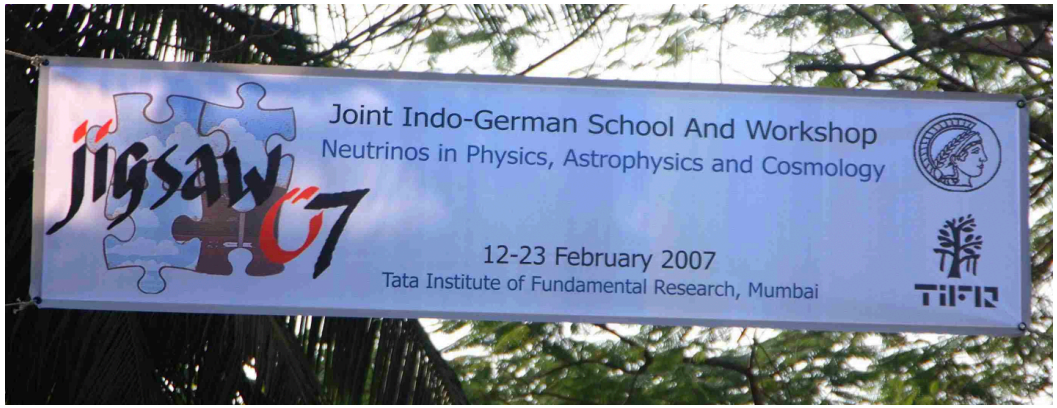
Fast oscillations in supernovae neutrinos: Beyond two flavors

Sovan Chakraborty

Indian Institute of Technology, Guwahati.

Current Topics in Astroparticle Physics,

10th November, 2022


jigsaw 07
Joint Indo-German School And Workshop
Neutrinos in Physics, Astrophysics and Cosmology
12-23 February 2007
Tata Institute of Fundamental Research, Mumbai
TIFR

Linearization:

MPP-2011-81, TIFR/TH/11-30

Linearized flavor-stability analysis of dense neutrino streams

Arka Banerjee,[1] Amol Dighe,[1] and Georg Raffelt[2]

[1] *Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India*
[2] *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany*

(Dated: July 13, 2011)

Neutrino-neutrino interactions in dense neutrino streams, like those emitted by a core-collapse supernova, can lead to self-induced neutrino flavor conversions. While this is a nonlinear phenomenon, the onset of these conversions can be examined through a standard stability analysis of the linearized equations of motion. The problem is reduced to a linear eigenvalue equation that involves the neutrino density, energy spectrum, angular distribution, and matter density. In the single-angle case, we reproduce previous results and use them to identify two generic instabilities: The system is stable above a cutoff density ("cutoff mode"), or can approach an asymptotic instability for increasing density ("saturation mode"). We analyze multi-angle effects on these generic types of instabilities and find that even the saturation mode is suppressed at large densities. For both types of modes, a given multi-angle spectrum typically is unstable when the neutrino and electron densities are comparable, but stable when the neutrino density is much smaller or much larger than the electron density. The role of an instability in the SN context depends on the available growth time and on the range of affected modes. At large matter density, most modes are off-resonance even when the system is unstable.

PACS numbers: 14.60.Pq, 97.60.Bw

I. INTRODUCTION

Neutrino flavor oscillations in a supernova (SN) are strongly suppressed by matter effects [1] until the neutrinos pass through the usual MSW region [2–5] far out in the envelope of the collapsing star. However, neutrino-neutrino interactions [6, 7], through a flavor off-diagonal refractive index, can trigger self-induced flavor conversions [8–13]. This collective effect tends to occur between the neutrino sphere and the MSW region and can lead to strongly modified neutrino spectra, showing features such as spectral swaps and splits [14–19]; for a review see Ref. [20]. The overall scenario, supported by heuristic ar-

dynamics. Recent studies dedicated to the SN accretion phase, under simplifying assumptions, once more confirm this picture [24, 25].

However, what is missing is a systematic approach to decide, without solving the equations of motion, if self-induced flavor conversions occur for given neutrino spectra (flavor-dependent energy and angular distribution), overall neutrino density, and matter density. Formal stability criteria exist only in the "single-angle approximation" where it is assumed that all neutrinos feel the same neutrino-neutrino refractive effect. In this case the analytic pendulum solution has been found and its existence and parameters can be calculated from the neutrino spec-

Linearized System

Liouville like equations

$$v^\beta \partial_\beta \rho = -i[H_p, \rho_p]$$

$$v^\beta = (1, \mathbf{v})$$
$$\beta = 0, \ldots, 3$$

$$\rho_{r,E,v} = g_{1E,v} \begin{pmatrix} s & S_1 \\ S_1^* & -s \end{pmatrix}_{r,E,v} \leftarrow |S| \ll 1$$

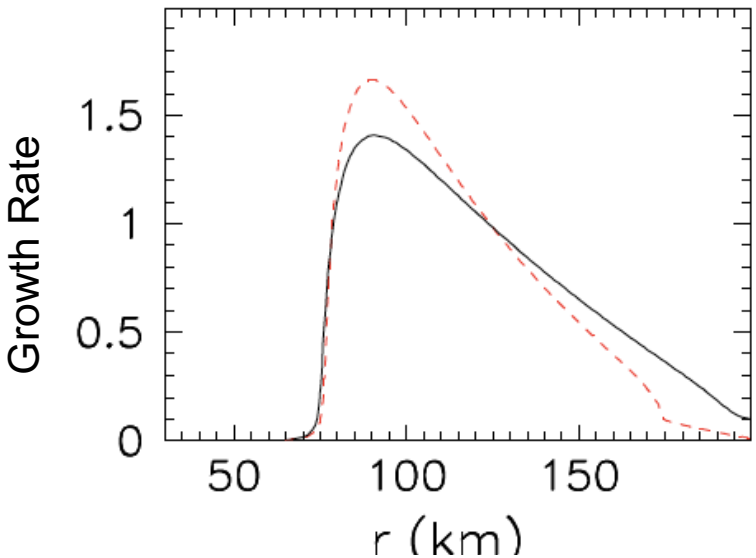
$$i\, v^\beta \partial_\beta S_{1E,\mathbf{v}} = (\omega_{12} + v^\beta \lambda_{1\beta}) S_{1E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

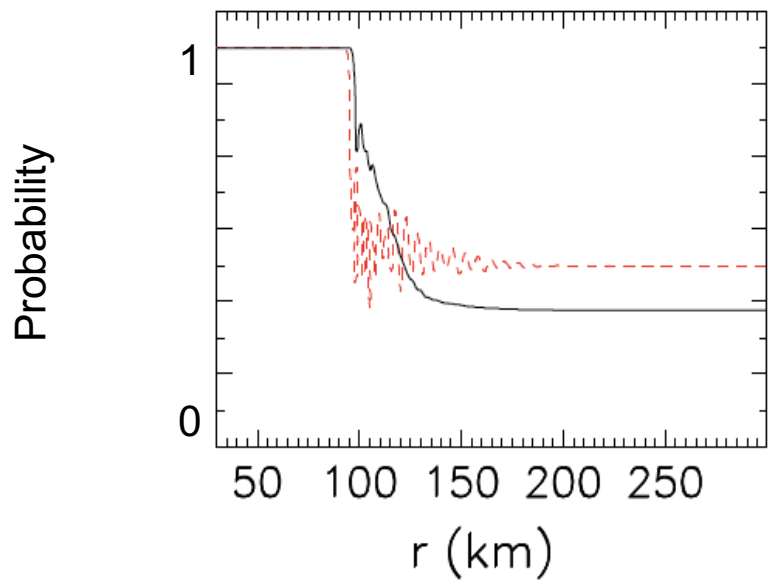
Spectrum:

