Davide Cadamuro

Max-Planck-Institut für Physik

The Strong CP Problem and its Axion Solution

IMPRS Particle Physics School Colloquium

15-04-2010

Outline

- Introducing the Strong CP violation
- What experiments tell us about the Strong CP
- The Axion: a solution to the Strong CP problem

Introducing the Strong CP violation

The complete QCD lagrangian is

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{Q}\left(i\not{D}\right) Q + \\ - \left({Q_{\mathrm{L}}}^\dagger M Q_{\mathrm{R}} + {Q_{\mathrm{R}}}^\dagger M^\dagger Q_{\mathrm{L}} \right) + \\ -\theta \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_a^{\mu\nu} F_a^{\rho\sigma}$$

Introducing the Strong CP violation

The complete QCD lagrangian is

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{Q}\left(i\not{D}\right) Q + \\ - \left({Q_{\mathrm{L}}}^{\dagger} M Q_{\mathrm{R}} + {Q_{\mathrm{R}}}^{\dagger} M^{\dagger} Q_{\mathrm{L}} \right) + \\ -\theta \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_a^{\mu\nu} F_a^{\rho\sigma}$$

"Wait a moment,
who is this guy!?!"

The θ-term

$$\mathcal{L}_\theta = -\theta \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_a^{\mu\nu} F_a^{\rho\sigma}$$

is a perfectly legitimate Lorentz-invariant term.

It is CP violating! → $E \cdot B$

This term is originated by the topological structure of the QCD vacuum, let's see how

Just few hints, because the matter is complicated!

Under a gauge transformation, the vector gauge field transform as

$$\tau_a A_a^\mu \equiv A^\mu \to \Omega(x) A^\mu \Omega^{-1}(x) + \frac{i}{g}\Omega(x)\partial^\mu \Omega^{-1}(x)$$

Just few hints, because the matter is complicated!

Under a gauge transformation, the vector gauge field transform as

$$\tau_a A_a^\mu \equiv A^\mu \rightarrow \Omega(x) A^\mu \Omega^{-1}(x) + \frac{i}{g}\Omega(x)\partial^\mu \Omega^{-1}(x)$$

Therefore the vacuum of the theory A=0 is transformed into the pure gauge quantity which can also be different from zero.

Defining as usual

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$$

$$\tilde{F}_{a\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$

The key quantity in L_θ is a total divergence:

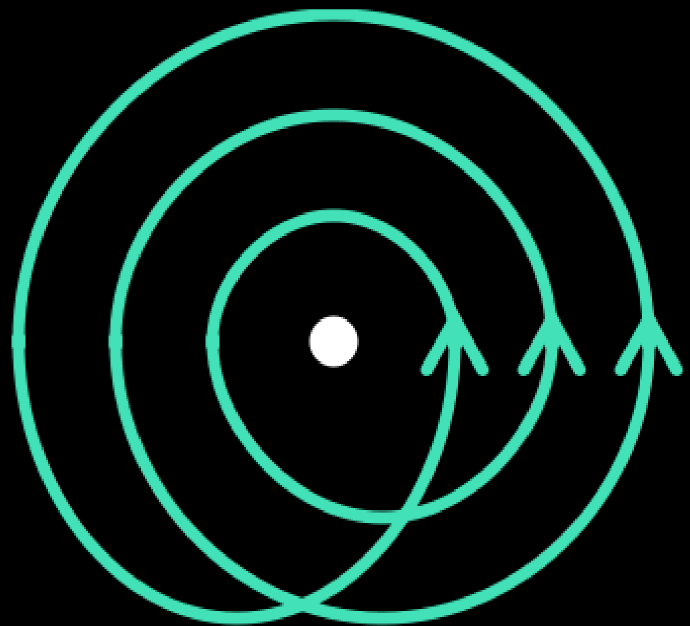
$$\mathrm{Tr}\tilde{F}F = \partial_\mu K^\mu$$

We have to check how $A_\mu = \frac{i}{g}\Omega\,\partial_\mu\,\Omega^{-1}$ behaves at ∞!!!

