Fuzzy Dark Matter within Kinetic Field Theory

Master‘s thesis research by Laurin Söding

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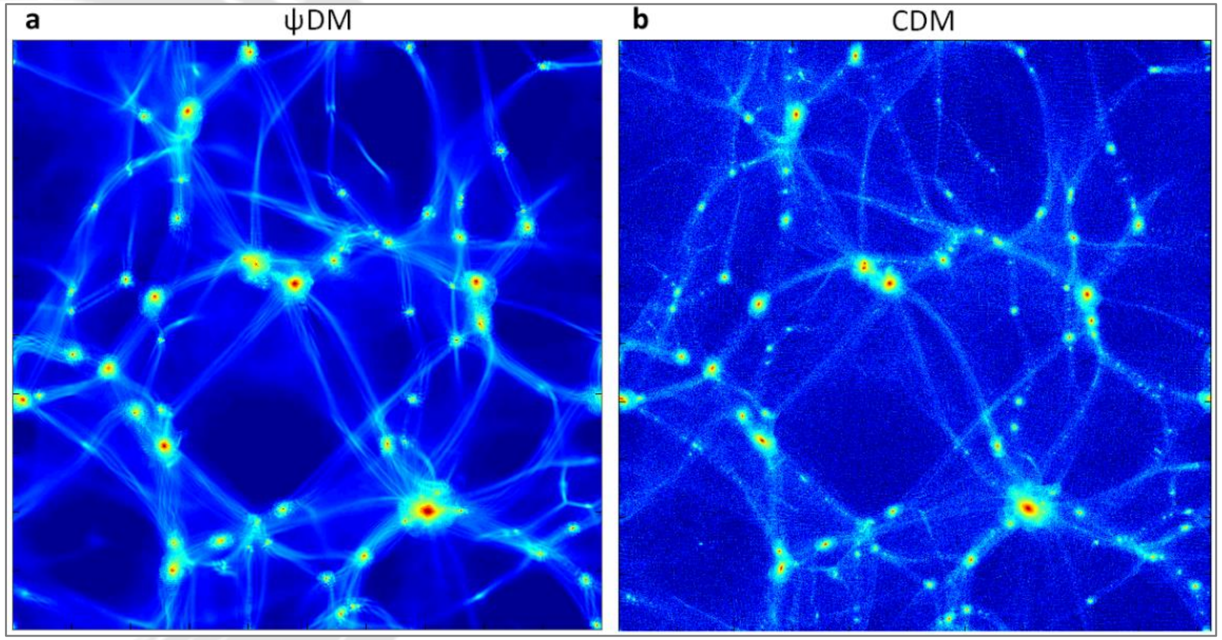
FDM as a Dark Matter Model

- What?
  - Ultra-light particles (mass $\sim 10^{-22}$eV)

  → De Broglie-wavelength (at typical cosmological velocities): λ_{dB}~kpc today

  →“Smears out“ small structures
- Why?
  - Cuspy halo problem
  - Missing satellites problem
- Existence motivated from many SM-extensions