$$g_{1E,\mathbf{v}} = \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

$$\int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\mathbf{v}}{4\pi},$$

LINEARIZED STABILITY ANALYSIS

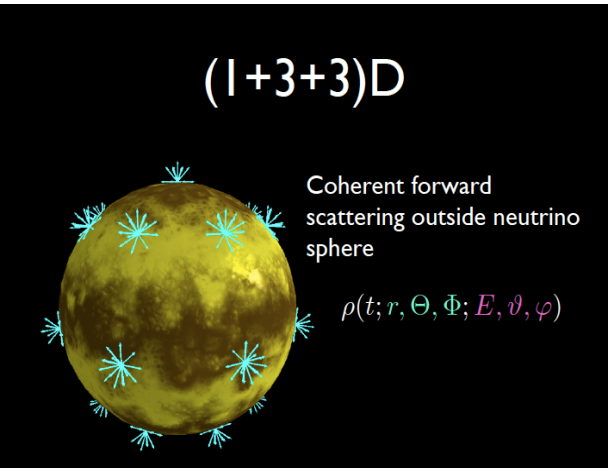


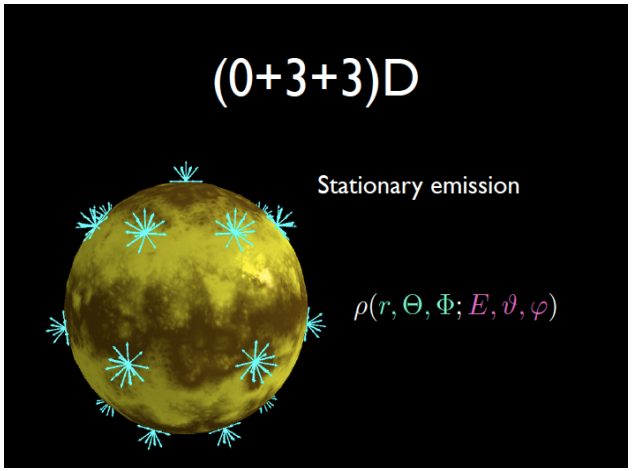


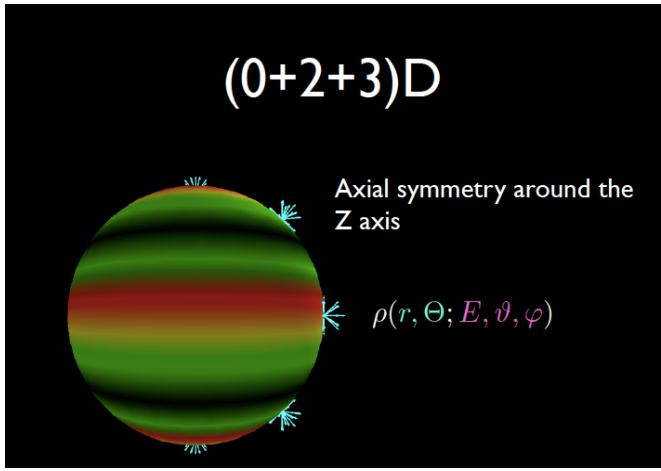
Onset of the conversion:
Peak of the growth rate curve

