Soft-collinear Gravity for Fermions

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General approach in SCET

- All fields and the according gauge transformations are decomposed into collinear and soft parts
  - a collinear gauge symmetry (and fields) for each collinear sector
  - a single, soft background
- Then a multipole-expansion of the soft fields follows $\phi_s(x) = \phi_s(x_-) + ...$
- Field redefinitions are introduced, such that the soft gauge symmtry respects the multipole-expansion
- The redefined fields get inserted into the Lagrangian

Notation

- We introduce reference verctors $n_\pm$, $n_+ n_- = 2, \quad n_\pm^2 = 0$
- A collinear momentum p^μ is then given by:

$$p^\mu = (n_+ \cdot p)\frac{n_-^\mu}{2} + (n_- \cdot p)\frac{n_+^\mu}{2} + p_\perp^\mu \equiv p_+ \cdot \frac{n_-^\mu}{2} + p_- \cdot \frac{n_+^\mu}{2} + p_\perp^\mu,$$

- and x^μ as :

$$x^\mu = (n_+ \cdot x)\frac{n_-^\mu}{2} + (n_- \cdot x)\frac{n_+^\mu}{2} + x_\perp{}^\mu = x_-^\mu + x_+^\mu + x_\perp^\mu$$

- Each components scales as follows with $\lambda = p_\perp / n_+ p$:

$$(p_+, p_\perp, p_-) \sim (1, \lambda, \lambda^2) \quad (x_+, x_\perp, x_-) \sim (1, \frac{1}{\lambda}, \frac{1}{\lambda^2}).$$

- The collinear quark field is decomposed as:

$$\psi_c(x) = \xi(x) + \eta(x), \quad \xi(x) = \frac{\not{n}_- \not{n}_+}{4}\psi_c(x),$$

Vierbein formalism

Common approach to describe interactions of gravity with fermions does not work for fermions:

- We introduce a local coordinate system and vierbeins ${e^a}_\mu$:

$$g_{\mu\nu} = {e^a}_\mu {e^b}_\nu \eta_{ab} \quad g^{\mu\nu} = {E^\mu}_a {E^\nu}_b \eta^{ab}$$

- Arbitrary vectors under GCT can be transformed into scalars using vierbeins:

$$ {B^a}_b = {e^a}_\mu {E^\nu}_b {B^\mu}_\nu $$

  ${B^a}_b$ scalar under GCT, tensor under LLT
- Now fermions can be treated as any other particle
- Two gauge transformations: GCT *and* LLT
- Covariant derivative $D_\mu \psi = (\partial_\mu - i\Omega_\mu)\psi$, $\Omega_\mu = \sigma^{ab}\omega_{\mu ab} = \frac{1}{2}\sigma^{ab}{E^\nu}_a e_{b\nu;\mu}$
-

$$S_m = \int d^4x \mathcal{L}_D = \frac{1}{2}\int d^4x \sqrt{-g}\left[\bar{\psi}{E^\mu}_a \gamma^a \left(i\overrightarrow{D_\mu}\psi\right) - \left(\bar{\psi}i\overleftarrow{D_\mu}\right){E^\mu}_a \gamma^a \psi\right].$$

Weak-field expansion

We expand the metric around a soft background:

$$
\begin{aligned}
g_{\mu\nu} &= g_{s\mu\nu} + h_{\mu\nu} \\
e^a{}_\mu &= e_s{}^a{}_\mu + E_s{}^{\rho a} e_{c\rho\mu},
\end{aligned}
$$

with $e_s{}^a{}_\mu e_s{}^b{}_\mu \eta_{ab} = g_{s\mu\nu}$.

