Quantum phase transitions in holographic superfluids

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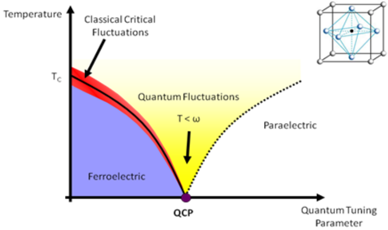
In collaboration with: J. Erdmenger, V. Grass and P. Kerner, arXiv:1101.xxxx

Motivations

Question 1: What are quantum phase transitions in condense matter physics?

- phase transitions at zero temperature which are driven by quantum fluctuations instead of thermal fluctuations
- realized by varying physical parameters (pressure, magnetic field, chemical composition,...)

- quantum critical point, region and finite temperature quantum criticality
- effects quantum criticality
  $\longrightarrow$ first observed in ferroelectrics
- "non-Fermi liquid" or a "strange metal" behavior in the QCR
- strongly interacting region



Ferroelectric quantum critical point
[www.phy.cam.ac.uk/research/qm/ferroelectrics.php]

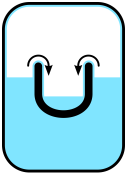
Motivations

Question 2: What are superconductors/ superfluids?

- superconductivity: electrical resistivity drops suddenly to zero below T_c
- superfluidity: viscosity vanishes and thermal conductivity becomes infinite below T_c

History

- SC and SF first observed in the 1930's
- 1935 London's equations
- 1950 Landau- Ginzburg theory
- 1957 Bardeen- Cooper- Schrieffer theory



Creeping Helium II
[Aarchiba, wikipedia.org/wiki/Superfluid]

New classes of superconductors

- 1986 cuprates as a new class of high-T_c superconductors ($T_c \sim 160K$)
- 2008 iron pnictides as a new class of high-T_c superconductors ($T_c \sim 56K$)

Unlike BCS theory, the pairing mechanism of high-T_c superconductors at strong coupling is not well understood!

Motivations

Question 3: Why holographic?

- conventional methods provide very few tools to study physical systems at strong coupling
- holographic method (gauge/gravity duality) is a powerful tool to study strongly coupled field theories, in particular for computing dynamical transport properties of strongly coupled systems at finite temperature

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The gauge/gravity duality as a tool

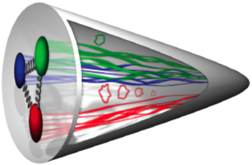
Classical gravitational theory in $d+1$ spacetime dimensions $\leftrightarrow$ Large N_c gauge theory in d spacetime dimensions

Duality realized by the matching

- of the parameters of the two theories;
- the partition function (gravity) with generating functional for correlation functions (gauge) .

Some properties

- The field theory lives on the boundary of the gravity theory
- The gauge/gravity duality describes weak/strong coupling duality



Layout

1. Introduction
   - Motivations
   - Constructing holographic superfluids
2. QPT in holographic superfluids at finite baryon and isospin chemical potential
   - Holographic superfluids in Einstein Yang-Mills theory
   - Holographic superfluids with D3/D7 model
3. Summary

Ingredients for constructing holographic superfluids

Landau- Ginzburg- Theory

- second order phase transition with complex scalar field ψ as order parameter
- the density of the superconducting charges is given by $|\psi(x)|^2$

The contribution of ψ to free energy is *assumed* to take the form

$$F = \alpha(T - T_c)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \cdots$$

with positive constants α and β. Minimizing the free energy F

$$\begin{cases} T > T_c & F_{min} = F(\psi = 0) \\ T < T_c & F_{min} = F(\psi \neq 0) \end{cases} \rightarrow \text{ associated with spontaneous breaking of U(1)}$$

local U(1) $\rightarrow$ superconductivity, global U(1) $\rightarrow$ superfluidity

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List of ingredients

- temperature
- charged field which can condense
- spontaneous breaking of an global U(1) symmetry
- *tuning parameters (magnetic field, chemical potentials,...)*