At $\mathbf{r} \to \infty$, the only relevant quantity is *n*, which describes the way Ω goes to the identity

$$\Omega_n \to e^{2\pi \mathrm{in}} \;\; \text{as } r \to \infty, \quad n = 0, \pm 1, \pm 2 \ldots$$

Practically we count how many times the transformation "winds" around the space

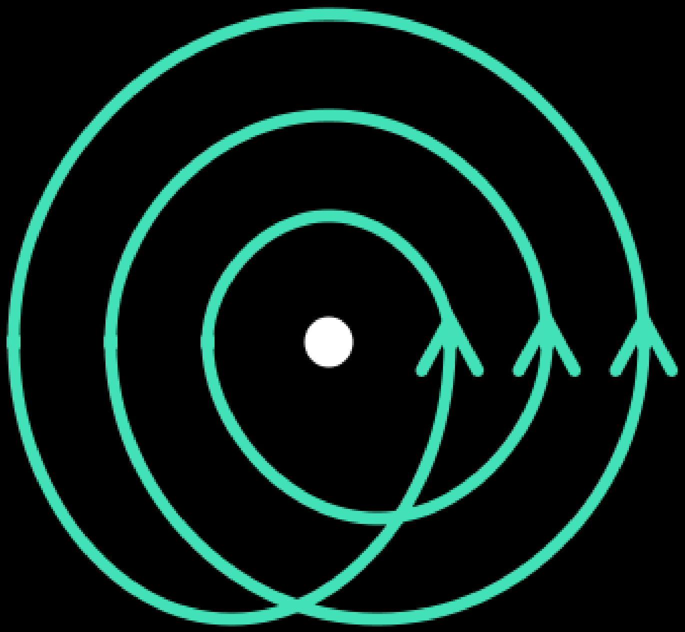


We can now classify the many vacua of the theory according to the winding number *n*

$$\ldots, |-2\rangle, |-1\rangle, |\ 0\rangle, |+1\rangle, \ldots$$

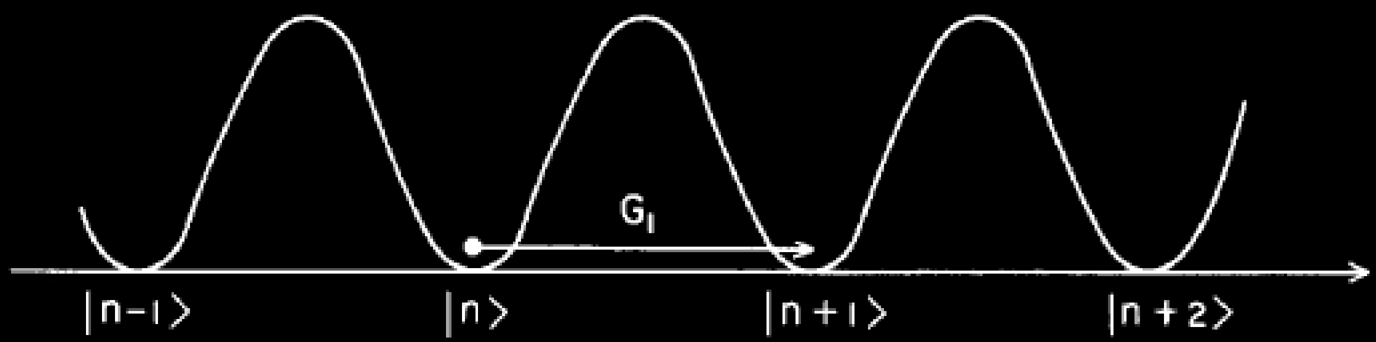
The winding number *n* is given by the expression