- We expand $g_{s\mu\nu} = \eta_{\mu\nu} + s_{\mu\nu}$ with $s_{\mu\nu} \sim \lambda^2$
- And the graviton field scales as:

$$
\begin{aligned}
&h_{++} \sim \lambda^{-1},\ h_{+\perp} \sim 1,\ h_{+-} \sim \lambda, \\
&h_{--} \sim \lambda^3,\ h_{-\perp} \sim \lambda^2,\ h_{\perp\perp} \sim \lambda.
\end{aligned}
$$

Field redefinitions

- Collinear Wilson-lines V_c, W_c, LLT and GCT light-cone gauge $h_{+\mu} = 0$,$\Omega_+ = 0$
- Soft Wilson-lines V_s, R, similar to Riemann normal coordinates for GCT and fixed-line gauge for LLT $(x - x_-)^\mu \, \Omega_\mu = 0$

$$
\begin{aligned}
\psi_c &= \left[R V_s \, V_c{}^{-1} W_c^{-1} \left(\hat{\xi} + \hat{\eta} \right) \right], \\
E_s{}^{\nu a}(x) e_{c\nu\mu} &= \left[R R_\mu{}^\alpha V_s{}^a{}_b \left[W^\rho{}_\alpha \, V_{cd}{}^b W_c{}^{-1} \left(\hat{e}_s{}^d{}_\rho(x) + \hat{E}_s{}^{\sigma d} \hat{e}_{c\sigma\rho} \right) \right] - \hat{e}_s{}^b{}_\alpha(x) \right], \\
\Omega_{c\mu} &= \left[R R_\mu{}^\alpha V_s \left(V_c^{-1} \left(i\partial_\alpha + \left[W^\rho{}_\alpha \, W_c^{-1} \hat{\Omega}_\rho \right] \right) V_c + \left(-i\partial_\alpha - \frac{n_{+\alpha}}{2} \hat{\Omega}_{s-}(x_-) \right) \right) V_s^{-1} \right]
\end{aligned}
$$

- Analogous to QCD: Left side is in light-cone gauge, "hat"–fields are in arbitrary gauge
- Left side transforms with $D(\Lambda(x))$, "hat"–fields with $D(\Lambda(x_-))$
- Soft background through soft vierbein $\hat{e}_s{}^a{}_\mu(x)$ and $\hat{\Omega}_{s-}(x_-)$

Soft-collinear Lagrangian

Above redefinitions are inserted into the Lagrangian:

$$\mathcal{L} = \sqrt{-\left(\hat{g}_s + \hat{h}\right)} \left(\bar{\hat{\xi}}(x) + \bar{\hat{\eta}}(x) + \left[W_c V_c^{-1} V_s \left[R^{-1} \bar{q}\right]\right](x)\right) \gamma^a i \left(\hat{E}^\rho{}_a + \tilde{E}_a^\rho(x)\right) \hat{D}_\rho(x)$$
$$\times \left(\hat{\xi}(x) + \hat{\eta}(x) + \left[W_c V_c V_s^{-1} \left[R^{-1} q(x)\right]\right]\right)$$

- $\tilde{E}_a^\rho(x)$ contains Riemann-tensors
- The collinear Wilson-lines vanish, when no soft quark fields or Riemann-tensors appear

Soft-collinear Lagrangian

- By integrating out $\eta(x)$ we arrive at:

$$\mathcal{L} = \frac{i}{2}\bar{\hat{\chi}} \not{n}_+ D_- \hat{\chi} + \frac{i}{2}\bar{\hat{\chi}} \not{\partial}_\perp \frac{1}{\partial_+} \not{\partial}_\perp \not{n}_+ \hat{\chi} + \mathcal{L}^{(1)} + \mathcal{O}(\lambda^2).$$

- Covariant derivative $D_- = \hat{E}_s{}^{\mu}{}_- \hat{D}_{s\mu}$
- Transparent structure, similar to QCD
- $\hat{\chi} = V_c^{-1} \left[W_c^{-1} \hat{\xi} \right]$

Building-blocks for N-jet operators

	collinear quark fields	collinear gauge fields	soft fields
QCD	$\chi \sim \lambda$	$\mathcal{A}_\perp \sim \lambda$	$q \sim \lambda^3, F^s{}_{\mu\nu} \sim \lambda^4$
Gravity	$\chi \sim \lambda$	$\mathfrak{h}_{\perp\perp} \sim \lambda$	$q \sim \lambda^3, R_{\mu\nu\rho\sigma} \sim \lambda^6$

- $\mathcal{A}_-$, $\mathfrak{h}_{\mu-}$ and the covariant derivatives are eliminated through the e.o.m.
- While $F^s{}_{\mu\nu} \sim \lambda^4$ in QCD, the Riemann-tensor already scales as $R_{\mu\nu\rho\sigma} \sim \lambda^6$
- In gravity the soft theorem has additional, twice suppressed universal contributions