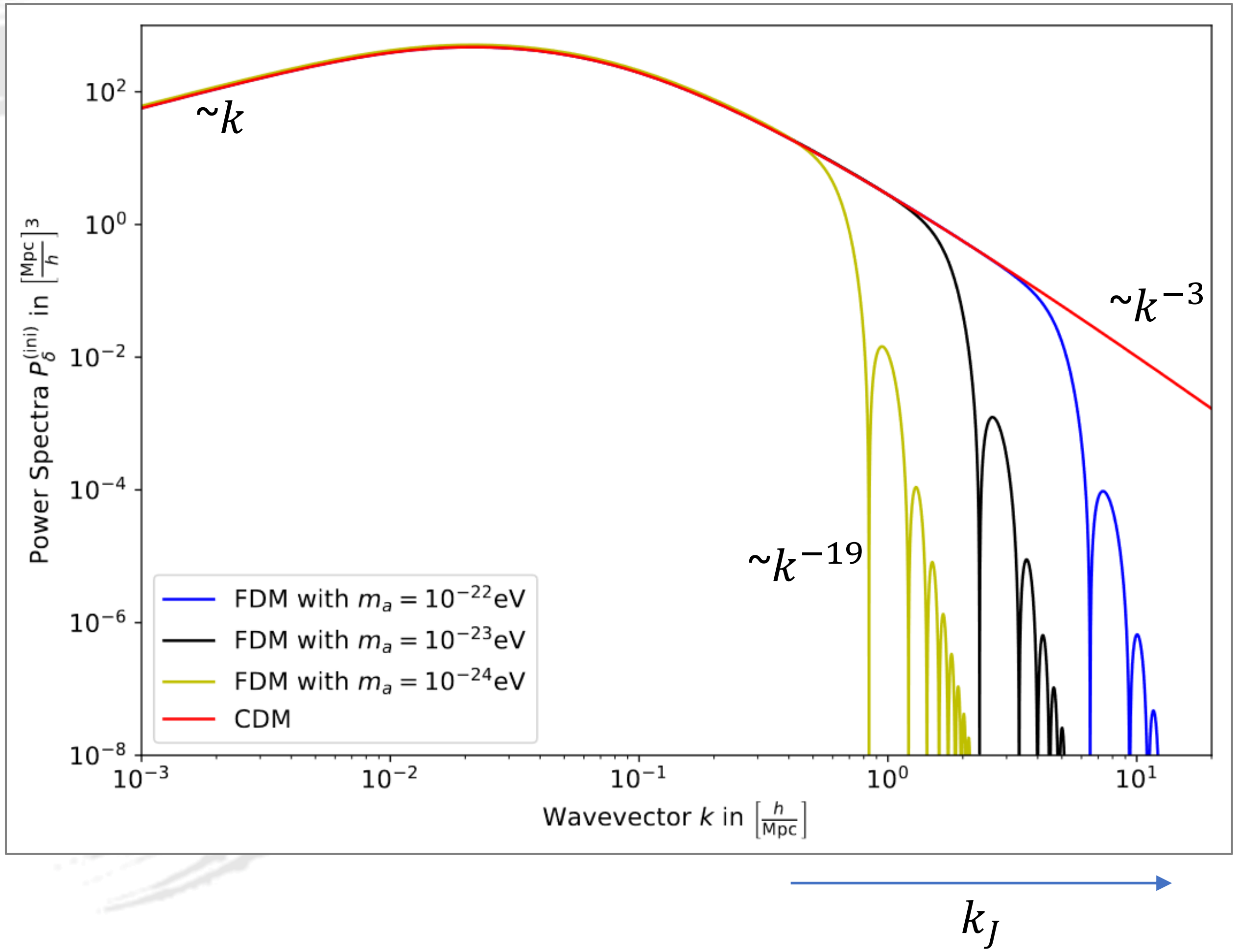
Schive et al. 2014: Cosmic Structure as the Quantum Interference of a Coherent Dark Wave

Problem: FDM Reduces Number of Small Structures

- Cut-off at Jeans scale $k_J \sim m^{1/2} a^{1/4}$
- Measurements today: Consistent with CDM (on intermediate scales)

→If DM were FDM, the power spectrum had to catch up to CDM case!

→Research question: How fast?

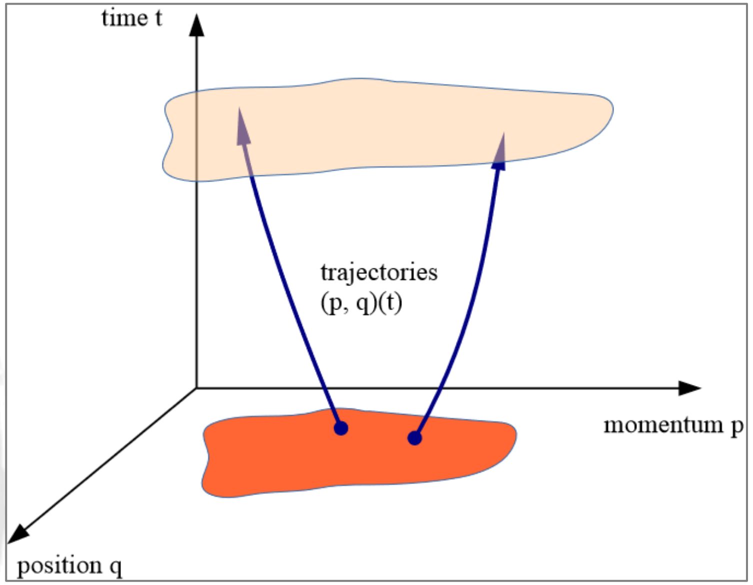


Method: Kinetic Field Theory

- What?
  - Statistical description of classical particle ensemble
  - In and out of equilibrium
  - Basis: Hamilton's equations:

$$\dot{x}(t) = -\begin{pmatrix} 0 & \mathbb{1}_3 \\ \mathbb{1}_3 & 0 \end{pmatrix} \nabla H(x(t), t) = E_0(x(t)) + E_I(x(t))$$