Constructing holographic superfluids

Minimal ingredients: (i) temperature;
(ii) field which can condense and breaks U(1) symmetry spont.;

The original AdS/CFT correspondence [Maldacena, 1997]

Type IIB superstring theory in $AdS_5 \times S^5$ spacetime $\leftrightarrow$ $\mathcal{N} = 4$ SYM on the 4d conformal boundary of AdS

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Extension to finite temperature [Witten, 1998] at large N_c and large $\lambda_{'t\ Hooft}$−coupling

classical gravity theory in AdS black hole spacetime $\leftrightarrow$ finite temperature field theory on the boundary of AdS black hole

The black hole horizon radius is directly related to the temperature on the FT side.

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One possibility for realizing (ii) is using the AdS/CFT dictionary

$$\mathcal{Z}_{string}\left[\phi\left(\vec{x}, r\right)|_{r\to\infty} = \phi_0\left(\vec{x}\right)\right] \equiv \left\langle e^{\int d^4x \phi_0(\vec{x})\mathcal{O}(\vec{x})}\right\rangle_{FT}$$

to relate the operator $\mathcal{O}\left(\vec{x}\right)$ describing the condensate with some field $\phi\left(\vec{x}, r\right)$ coupled to gravity in the bulk, and the task is finding stable AdS black hole with ϕ condensing at low temperature [Gubser, 2008], [Hartnoll, Herzog, Horowitz, 2008].

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Another possibility for realizing (ii) is using flavor D-branes to introduce fundamental matter which condenses at low temperature. This so called *top down* approach (dual field theory is known) using D3/D7 model is the first stringy realization of holographic $p-$wave superfluid [Ammon, Erdmenger, Kerner and Kaminski, 2008].

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Extra ingredients: (iii) magnetic field;
(iv) chemical composition (isospin, baryon chemical potential);

Superfluids in Einstein Yang-Mills theory

Minimal ingredients: (i) temperature;
(ii) field condenses at low T and breaks U(1) spontaneously;
Extra ingredients: (iii) isospin and (iv) baryon chemical potential;

Consider the $U(2)$ Einstein-YM theory in $(4+1)$-dimensional asymptotically AdS space

$$S = \int \mathrm{d}^5 x \frac{\sqrt{-g}}{\kappa_5^2}\left[\frac{1}{2}(\mathcal{R}-\Lambda) - \frac{\hat{\alpha}_{\mathrm{YM}}^2}{4} F^a_{\mu\nu}F^{a\mu\nu} - \frac{\hat{\alpha}_{\mathrm{MW}}^2}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right]$$

$$SU(2)\text{-field strength}: \; F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu \,,$$

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Taking following ansatz for constructing charged black hole solutions with a vector hair

$$A = \phi(r)\tau^3 \mathrm{d}t + w(r)\tau^1 \mathrm{d}x \, , \qquad \mathcal{A} = \psi(r)\mathrm{d}t \, ,$$

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2 \mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2 f(r)^{-4}\mathrm{d}x^2 + r^2 f(r)^2(\mathrm{d}y^2 + \mathrm{d}z^2) \, .$$

The **gauge/gravity dictionary** relates quantities from both sides.

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The **gauge/gravity dictionary** relates quantities from both sides:

- temperature T $\leftrightarrow$ r_h with $N(r_h)\sigma(r_h)^2 = 0$
- conserved current $J_\mu \leftrightarrow A_\mu$ in the sense $A_\mu(r_\infty) = s_\mu + e_\mu r^{-1} + \mathcal{O}(r^{-2})$
  The time-component of the gauge field
  $\psi(r)$ gives the $U(1)_B$ baryon chemical potential μ_B and the density $\langle J_t^0 \rangle$
  $\phi(r)$ gives the $SU(2)_I$ isospin chemical potential μ_I and the density $\langle J_t^3 \rangle$
- demanding the boundary condition $w(r_\infty) = 0 + \langle J_x^1 \rangle r^{-1} + \mathcal{O}(r^{-2})$
  $\longrightarrow$ condensate with no source term, i.e. spontaneous symmetry breaking
- gauge symmetries in the bulk correspond to global symmetries in the dual FT
  $\longrightarrow$ holographic superfluid