Summary

- The same structure as in QCD emerges in gravity
  - soft vierbein
  - soft spin-connection
- Interactions expressed with covariant derivatives and Riemann-tensors
- Only few building-blocks for N-jet operators
- Soft-theorem in gravity is valid up sub-sub-leading order

SCET Lagrangian

Above redefinitions are inserted into the Lagrangian:

$$\mathcal{L} = \left(\bar{\hat{\xi}}(x) + \bar{\hat{\eta}}(x) + \bar{q}(x)R(x)W_c^\dagger(x)\right)\gamma^\rho\left(i\hat{D}_\rho(x) + g\tilde{F}_\rho(x)\right)$$
$$\times\left(\xi(x) + \eta(x) + W_c(x)R^\dagger(x)q(x)\right)$$

- $\tilde{F}_\rho(x)$ contains field-strength tensor terms
- The collinear Wilson lines vanishes, when no soft quark fields or field-strength tensors appear

- By integrating out $\eta(x)$ we arrive at:

$$\mathcal{L} = \bar{\xi}\left(iD_- + i\not{D}_{\perp c}\frac{1}{iD_{+c}}i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\xi + \bar{q}(x)\not{D}_s(x)q(x) + \mathcal{L}_{\xi}^{(1)} + \mathcal{L}_{\xi}^{(2)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\xi q}^{(2)}$$

  - Transparent structure : We have a soft, covariant derivativeD_-. Higher suppressed interactions to soft fields are expressed via the field-strength tensor

  - Each term is homogeneous in λ

Diffeomorphism (GCT)

- We consider:

$$x^\mu \longrightarrow x^\mu + \epsilon^\mu(x),$$

  as exact.
- Fields transform as:

$$\phi(x) \longrightarrow U\phi(x)$$
$$A^{\mu\nu}{}_\alpha(x) \longrightarrow \left[U U^\mu{}_\lambda U^\nu{}_\rho U_\alpha{}^\beta A^{\lambda\rho}{}_\beta\right](x),$$

- where $U^\mu{}_\nu$, $U_\nu{}^\mu$ und U gegeben sind als:

$$U^\mu{}_\nu \equiv \frac{\partial y^\mu}{\partial x^\nu} = \delta^\mu{}_\nu + \partial_\nu \epsilon^\mu(x)$$
$$U_\nu{}^\mu \equiv \frac{\partial x^\mu}{\partial y^\nu} = \delta^\mu{}_\nu - \partial_\nu \epsilon^\mu + \partial_\nu \epsilon_\beta \partial^\beta \epsilon^\mu + \mathcal{O}(\epsilon^3).$$
$$U = 1 - \epsilon^\alpha \partial_\alpha + \frac{1}{2}\epsilon^\alpha \epsilon^\beta \partial_\alpha \partial_\beta + \epsilon^\alpha \left[\partial_\alpha \epsilon^\beta\right] \partial_\beta + \mathcal{O}(\epsilon^3)$$

