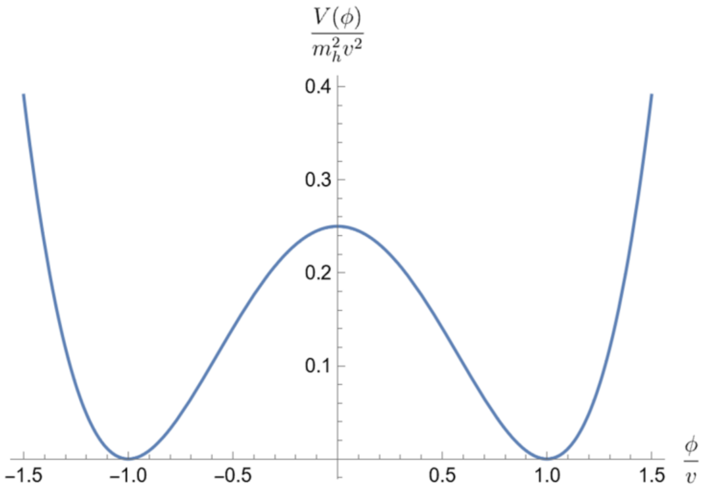
Radiation Emission during the Erasure of Magnetic Monopoles

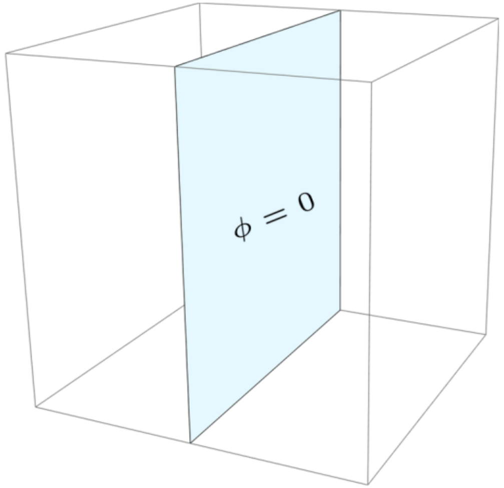
Maximilian Bachmaier
In collaboration with: Prof. Dr. Gia Dvali,
Juan Sebastian Valbuena Bermudez

18. July 2022
MPP PhD Recruiting Workshop

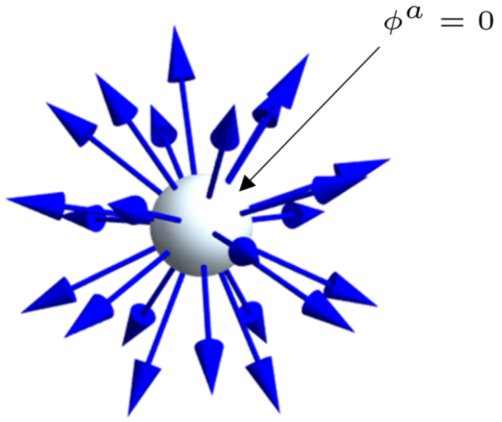
LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MAX-PLANCK-INSTITUT FÜR PHYSIK

Introduction





Domain Wall



Magnetic Monopole

Dvali, Liu and Vachaspati 1997: Domain Walls erase Magnetic Monopoles

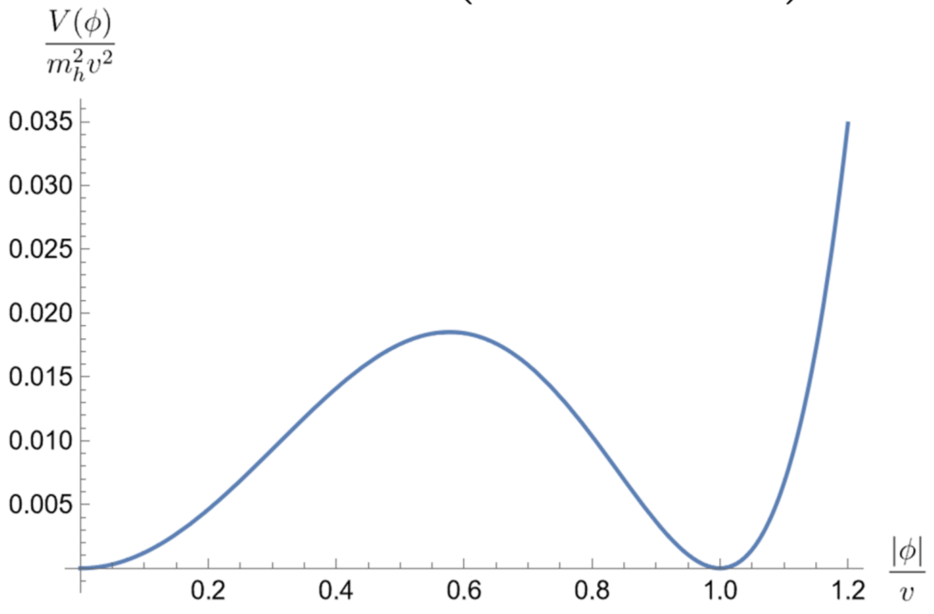
Zeldovich and Khlopov 1978, Preskill 1979: Magnetic Monopole Problem

The Model

We consider a $SU(2)$ gauge theory with ϕ^6 potential

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + \mathrm{Tr}((D_\mu\phi)^\dagger(D^\mu\phi)) - V(\phi)$$

$$V(\phi) = \lambda\left(\mathrm{Tr}(\phi^\dagger\phi) - \frac{v^2}{2}\right)^2 \mathrm{Tr}(\phi^\dagger\phi)$$



Dvali, Nielsen, Tetradis 2008

$$\langle\phi^a\phi^a\rangle = \begin{cases} 0 \rightarrow \text{unbroken vacuum} \\ v^2 \rightarrow \text{broken vacuum} \end{cases}$$

Disconnected Vacuum Manifold
$\rightarrow$ Domain Walls

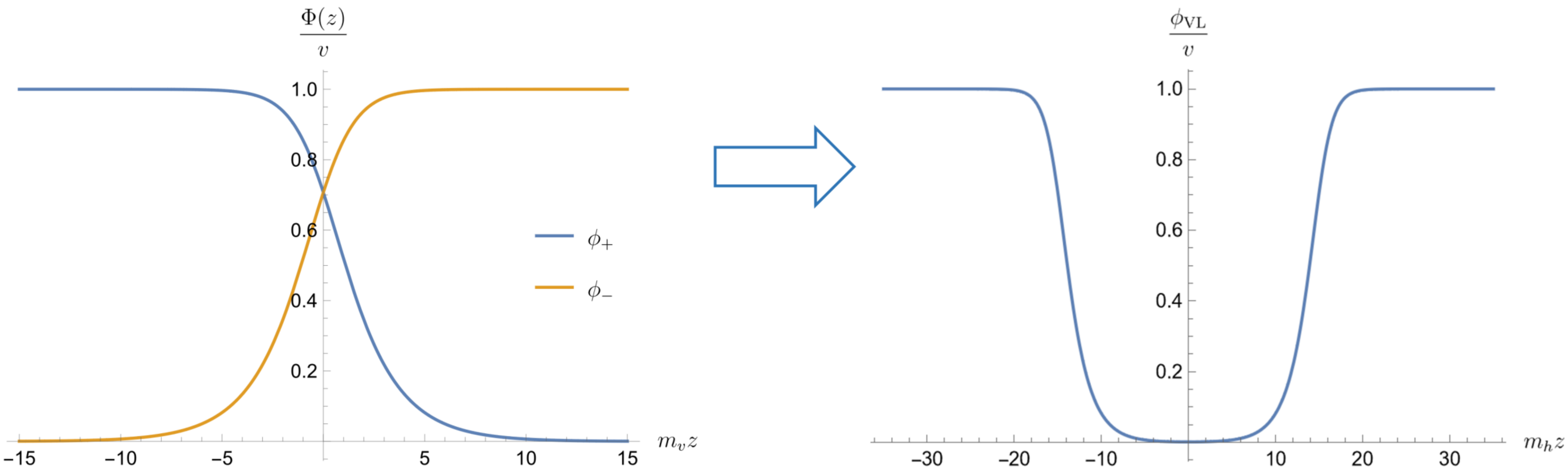
$\langle\phi^a\phi^a\rangle = v^2 \rightarrow SU(2)$ breaks down to $U(1)$
$\rightarrow$ Magnetic Monopoles