Superfluids in Einstein Yang-Mills theory

Results for the fully back-reacted case $\alpha_{YM} \neq 0$, $\alpha_{MW} \neq 0$

- For $w(r) = 0$ the AdS Reissner Nordström black hole solves the EoM
  $\rightarrow$ dual to the normal conducting state

$$ds^2 = -N(r)\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2\mathrm{d}\vec{x}^2\,, \quad \phi(r) = \mu_I - \frac{q_I}{r^2}\,, \quad \psi = \mu_B - \frac{q_B}{r^2}\,,$$

$$N(r) = r^2 - \frac{2}{r^2}\left(\frac{r_h^4}{2} + \frac{\alpha_{\mathrm{YM}}^2 q_I^2 + \alpha_{\mathrm{MW}}^2 q_B^2}{3r_h^2}\right) + \frac{2(\alpha_{\mathrm{YM}}^2 q_I^2 + \alpha_{\mathrm{MW}}^2 q_B^2)}{3r^4}\,,$$

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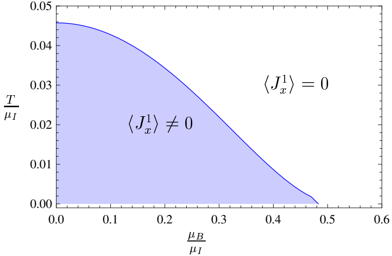
- For $w(r) \neq 0$ there is no analytical solution for the metric, ϕ and ψ $\rightarrow$ numerics



Phase diagram for $\alpha_{\mathrm{MW}} = 1$ and $\alpha_{\mathrm{YM}} = 0.1$

- Instability of the RN solution

$$\left(\frac{\mu_B}{\mu_I}\right)_c = \frac{\sqrt{1 - 3\alpha_{\mathrm{YM}}^2}}{\sqrt{3}\,\alpha_{\mathrm{MW}}}$$

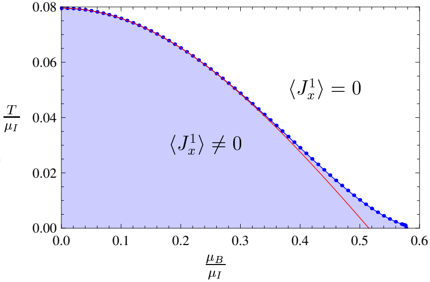
- Semi back-reacted case $\alpha_{YM} = 0$

Superfluids in Einstein Yang-Mills theory

Results for the semi back-reacted case $\alpha_{YM} = 0$, $\alpha_{MW} \neq 0$

- For $w(r) \neq 0$, analytical solutions can be obtained via expansion in small $\delta \equiv \mu_B$ and $\epsilon \propto \langle J_x^1 \rangle$

$$\begin{aligned} \phi(r) &= \phi_{0,0}(r) + \delta^2\, \phi_{2,0}(r) + \delta^4\, \phi_{4,0}(r) + \dots \\ &\quad + \epsilon^2 \left(\phi_{0,2}(r) + \delta^2\, \phi_{2,2}(r) + \dots \right) \\ &\quad + \epsilon^4\, \phi_{0,4}(r) + \dots \\ w(r) &= \epsilon \left(w_{0,1}(r) + \delta^2\, w_{2,1}(r) + \delta^4\, w_{4,1}(r) + \dots \right) \\ &\quad + \epsilon^3 \left(w_{0,3}(r) + \delta^2\, w_{2,3}(r) + \dots \right) \\ &\quad + \dots \end{aligned}$$