Collinear Lagrangian I

$$\begin{aligned}
\mathcal{L}^{(2)} =& - i\bar{\xi}\frac{\not{n}_+}{2}\Big(\frac{1}{2}\{\mathcal{G}_-^{\mu}, \partial_\mu\} + \frac{1}{2}\Big\{\not{\partial}_\perp \frac{1}{n_+\partial}, \gamma_\perp^{\nu}\{\mathcal{G}_\nu^{\mu}, \partial_\mu\}\Big\} \\
&+ \frac{1}{4}\Big(\not{\partial}_\perp\{\frac{1}{n_+\partial}, \mathfrak{h}\}\gamma_\perp^{\nu}\{\mathcal{H}_\nu^{\alpha}, \partial_\alpha\} + \gamma_\perp^{\nu}\{\mathcal{H}_\nu^{\alpha}, \partial_\alpha\}\{\frac{1}{n_+\partial}, \mathfrak{h}\}\not{\partial}_\perp\Big) \\
&- \frac{1}{4}\{\gamma_\perp^{\nu}\mathcal{H}_\nu^{\mu}, \partial_\mu\}\frac{1}{n_+\partial}\{\mathcal{H}_\beta^{\alpha}, \partial_\alpha\}\gamma_\perp^{\beta} + \frac{1}{16}\not{\partial}_\perp\{\frac{1}{n_+\partial}, \mathfrak{h}^2 - 2\mathfrak{h}^{\alpha\beta}\mathfrak{h}_{\alpha\beta}\}\not{\partial}_\perp \\
&- \frac{1}{16}\not{\partial}_\perp(\{\frac{1}{n_+\partial}, \mathfrak{h}^2\} + \frac{3}{2}\mathfrak{h}^2\frac{1}{n_+\partial} + \mathfrak{h}\{\frac{1}{n_+\partial}, \mathfrak{h}\})\not{\partial}_\perp \\
&- i\frac{1}{16}\big(\mathfrak{h}_\alpha^{\lambda}[\partial_\beta\mathfrak{h}_{\mu\lambda}]\varepsilon^{\alpha\beta\mu-}\gamma_5 + \Big[\not{\partial}_\perp\frac{1}{n_+\partial}, \mathfrak{h}_\alpha^{\lambda}[\partial_\beta\mathfrak{h}_{\mu\lambda}]\varepsilon^{\alpha\beta\mu\nu}\gamma_{\perp\nu}\gamma_5\Big] \\
&+ \not{\partial}_\perp\frac{1}{n_+\partial}\mathfrak{h}_\alpha^{\lambda}[\partial_\beta\mathfrak{h}_{\mu\lambda}]\varepsilon^{\alpha\beta\mu+}\gamma_5\frac{1}{n_+\partial}\not{\partial}_\perp)\Big)\xi \\
&+ {\mathcal{L}^{(2)}}_{as},
\end{aligned}$$

Collinear Lagrangian II

$$
\begin{aligned}
\mathscr{L}^{(2)}{}_{as} = &- \frac{i}{16}\bar{\xi}\left(\left\{a^{\mu}{}_{-}, \partial_{\mu}\right\} \not{n}_{+} + \left\{\gamma^{\mu}, \sigma^{ab}[\partial_{\mu} a_{ba}]\right\}\right)\xi \\
&- \frac{i}{16}\bar{\xi}\left(\gamma_{\perp}{}^{\mu}\left\{a^{\rho}{}_{\mu}, \partial_{\rho}\right\} + \frac{i}{2}\left\{\gamma^{\mu}, \sigma^{ab}[\partial_{\mu} a_{ba}]\right\}\right)\frac{1}{\partial_{+}}\not{\partial}_{\perp}\not{n}_{+}\xi \\
&- \frac{i}{16}\bar{\xi}\not{\partial}_{\perp}\frac{1}{\partial_{+}}\left(\frac{i}{4}\not{n}_{+}\left\{\gamma^{\mu}, \sigma^{ab}[\partial_{\mu} a_{ba}]\right\}\not{n}_{+}\right)\frac{1}{\partial_{+}}\not{\partial}_{\perp}\xi \\
&- \frac{i}{16}\bar{\xi}\not{n}_{+}\not{\partial}_{\perp}\frac{1}{\partial_{+}}\left(\gamma_{\perp}{}^{\mu}\left\{a^{\rho}{}_{\mu}, \partial_{\rho}\right\} + \frac{i}{2}\left\{\gamma^{\mu}, \sigma^{ab}[\partial_{\mu} a_{ba}]\right\}\right)\xi,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{H}^{\mu}{}_{\nu} &= \frac{1}{2}\left(\mathfrak{h}^{\mu}{}_{\nu} - \mathfrak{h}\delta^{\mu}{}_{\nu}\right) \\
\mathscr{G}^{\mu}_{\nu} &= -\frac{1}{8}\Big(3\mathfrak{h}^{\mu\alpha}\mathfrak{h}_{\alpha\nu} - 2\mathfrak{h}\mathfrak{h}^{\mu}_{\nu} + (\mathfrak{h}^{2} - 2\mathfrak{h}^{\alpha\beta}\mathfrak{h}_{\alpha\beta})\delta^{\mu}_{\nu}\Big).
\end{aligned}
$$