- How?
  - Generating functional $Z[J, K] = e^{\hat{S}_I} \int \mathrm{d}\Gamma e^{i\langle J, x_0[K] \rangle}$



Bartelmann et al. 2019: Cosmic Structure Formation with Kinetic Field Theory

Interaction operator: $e^{\hat{S}_I} = e^{\frac{\delta}{i\delta K} \cdot E_I\left(\frac{\delta}{i\delta J}\right)}$

Perturbation series: $e^{\hat{S}_I} = 1 + \hat{S}_I + \frac{\hat{S}_I^2}{2} + \cdots$

Weighed integral over initial states
→Initial ensemble statistics
→Correlated particle positions and momenta

"free" particle trajectories:
Chosen as Zel'dovich trajectories
→Mimic linear growth (and possibly a little further)

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$$\rho(q,t) = \sum_{j=1}^{N} \delta_D\left(q - q_j(t)\right) \longrightarrow \hat{\rho}(k,t) = \sum_{j=1}^{N} \exp\left(-ik \cdot \left[-i \frac{\partial}{\partial J_{q_j}(t)}\right]\right) = \sum_{j=1}^{N} \hat{\rho}_j(k,t)$$

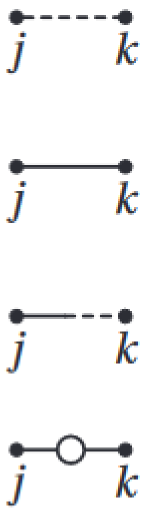
Density operator allows for computation of 2-pt. cumulant by successive application:

$$\langle \rho\rho \rangle = \hat{\rho}\hat{\rho} Z[J,K] \Big|_{J=K=0}$$

Challenges and Solutions 1/2: Calculations

- Computation method:
  - Started with direct computation of integral expressions for 1st order
    → Many terms: including all initial correlations made things very difficult
- Changed direction: Used work of former PhD-student
  - Clustering method
  - Set of Feynman rules

→ Computed, simplified and classified diagrams



$$= \mathscr{P}_{p_j p_k}\big(\mathscr{K}^{(\mathrm{i})}_{jk}\big) := \int_{q^{(\mathrm{i})}_{jk}} \mathrm{e}^{-\mathrm{i}\theta_q}\left(\mathrm{e}^{-\theta_p}-1\right),$$

$$= \mathscr{P}_{\delta_j \delta_k}\big(\mathscr{K}^{(\mathrm{i})}_{jk}\big) := \int_{q^{(\mathrm{i})}_{jk}} C_{\delta_j \delta_k}\mathrm{e}^{-\theta},$$

$$= \mathscr{P}_{\delta_j p_k}\big(\mathscr{K}^{(\mathrm{i})}_{jk}\big) := \int_{q^{(\mathrm{i})}_{jk}} \left(-\mathrm{i}C_{\delta_j p_k}\cdot L_{p,\mathrm{I}_k}\right)\mathrm{e}^{-\theta},$$

$$= \mathscr{P}_{(\delta p)^2_{jk}}\big(\mathscr{K}^{(\mathrm{i})}_{jk}\big)$$

$$:= \int_{q^{(\mathrm{i})}_{jk}} \left(-\mathrm{i}C_{\delta_j p_k}\cdot L_{p,\mathrm{I}_k}\right)\left(-\mathrm{i}C_{\delta_k p_j}\cdot L_{p,\mathrm{I}_j}\right)\mathrm{e}^{-\theta},$$