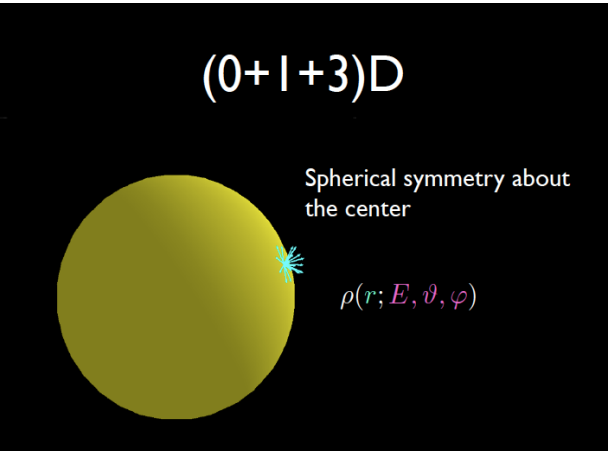
S.C & Mirizzi, PRD, 2014

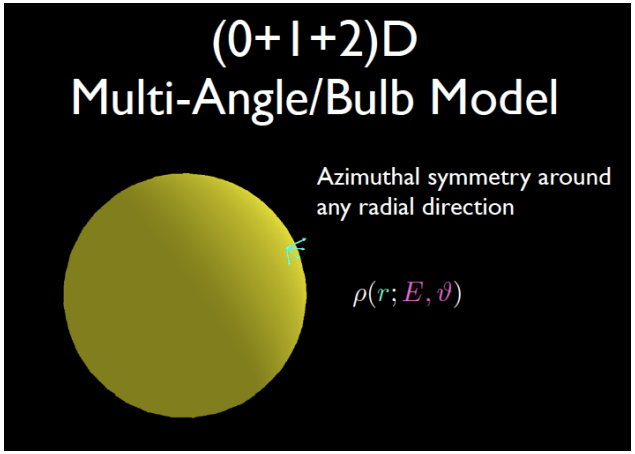
NEUTRINO TRANSPORT & FLAVOR OSCILLATIONS: 7D PROBLEM

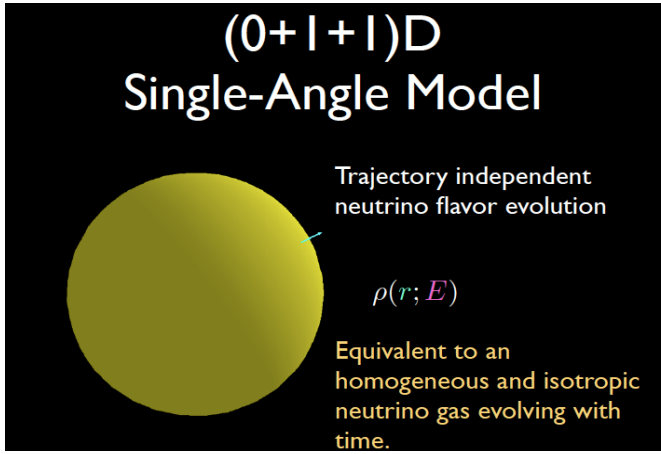










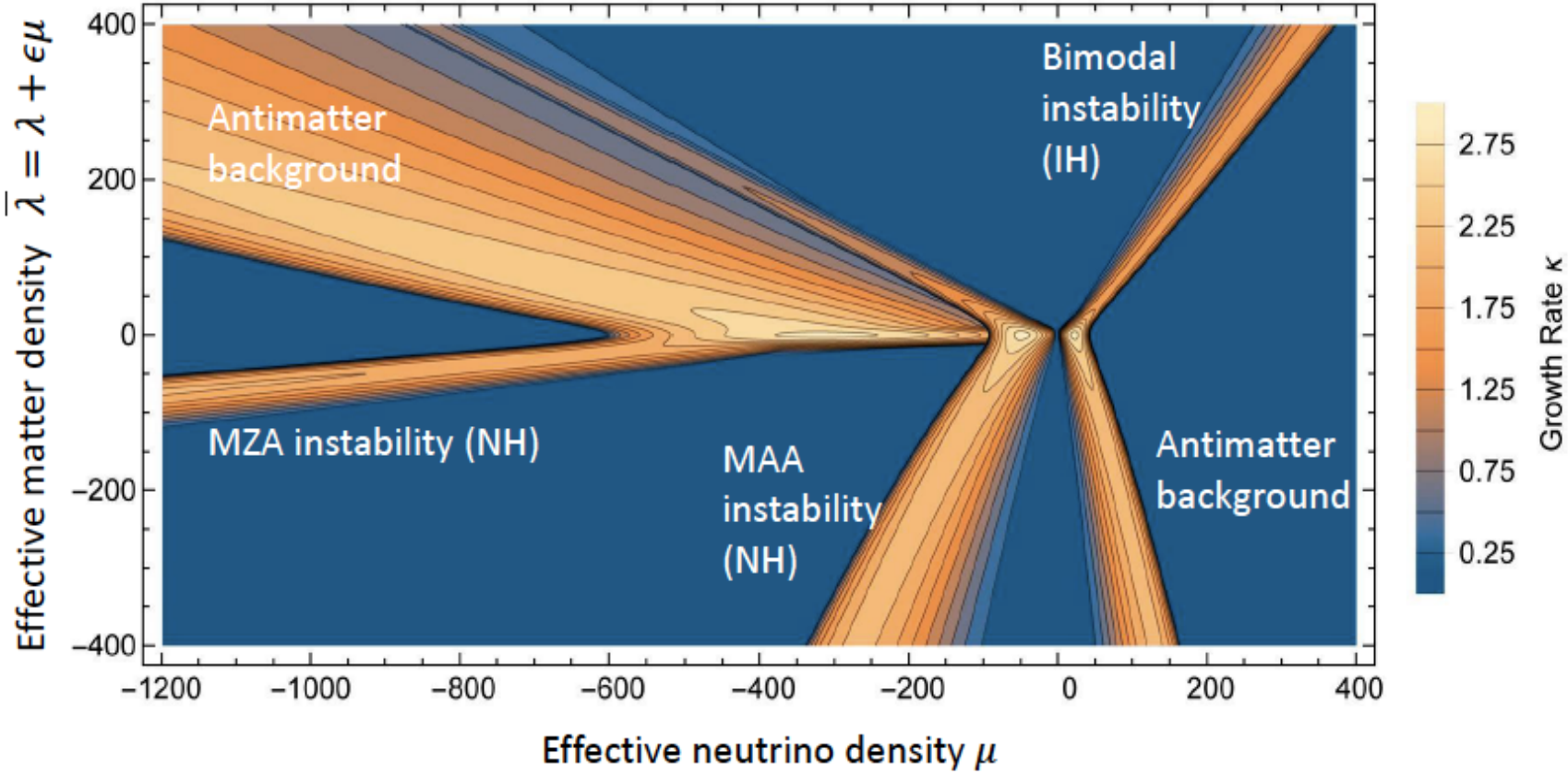


slides from H. Duan

Duan & Shalgar, PLB 2015
Mirizzi, Mangano & Saviano, PRD 2015

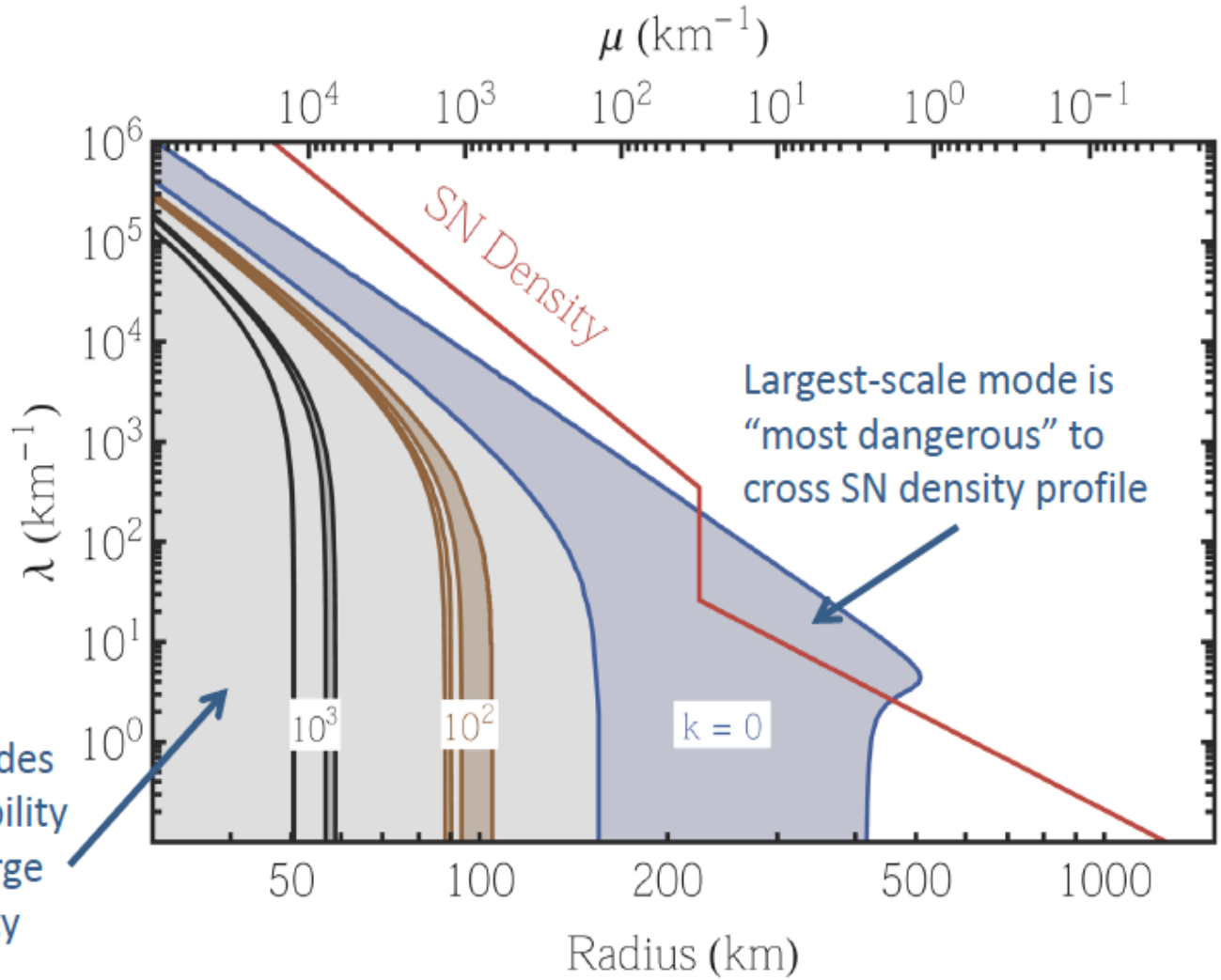
LINEARIZED STABILITY ANALYSIS (0+3+3)

Growth rate in 2D parameter space of effective matter and neutrino density
("Butterfly diagram")



S.C, Hansen, Izaguirre & Raffelt, JCAP 2015

LINEARIZED STABILITY ANALYSIS: INHOMOGENEITY IN TRANSVERSE PLANE (0+3+3)



Small-scale modes "fill in" the stability footprint for large neutrino density

S.C, Hansen, Izaguirre & Raffelt, JCAP 2015

FAST INSTABILITY (0+1+3)

PHYSICAL REVIEW D **72,** 045003 (2005)

Speed-up of neutrino transformations in a supernova environment

R. F. Sawyer
Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106, USA
(Received 8 April 2005; published 5 August 2005)

When the neutral current neutrino-neutrino interaction is treated completely, rather than as an interaction among angle-averaged distributions, or as a set of flavor-diagonal effective potentials, the result can be flavor mixing at a speed orders of magnitude faster than that one would anticipate from the measured neutrino oscillation parameters. It is possible that the energy spectra of the three active species of neutrinos emerging from a supernova are nearly identical.

PACS numbers: 95.30.Cq, 97.60.Bw

10.1103/PhysRevD.72.045003

oscillation length of the order of a kilometer for a 20 MeV neutrino, and uninteresting in any case by virtue of the near ... of their spectra and angular distributions. The ν_e ... vacuum is longer by a factor of 40 or ... tron density ensures that Thus there is

PRL **116,** 081101 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Neutrino Cloud Instabilities Just above the Neutrino Sphere of a Supernova

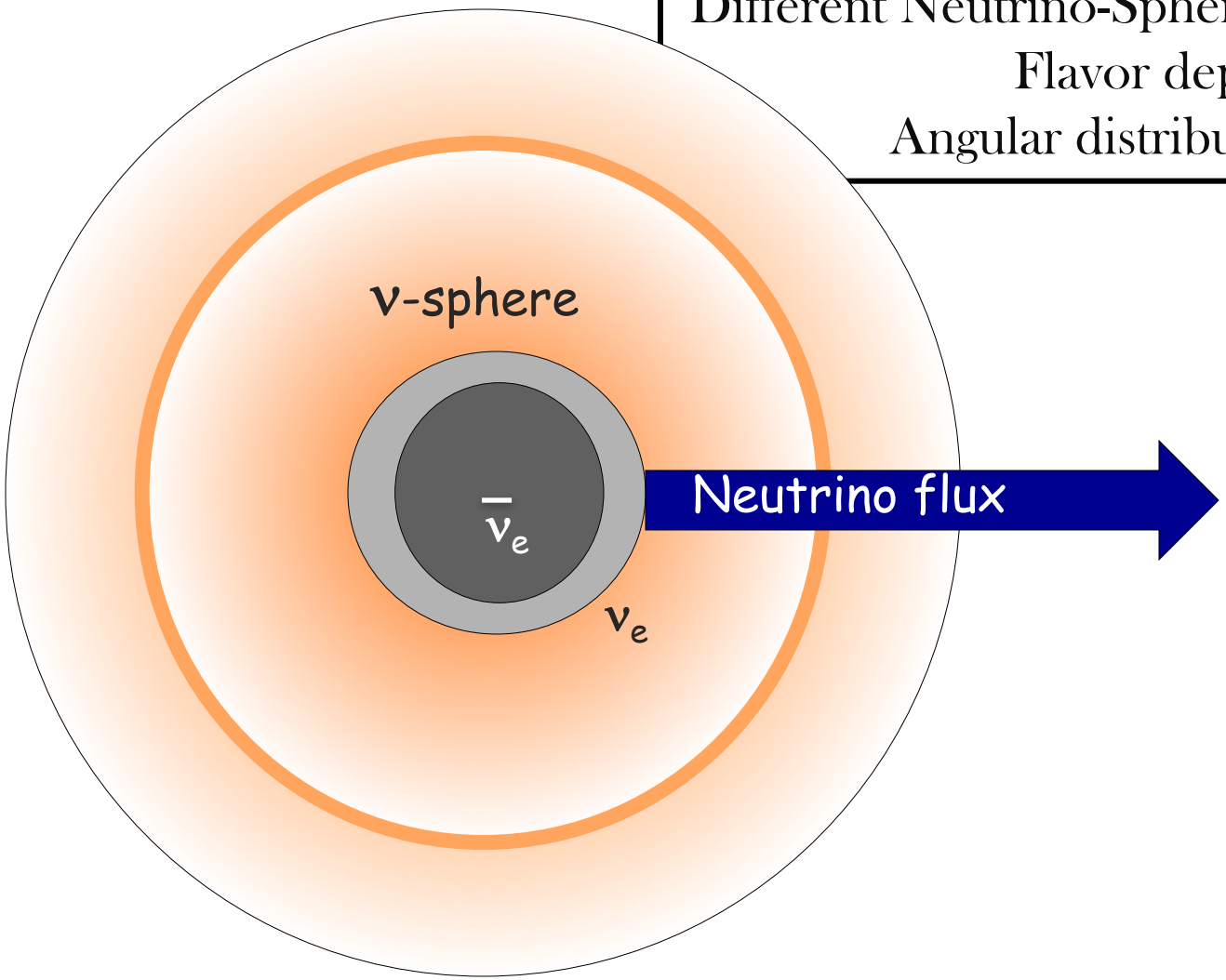
R. F. Sawyer
Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106, USA
(Received 7 September 2015; revised manuscript received 2 January 2016; published 25 February 2016)

Most treatments of neutrino flavor evolution, above a surface of the last scattering, take identical angular distributions on this surface for the different initial (unmixed) flavors, and for particles and antiparticles. Differences in these distributions must be present, as a result of the species-dependent scattering cross sections lower in the star. These lead to a new set of nonlinear equations, unstable even at the initial surface with respect to perturbations that break all-over spherical symmetry. There could be important consequences for explosion dynamics as well as for the neutrino pulse in the outer regions.