Phase diagram for $\alpha_{\rm MW} = 1$ and $\alpha_{\rm YM} = 0$

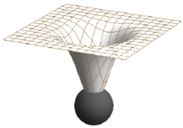
- Difference in free energy

$$F_{\rm sf} - F_{\rm nf} = \frac{r_H^4}{4}\left(-\frac{71}{53{,}760}\epsilon^4 + \mathcal{O}(\delta^p \epsilon^q) \right) \sim \left(T_c\left(\delta\right) - T\right)^2$$

Holographic superfluids with D3/D7 model

- Two description of D_p-branes
  - $p-$spatial extended objects where open strings can end
  - massive objects as solution of supergravity
- Extending the AdS/CFT by flavor probe D_p-branes [Karch, Katz, 2002]

	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
N_c D3 branes:	X	X	X	X						
N_f D7 branes:	X	X	X	X	X	X	X	X		



- Field theory at the probe limit $N_f \ll N_c$, $N_c \gg 1$
  - $\mathcal{N} = 4$ gauge multiplet $\rightarrow$ massless open string modes on the D3 branes
  - $\mathcal{N} = 2$ SYM hypermultiplets $\rightarrow$ strings between the D3 and D7 branes
- The dynamics of the flavor branes are described by the DBI action

$$S_{D7} = -N_f T_{D7} \int d^8\xi \sqrt{-det(g_{ab} + (2\pi\alpha')F_{ab})}$$

Holographic superfluids with D3/D7 model

Minimal ingredients: (i) temperature;
(ii) field condenses at low T and breaks U(1) spontaneously;
Extra ingredients: (iii) isospin and (iv) baryon chemical potential;

- Stack of D3 and temperature

$$ds^2 = \frac{\varrho^2}{2R^2}\left(-\frac{f^2}{\tilde{f}}\mathrm{d}t^2 + \tilde{f}\mathrm{d}\vec{x}^2\right) + \left(\frac{R}{\varrho}\right)^2 (\mathrm{d}\varrho^2 + \varrho^2 \mathrm{d}\Omega_5^2)\,,$$

$$f(\varrho) = 1 - \frac{\varrho_H^4}{\varrho^4}\,, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho_H^4}{\varrho^4}$$

- Introducing two coincident flavor probe $D7$-branes
$\rightarrow$ world volume field theory $U(2) = U(1)_B \times SU(2)_I$
- Introducing baryon and isospin chemical potential with $A_t^0(\varrho)\tau^0\mathrm{d}t$ and $A_t^3(\varrho)\tau^3\mathrm{d}t$

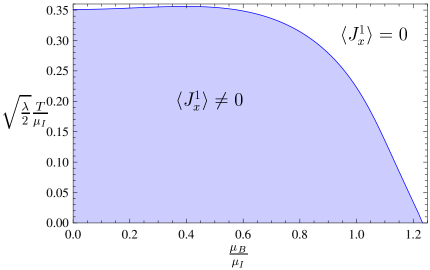
$$\mu_B = \lim_{\varrho\to\infty} A_t^0(\varrho)\,, \qquad \mu_I = \lim_{\varrho\to\infty} A_t^3(\varrho)$$

- Introducing the field $A_x^1\tau^1\mathrm{d}x$ which condenses and breaks the $U(1)_3$
- Solving the non-Abelian DBI action

Holographic superfluids with D3/D7 model

Results for the D3/D7 model
Phase transition and phase diagram

- critical temperature first increases with increasing μ_B/μ_I
- transition temperature is zero at $\mu_B/\mu_I \approx 1.23$
- second order phase transition



Phase diagram for D3/D7 model at finite μ_B and μ_I

Summary

Conclusion

- Constructed holographic superfluids using back-reacted Einstein-Yang-Mills theory and D3/D7 model at finite baryon and isospin chemical potential
- In both cases there are quantum phase transitions
- The quantum phase transitions in the two models may belong to different classes
- Still no analytical proof that such models might provide descriptions for quantum critical points

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