Domain Wall Solution

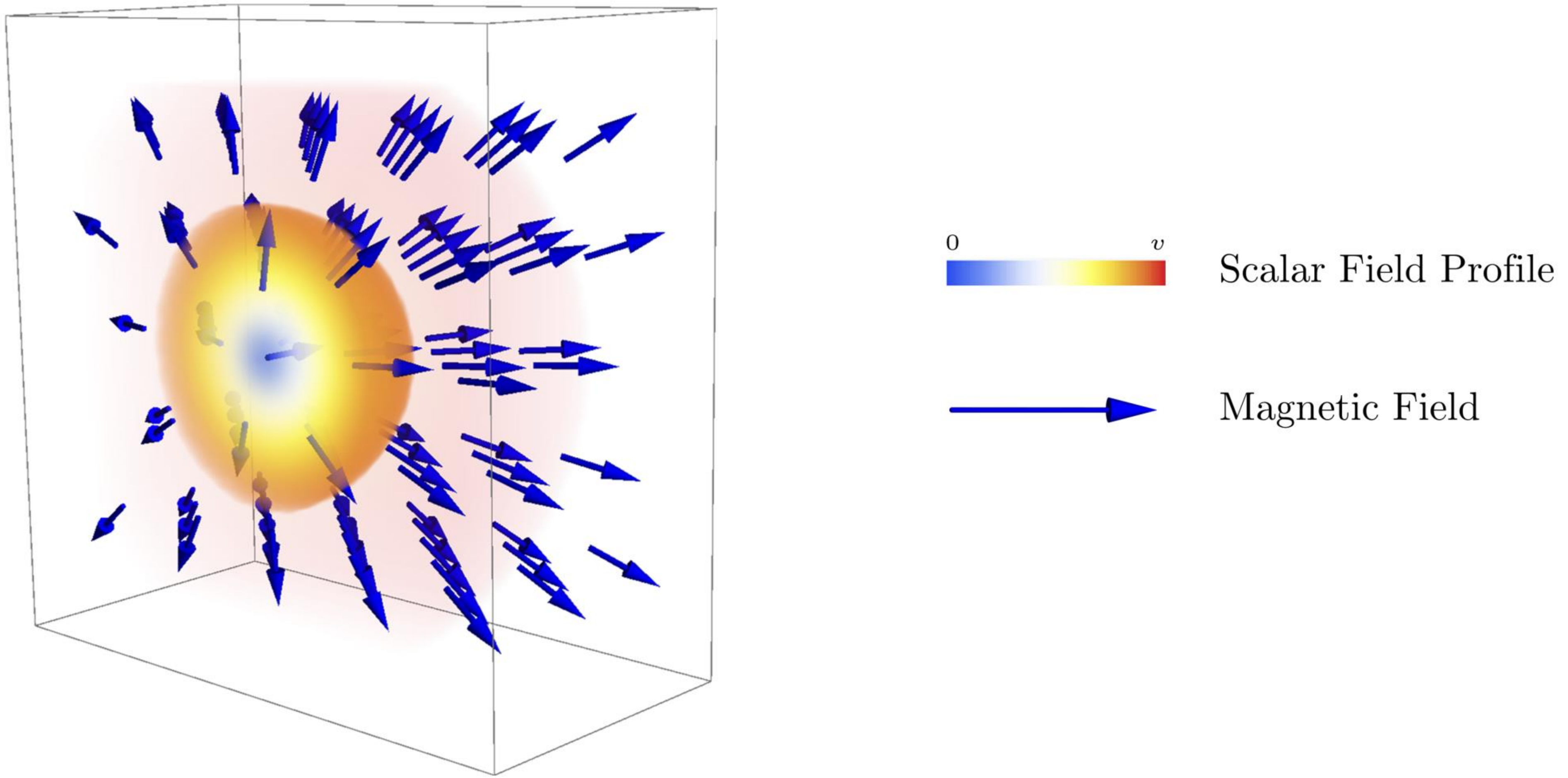
Two domain wall solutions are

$$\phi_\pm = \frac{v}{\sqrt{1 + e^{\pm m_h z}}}$$

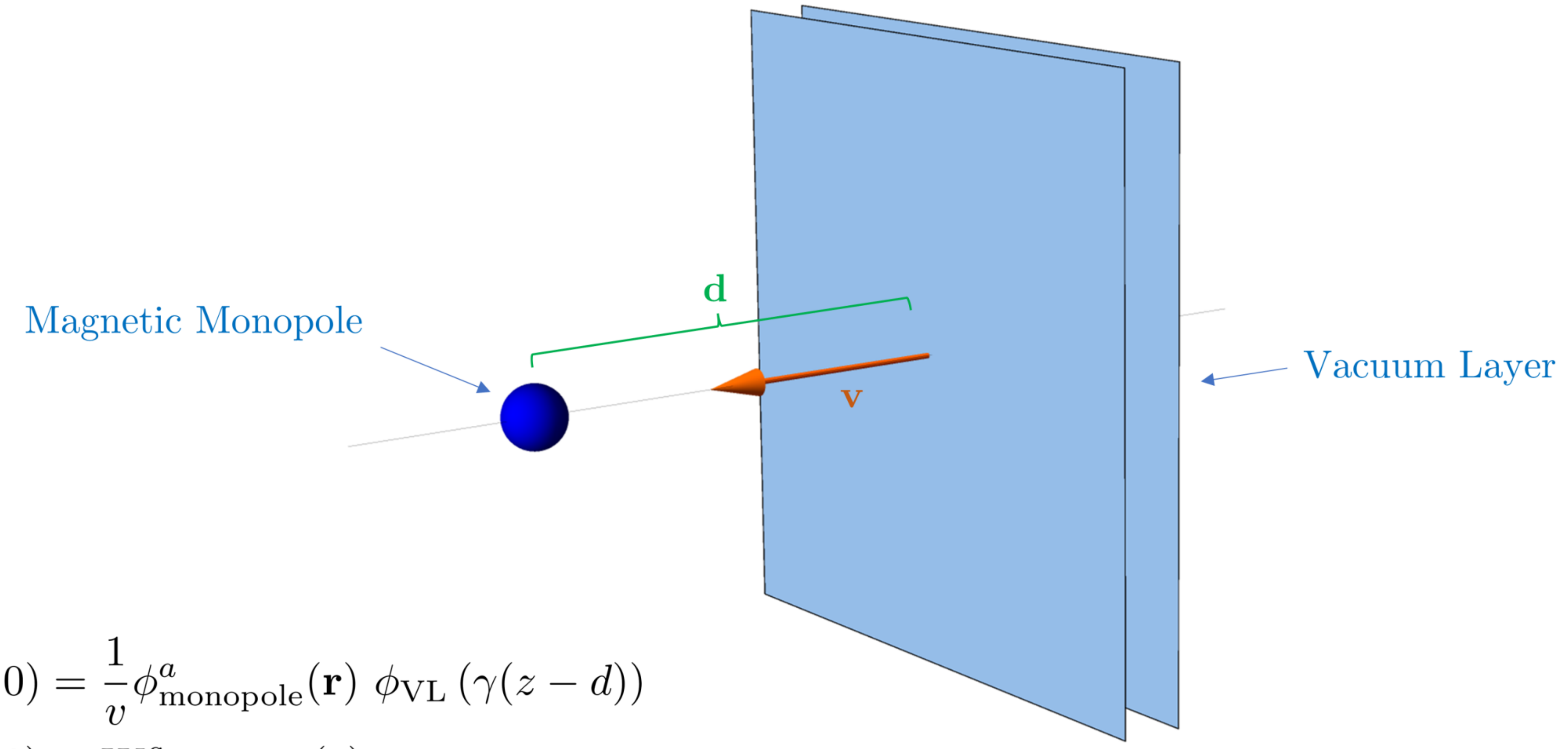
SU(2) Invariant Vacuum Layer



‘t Hooft – Polyakov Magnetic Monopole



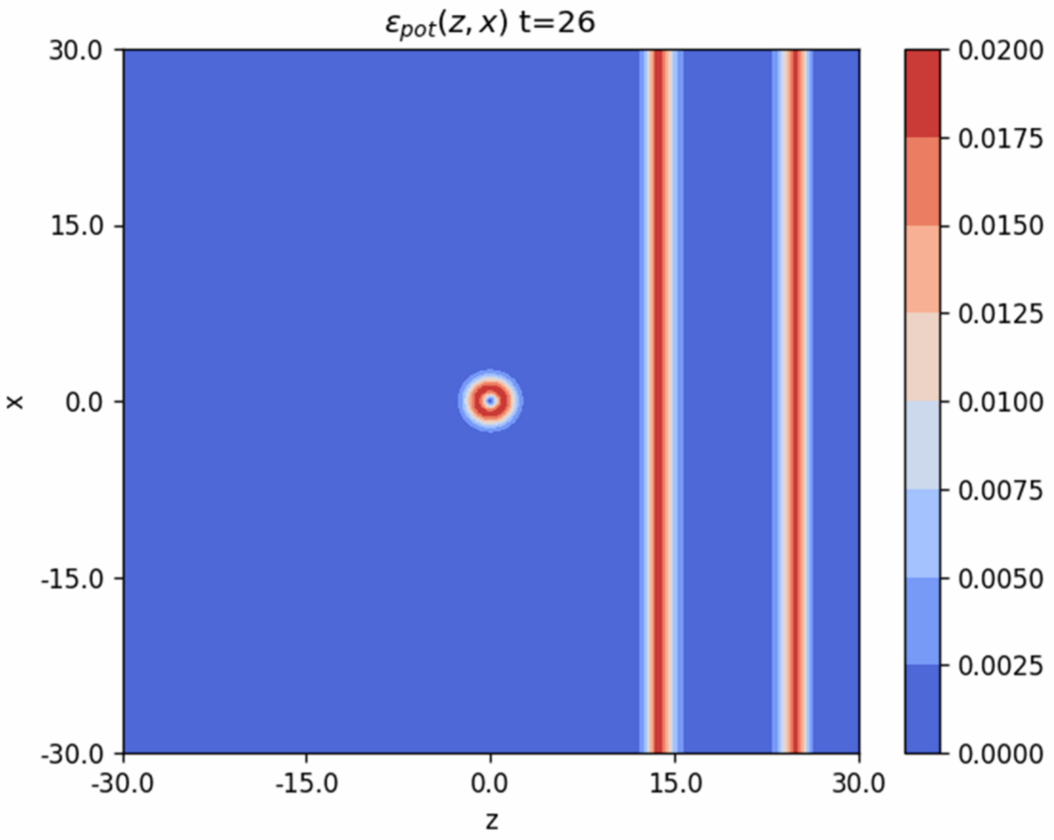
Initial Configuration



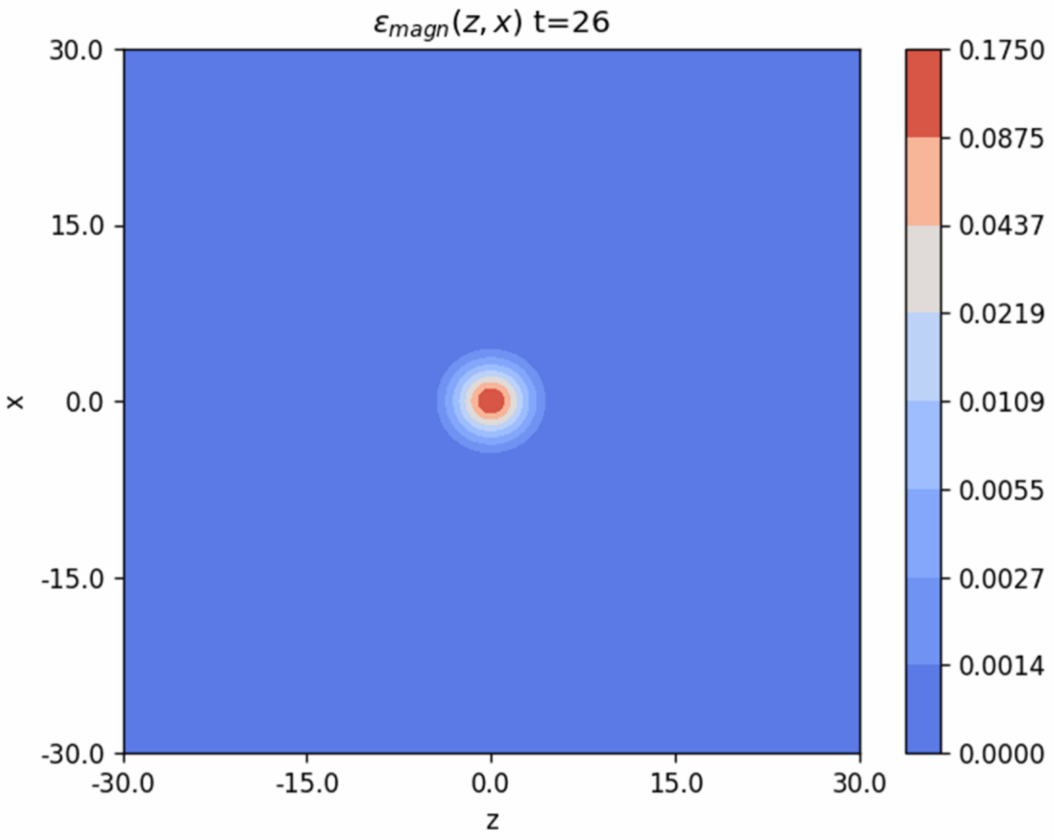
$$\phi^a(\mathbf{r}, t=0) = \frac{1}{v}\phi^a_{\text{monopole}}(\mathbf{r})\ \phi_{\text{VL}}\left(\gamma(z-d)\right)$$
$$W^a_i(\mathbf{r}, t=0) = W^a_{\text{monopole},i}(\mathbf{r})$$