DOI: 10.1103/PhysRevLett.116.081101

SN ν FLAVOR TRANSITIONS: COLLECTIVE OSCILLATION

Different Neutrino-Sphere for different flavors :
Flavor dependent
Angular distribution / spectra



FAST INSTABILITY: ORDER μ GROWTH

$$i\, v^\beta \partial_\beta S_{1E,\mathbf{v}} = (\omega_{12} + v^\beta \lambda_{1\beta}) S_{1E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

- Unstable modes grow with rates of order μ instead of ω (μ >> ω)
- This requires different angle distribution for different flavors.
- The difference spectrum $g_{\omega,v}$ is flavor dependent

S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

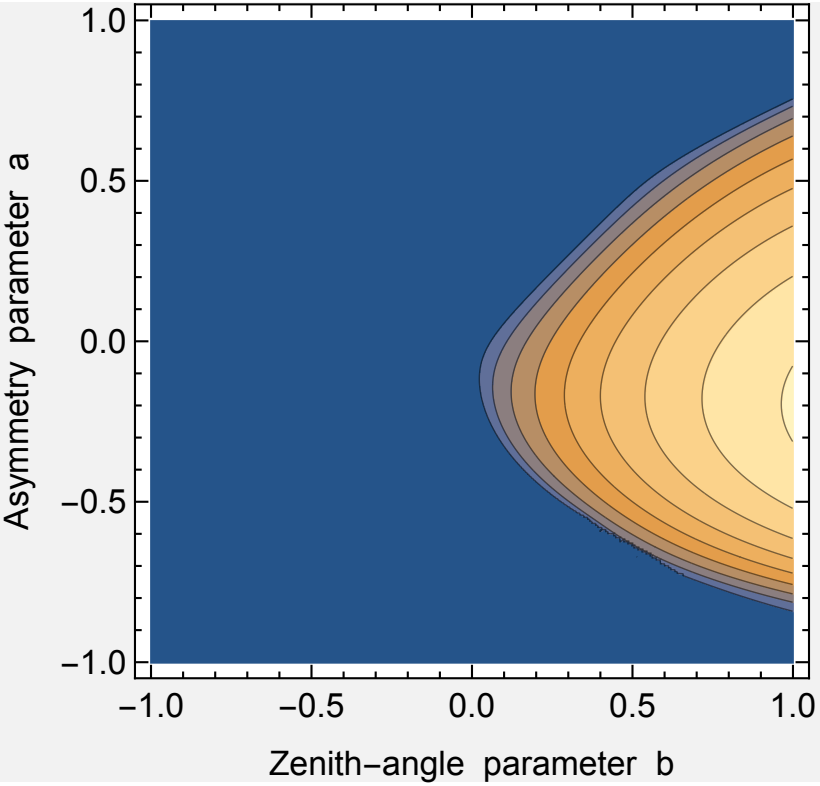
S.C, Hansen, Izaguirre & Raffelt, Nucl.Phys.B 2016

FAST INSTABILITY (0+1+3)

SN: $a > 0, b > 0$

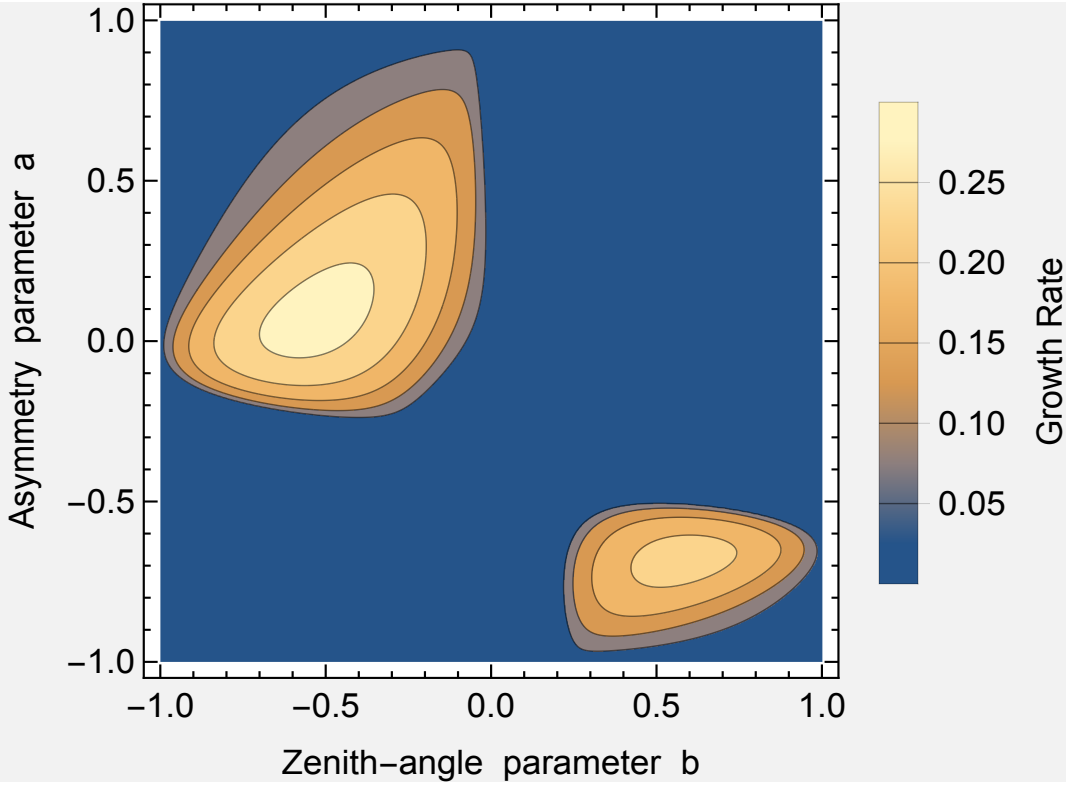
Axially symmetric solution

(0+1+2)



Axial symmetry broken

(0+1+3)



S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

Fast Oscillations: Three Flavor

$$i\, v^\beta \partial_\beta S_{1E,\mathbf{v}} = (\omega_{12} + v^\beta \lambda_{1\beta}) S_{1E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

$$S_{1\Gamma,\mathbf{r}} = Q_{1\Gamma,K}\, e^{-i(K_1^0 t - \mathbf{K}_1 \cdot \mathbf{r})}$$

Izaguirre, Raffelt and Tamborra, PRL 2016

In the fast oscillation limit, $\omega \to 0$,

$$\Pi^{\alpha\beta}_{j,K} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{j,\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_j^\gamma - \lambda_j^\gamma)}$$

j =1

However, in the three flavor case: two more such tensors,

$$\det \Pi^{\alpha\beta}_{j,K} = 0$$

Dispersion relations:
Connecting K_j^0 & K_j^i

j = 1,2,3

Three decoupled dispersion relations!