Transformation of fields I

Under collinear GCT:

$$
\begin{aligned}
h_{\mu\nu} &\to \left[U_c U_{c\mu}{}^{\alpha} U_{c\nu}{}^{\beta} \left(g_{s\alpha\beta} + h_{\alpha\beta}\right)\right] - g_{s\alpha\beta}, \\
\psi_c &\to \left[U_c \psi_c\right], \\
E_s{}^{\rho a} e_{c\rho\mu} &\to \left[U_c U_{c\mu}{}^{\alpha} \left(e_s{}^a{}_\alpha + E_s{}^{\rho a} e_{c\rho\alpha}\right)\right] - e_s{}^a{}_\mu, \\
s_{\mu\nu} &\to s_{\mu\nu}, \\
e_s{}^a{}_\mu &\to e_s{}^a{}_\mu, \\
q &\to q,
\end{aligned}
$$

Transformation of fields II

Under soft GCT

$$
\begin{aligned}
h_{\mu\nu} &\to \left[U_s U_{s\mu}{}^{\alpha} U_{s\nu}{}^{\beta} h_{\alpha\beta}\right], \\
\psi_c &\to \left[U_s \psi_c\right], \\
E_s{}^{\rho a} e_{c\rho\mu} &\to \left[U_s U_{s\mu}{}^{\alpha} E_s{}^{\rho a} e_{c\rho\alpha}\right], \\
g_{s\mu\nu} &\to \left[U_s U_{s\mu}{}^{\alpha} U_{s\nu}{}^{\beta} g_{s\alpha\beta}\right], \\
e_s{}^{a}{}_{\mu} &\to \left[U_s U_{s\mu}{}^{\alpha} e_s{}^{a}{}_{\alpha}\right], \\
q &\to \left[U_s q\right].
\end{aligned}
$$

Transformation of fields III

Collinear LLT:

$$\begin{aligned} h_{\mu\nu} &\to h_{\mu\nu}, \\ \psi_c &\to D_c \psi_c, \\ E_s{}^{\rho a} e_{c\rho\mu} &\to \Lambda_c{}^a{}_b \left(e_s{}^b{}_\mu + E_s{}^{\rho b} e_{c\rho\alpha} \right) - e_s{}^a{}_\mu, \\ s_{\mu\nu} &\to s_{\mu\nu}, \\ e_s{}^a{}_\mu &\to e_s{}^a{}_\mu, \\ q &\to q, \end{aligned}$$

Transformation of fields IV

and soft LLT:

$$\begin{aligned}
h_{\mu\nu} &\to h_{\mu\nu}, \\
\psi_c &\to D_s \psi_c, \\
E_s{}^{\rho a} e_{c\rho\mu} &\to \Lambda_s{}^a{}_b E_s{}^{\rho b} e_{c\rho\mu}, \\
g_{s\mu\nu} &\to g_{s\mu\nu}, \\
e_s{}^a{}_\mu &\to \Lambda_s{}^a{}_b e_s{}^b{}_\mu, \\
q &\to D_s q.
\end{aligned}$$

$$\begin{aligned}
&\Omega_{s\mu} \to D_s \Omega_{s\mu} D_s^{-1} + i D_s \left[\partial_\mu D_s^{-1} \right], \\
&\Omega_{c\mu} \to D_s \Omega_{s\mu} D_s^{-1},
\end{aligned}$$

Soft-collinear Lagrangian I

$$\tilde{E}^{\rho}(x)_a = \left[W_c W_{c\sigma}{}^{\rho} V_{ca}{}^{b} \mathfrak{E}_s{}^{\sigma}{}_b \right].$$

$$\mathcal{L}^{(1)} = -\frac{i}{8}\bar{\hat{\chi}}\left\{n_-{}^a \mathfrak{h}_a{}^{\mu}, \partial_\mu\right\} \not{n}_+ \hat{\chi} + \frac{i}{8}\left[\bar{\hat{\chi}}\left\{D_-, \mathfrak{h}\not{n}_+\right\}\hat{\chi}\right] + \frac{i}{8}\mathfrak{h}\left[\bar{\hat{\chi}}\not{\partial}_\perp \frac{1}{\partial_+}\not{\partial}_\perp \not{n}_+ \hat{\chi} + h.c.\right]$$
$$- \frac{i}{8}\bar{\hat{\chi}}\gamma_\perp{}^a \left\{\mathfrak{h}_a{}^{\rho}, \partial_\rho\right\}\frac{1}{\partial_+}\not{\partial}_\perp \not{n}_+ \hat{\chi} - \frac{i}{8}\bar{\hat{\chi}}\not{\partial}_\perp \frac{1}{\partial_+}\gamma_\perp{}^a\left\{\mathfrak{h}_a{}^{\rho}, \partial_\rho\right\}\not{n}_+\hat{\chi}.$$