Just for illustration – this is how the lines look and what each line stands for

Challenges and Solutions 2/2: Numerics

- Many nested integrals
  - Partly strongly oscillating

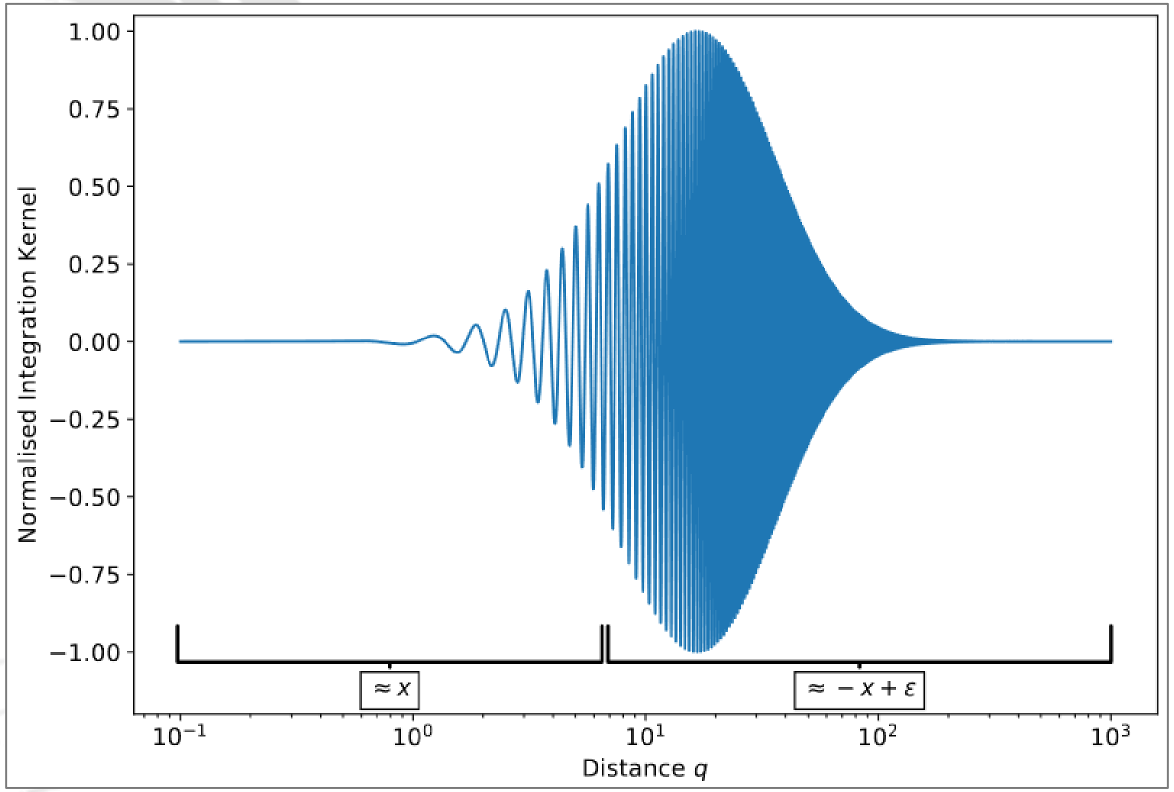
→Numerically challenging
  - Expansion into infinite sum of integrals