Erasure of the Monopole

Potential Energy Density

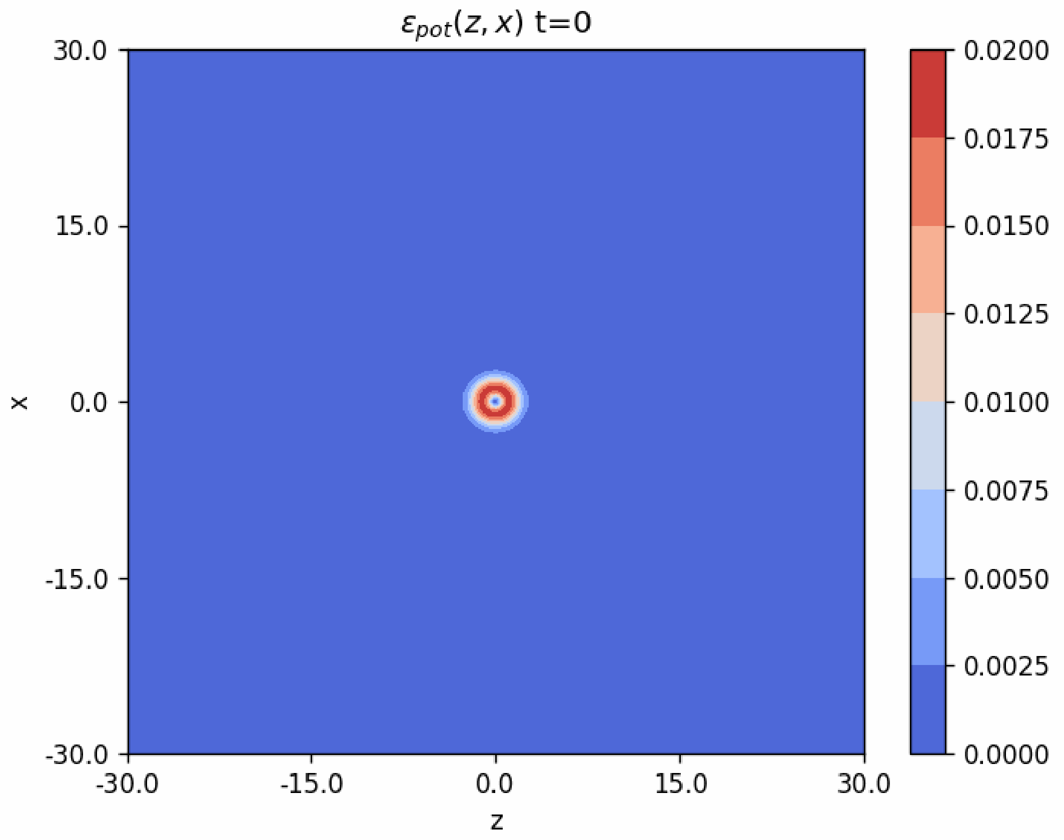


Magnetic Energy Density

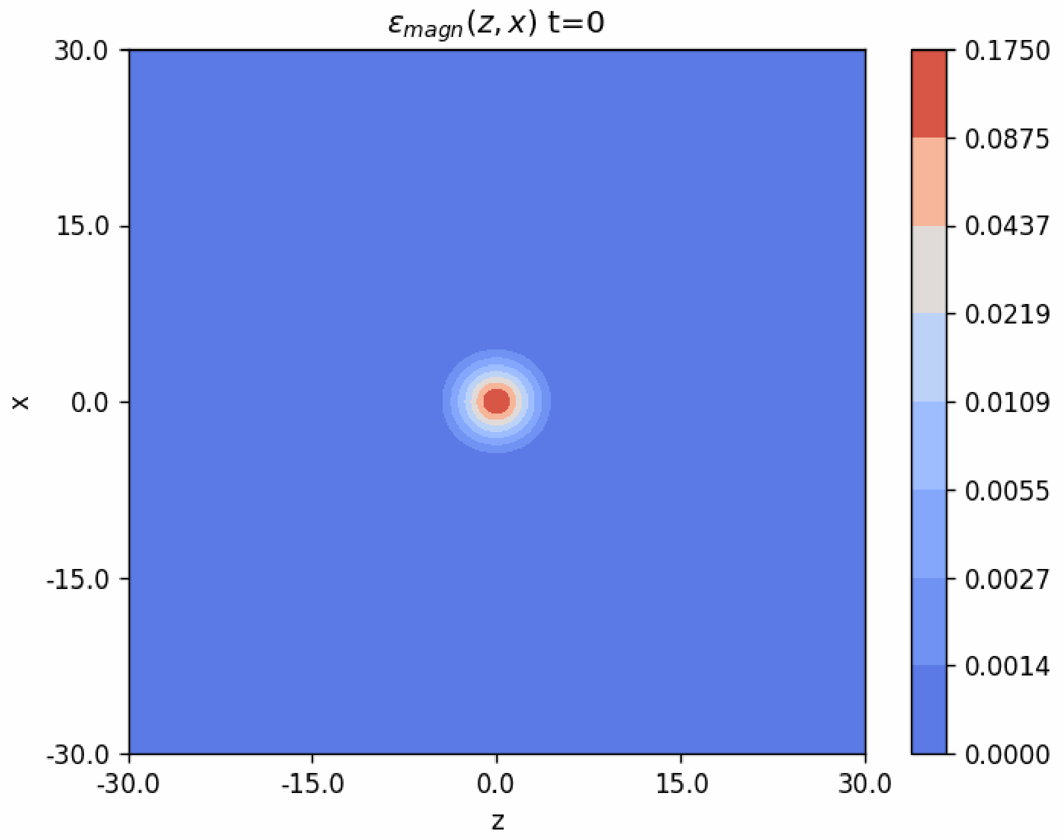


Erasure of the Monopole

Potential Energy Density

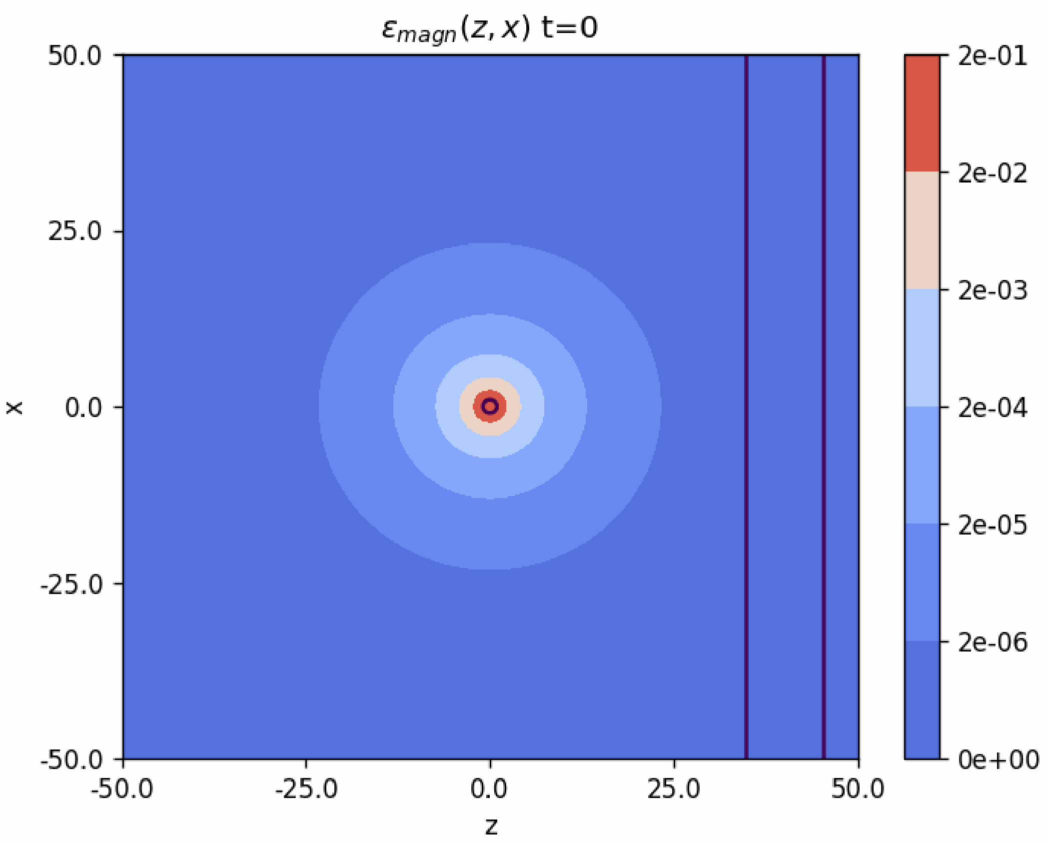


Magnetic Energy Density

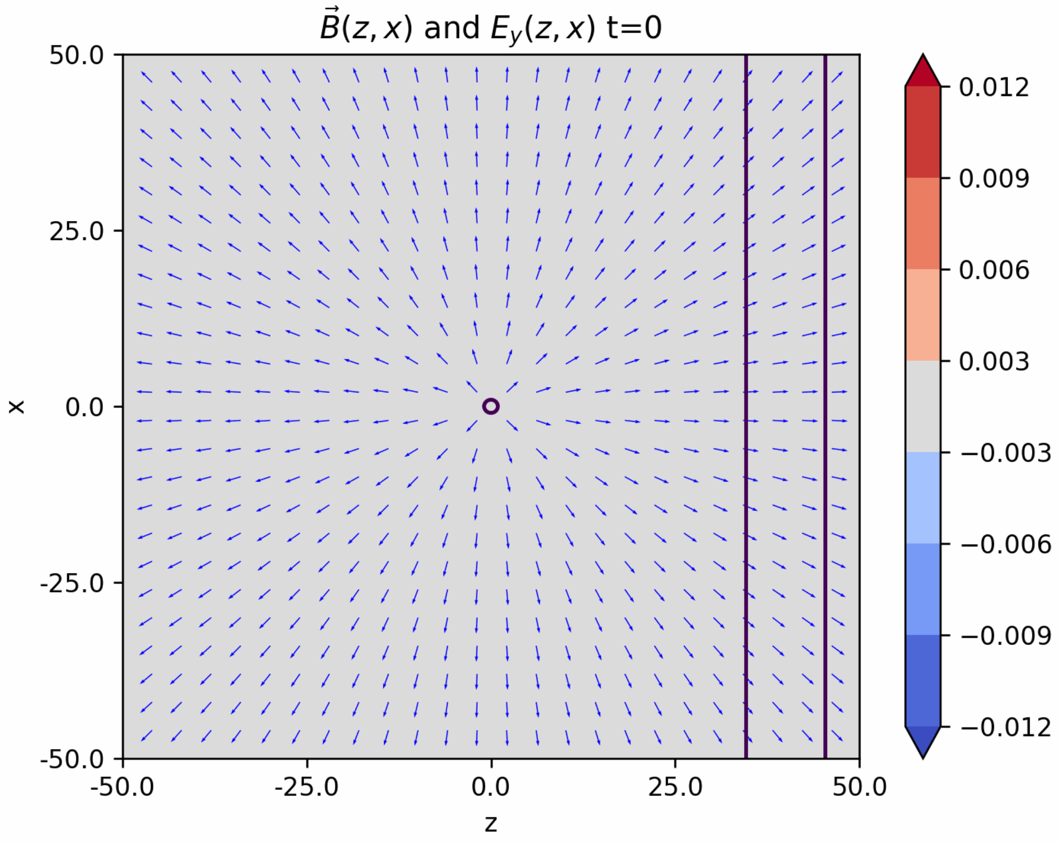


Erasure of the Monopole

Magnetic Energy Density

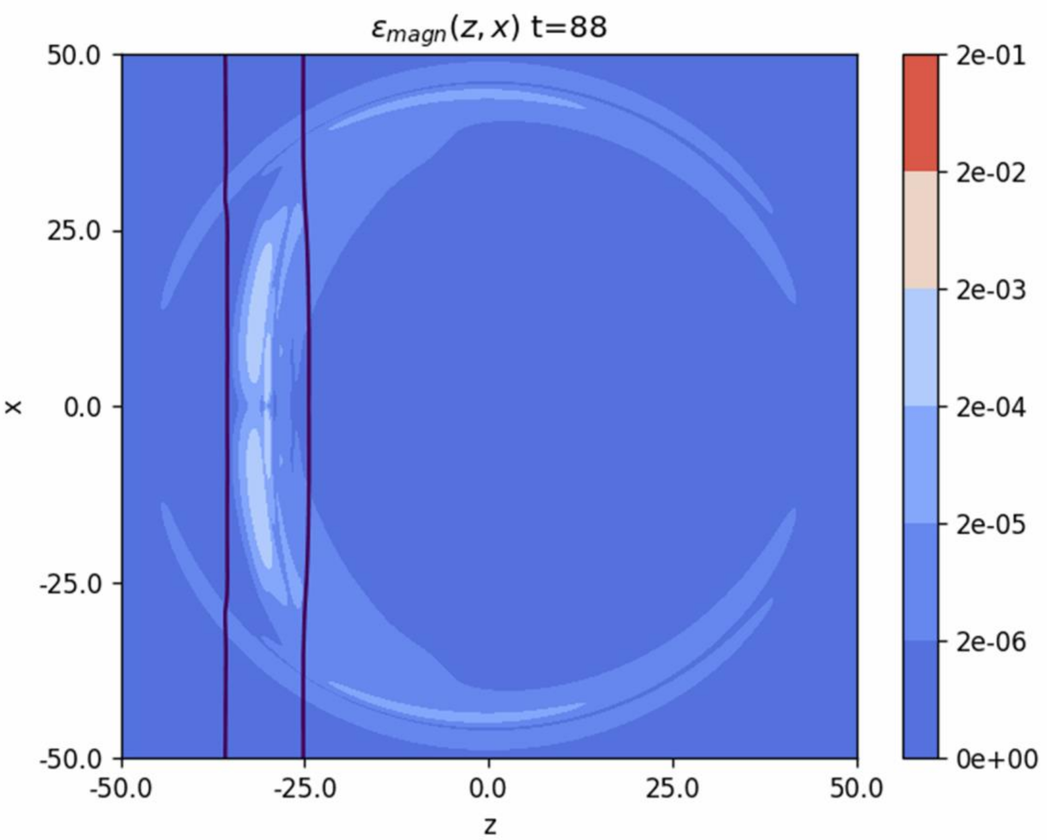


Magnetic and Electric Field

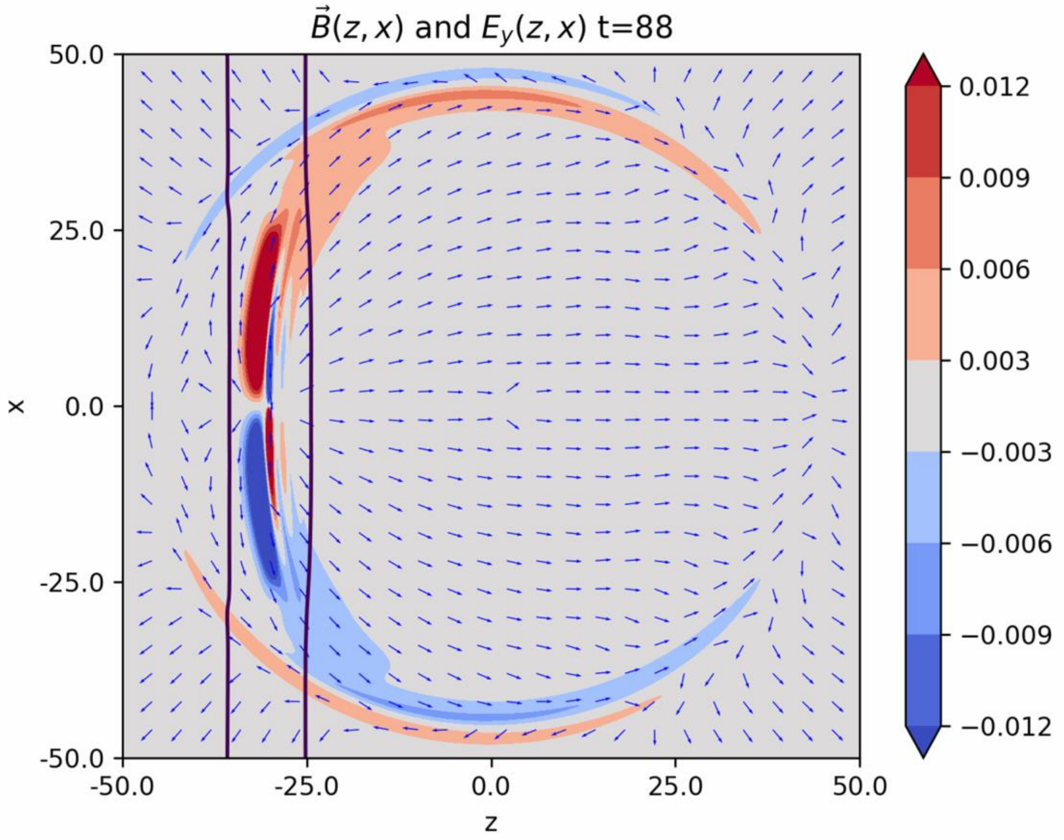


Erasure of the Monopole

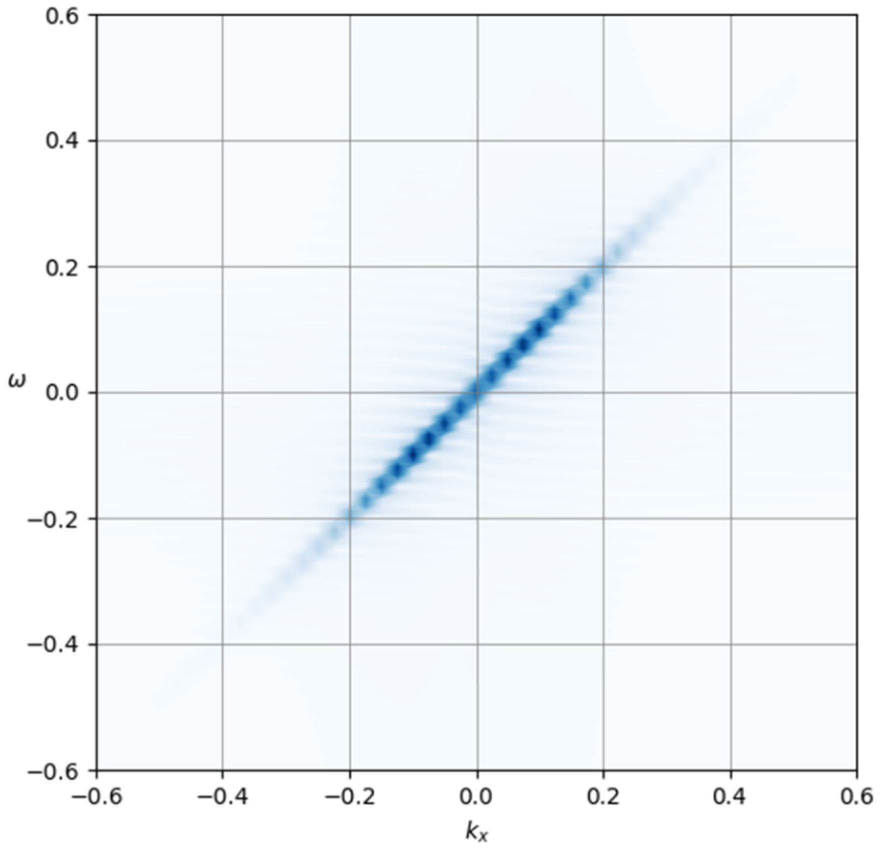