Wilson-lines

- Soft GCT Wilson-line:

$$R^{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} {\theta_s}^n(x) \cdot \partial^n = 1 + {\theta_s}^\alpha \partial_\alpha + \frac{1}{2} {\theta_s}^\alpha {\theta_s}^\beta \partial_\alpha \partial_\beta + \mathcal{O}({\theta_s}^3),$$

  - θ_s defined with symmetric, soft vierbein ${F^\mu}_\alpha$;

$${\theta_s}^\mu(x) = \left({F^\mu}_\alpha - {\delta^\mu}_\alpha\right)(x - x_-)^\alpha - \frac{1}{2}(x - x_-)^\alpha (x - x_-)^\beta {F^\nu}_\alpha {F^\rho}_\alpha {\Gamma^\mu}_{\rho\nu} + \ldots$$

  - Analogous to Riemann normal coordinates
  - $\check{g}_{s\mu\nu}(x) = \hat{g}_{s\mu\nu}(x) + \mathfrak{g}_{\mu\nu}(x)$
  - Residual transformation:

$$x^\mu \to x^\mu + \epsilon^\mu(x_-) + \frac{1}{2}\left(\partial^\mu \epsilon_\nu - \partial_\nu \epsilon^\mu\right)(x_-)(x - x_-)^\nu + \ldots\ .$$

- Soft LLT Wilson-line

$$V_s = \mathbf{P}exp\left(+i\int_{x_1}^{x_2} dy^\mu [R^\nu{}_\mu R^{-1}\Omega_{s\mu}](y)\right)$$

  - Takes us into fixed-line gauge $(x-x_1)^\mu\,\check{\Omega}_{s\mu}(x)=0$
  - Residual transformation only depends on x_-
  -

$$\check{e}_s{}^a{}_\mu(x) = \overbrace{\delta^a{}_\mu + \frac{1}{4}n_{+\mu}s^a{}_- + \frac{1}{2}n_{+\mu}y^\rho\omega_-{}^a{}_\rho - \frac{1}{16}n_{+\mu}s_{-\beta}s^{\beta a}}^{\hat{e}_s{}^a{}_\mu} \underbrace{-\frac{1}{6}\left(x-x_-\right)^\rho\left(x-x_-\right)^\alpha R^a{}_{\rho\mu\alpha} - \frac{1}{6}\left(x-x_-\right)^\rho\left(x-x_-\right)^\alpha n_{+\mu}R^a{}_{\rho-\alpha}}_{\mathfrak{e}_s{}^a{}_\mu} + \mathcal{O}(\lambda^5).$$

- Collinear GCT-Line:

$$W_c^{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \theta_{LC}^n(x) \cdot \partial^n,$$

  - θ_{LC} is given by:

$$\theta_{LC}^{\alpha} = -\frac{1}{(\partial_+)^2} \hat{\Gamma}^{\alpha}{}_{++} + \frac{1}{(\partial_+)^2} \left(2\hat{\Gamma}^{\alpha}{}_{\beta +} \frac{1}{\partial_+} \hat{\Gamma}^{\beta}{}_{++} + \partial_{\beta} \hat{\Gamma}^{\alpha}{}_{++} \frac{1}{(\partial_+)^2} \hat{\Gamma}^{\beta}{}_{++} \right) + \ldots$$

  - Takes us into GCT light-cone gauge:

$$\mathfrak{h}_{+\mu} = 0$$

  - $\mathfrak{h}_{\mu\nu}$ *not* homogeneous in λ

- Collinear LLT line:

$$V_c(x) = \mathbf{P}exp\left(i\int_{-\infty}^{0} dsn_+{}^{\mu}\left[W^{\rho}{}_{\mu}W^{-1}{}_{c}\hat{\Omega}_{\rho}\right](x+sn_+)\right).$$

$$V_c{}^{a}{}_{b}(x) = \left(\mathbf{P}exp\left(i\int_{-\infty}^{0} dsn^{\mu}{}_{+}\left[W^{\rho}{}_{\mu}W_c^{-1}\hat{\Omega}_{\rho}\right](x+sn_+)\right)\right)^{a}{}_{b}$$