$$I(k) = \sum_{l}^{\infty} \int \mathrm{d}\, q\, W_l(k, q) j_l(k \cdot q)$$

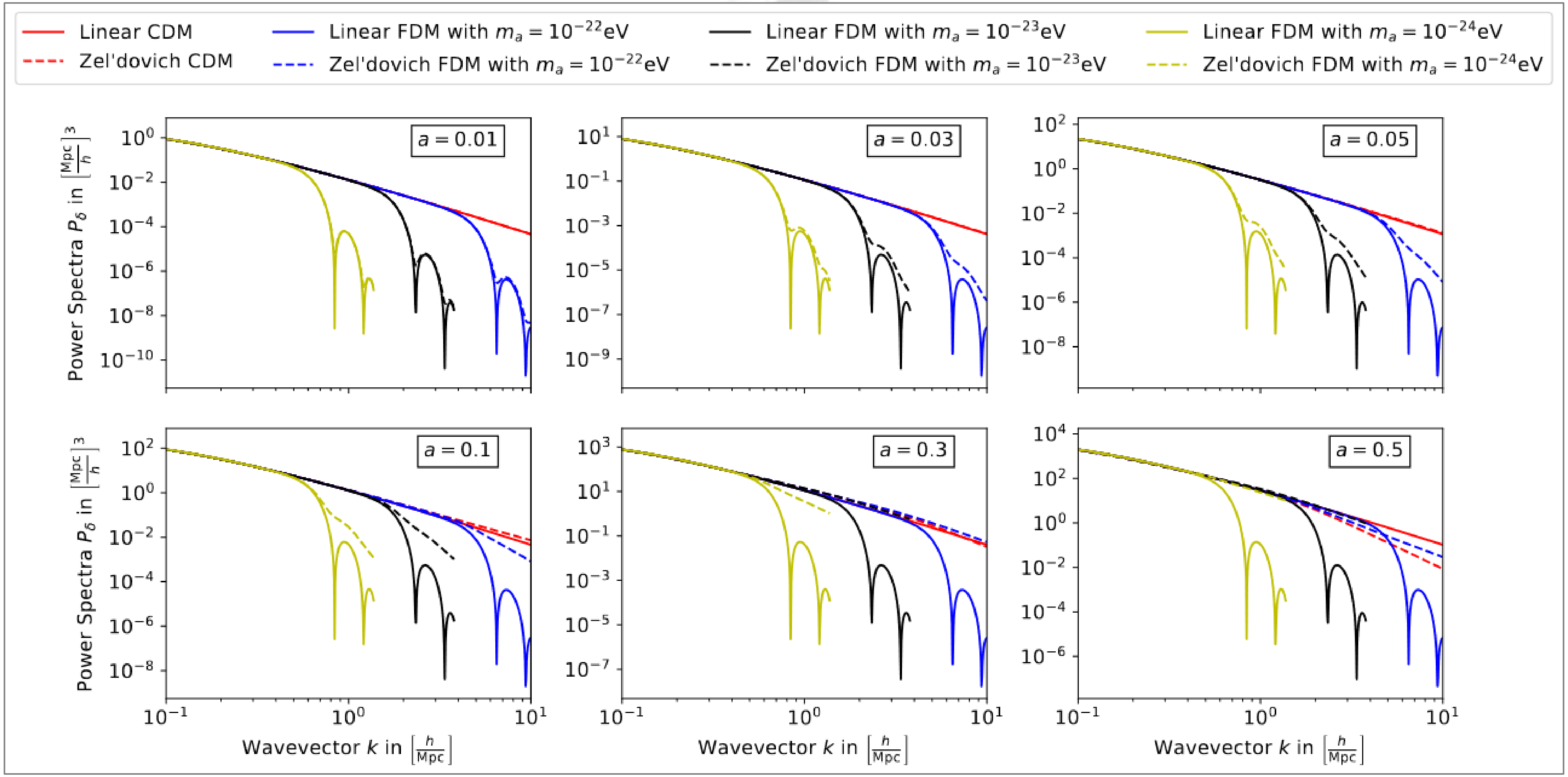
→Lewin Collocation

→Create large look-up-tables