Magnetic Energy Density



Magnetic and Electric Field



Analysing the Radiation



Fourier analysis reveals the dispersion relation

$$\omega = k$$

Summary

erasure mechanism checked for the $SU(2)$ gauge theory with ϕ^6 potential
$\rightarrow$ solution for magnetic monopole problem
$\rightarrow$ phenomenon relevant for cosmology beside monopole problem

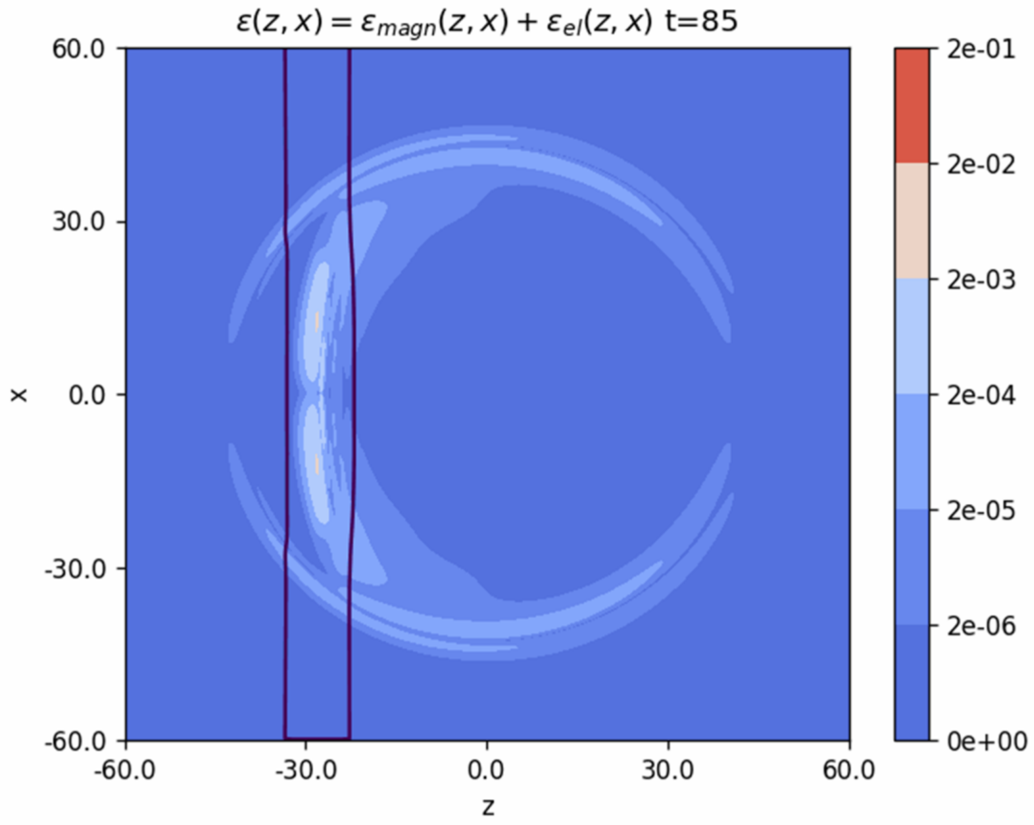
erasure leads to electromagnetic radiation emission
$\rightarrow$ observable for future experiments?

Thank you!