$$n = \frac{g^2}{32\pi^2}\int d^4x \tilde{F}_a^{\mu\nu} F_{a\mu\nu}$$



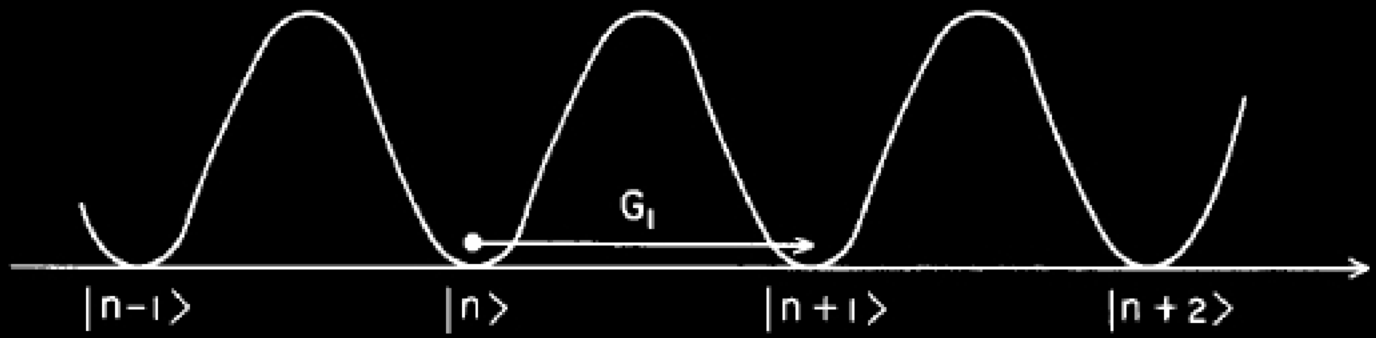
which strongly resembles our guy!

The true vacuum of the theory must be a superposition of the different vacua because of tunnelling effect



The true vacuum must be invariant to any transformation Ω_m

The true vacuum of the theory must be a superposition of the different vacua because of tunnelling effect



The true vacuum must be invariant to any transformation Ω_m

$$|\theta\rangle = \sum_{n=-\infty}^{+\infty} e^{-in\theta}|n\rangle$$

"The θ-vacuum"

The transition amplitude between two distinct θ-vacua gifts us our beloved $\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} \mathrm{Tr}\tilde{F}F$:

$$\langle\theta'|e^{-iHt}|\theta\rangle = \sum_{m'}\sum_{m} e^{im'\theta'-im\theta}\langle m'|e^{-iHt}|m\rangle =$$

$$=\sum_{m} e^{-im(\theta-\theta')}\sum_{n=m-m'}\int[\mathfrak{D}A_\mu \ldots]_n\, e^{-in\theta-i\int d^4x\mathcal{L}} =$$

$$= \delta(\theta'-\theta)\int[\mathfrak{D}A_\mu \ldots]\, e^{-i\int d^4x(\mathcal{L}_\theta+\mathcal{L})}$$

The transition amplitude between two distinct θ-vacua gifts us our beloved $\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} \mathrm{Tr}\tilde{F}F$:

$$\langle\theta'|e^{-iHt}|\theta\rangle = \sum_{m'}\sum_{m} e^{im'\theta' - im\theta}\langle m'|e^{-iHt}|m\rangle =$$

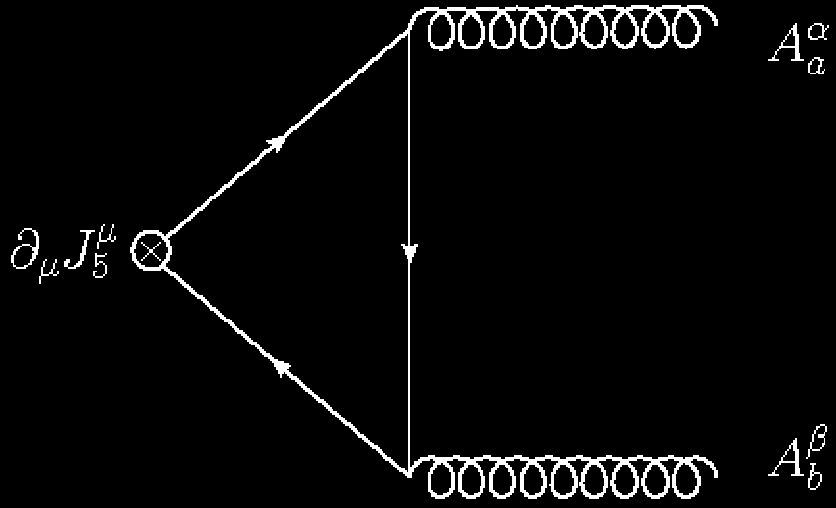
$$= \sum_{m} e^{-im(\theta-\theta')} \sum_{n=m-m'} \int [\mathfrak{D}A_\mu \ldots]_n \, e^{-in\theta - i\int d^4x\mathcal{L}} =$$

$$= \delta(\theta' - \theta) \int [\mathfrak{D}A_\mu \ldots] \, e^{-i\int d^4x(\mathcal{L}_\theta + \mathcal{L})}$$

