Axions in Cosmology

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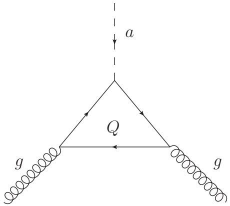
The Hadronic Axion

- Axion coupling:

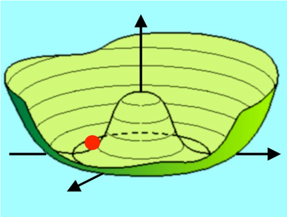
$$\mathcal{L}_a = a\frac{\alpha_s}{8\pi}\frac{1}{f_{\mathrm{PQ}}}G^{b\ \mu\nu}\widetilde{G}^b_{\mu\nu}$$

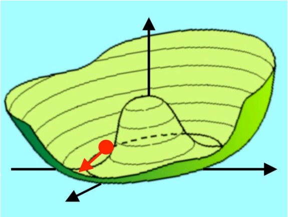
- Idea: Construct a model such that interaction imply the counterterm
- A complex scalar field ϕ and an exotic heavy quark with mass $M_Q = h\langle\phi\rangle = hf_{\mathrm{PQ}}/\sqrt{2}$
- Coupling to gluons give effective term in the Lagrangian



The Potential

- Mexican hat
- Hat gets tilted by instanton effects at $T \approx \Lambda_{\mathrm{QCD}}$: axion acquires a mass
- Lowest point is the CP conserving value: $\langle a \rangle = -\bar{\theta} f_{\mathrm{PQ}}$





Axion Properties

- Coupling:

$$\mathcal{L}_a = a\frac{\alpha_s}{8\pi}\frac{1}{f_{\rm PQ}}G^{b\ \mu\nu}\widetilde{G}^b_{\mu\nu}$$

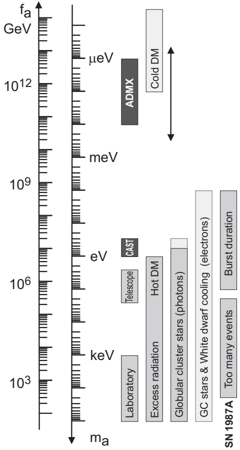
- Axion mass:

$$m_a \simeq 0.60\ \text{meV}\left(\frac{10^{10}\ \text{GeV}}{f_{\rm PQ}}\right)$$

- All limits imply

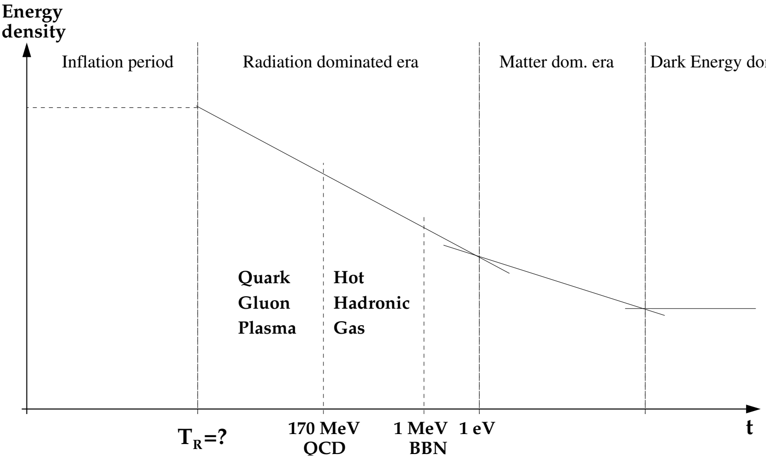
$$f_{\rm PQ} \gtrsim 6\times 10^8\ \text{GeV}$$

$$m_a \lesssim 10\ \text{meV}$$



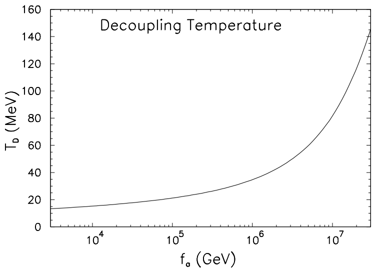
[Raffelt,'06]

Cosmological Setting



Here we assume $f_{\mathrm{PQ}} > T_{\mathrm{R}}$

Axions in the Primordial Hot Hadronic Gas



[Hannestad,Mirizzi,Raffelt,'05]

- $T_{\mathrm{R}} > T_{\mathrm{D}}$: Axions were in thermal equilibrium for $T > T_{\mathrm{D}}$

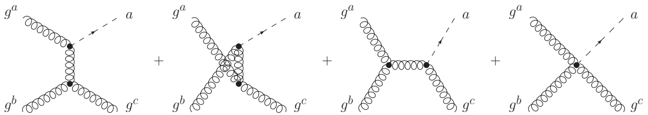
$$\pi + \pi \leftrightarrow \pi + a \quad \text{and} \quad N + \pi \leftrightarrow N + a$$