Backup and more Details

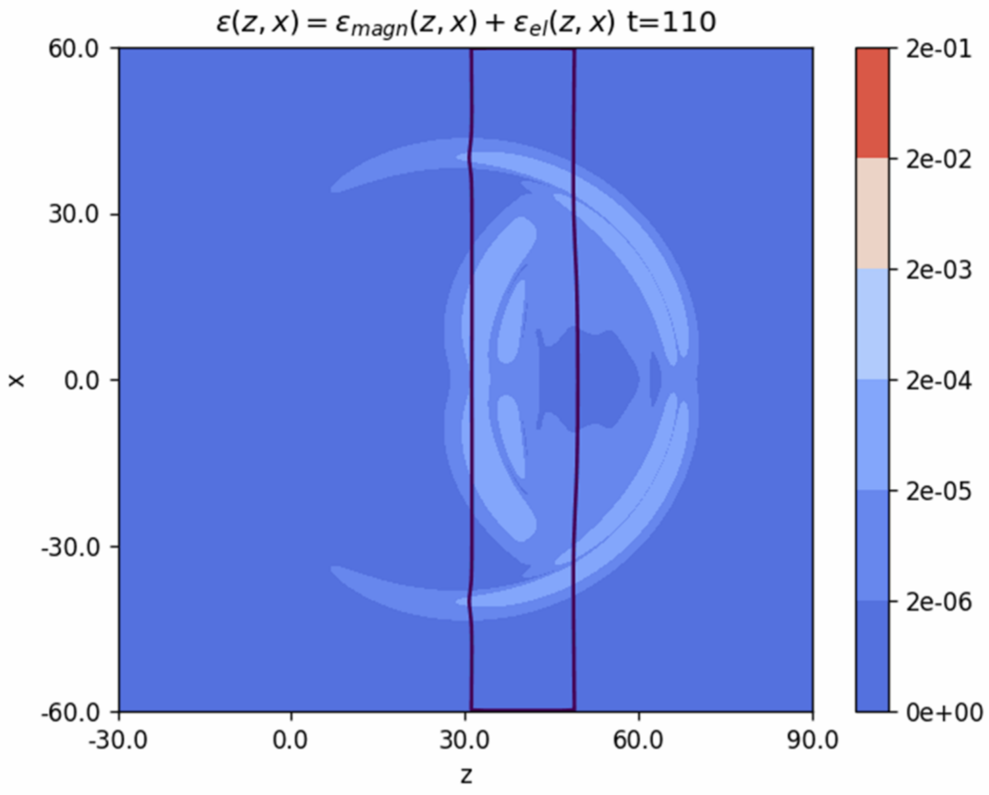
Direction of Radiation Emission

$v_{\text{Monopole}} = 0$ and $v_{\text{VL}} = -0.8$

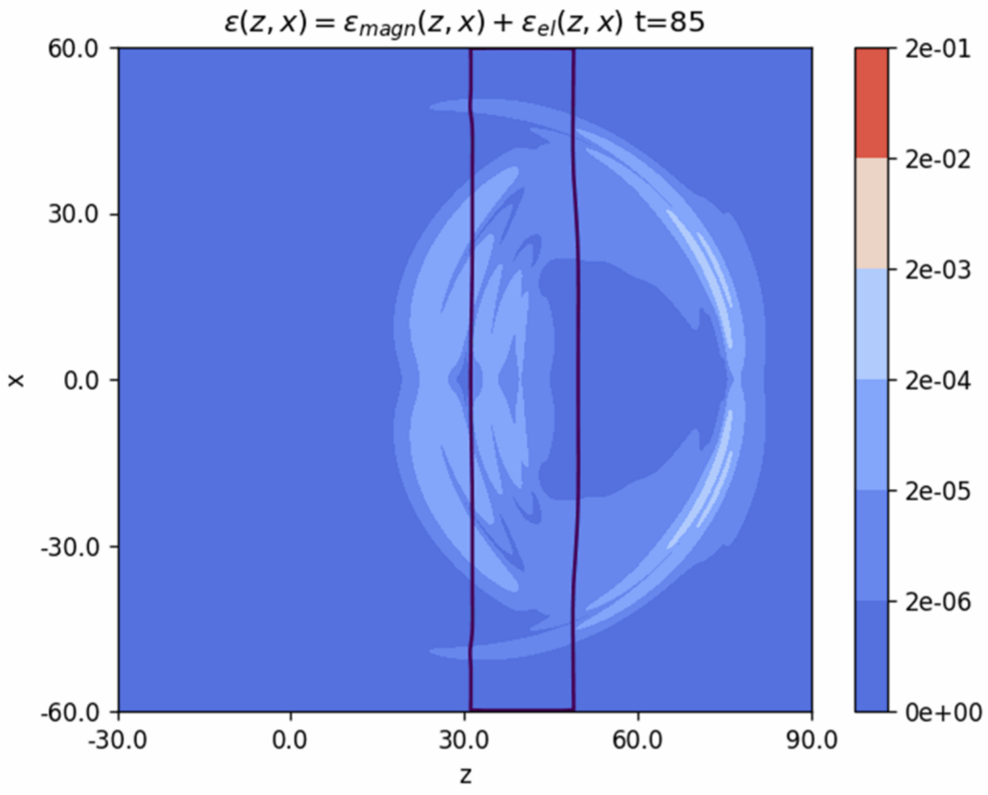


Direction of Radiation Emission

$v_{\text{Monopole}} = 0.4$ and $v_{\text{VL}} = 0$

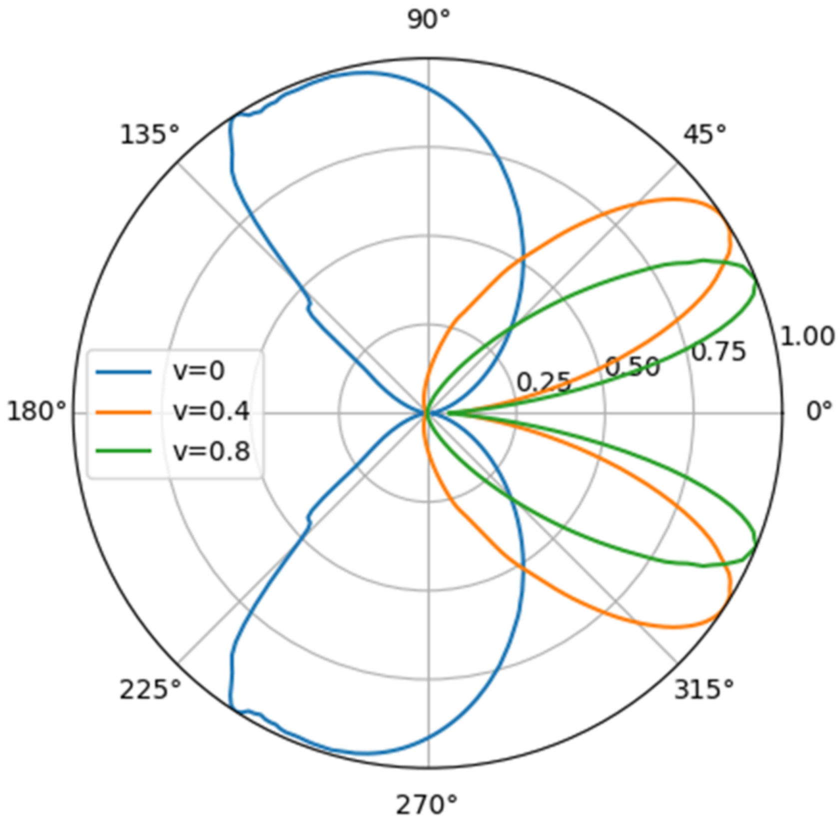


$v_{\text{Monopole}} = 0.8$ and $v_{\text{VL}} = 0$

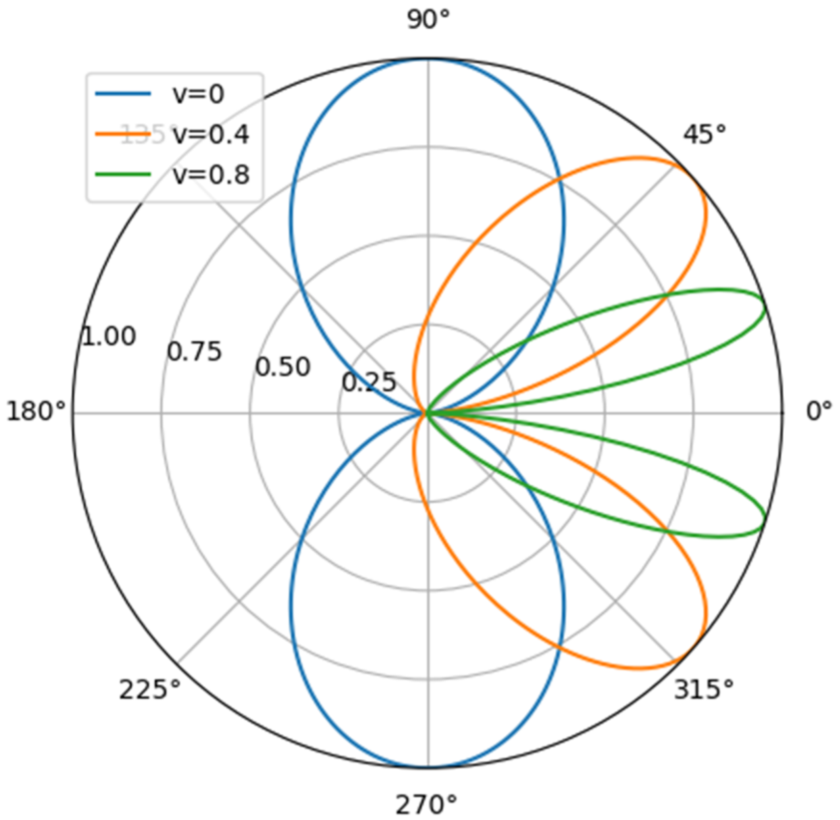


Direction of Radiation Emission

Erasure of Magnetic Monopole



Acceleration of Magnetic Monopole (Theory)



Magnetic Monopole Profile

We can use the 't Hooft magnetic monopole ansatz to solve the field equations[1]

$$W_i^a = \varepsilon_{aij} \frac{r_j}{r^2} \frac{1}{g} (1 - K(r))$$
$$W_0^a = 0$$
$$\phi^a = \frac{r^a}{r^2} \frac{1}{g} H(r)$$

I found the boundary conditions

$$K(r) \xrightarrow{r \to 0} 1 \qquad K(r) \xrightarrow{r \to \infty} 0$$
$$K'(r) \xrightarrow{r \to 0} 0$$
$$\frac{H(r)}{m_v r} \xrightarrow{r \to 0} 0 \qquad \frac{H(r)}{m_v r} \xrightarrow{r \to \infty} 1$$

[1]'t Hooft – Magnetic Monopoles in Unified Gauge Theories and
[2]Polyakov – Particle Spectrum in Quantum Field Theory

Magnetic Monopole Profile

We can use the 't Hooft magnetic monopole ansatz to solve the field equations

$$W_i^a = \varepsilon_{aij} \frac{r_j}{r^2} \frac{1}{g}(1 - K(r))$$
$$W_0^a = 0$$
$$\phi^a = \frac{r^a}{r^2} \frac{1}{g} H(r)$$