And this is not the whole story!!!

Also the anomalous chiral current gives its contribution:

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{Q}\left(i\not{D}\right) Q + \\ - \left({Q_{\mathrm{L}}}^{\dagger} M Q_{\mathrm{R}} + {Q_{\mathrm{R}}}^{\dagger} M^{\dagger} Q_{\mathrm{L}}\right)$$

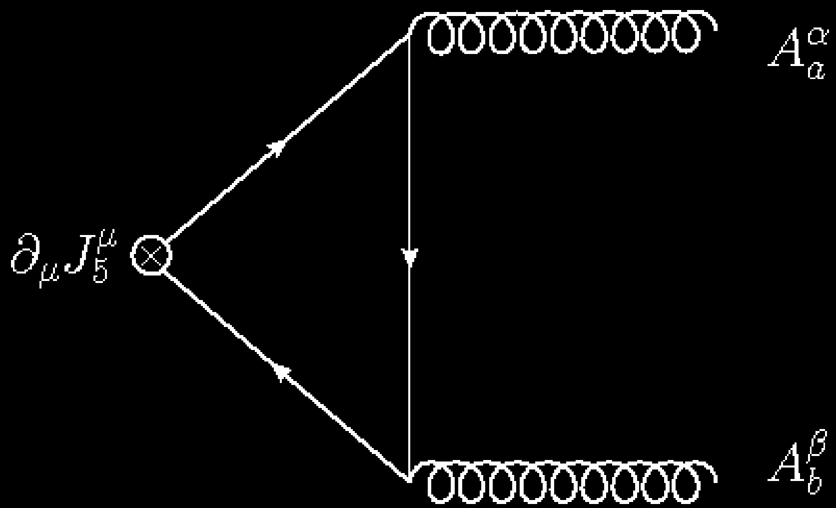


$$M = |M| e^{i\alpha}$$

And this is not the whole story!!!

Also the anomalous chiral current gives its contribution:

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{Q}\left(i\not{D}\right) Q +$$
$$-\left({Q_{\mathrm{L}}}^{\dagger} M Q_{\mathrm{R}} + {Q_{\mathrm{R}}}^{\dagger} M^{\dagger} Q_{\mathrm{L}}\right)$$



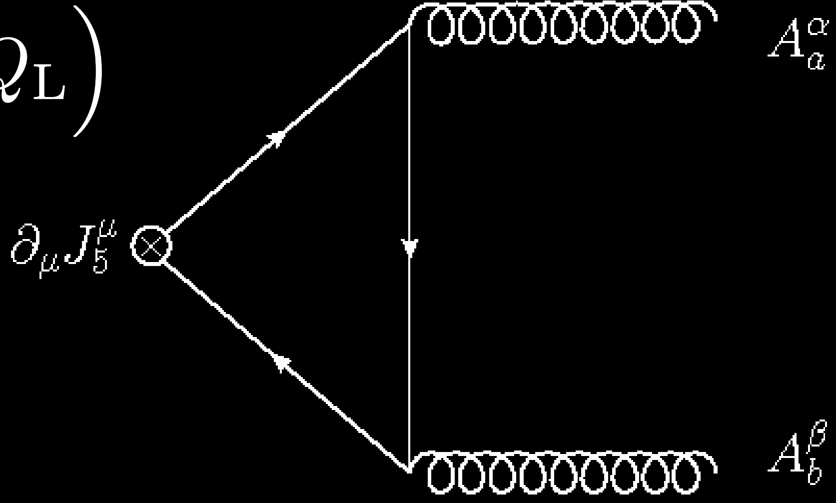
$$M = |M| e^{i\alpha}$$

$$\mathcal{L}_m = -|M| \left({Q_{\mathrm{L}}}^{\dagger} e^{i\alpha} Q_{\mathrm{R}} + {Q_{\mathrm{R}}}^{\dagger} e^{-i\alpha} Q_{\mathrm{L}}\right)$$

And this is not the whole story!!!