  - Takes us into LLT light-cone gauge: $\check{\Omega}_+ = 0$.
  - $\hat{\Omega}_\rho$ wird gebildet mit $\hat{e}^a_\mu = \hat{e}_s{}^a{}_\mu + \hat{E}_s{}^{\rho a}\hat{e}_{c\rho\mu}$

Soft-theorem

$$\mathcal{A}_{rad} = -\frac{1}{2}\sum_i \left(\frac{\epsilon_{\mu\nu}(k) p_i{}^{\mu} p_i{}^{\nu}}{p_i \cdot k} - \frac{\epsilon_{\mu\nu}(k) p_i{}^{\mu} k_\rho J_i{}^{\nu\rho}}{p_i \cdot k} + \frac{1}{2}\frac{\epsilon_{\mu\nu}(k) k_\rho k_\sigma J_i{}^{\rho\mu} J_i{}^{\sigma\nu}}{p_i \cdot k} \right) \mathcal{A}.$$

$$J_i{}^{\nu\rho} = L_i{}^{\nu\rho} + S_i{}^{\nu\rho}.$$

Orbitaler angular momentum:

$$L_i{}^{\nu\rho} = p_i{}^{[\nu} \frac{\partial}{\partial p_{\rho]i}},$$

Spin angular momentum:

$$S_i{}^{\nu\rho} = -i\sigma^{\nu\rho} = \frac{1}{4}\left[\gamma^\nu, \gamma^\rho\right].$$

Exemplary calculation Soft-theorem I

The Lagrangian $\mathcal{L}^{(1)}$:

$$\mathcal{L}^{(1)} = -\frac{i}{4}\bar{\xi}\left(\left(x_{\perp}{}^{\mu}\omega_{--\mu}\partial_{+}\right) + s_{-\mu}\partial_{\perp}{}^{\mu}\right)\not{n}_{+}\xi.$$

Dann erhalten wir für $\left\{C^{A1}, \mathcal{L}^{(1)}\right\}$ folgenden Ausdruck:

$$-\bar{\xi}\frac{i^2}{4}\int\frac{dq^4}{(2\pi)^4}\left(i\frac{X_{\perp}{}^{\mu}}{2}\left(-ik_{\mu}\epsilon_{--}(k) + ik_{-}\epsilon_{\mu-}(k)(-iq_{+})\right) + \delta^4(q^{\mu} + k_{-}{}^{\mu} - p_{+}{}^{\mu})\epsilon_{-\mu}(k)\right)$$
$$\times(-iq_{\perp}{}^{\mu})\frac{1}{2}\frac{i\not{n}_{+}\not{n}_{-}q_{+}}{q^2 + i\epsilon}q_{\perp}{}^{\sigma}C_{\sigma}^{A1},$$

Exemplary calculation Soft-theorem II

with X^μ given by:

$$X^\mu = \partial^\mu \left[(2\pi)^4 \delta^4 \left(\sum p_{in} - p_{out} \right) \right].$$

The second term vanishes, due to the delta-distribution and the choice of aour reference frame, $p_\perp = 0$. Integrating by parts leads to:

$$\bar{\xi} \frac{-i}{8} \int dq^4 \delta^4 (q^\mu + k_-{}^\mu - p_+{}^\mu) \left(-i k_\mu \epsilon_{--}(k) + i k_- \epsilon_{\mu -}(k)(-i q_+) \right) \frac{1}{2} \frac{i \not{n}_+ \not{n}_- q_+}{q^2 + i\epsilon} \left(\frac{\partial}{\partial q_{\perp\mu}} q_\perp{}^\sigma \right) \times C_\sigma^{A1}.$$

and then to:

$$-\frac{1}{4} \frac{p_+}{k_-} \bar{\xi} \left(k_- \epsilon_{\mu -}(k) - k_\mu \epsilon_{--}(k)) \right) \eta_\perp{}^{\sigma\mu} C_\sigma^{A1}.$$