With this the field equations become

$$K''(r) = \frac{1}{r^2} \left(K(r)^3 - K(r) + H(r)^2 K(r) \right)$$
$$H''(r) = \frac{2}{r^2} H(r) K(r)^2 + m_h^2 \left(\frac{3}{4} \frac{1}{r^4 m_v^4} H(r)^5 - \frac{1}{r^2 m_v^2} H(r)^3 + \frac{1}{4} H(r) \right)$$

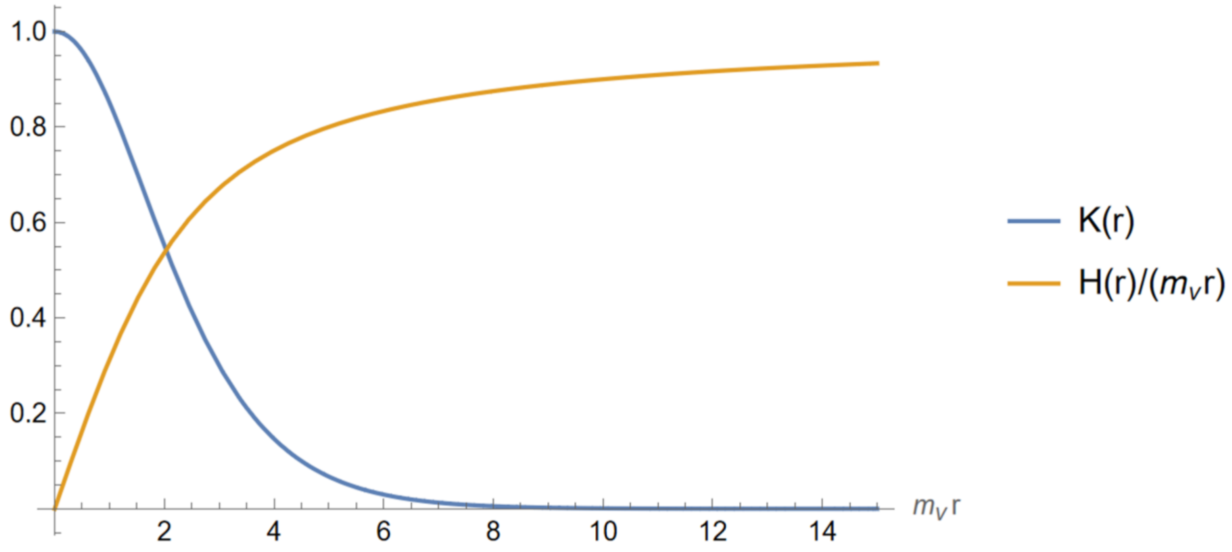
[1]'t Hooft – Magnetic Monopoles in Unified Gauge Theories and
[2]Polyakov – Particle Spectrum in Quantum Field Theory

Magnetic Monopole Profile

The equations can be solved exactly in the BPS limit[1] ($m_h \to 0$) by

$$K(r) = \frac{m_v r}{\sinh(m_v r)} \quad (1)$$

$$H(r) = \frac{m_v r}{\tanh(m_v r)} - 1 \quad (2)$$

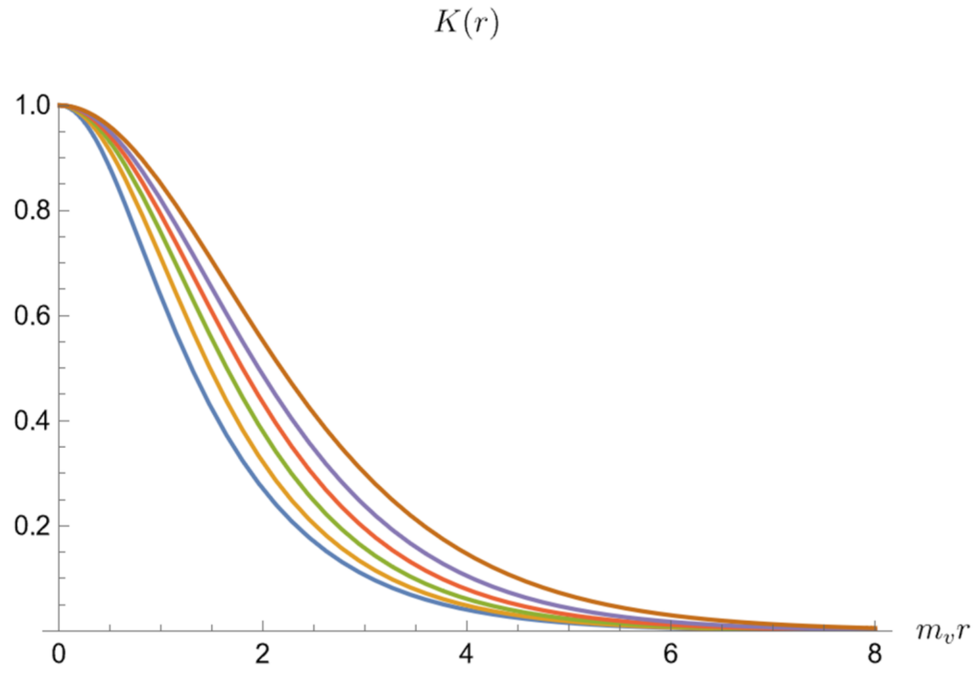


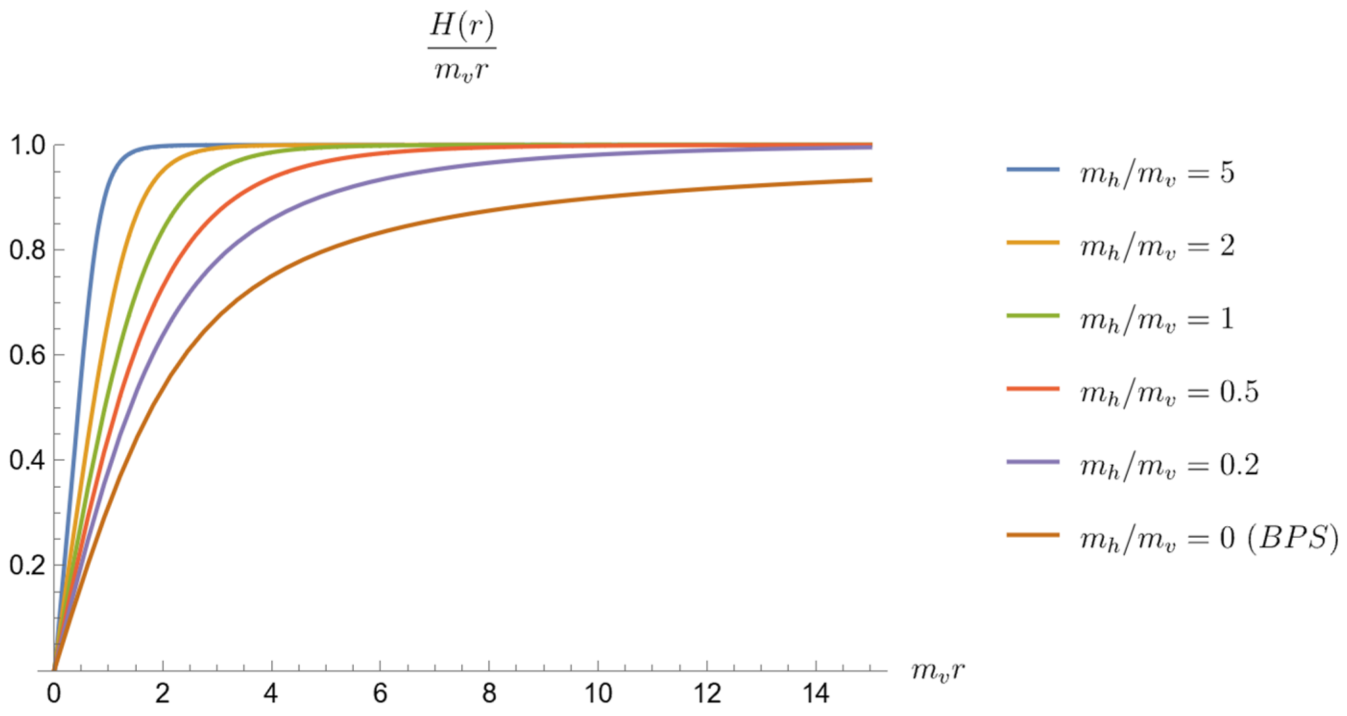
[1]Prasad, Sommerfield – Exact Solution for the t Hooft Monopole and Julia-Zee Dyon

Magnetic Monopole Profile

I tried two numerical methods to find the profile functions for $m_h \neq 0$:

- Shooting Method
- Iterative Method (starting with BPS limit)