Also the anomalous chiral current gives its contribution:

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{Q}\left(i\not{D}\right)Q + \\ -|M|\left(Q_{\mathrm{L}}{}^{\dagger} e^{i\alpha} Q_{\mathrm{R}} + Q_{\mathrm{R}}{}^{\dagger} e^{-i\alpha} Q_{\mathrm{L}}\right)$$



$$Q \to e^{i\frac{\alpha}{2}\gamma_5} Q \Rightarrow J_5^\mu = \bar{Q}\gamma^\mu\gamma_5 Q$$

$$\delta\mathcal{L} = \alpha\partial_\mu J_5^\mu = -\alpha|M|\bar{Q}\gamma_5 Q + n_Q \frac{\alpha g^2}{32\pi^2} \mathrm{Tr}\tilde{F}F$$

And this is not the whole story!!!

Also the anomalous chiral current gives its contribution:

$$\delta\mathcal{L} = \alpha\partial_\mu J_5^\mu = -\alpha|M|\bar{Q}\gamma_5 Q + n_Q \frac{\alpha g^2}{32\pi^2}\mathrm{Tr}\tilde{F}F$$

If one Q has m=0, then θ is unphysical,

otherwise we have

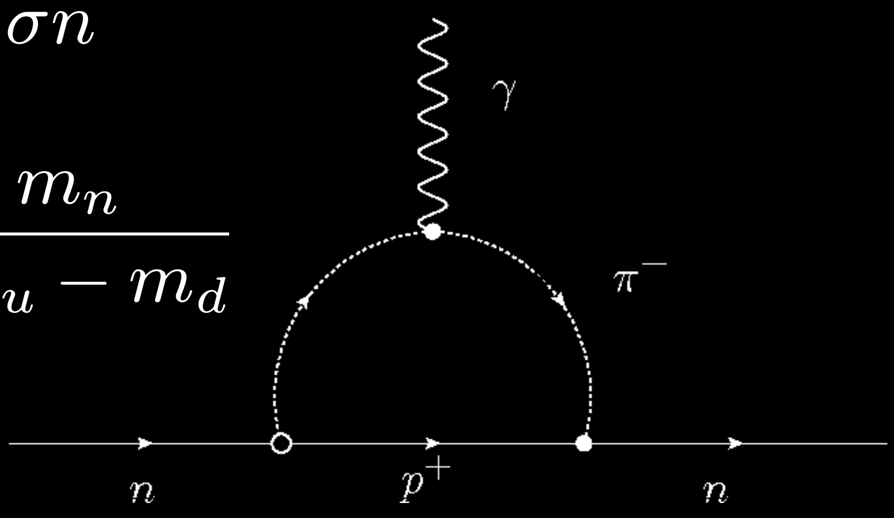
$$\bar{\theta} = \theta_{\mathrm{vac}} + \arg\left(\det\left(M\right)\right)$$

The neutron electric dipole moment (nEDM)

The effective mixing of the θ parameter in the quark field via the anomalous chiral current produce a CP violating term in the π-n interaction lagrangian

$$\mathcal{L}_{\pi n} = \pi \cdot \bar{n} \left(i\gamma_5 g_{nn\pi} + g^{\theta}_{nn\pi} \right) \sigma n$$

$$g^{\theta}_{nn\pi} = -\frac{\bar{\theta}}{f_\pi} \frac{m_u m_d}{m_u + m_d} \frac{m_\Xi - m_n}{2m_s - m_u - m_d}$$

