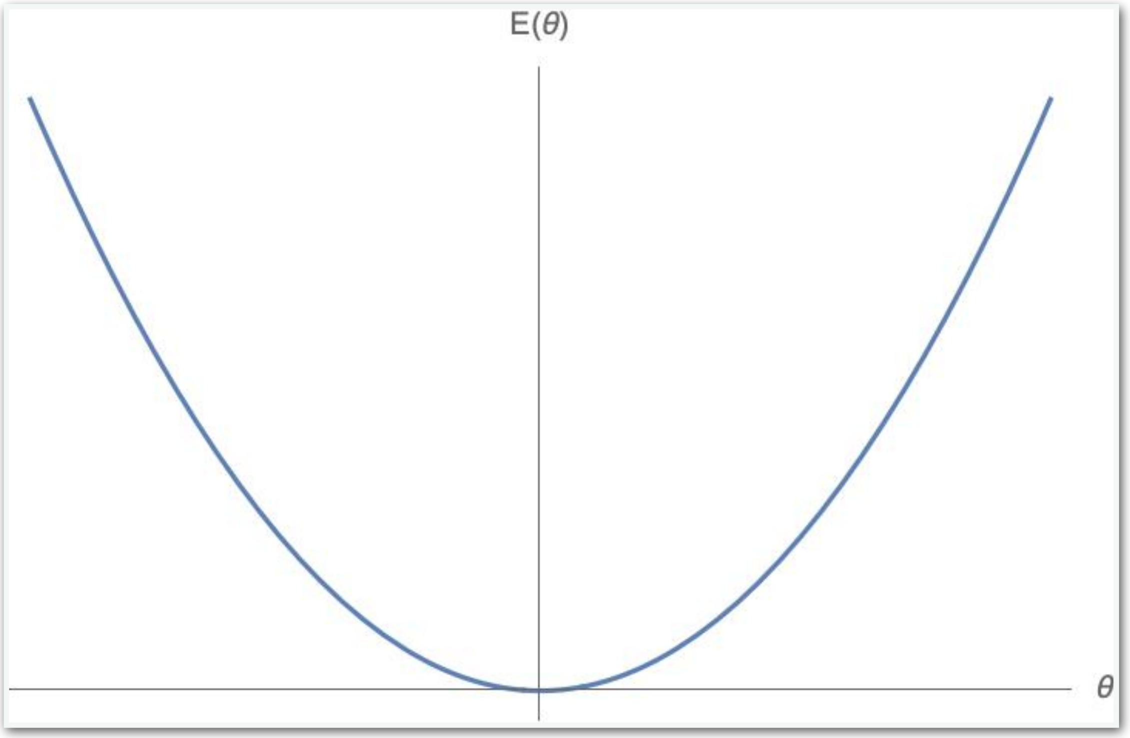
$\theta \neq 0 \implies$ positive vacuum energy
(positive cosmological constant; de Sitter)

However: There are arguments from quantum gravity that de Sitter is inconsistent
S-matrix arguments, Quantum breaking, Swampland conjecture, …

$$\boxed{\implies \theta \overset{!}{=} 0}$$

Mechanism that achieves this is necessary!

Idea



If θ dynamical, then vacuum at $\theta = 0$. But how to achieve this?

PQ Mechanism

Introduce: $\underbrace{\text{non-linearly realised}}_{\text{spontaneously broken}}\ \underbrace{\text{axial}}_{\text{treats left- and right handed fermions differently}}\ \text{U(1) symmetry that is}\ \underbrace{\text{anomalous w.r.t. QCD}}_{\text{explicitly broken by non-perturbative QCD effects}}$

$$\boxed{\text{Axion is the pseudo Goldstone-boson of this } U(1)_{PQ}}$$

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EFT: $$\mathscr{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \mathscr{L}(\partial_\mu a, \psi) + \underbrace{\left(\frac{a}{f_a} + \theta\right)\frac{g^2}{32\pi^2}\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}}\,, \qquad a \longrightarrow a + \kappa f_a$$

$\kappa =$ const

Low energy: $$V(a) = \Lambda^4_{\mathrm{QCD}}\left(1 - \cos\left(\frac{a}{f_a}\right)\right) = \frac{1}{2}\frac{\Lambda^4_{\mathrm{QCD}}}{f_a^2}a^2 + \cdots$$

Vacuum: $$\left\langle \frac{a}{f_a} \right\rangle \equiv \langle \theta(x) \rangle = 0$$

An Axion inside QCD?