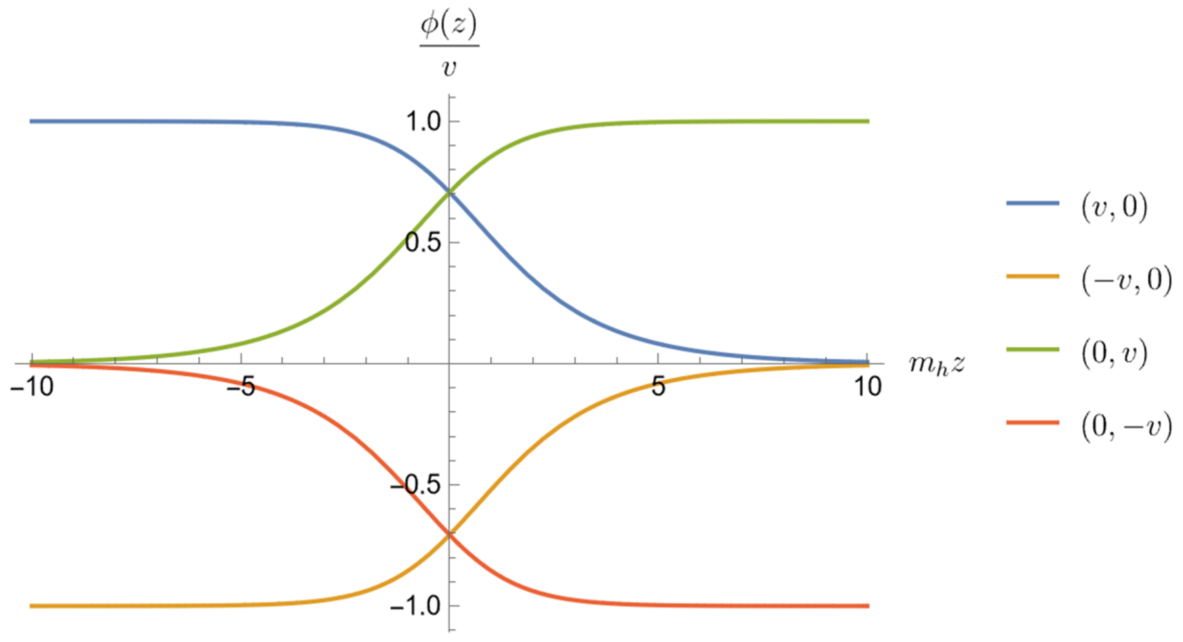


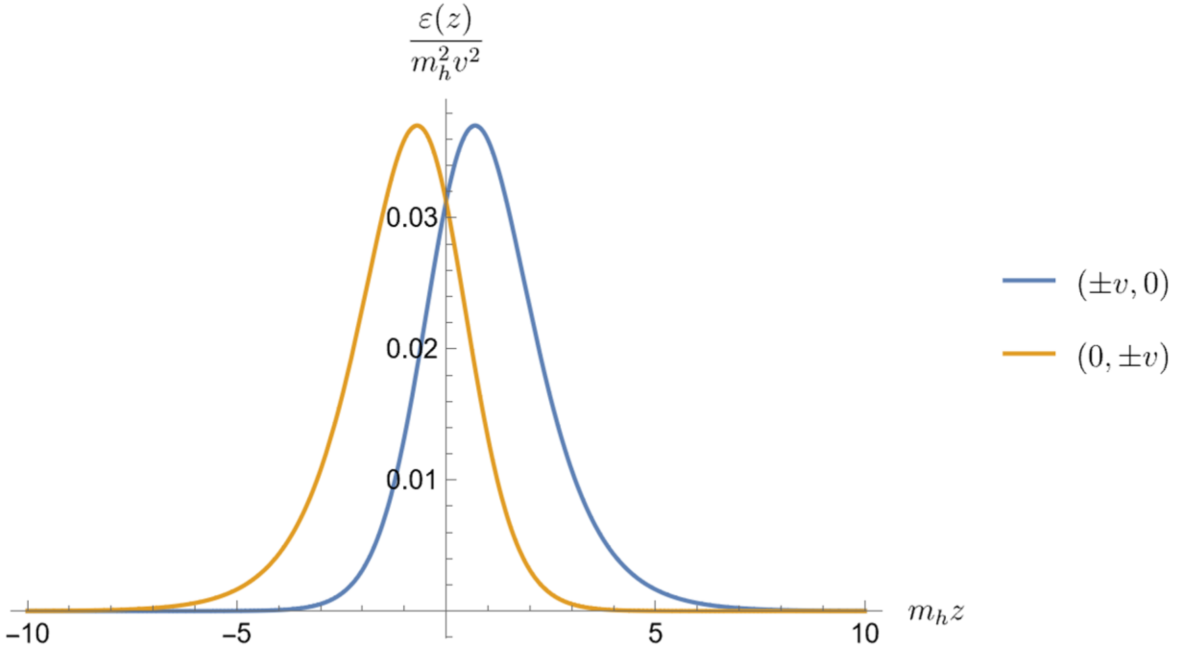


Domain Wall Solution

The domain wall solutions are

$$\phi_{(\pm v,0)} = \frac{\pm v}{\sqrt{1+e^{m_h z}}} \qquad \phi_{(0,\pm v)} = \frac{\pm v}{\sqrt{1+e^{-m_h z}}}$$

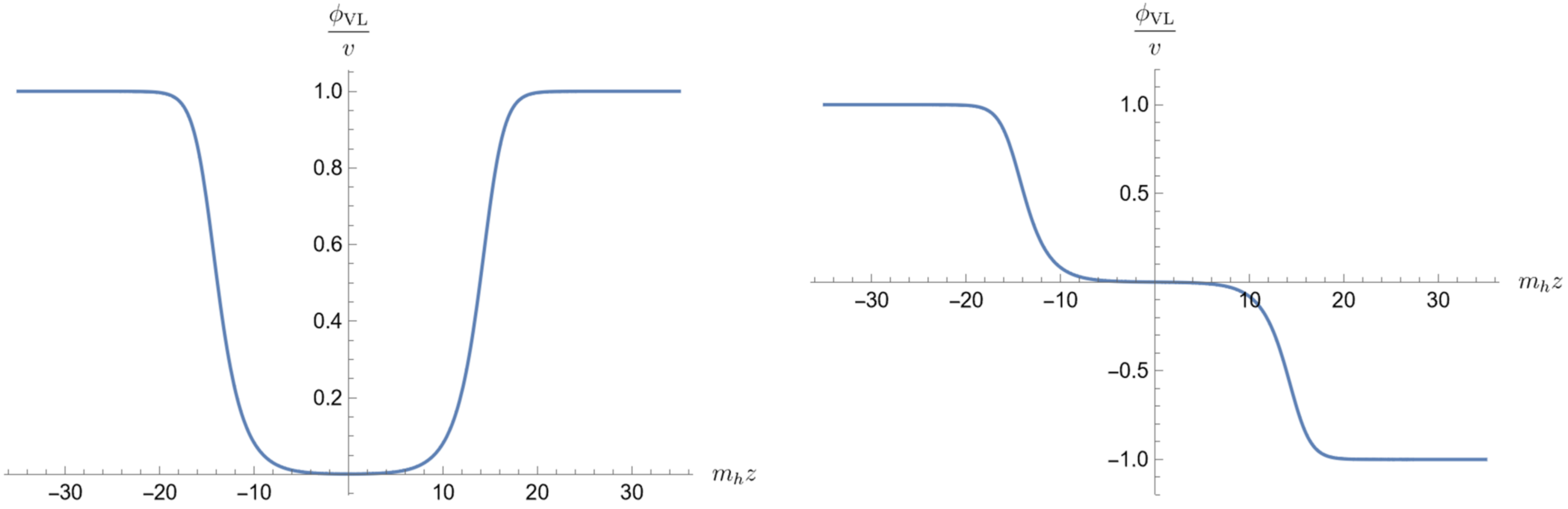




Domain Wall Solution

With the domain wall solutions we can build a $SU(2)$ invariant vacuum layer

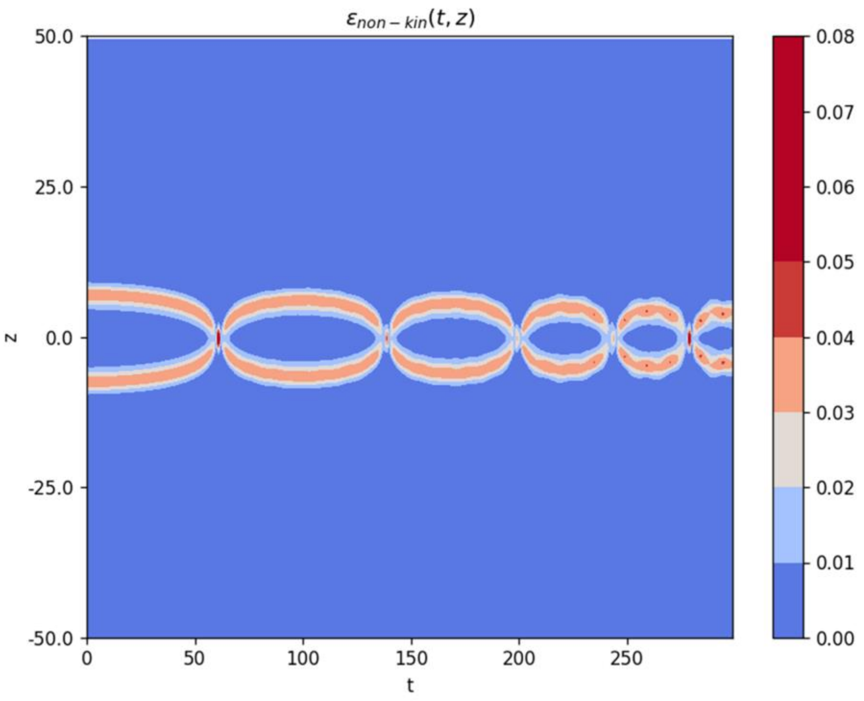
$$\phi_{(v,0,v)}(z) = \phi_{(v,0)}(z+h) + \phi_{(0,v)}(z-h)$$
$$\phi_{(v,0,-v)}(z) = \phi_{(v,0)}(z+h) + \phi_{(0,-v)}(z-h)$$



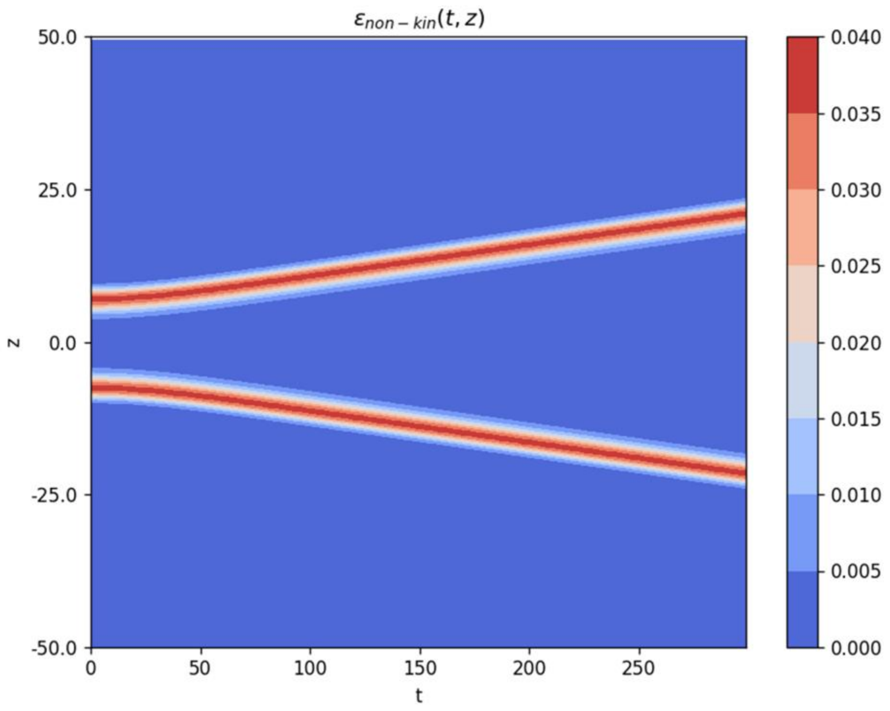
Domain Wall Solution

Vacuum layers are unstable (attracting/repelling walls), which I demonstrated in a numerical simulation

(v,0,v)



(v,0,-v)



In the later simulations we need to choose a large enough distance to avoid this behavior