- Freeze out if interaction rate becomes too slow compared to expansion rate of the Universe: $\Gamma_a \approx H$ at $T \approx T_{\mathrm{D}}$
- $T_{\mathrm{R}} < T_{\mathrm{D}}$: Axions are thermally produced via

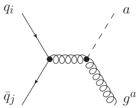
$$\pi + \pi \rightarrow \pi + a \quad \text{and} \quad N + \pi \rightarrow N + a$$

Thermal Axion Production in the Quark Gluon Plasma

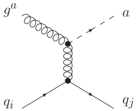
- $g^a + g^b \longrightarrow g^c + a$



- $q_i + \bar{q}_j \longrightarrow g^a + a$



- $q_i + g^a \longrightarrow q_j + a$



Results for Squared Matrix Elements

Label i	Process i	$\lvert M_i\rvert^2 / \left(\frac{\alpha_s^3}{f_{\mathrm{PQ}}^2}\frac{1}{\pi}\right)$
A	$g^a + g^b \to g^c + a$	$-2\frac{(s^2+st+t^2)^2}{st(s+t)}\lvert f^{abc}\rvert^2$
B	$q_i + \bar{q}_j \to g^a + a$	$\frac{1}{2}\left(\frac{2t^2}{s} + 2t + s\right)\lvert T^a_{ij}\rvert^2$
C	$q_i + g^a \to q_j + a$	$\frac{1}{2}\left(-\frac{2s^2}{t} - 2s - t\right)\lvert T^a_{ij}\rvert^2$

depending on $s = (P_1 + P_2)^2$ and $t = (P_1 - P_3)^2$

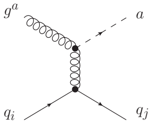
IR divergences

Processes A and C: logarithmic IR divergence

Regularization using QCD Debye mass done by [Masso,Rota,Zsembinszki,'02] $\longleftarrow$ gauge-dependent

HTL Resummation and Braaten-Yuan ['91] Prescription

introduce $g_s T \ll k_{\rm cut} \ll T$ with $g_s \ll 1$



$$\frac{dW_a}{d^3p}\Big|_{\rm hard} = \frac{1}{2(2\pi)^3 E}\int \frac{d\Omega_p}{4\pi}\int\left[\prod_{j=1}^{3}\frac{d^3p_i}{(2\pi)^3 2E_i}\right] \times (2\pi)^4\delta^4(P_1+P_2-P_3-P) \times \{f_1(E_1)f_2(E_2)[1\pm f_3(E_3)] \times [1+f_a(E)]|M_{1+2\to 3+{\rm a}}|^2\}_{k>k_{\rm cut}}$$

$$\frac{dW_a}{d^3p}\Big|_{\rm hard} = A_{\rm hard} + B\log\left(\frac{T}{k_{\rm cut}}\right)$$

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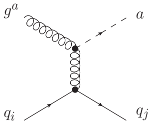
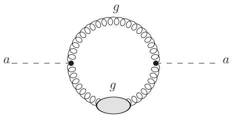




$$\frac{dW_a}{d^3p}\bigg|_{\rm hard} = \frac{1}{2(2\pi)^3 E}\int\frac{d\Omega_p}{4\pi}\int\left[\prod_{j=1}^{3}\frac{d^3p_i}{(2\pi)^3 2E_i}\right] \times (2\pi)^4\delta^4(P_1+P_2-P_3-P) \times\{f_1(E_1)f_2(E_2)[1\pm f_3(E_3)] \times [1+f_a(E)]|M_{1+2\to 3+{\rm a}}|^2\}_{k>k_{\rm cut}}$$

$$\frac{dW_a}{d^3p}\bigg|_{\rm hard} = A_{\rm hard} + B\log\left(\frac{T}{k_{\rm cut}}\right)$$

$$\frac{dW_a}{d^3p}\bigg|_{\rm soft} = -\frac{1}{(2\pi)^3}\frac{f_{\rm B}(E)}{E}\,{\rm Im}\,\Sigma_a(E+i\epsilon,\vec{p})_{k<k_{\rm cut}}$$

$$\frac{dW_a}{d^3p}\bigg|_{\rm soft} = A_{\rm soft} + B\log\left(\frac{k_{\rm cut}}{m_g^{\rm th}}\right)$$

Result is independent of $k_{\rm cut}$: $\dfrac{dW_a}{d^3p} = \dfrac{dW_a}{d^3p}\bigg|_{\rm hard} + \dfrac{dW_a}{d^3p}\bigg|_{\rm soft}$