Introduce: global non-linearly realised axial U(1) Symmetry that is anomalous w.r.t. QCD

- non-linearly realised: spontaneously broken
- axial: treats left- and right handed fermions differently
- anomalous w.r.t. QCD: explicitly broken by non-perturbative QCD effects

$\boxed{\eta' \text{ is the pseudo Goldstone-boson of this } U(1)_A}$

An Axion inside QCD?

Introduce: global $\underbrace{\text{non-linearly realised}}_{\text{spontaneously broken}}$ $\underbrace{\text{axial}}_{\text{treats left- and right handed fermions differently}}$ U(1) Symmetry that is $\underbrace{\text{anomalous w.r.t. QCD}}_{\text{explicitly broken by non-perturbative QCD effects}}$

η' is the pseudo Goldstone-boson of this U(1)$_A$

However: Additional (large) explicit breaking by quark masses

$$\mathscr{L}_{\text{QCD}} \supset \operatorname{Tr} \bar{q}_L M q_R$$

Vacuum: $\left\langle \frac{\eta'}{f_{\eta'}} \right\rangle \equiv \langle \theta(x) \rangle \neq 0$ reintroduces θ

If explicit breaking small then strong CP puzzle can still be solved.
But for strong CP problem no other explicit breaking is allowed!

Strong CP in Topological Higher-Form Language

Topological Susceptibility: $\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0} \equiv \lim_{q\to 0} \int d^4x \, e^{iqx} \langle T[G\tilde{G}(x) G\tilde{G}(0)]\rangle = \text{const} \neq 0$

Non-zero value means that θ is physical!

Strong CP in Topological Higher-Form Language

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Chern-Simon 3-Form: $C_{\mu\nu\rho} = \text{tr}\left[A_{[\mu}\partial_\nu A_{\rho]} - \frac{3}{2}A_{[\mu}A_\nu A_{\rho]}\right]$

Chern-Simon current: $\tilde{C}^\mu = \epsilon^{\mu\nu\rho\sigma}C_{\nu\rho\sigma}$

Pontryagin density: $E = \partial_\mu \tilde{C}^\mu = G_{\alpha\beta}\tilde{G}^{\alpha\beta}$

Pontryagin index: $n = \frac{g^2}{32\pi^2}\int \tilde{C}^0(x)\ d^3x$

Rewrite constant topological susceptibility: $\langle CC\rangle_{q\to 0} \sim \frac{1}{q^2}$ massless 3-form!

Removing this pole makes θ unphysical!

Uniquely achieved in gauge invariant way by making C massive.

Effective Field Theory

Without Axion: $$\mathscr{L}_{\text{eff}} = -\frac{1}{\Lambda_{\text{QCD}}^4}(F_{\mu\nu\rho\sigma})^2 + \theta\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} + \cdots$$

$$= -\frac{1}{\Lambda_{\text{QCD}}^4}E^2 + \theta E + \cdots \qquad \Longrightarrow \qquad \langle E \rangle = \Lambda_{\text{QCD}}^4\theta$$

Solve eom: $E = \Lambda_{\text{QCD}}^4(\theta + \kappa)$ background constant field

Effective Field Theory

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Solve eom: $E = \Lambda_{\text{QCD}}^4(\theta + \kappa)$ background constant field