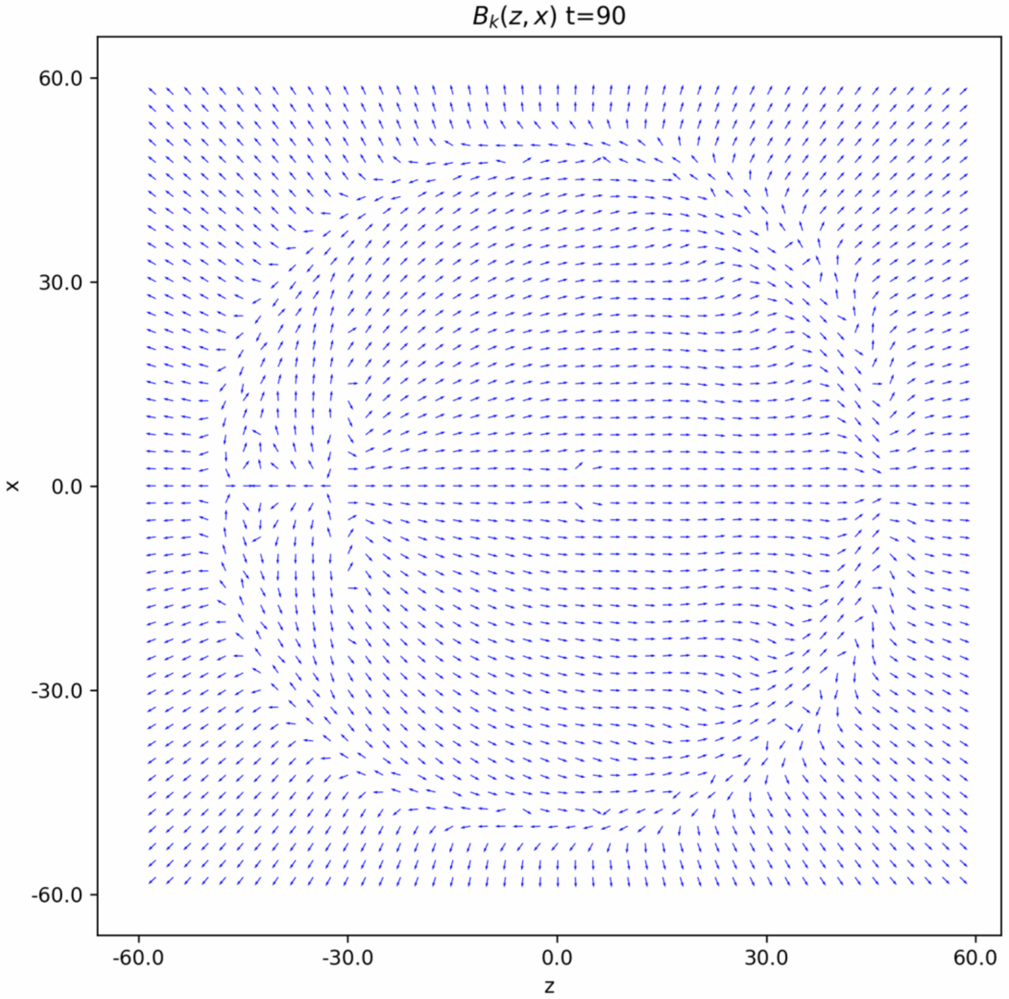
Initial Configuration

For the simulations we chose the time gauge $W_t^a = 0$ and the following initial ansatz

$$
\begin{aligned}
\phi^a(\mathbf{r}, t=0) &= \frac{1}{v}\phi^a_{\text{monopole}}(\mathbf{r})\ \phi_{\text{VL}}\left(\gamma(z-d)\right) \\
\partial_t\phi^a(\mathbf{r}, t=0) &= \frac{1}{v}\phi^a_{\text{monopole}}(\mathbf{r})\ \partial_t\phi_{\text{VL}}(\gamma(z-d-ut))\Big|_{t=0} \\
W_i^a(\mathbf{r}, t=0) &= W^a_{\text{monopole},i}(\mathbf{r}) \\
\partial_t W_i^a(\mathbf{r}, t=0) &= 0
\end{aligned}
$$

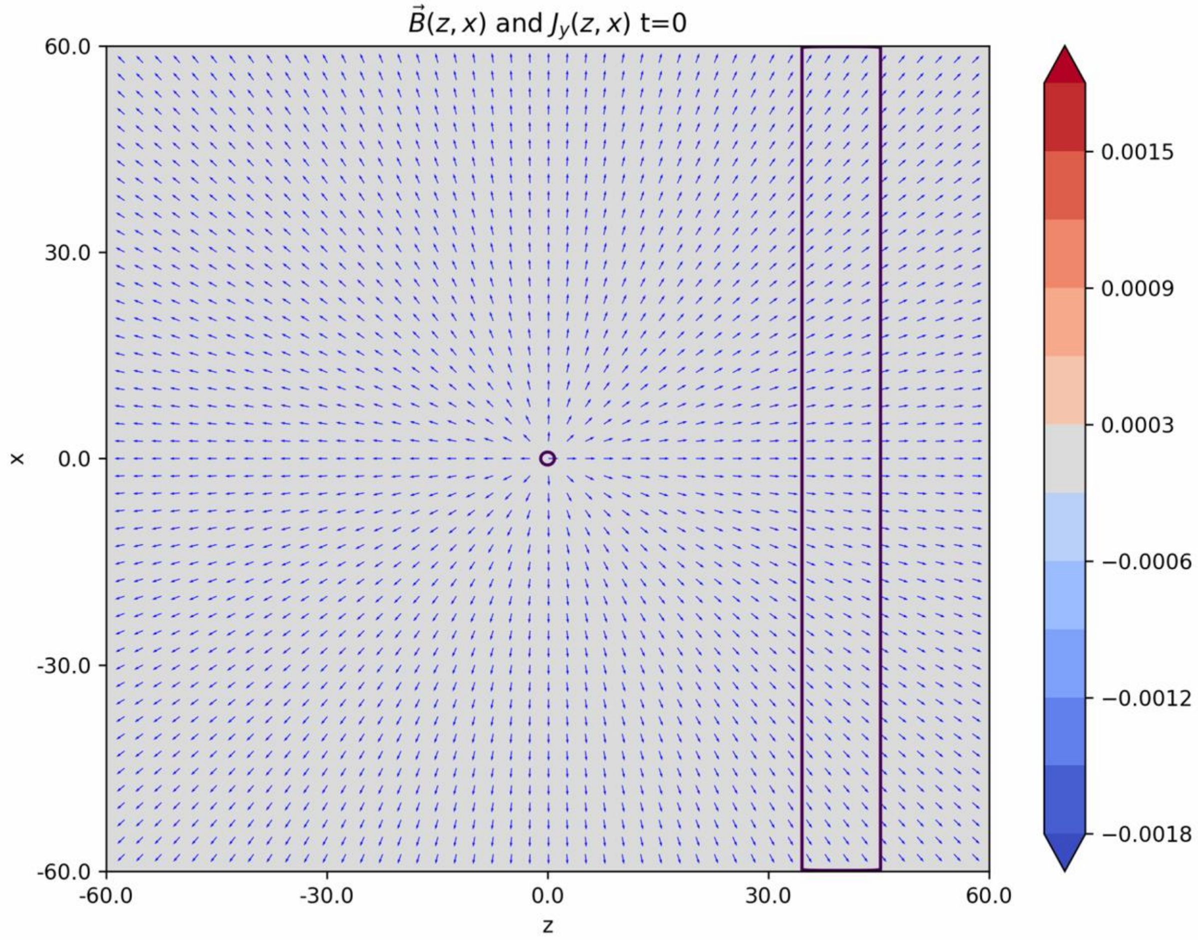
u: velocity of VL
γ: Lorentz factor of VL
d: distance between monopole and centre of VL

Direction of the Magnetic Field



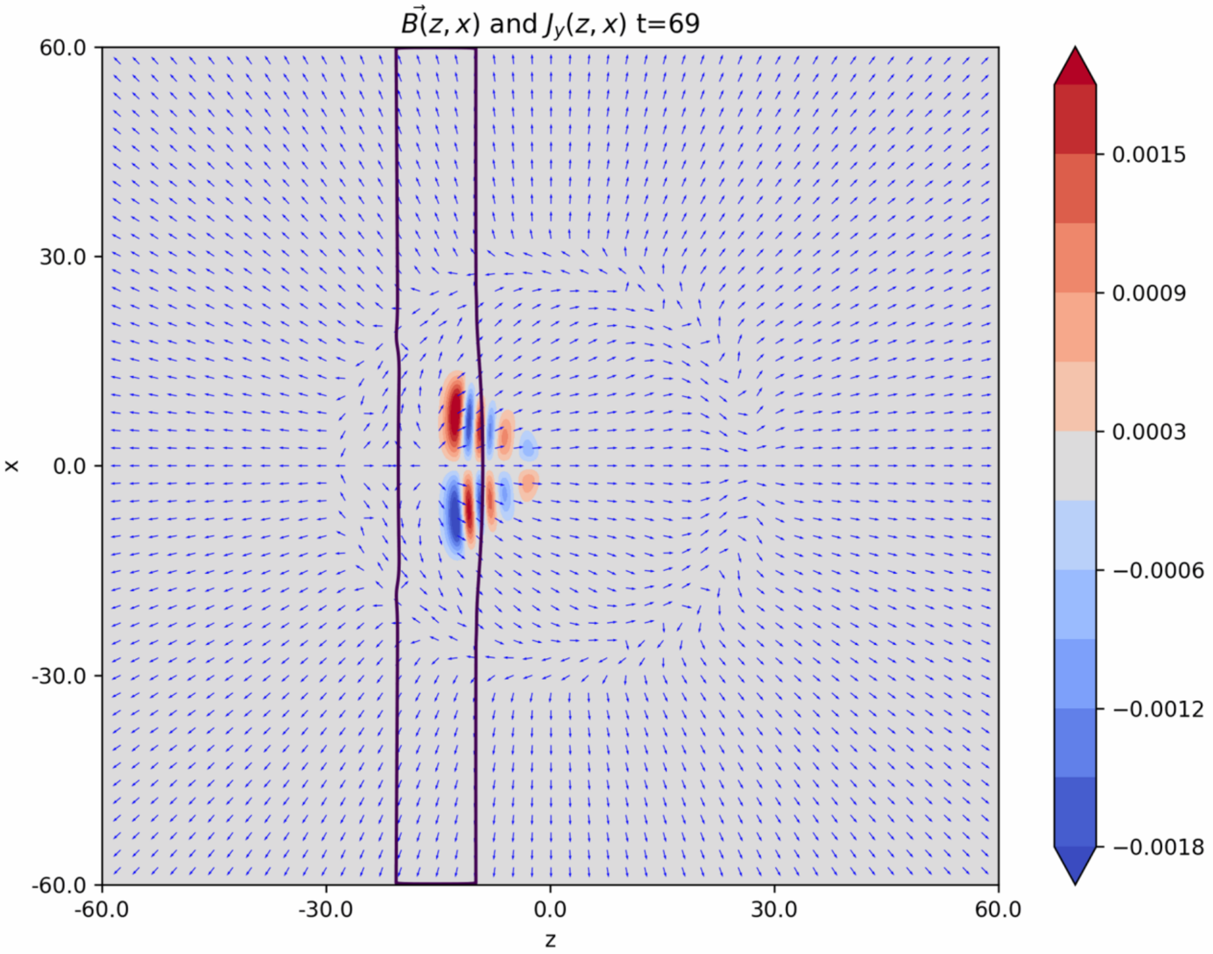
Why the magnetic field is perpendicular to the domain wall after the collision?

Direction of the Magnetic Field



$$J_i^{\mathrm{U}(1)} \sim \varepsilon^{abc} W_j^b G_{ji}^c \frac{\Phi^a}{\sqrt{\Phi^b \Phi^b}}$$

Direction of the Magnetic Field



In the y=0 plane, the current points completely in the y-direction (other components are zero)

→ current flows in circles parallel to the DW

→ explains why arrows are perpendicular to the DW

References

Dvali, Liu, Vachaspati – Sweeping Away the Monopole Problem

Dvali, Nielsen, Tetradis – Localization of Gauge Fields and Monopole Tunnelling

‘t Hooft – Magnetic Monopoles in Unified Gauge Theories and

Polyakov – Particle Spectrum in Quantum Field Theory

Prasad, Sommerfield – Exact Solution for the t Hooft Monopole and Julia-Zee Dyon

Preskill – Cosmological Production of Superheavy Magnetic Monopoles

Shnir – Topological and Non-topological solitons in scalar field theories

Valbuena Bermudez – Erasure of Defects: Vortex Unwinding by Domain Wall Sweeping

Zeldovich, Khlopov – On the Concentration of Relic Magnetic Monopoles in the Universe