# Precision Flavour Physics with $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$

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in collaboration with
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based on arXiv:0909.1512v1

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# Motivation

## $b \to s\nu\bar{\nu}$ transitions

- Induced at short distances (weak scale)
- FCNC processes in the SM only beyond the tree level
- Further suppressed by the off-diagonal entries of the CKM matrix
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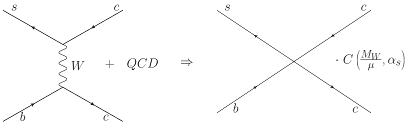
## Combined analysis of the processes $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$

- Both decays feature similar long distance dynamics
- Does this allow us to construct precision observables?

# Outline

1. Review: Effective Weak Hamiltonian
2. Hadronic Physics
   - QCD Factorization
   - Form factors
   - Parametrization
3. Numerical Results
   - Integrated branching fractions
   - Precision observables
   - Experimental status
4. Summary

# Effective Weak Hamiltonian

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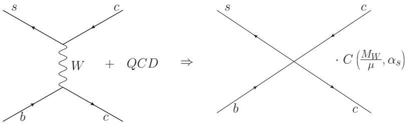


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- Along with the corresponding coupling constants, the Wilson Coefficients $C_i(\mu)$, the effective Hamiltonian is defined

$$H_{eff} \;=\; -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[ \sum_{i=1}^{10} \; C_i Q_i + C_\nu Q_\nu \right] \;+\; \mathcal{O}(p^2/M_W^2)$$

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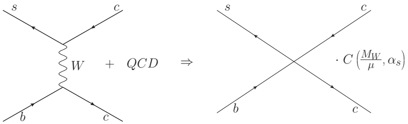


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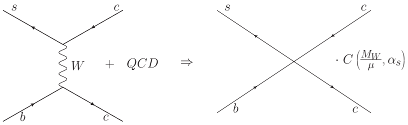


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- This procedure separates short-distance (coefficients) and long-distance physics (operator matrix elements) at the scale $\mu$.

# QCD Factorization

## Factorization Formula

$$\langle \bar{K}\ l^+ l^- | H_{eff} | \bar{B} \rangle \ = \ C \cdot f + \Phi_B \otimes T \otimes \Phi_K \ + \ \mathcal{O}(\Lambda_{QCD}/m_B)$$

- In the heavy-quark limit short-distance contributions $\mathcal{O} \sim m_B$ and long-distance contributions $\mathcal{O} \sim \Lambda_{QCD}$ are disentangled.

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- In the kinematical region of high $q^2 \sim m_b^2$ the theoretical framework to address the non-local terms is an operator product expansion.

## Hadronic Matrix Elements for $B \to K$ transitions

$$\langle K(p_K)|V^\mu|B(p)\rangle = f_+(q^2)\left[p^\mu + p_K^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu\right] + f_0(q^2)\,\frac{m_B^2 - m_K^2}{q^2} q^\mu$$

$$q_\nu \langle K(p_K)|T^{\mu\nu}|B(p)\rangle = i\,\frac{f_T(q^2)}{m_B + m_K}\left[q^2(p + p_K)^\mu - (m_B^2 - m_K^2)q^\mu\right]$$

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$\Rightarrow$ The entire hadronic physics can be expressed in terms of $f_+(q^2)$.

## Parametrization

$$f_+(s) = f_+(0)\frac{1+(a_0 b_0 - b_0 - b_1)s}{(1-b_0 s)(1-b_1 s)} \qquad s = q^2/m_B^2$$

- In this parametrization the parameter $b_0$ represents the position of the $B_s^*$ pole and will be treated as fixed at $b_0 = \frac{m_B^2}{m_{B_s^*}^2} \approx 0.95$.

  [D. Becirevic and A. B. Kaidalov, Phys. Lett. B 478, 2000]

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  [D. Becirevic and A. B. Kaidalov, Phys. Lett. B 478, 2000]
- The remaining parameters have been calculated with QCD sum rules [P. Ball and R. Zwicky, Phys. Rev. D 71, 2005].
- Combined with recent data from Belle [J. T. Wei and P.Chang, arXiv:0904.0770v1] the following parameter space seems reasonable.

## Range of parameter space

$$f_+(0) = 0.304 \pm 0.042, \qquad a_0 = 1.6 \pm 0.2, \qquad b_1/b_0 = 1.0^{+0.0}_{-0.5}$$

## Differential Branching Fractions

$$\frac{dB(B \to K l^+ l^-)}{ds} = \tau_B \frac{G_F^2 \alpha^2 m_B^5}{1536\pi^5} |V_{ts} V_{tb}|^2 \lambda_k^{\frac{3}{2}}(s) f_+^2(s) (|\tilde{C}_{10}|^2 + |C_9^{eff}(s)|^2)$$

$$\frac{dB(B \to K \nu \overline{\nu})}{ds} = \tau_B \frac{G_F^2 \alpha^2 m_B^5}{256\pi^5} |V_{ts} V_{tb}|^2 \lambda_k^{\frac{3}{2}}(s) f_+^2(s) |\tilde{C}_\nu|^2$$