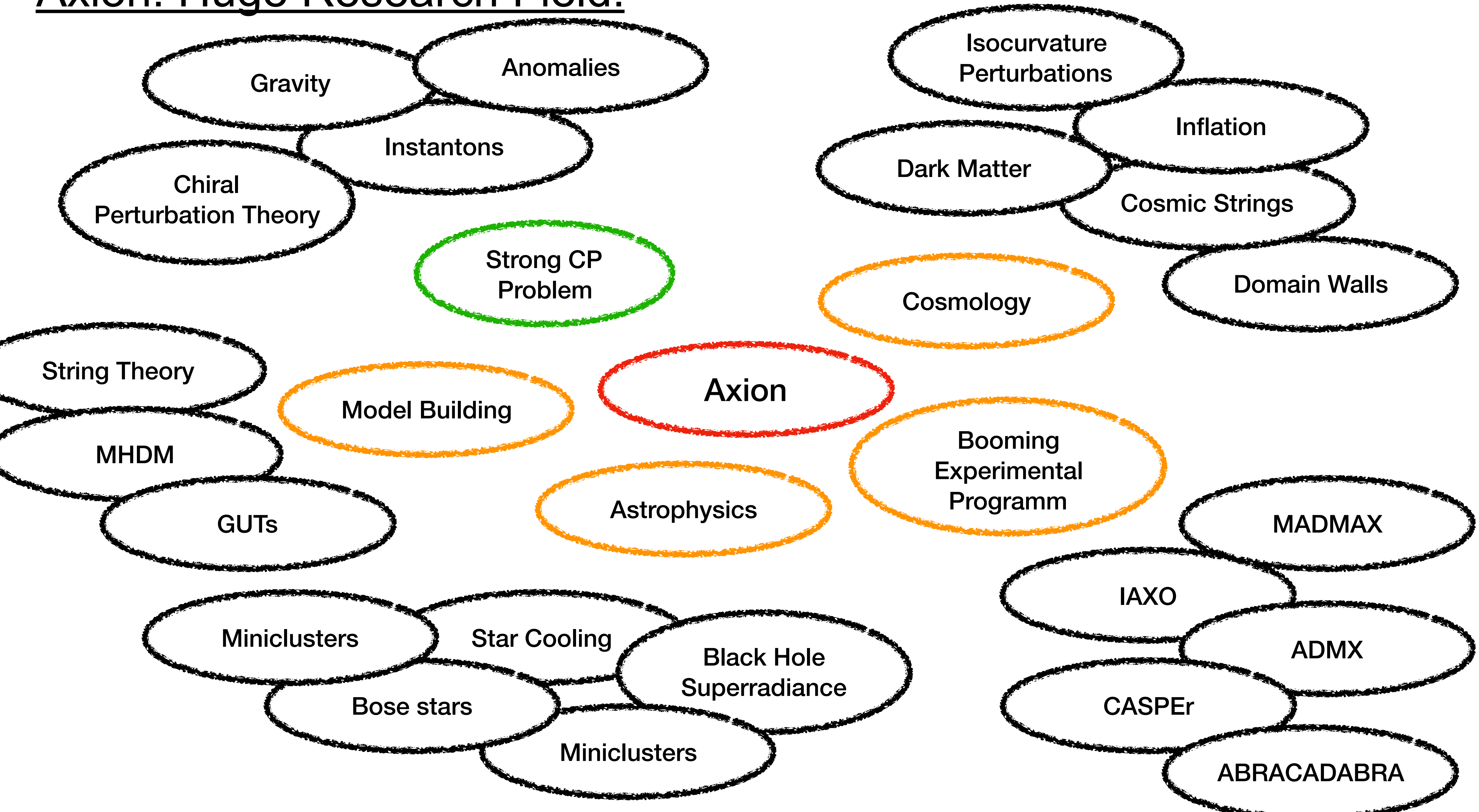
With Axion: $$\mathscr{L}_{\text{eff}} = -(F_{\mu\nu\rho\sigma})^2 + m^2\left(C_{\mu\nu\rho} - \partial_{[\mu}B_{\nu\rho]}\right)^2$$

Background constant field is no longer solution of eom

Hodge-Dual: $$\mathscr{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)^2 - \frac{m^2}{2}(a-\kappa)^2 \ ,$$

Key-point: If axion introduced as 2-form, PQ symmetry is gauged,
thus protected against explicit breaking

Axion: Huge Research Field!



Conclusion

QCD has a non-trivial vacuum structure described by θ- vacua

These contribute to the unobserved nEDM $\Longrightarrow \theta < 10^{-10}$ or so

Strong CP puzzle: Why is θ so small?

Quantum Gravity arguments demand $\theta = 0$

Strong CP

PQ mechanism: Introduce non-linearly realised axial U(1) symmetry that is anomalous w.r.t. QCD

Axion is the pseudo Goldstone-boson of this $U(1)_{PQ}$

When introduced as 2-form PQ symmetry is gauged

Axion

Thank you for your attention!

Instantons- QCD

$$\mathscr{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

Field-Strength tensor: $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$

Gauge Transformation: $A_\mu \longrightarrow U A_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1}$

Vacuum: $\cancel{A_i(x)|_{\mathrm{vac}}} = 0$ $\quad A_i(x)|_{\mathrm{vac}} = \frac{i}{g} U(x) \partial_\mu U(x)^{-1}$

Finite action: $\lim_{x\to\infty} U(x) = \mathrm{const}$ $\longrightarrow$ space topologically equivalent to 3-sphere!

Mapping: $U : S^3 \longrightarrow S^3$ $\Longrightarrow$ winding number $|n\rangle$