S.C & **M. Chakrabotry**, **JCAP, (2019)**

Fast Oscillations: Three Flavor

$$\Pi^{\alpha\beta}_{j,K} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{j,\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K^\gamma_j - \lambda^\gamma_j)}$$

S.C & M. Chakrabotry, JCAP, (2019)

There are three effective 'lepton numbers' not only ELN

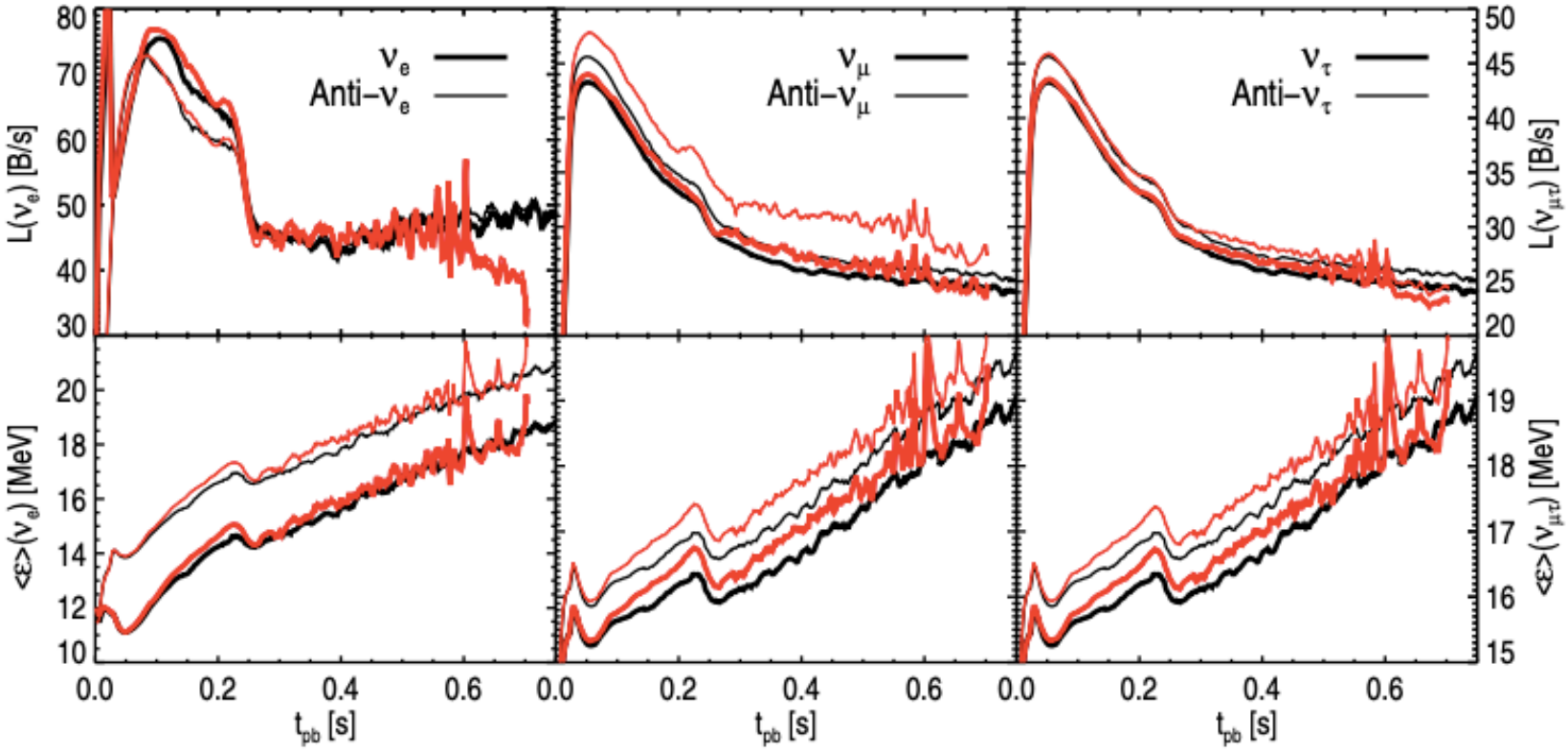
$$G_{1,\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{1,E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^\infty \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + f_{\bar{\nu}_\mu,\mathbf{p}} \right),$$

$$G_{2,\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{2,E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^\infty \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}} \right),$$

$$G_{3,\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{3,E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^\infty \frac{E^2 dE}{2\pi^2} \left(f_{\nu_\mu,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}} \right),$$

Neglecting the three flavor effect is
non-trivial than simply assuming that
fluxes of the mu and tau neutrinos are equal.

Full flavor analysis: the rise of mu-tau neutrinos



Asymmetry between
Muon neutrinos
and Muon antineutrinos,
mith Muon creation from pair
production

R. Bollig, H.-T. Janka, A. Lohs,
G. Martnez-Pinedo, C.J. Horowitz,
and T. Melson,
PRL 119, 242702 (2017)

Fast oscillations: Three flavor effects

Nonlinear Results: SN like toy problems

Full flavor Test: Set Up

- Both axial and zenith angle breaking instabilities are allowed

- The angular information of the 'muon' simulations were not sufficient

Probe by toy examples

F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRL 2020

F. Capozzi, S. Abbar, R. Bollig, H. -T. Janka PRD 2021

Full flavor Test: Set Up

Probe by toy examples

ELN

$$G_{\mathbf{v}}^{e} = \sqrt{2}G_F \int_0^\infty \frac{dEE^2}{2\pi^2}[\rho_{ee}(E,\mathbf{v}) - \bar{\rho}_{ee}(E,\mathbf{v})].$$

MuLN

$$G_{\mathbf{v}}^{\mu} = \sqrt{2}G_F \int_0^\infty \frac{dEE^2}{2\pi^2}[\rho_{\mu\mu}(E,\mathbf{v}) - \bar{\rho}_{\mu\mu}(E,\mathbf{v})],$$

TauLN

$$G_{\mathbf{v}}^{\tau} = \sqrt{2}G_F \int_0^\infty \frac{dEE^2}{2\pi^2}[\rho_{\tau\tau}(E,\mathbf{v}) - \bar{\rho}_{\tau\tau}(E,\mathbf{v})].$$

Effective lepton numbers

$$G_{\mathbf{v}}^{e\mu} = G_{\mathbf{v}}^{e} - G_{\mathbf{v}}^{\mu},$$

$$G_{\mathbf{v}}^{e\tau} = G_{\mathbf{v}}^{e} - G_{\mathbf{v}}^{\tau},$$

$$G_{\mathbf{v}}^{\mu\tau} = G_{\mathbf{v}}^{\mu} - G_{\mathbf{v}}^{\tau}.$$

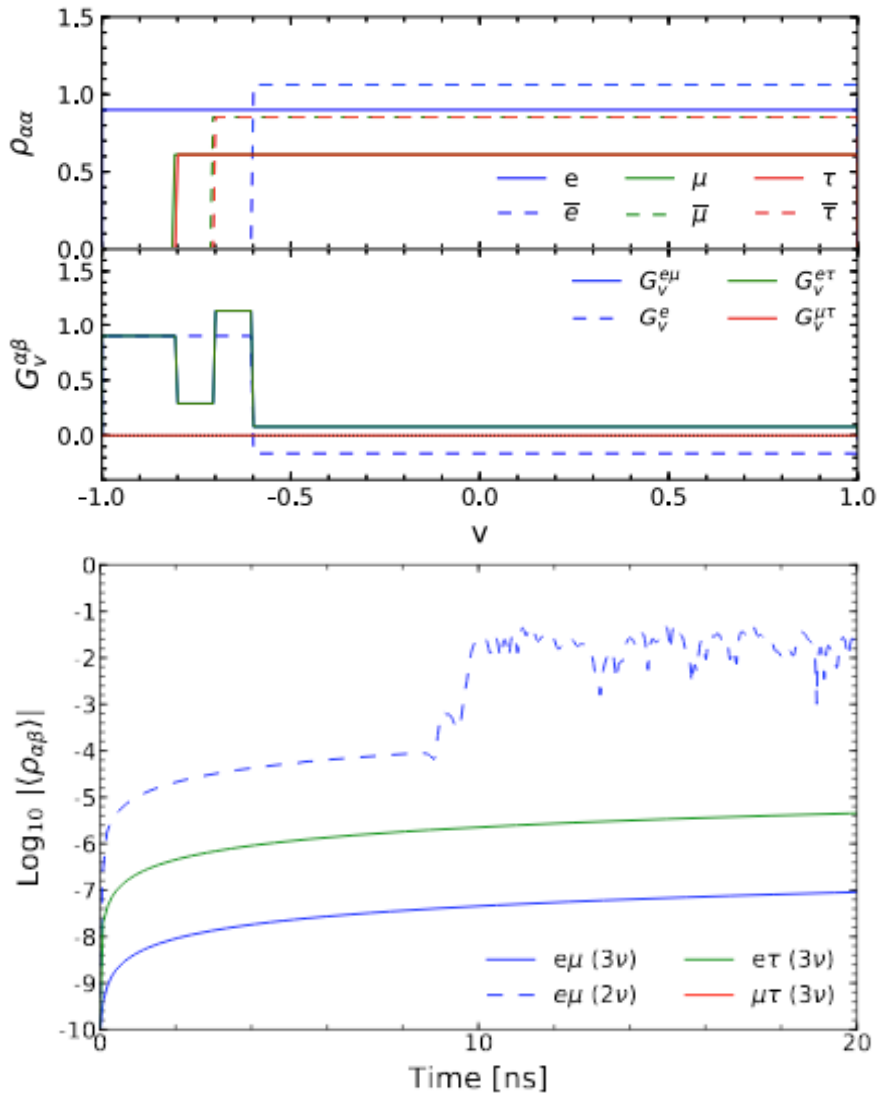
F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRL 2020