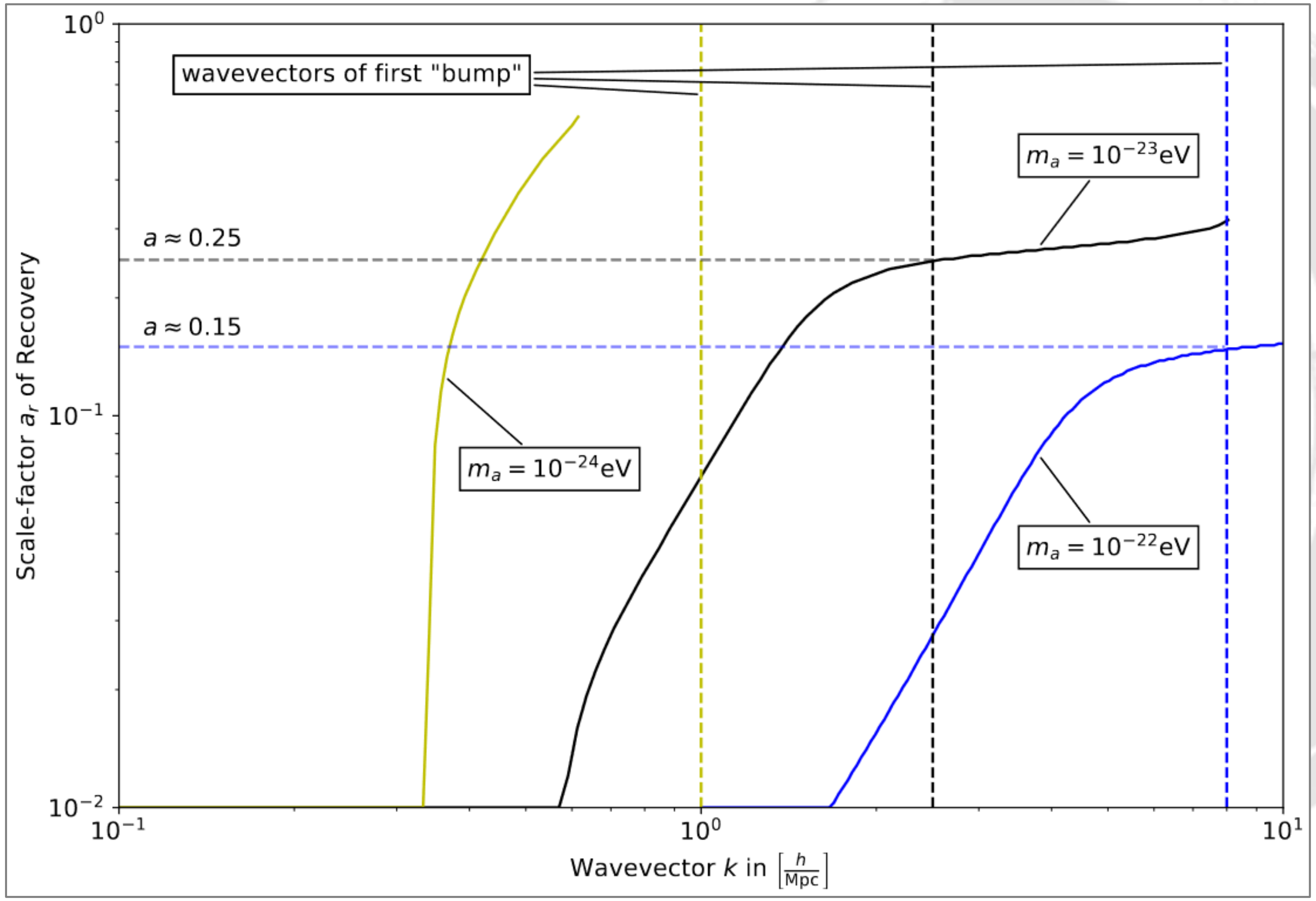
→Use Monte-Carlo integration for the remaining integrals

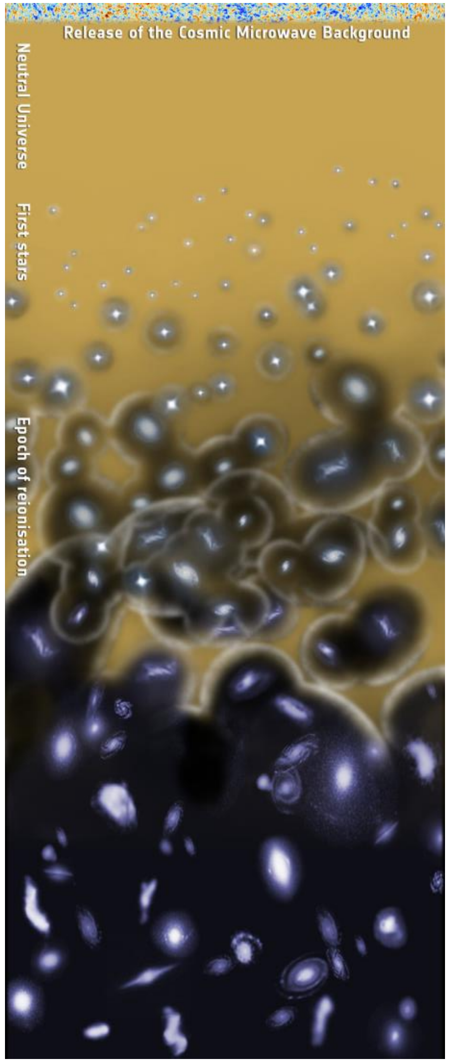


Results 1/2: First Order Corrections Are Negligible!