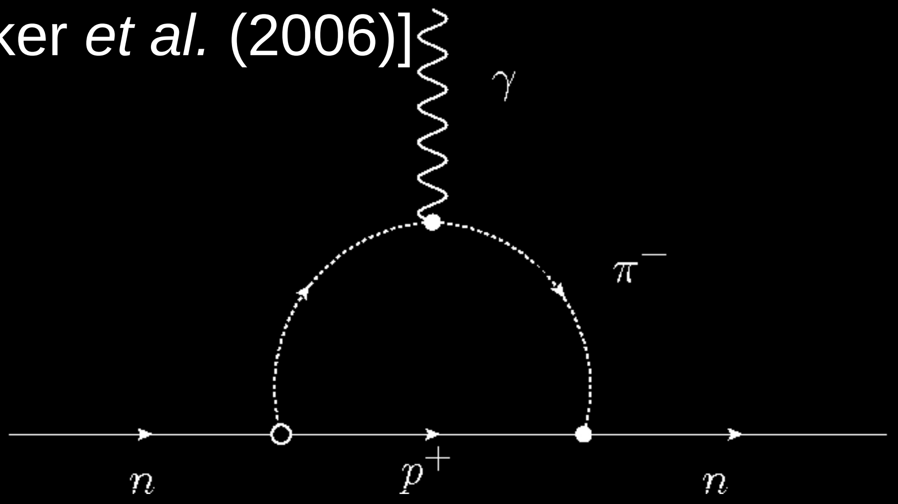


The neutron electric dipole moment (nEDM)

$d_n \simeq 10^{-16}\bar{\theta} e\,\mathrm{cm}$ Theoretical estimate

$\hbar\omega = 2|\mu_n B \pm d_n E|$ Larmor frequency to be measured on ultra-cold n

$|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$ [Baker *et al.* (2006)]



The neutron electric dipole moment (nEDM)

$$d_n \simeq 10^{-16}\bar{\theta} e\,\mathrm{cm}$$

Theoretical estimate

$$\hbar\omega = 2|\mu_n B \pm d_n E|$$

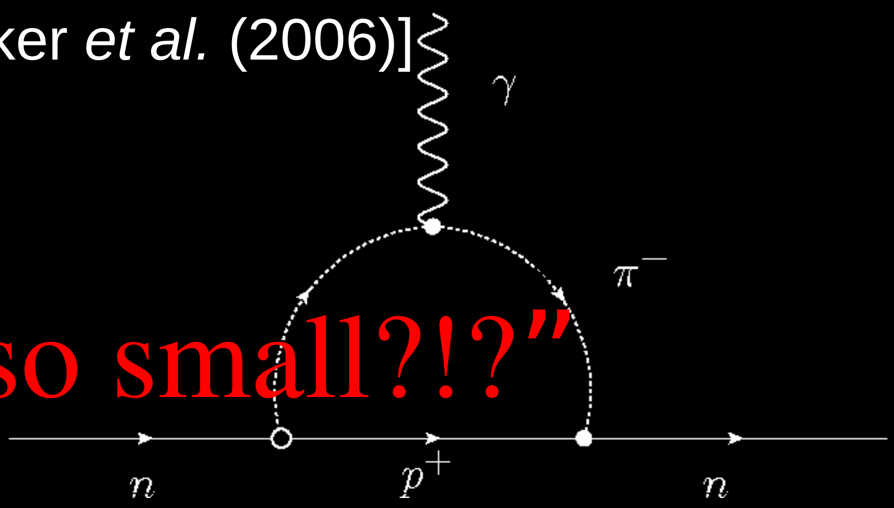
Larmor frequency to be measured on ultra-cold n

$$|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$$

[Baker *et al.* (2006)]

$$\bar{\theta} < 10^{-11}$$

"Why so small?!?"



The  solution

The anomaly of the chiral current gives us a way to escape from the Strong CP problem.

The basic idea is to have a U(1) chiral global symmetry which is spontaneously broken to a CP conserving minimum of the potential [Peccei & Quinn (1977)].

The Goldstone boson of this theory is the Axion [Weinberg (1978), Wilczek (1978)].