Full flavor Test: Results

Case I:

Fast Conversion
In 2 flavor

No Fast Conversion
In 3 flavor



F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRL 2020

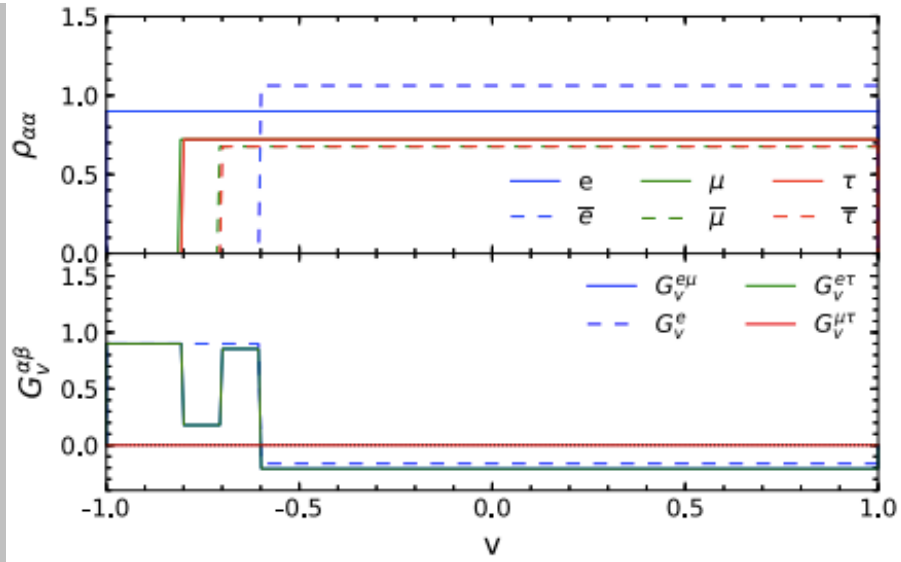
Full flavor Test: Results

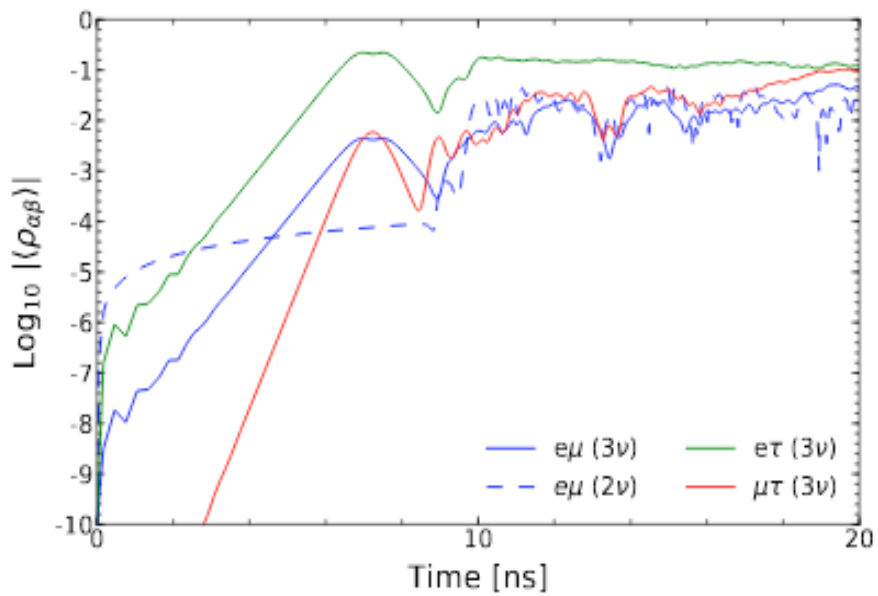
Case II:

Fast Conversion
In 2 flavor

Fast Conversion
in all flavors

Though, μ-τ sector
has no crossing
It participates in the
flavor conversion





F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRL 2020

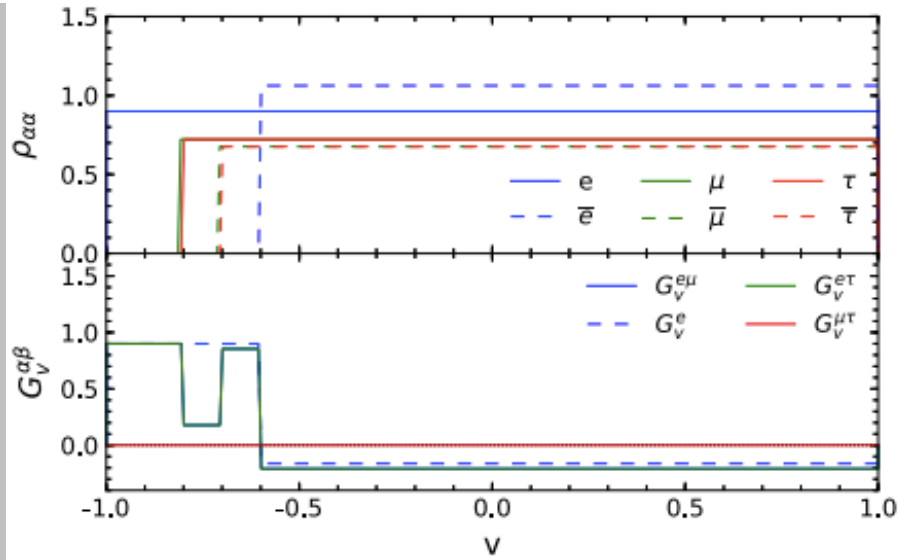
Full flavor Test: Results

Case II:

Fast Conversion
In 2 flavor

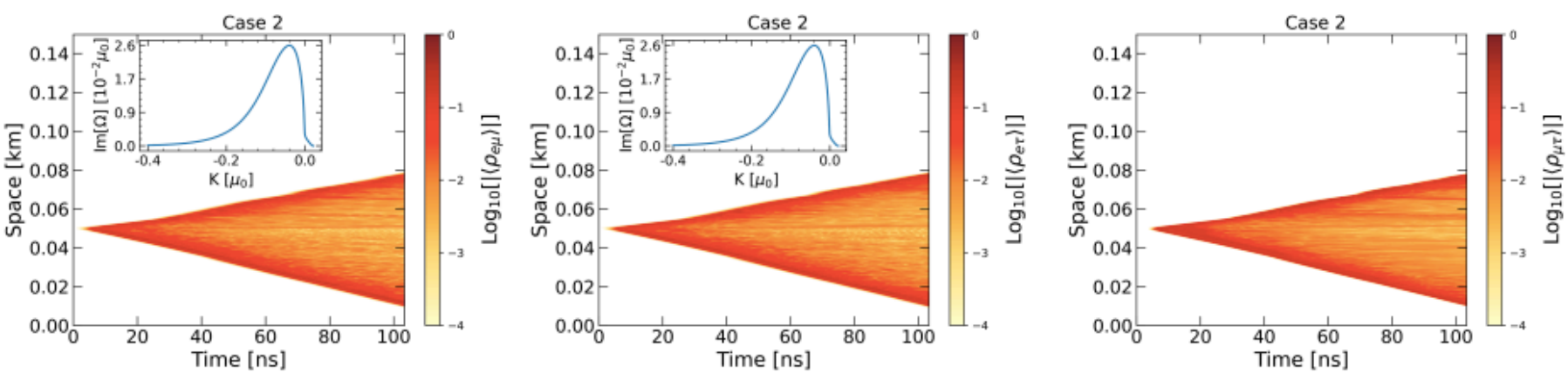
Fast Conversion
in all flavors

Though, μ-τ sector
has no crossing
It participates in the
flavor conversion



1 + 1D results

F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRD 2022



Beyond 3 Flavors: Sterile Neutrinos

$$\rho_{\mathbf{p}} = \begin{pmatrix} \rho_{\mathbf{p}}^{ee} & \rho_{\mathbf{p}}^{e\mu} & \rho_{\mathbf{p}}^{es} \\ \rho_{\mathbf{p}}^{\mu e} & \rho_{\mathbf{p}}^{\mu\mu} & \rho_{\mathbf{p}}^{\mu s} \\ \rho_{\mathbf{p}}^{se} & \rho_{\mathbf{p}}^{s\mu} & \rho_{\mathbf{p}}^{ss} \end{pmatrix}.$$

2 active + 1 sterile, 1 eV^2 mass squared difference

Linearized system: The off diagonals involving sterile evolve trivially in linear order

$$i\, v^\beta \partial_\beta S_{3E,\mathbf{v}} = [\omega_3 + v^\beta\, \lambda_\mu^\beta] S_{3E,\mathbf{v}} - \sqrt{2}\, G_F\, S_{2E,\mathbf{v}}\, \xi^{-1}\, v^\beta \int d\Gamma'\, v'_\beta\, g_{E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

Non linear results found no influence of sterile on evolution on Fast Oscillation.