Thermal Axion Number Density

$$\frac{dn_a}{dt} + 3Hn_a = C_a$$

$$C_a = \int d^3p \left[\frac{dW_a}{d^3p}\right] = \frac{\alpha_s^3}{f_{\rm PQ}^2} T^6 \zeta(3) \frac{1}{\pi^4} \left\{ \ln\left[\frac{T^2}{(m_g^{\rm th})^2}\right] + 0.406 \right\}$$

with the thermal gluon mass

$$(m_g^{\rm th})^2 = \frac{g_s^2 T^2}{9}\left(N + \frac{n_f}{2}\right)$$

for SU(N) color group and n_f quark flavors

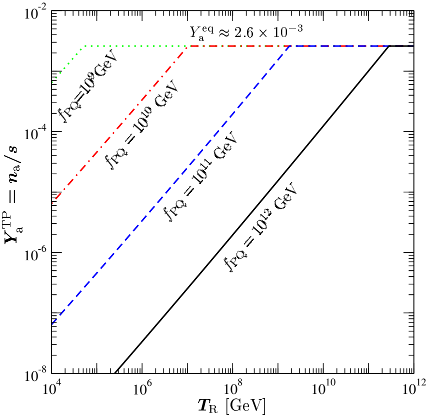
Axion Yield

$$Y_a(T_0) = \frac{n_a(T_0)}{s} \simeq 18.6 \times g_s^6 \ln\left(\frac{1.501}{g_s}\right)\left(\frac{10^{10}\ \mathrm{GeV}}{f_{\mathrm{PQ}}}\right)^2\left(\frac{T_{\mathrm{R}}}{10^{10}\mathrm{GeV}}\right)$$

- Note: Production only!
- Therefore: Yield of thermally produced axions
- Max: equilibrium yield

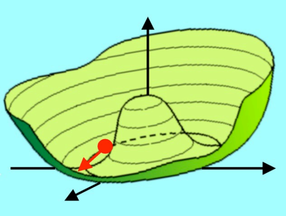
$$Y_a^{\mathrm{eq}} \approx 2.6 \times 10^{-3}$$

- Kink: decoupling temperature



Misalignment Production

- At $T \approx \Lambda_{\text{QCD}}$: Axion mass switches on
- Axion field performs coherent oscillations around CP conserving minimum
- Condensate of cold axions forms
- Possible candidate for Dark Matter



Resulting energy density depends on cosmological setting

Misalignment Production

- $T_{\rm R} > f_{\rm PQ}$: PQ-symmetry restored after inflation and broken subsequently
- Universe consists if many patches with different $\bar{\theta}_i$
- average value relevant: $\langle \bar{\theta}_i^2 \rangle = \pi^2/3$

$$\Omega_a h^2 \approx 0.6 \left(\frac{f_{\rm PQ}}{10^{12}\ \rm GeV} \right)^{7/6}$$

- $T_{\rm R} < f_{\rm PQ}$: PQ-symmetry broken before or during inflation and not restored afterwards
- one value of $\bar{\theta}_i$ for the whole observable Universe
- energy density dominated by a random variable

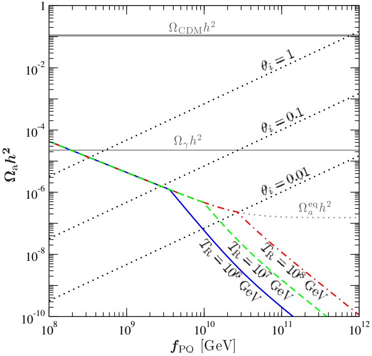
$$\Omega_a h^2 \approx 0.15 f(\bar{\theta}_i^2) \bar{\theta}_i^2 \left(\frac{f_{\rm PQ}}{10^{12}\ \rm GeV} \right)^{7/6}$$

Axion Energy Density

$$\Omega = \frac{\rho}{\rho_c} = \frac{\langle E \rangle n}{\rho_c}$$

$$\Omega_a^{\mathrm{TP/eq}} h^2 \simeq \sqrt{\langle p_{a,0} \rangle^2 + m_a^2}\, Y_a^{\mathrm{TP/eq}} s(T_0) h^2 / \rho_c$$

$$\Omega_a^{\mathrm{MIS}} h^2 \approx 0.15 f(\bar{\theta}_i^2) \bar{\theta}_i^2 \left(\frac{f_{\mathrm{PQ}}}{10^{12}\ \mathrm{GeV}} \right)^{7/6}$$



Conclusions

- Axions are a very light and weakly interacting particle species
- They are produced via scattering processes and can reach thermal equilibrium in the early Universe
- Thermal field theory allows for a gauge-invariant treatment of axion production
- Nonthermal axions can provide the dark matter