The simplest axion model is the one called KSVZ

[Kim (1979), Shifman,Vainstein & Zakharov (1980)]

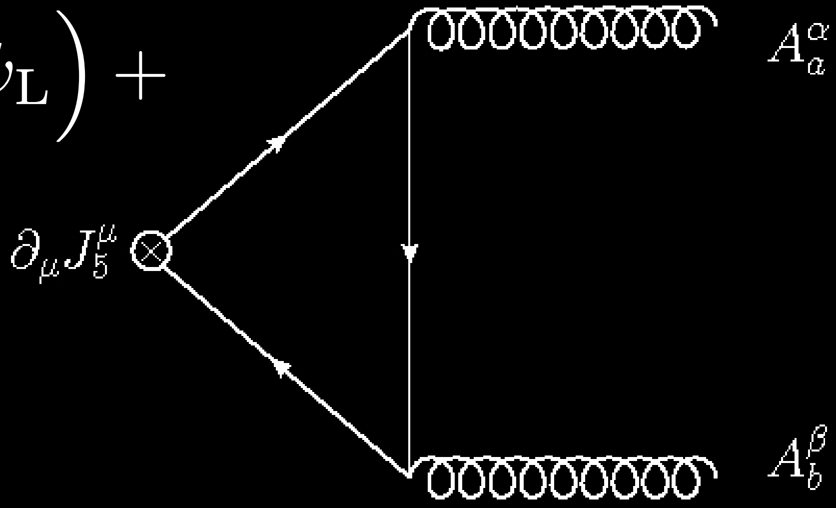
$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{\psi} \left(i \not{D} \right) \psi + \\ - \left({\psi_{\rm L}}^\dagger S \psi_{\rm R} + {\psi_{\rm R}}^\dagger S^\dagger \psi_{\rm L} \right) - \lambda \left(S^\dagger S - f_a^2 \right)^2$$

$S(x) = \rho(x) e^{i \frac{a(x)}{f_a}}$ a higgs boson singlet of SM gauge group

$\psi(x)$ a heavy quark, singlet of $SU(2)_L \times U(1)_Y$,
charged under $SU(3)_c$

When the U(1) symmetry is spontaneously broken at the very high energy scale $f_a \sim 10^{10}$ GeV, $S \rightarrow f_a \exp(ia/f_a)$

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{\psi} \left(i \not{D} \right) \psi + \\ -f_a \left({\psi_{\mathrm{L}}}^\dagger e^{i \frac{a}{f_a}} \psi_{\mathrm{R}} + {\psi_{\mathrm{R}}}^\dagger e^{-i \frac{a}{f_a}} \psi_{\mathrm{L}} \right) + \\ -\bar{\theta} \frac{g_s^2}{64\pi^2} \mathrm{Tr} \tilde{F} F$$



$$\psi \rightarrow e^{i \frac{a}{2f_a} \gamma_5} \psi \Rightarrow J_5^\mu = \bar{\psi} \gamma^\mu \gamma_5 \psi$$

$$\delta\mathcal{L} = \frac{a}{f_a} \partial_\mu J_5^\mu = -\frac{a}{f_a} \bar{\psi} \gamma_5 \psi + \partial_\mu a \partial^\mu a + \frac{a}{f_a} \frac{g^2}{32\pi^2} \mathrm{Tr} \tilde{F} F$$

Integrating out the heavy field ψ, at low energy the effective lagrangian remains

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \partial_\mu a \partial^\mu a + \left(\frac{a}{f_a} - \bar{\theta} \right) \frac{g_s^2}{32\pi^2} \mathrm{Tr} \tilde{F} F$$

The potential gets tilted to the CP conserving minimum $\langle a \rangle = \bar{\theta} f_a$

Redifining $a \equiv a - \bar{\theta} f_a$, we have now the SM lagrangian plus a new particle, the axion:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \partial_\mu a \partial^\mu a + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} {\rm Tr} \tilde{F} F$$

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 6.3\,{\rm meV} \left(\frac{10^9 {\rm GeV}}{f_a} \right)$$

$$g_{aX} \propto \frac{1}{f_a}$$

In summary

- QCD has a legitimate CP violating term proportional to the θ parameter
- Measurements tell that θ is very close to 0
- The Axion provides a solution to smallness of θ