Results 2/2: Delayed Structure Formation and Re-ionisation





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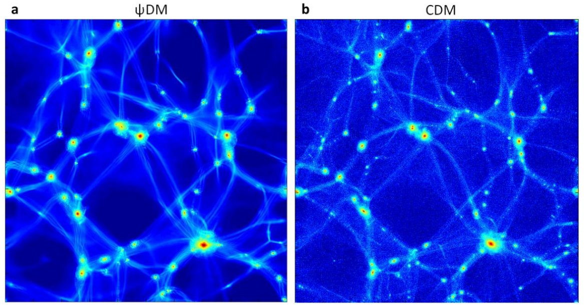
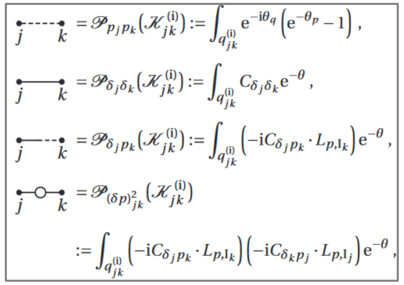
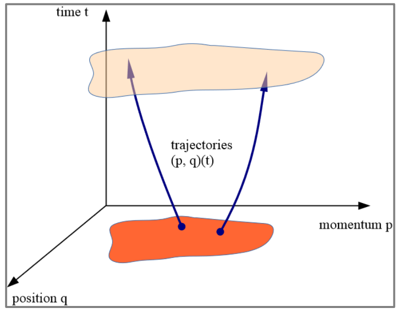
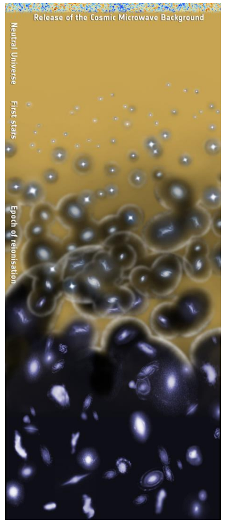
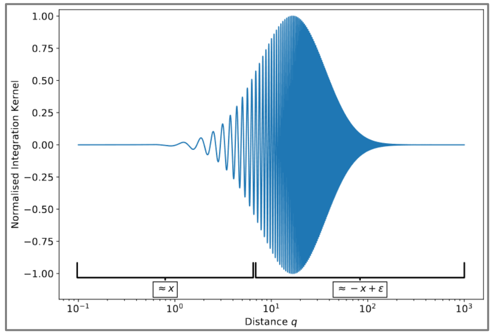
Questions?

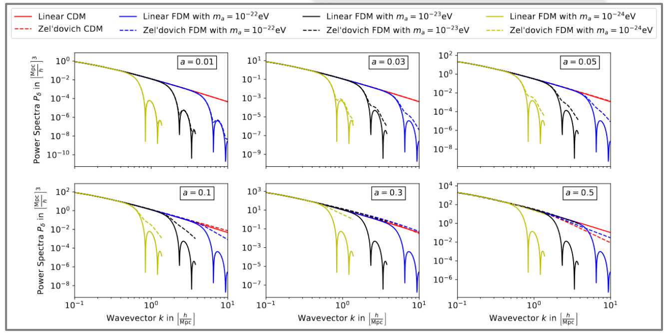


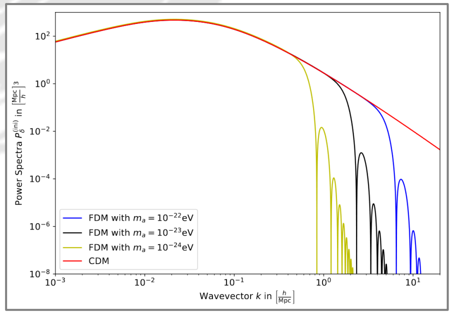


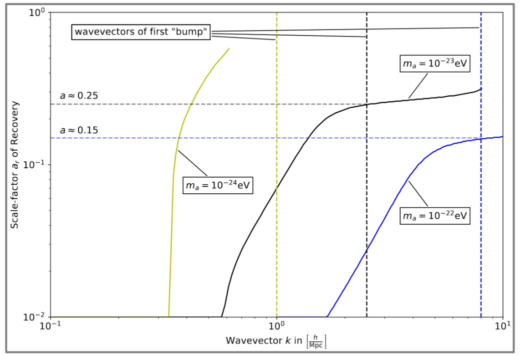












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