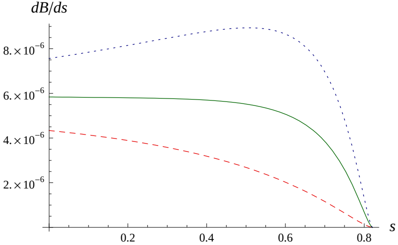


Figure: 1: $B \to K \nu \bar{\nu}$

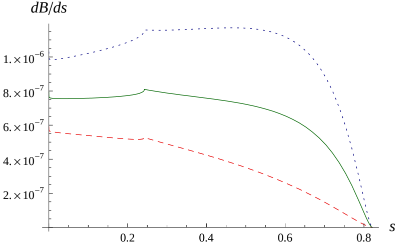


Figure: 2: $B \to K l^+ l^-$

# Numerical Results

## Integrated Branching Fractions

$$\mathcal{B}(B^- \to K^- \nu\bar{\nu})\left[10^{-6}\right] = 4.4\ {}^{+1.3}_{-1.1}(f_+(0))\ {}^{+0.8}_{-0.7}(a_0)\ {}^{+0.0}_{-0.7}(b_1)$$

$$\mathcal{B}(B^- \to K^- l^+ l^-)\left[10^{-7}\right] = 5.8\ {}^{+1.7}_{-1.5}(f_+(0))\ {}^{+1.0}_{-0.9}(a_0)\ {}^{+0.0}_{-0.9}(b_1)\ {}^{+0.4}_{-0.3}(\mu)$$

## Uncertainties from other Sources

- Contributions of relative size $\left|\frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}\right| < 0.02$ have been neglected.
- Weak annihilation diagrams contribute at leading order. Nevertheless the numerical impact is less than 1% and therefore negligible.
- The reaction $B^- \to \tau^- \bar{\nu}_\tau \to K^- \nu_\tau \bar{\nu}_\tau$ produces a background for the decay $B^- \to K^- \nu_\tau \bar{\nu}_\tau$. The resulting uncertainties will diminish to roughly 1% with increasing accuracy of the $B^- \to \tau^- \bar{\nu}_\tau$ measurement.

# Numerical Results

## Precision Observables

$$\mathcal{R} = \frac{\mathcal{B}(B^- \to K^-\nu\bar{\nu})}{\mathcal{B}(B^- \to K^- l^+ l^-)} = 7.59 \; {}^{+0.01}_{-0.01}(a_0) \; {}^{+0.00}_{-0.02}(b_1) \; {}^{-0.48}_{+0.41}(\mu)$$

$$\mathcal{R}_{25} = \frac{\mathcal{B}(B^- \to K^-\nu\bar{\nu})}{\mathcal{B}(B^- \to K^- l^+ l^-)} = 7.60 \; {}^{+0.00}_{-0.00}(a_0) \; {}^{+0.00}_{-0.00}(b_1) \; {}^{-0.43}_{+0.36}(\mu)$$

$$\mathcal{R}_{256} = \frac{\mathcal{B}(B^- \to K^-\nu\bar{\nu})}{\mathcal{B}(B^- \to K^- l^+ l^-)} = 14.60 \; {}^{+0.28}_{-0.38}(a_0) \; {}^{+0.10}_{-0.02}(b_1) \; {}^{-0.80}_{+0.62}(\mu)$$

The charmonium resonances $\Psi(1S)$ and $\Psi(2S)$ spoil the validity of perturbation theory in the kinematical region $0.25 \leq s \leq 0.6$. To obtain theoretically clean observables and still examine most of the spectrum the ratios $\mathcal{R}_{25}$ and $\mathcal{R}_{256}$ were defined in the following way:

$$\mathcal{R}_{25} \equiv \frac{\int_0^{0.25} ds\; d\mathcal{B}(B^- \to K^-\nu\bar{\nu})/ds}{\int_0^{0.25} ds\; d\mathcal{B}(B^- \to K^- l^+ l^-)/ds}$$

$$\mathcal{R}_{256} \equiv \frac{\int_0^{s_{max}} ds\; d\mathcal{B}(B^- \to K^-\nu\bar{\nu})/ds}{\int_0^{0.25} ds\; d\mathcal{B}(B^- \to K^- l^+ l^-)/ds + \int_{0.6}^{s_{max}} ds\; d\mathcal{B}(B^- \to K^- l^+ l^-)/ds}$$

# Experimental status

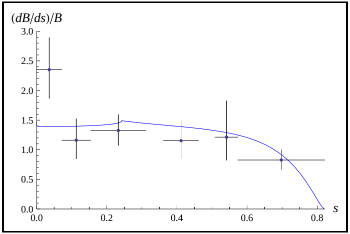


Figure: Shape of $B \to K l^+ l^-$ from Belle data (crosses) and theory with best-fit parameters (solid curve).

$$\mathcal{B}^{exp}\left(B^0 \to K^0 \nu\bar{\nu}\right) \leq 160 \cdot 10^{-6}$$
$$\mathcal{B}^{exp}\left(B^+ \to K^+ \nu\bar{\nu}\right) \leq 14 \cdot 10^{-6}$$
$$\mathcal{B}^{exp}\left(B^+ \to K^+ l^+ l^-\right) = 0.48^{+0.05}_{-0.04} \pm 0.03 \cdot 10^{-6}$$

$\Rightarrow$ Combining the experimental value for $\mathcal{B}\left(B^+ \to K^+ l^+ l^-\right)$ with the theoretical prediction of $\mathcal{R}$ one can improve the estimate for the neutrino mode: $\mathcal{B}(B^- \to K^- \nu\bar{\nu}) = (3.64 \pm 0.47) \cdot 10^{-6}$.

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- The hadronic uncertainties contained in the form factors are eliminated almost completely by considering suitable ratios of the integrated branching fractions. At the same time the sensitivity to physics beyond the SM is preserved.
- The remaining perturbative uncertainty of suitable ratios is roughly $\pm 5\%$ at next-to-leading (NLO). A further improvement by a NNLO analysis is achievable.