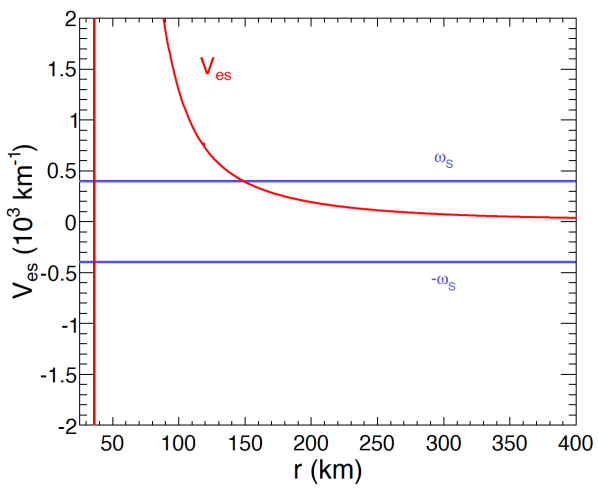
Active-Sterile MSW & Fast Oscillation

1) Fast Oscillation driven by,

S.C & M. Chakraborty, in preparation

$$G^e_{\mathbf{v}} = \sqrt{2}G_F \int_0^\infty \frac{dEE^2}{2\pi^2}[\rho_{ee}(E,\mathbf{v}) - \bar{\rho}_{ee}(E,\mathbf{v})].$$

2) For a given mass ordering MSW either for neutrino or antineutrino



I. Tamborra, G. Raffelt, L. Huedepohl & H-T Janka, JCAP 2011

$$\mathrm{H}^{\mathrm{m}+\nu\nu}_{ee} = \sqrt{2}G_{\mathrm{F}}\left[N_b\left(\frac{3}{2}Y_e - \frac{1}{2}\right) + 2(N_{\nu_e} - N_{\bar{\nu}_e}) + (N_{\nu_x} - N_{\bar{\nu}_x})\right]$$

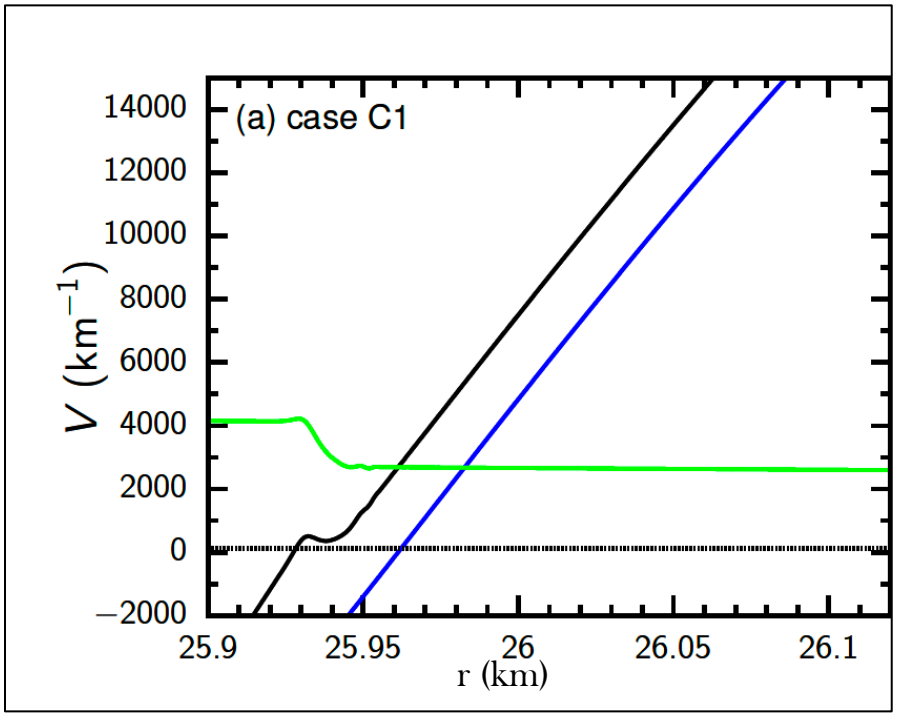
$$\mathrm{H}^{\mathrm{m}+\nu\nu}_{xx} = \sqrt{2}G_{\mathrm{F}}\left[N_b\left(\frac{1}{2}Y_e - \frac{1}{2}\right) + (N_{\nu_e} - N_{\bar{\nu}_e}) + 2(N_{\nu_x} - N_{\bar{\nu}_x})\right]$$

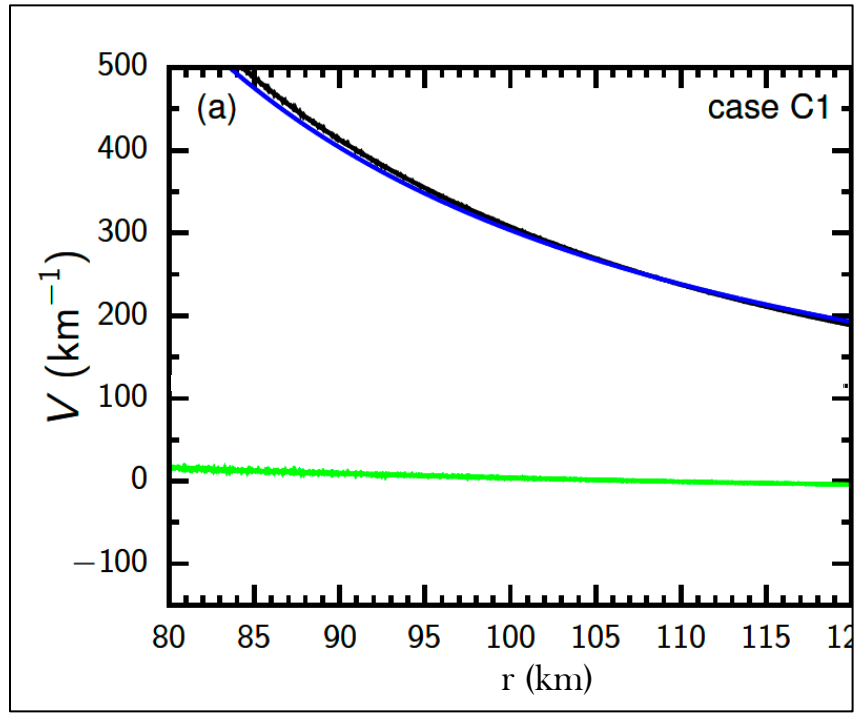
Oscillation between e-s sector, only.

3) ELN (hence Fast oscillation) can be modified by Active-Sterile MSW

Beyond 3 Flavors: Sterile Neutrinos

S.C & M. Chakraborty, in preparation





Inner resonance: few 10s km

Outer resonance: 50-100 km

Z. Xiong, Meng-Ru Wu & Y-Z. Qian, APJ 2019

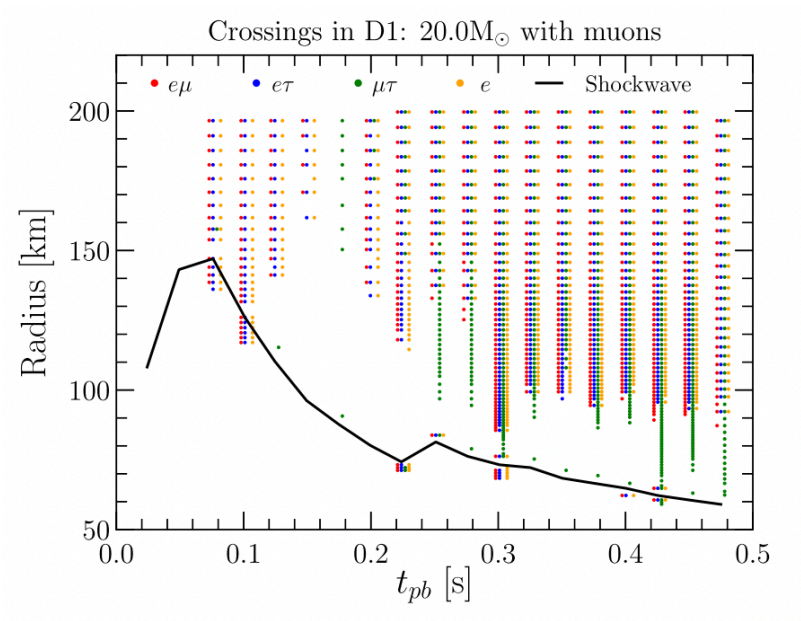
Beyond 3 Flavors: Sterile Neutrinos

S.C & M. Chakraborty, in preparation

Scenario I: Fast oscillation before active-sterile MSW

Relatively straight forward implication
Possible effect on nucleosynthesis or neutrino spectra at earth

Scenario II: Fast oscillation after active-sterile MSW

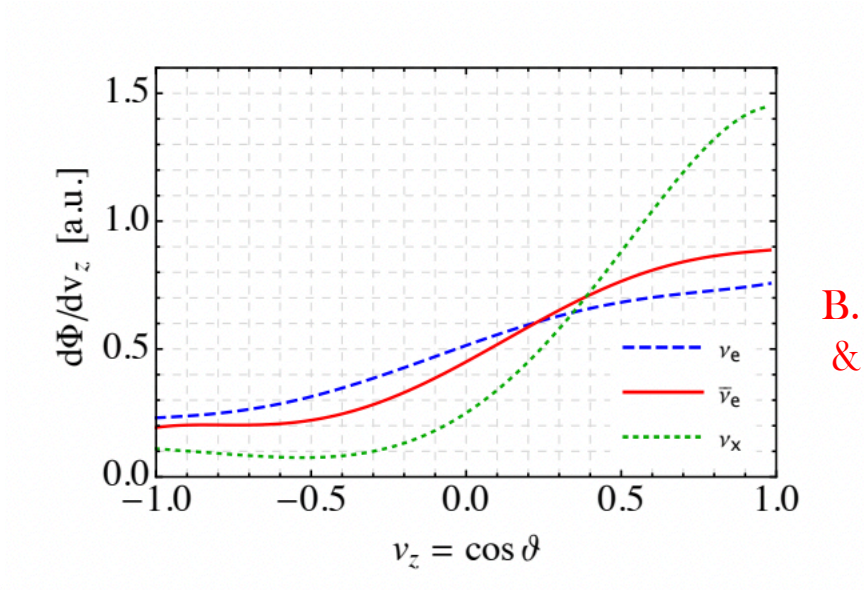


F. Capozzi, S. Abbar, R. Bollig, H. -T. Janka PRD 2021

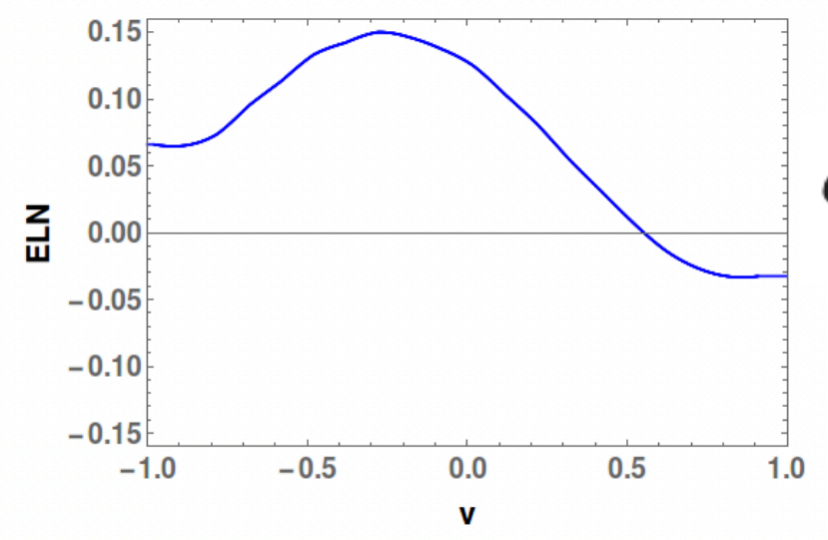
Beyond 3 Flavors: Sterile Neutrinos

Toy Model : Oscillation in e-s sector, only + NO in e-s.

S.C & M. Chakraborty, in preparation



B. Dasgupta, A. Mirizzi & M. Sen, JCAP 2017

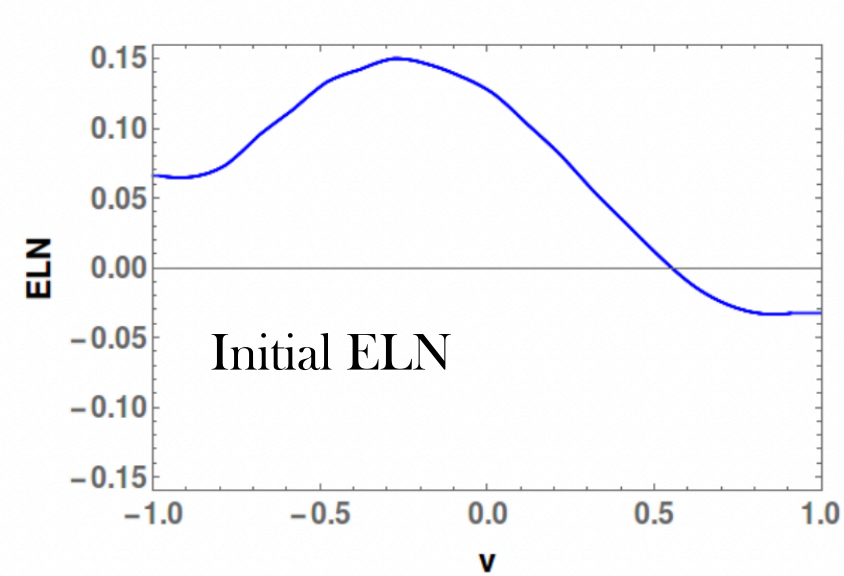


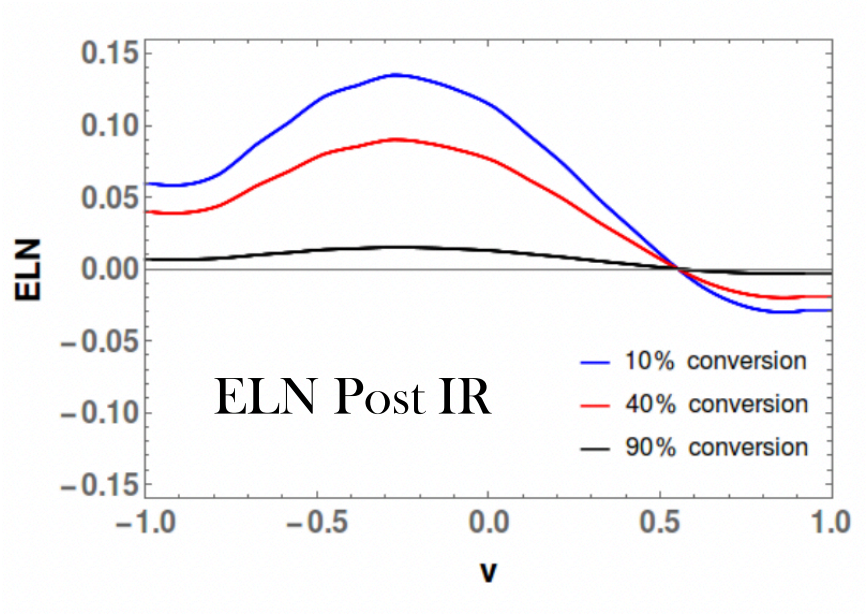
$$G_{\mathbf{v}}^{e} = \sqrt{2} G_F \int_0^\infty \frac{dE E^2}{2\pi^2} [\rho_{ee}(E, \mathbf{v}) - \bar{\rho}_{ee}(E, \mathbf{v})].$$

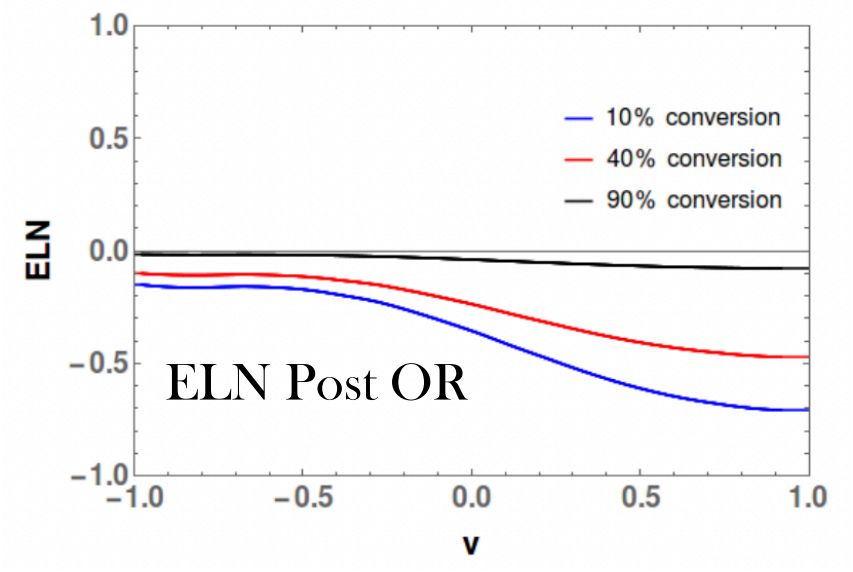
Beyond 3 Flavors: Sterile Neutrinos

Toy Model:

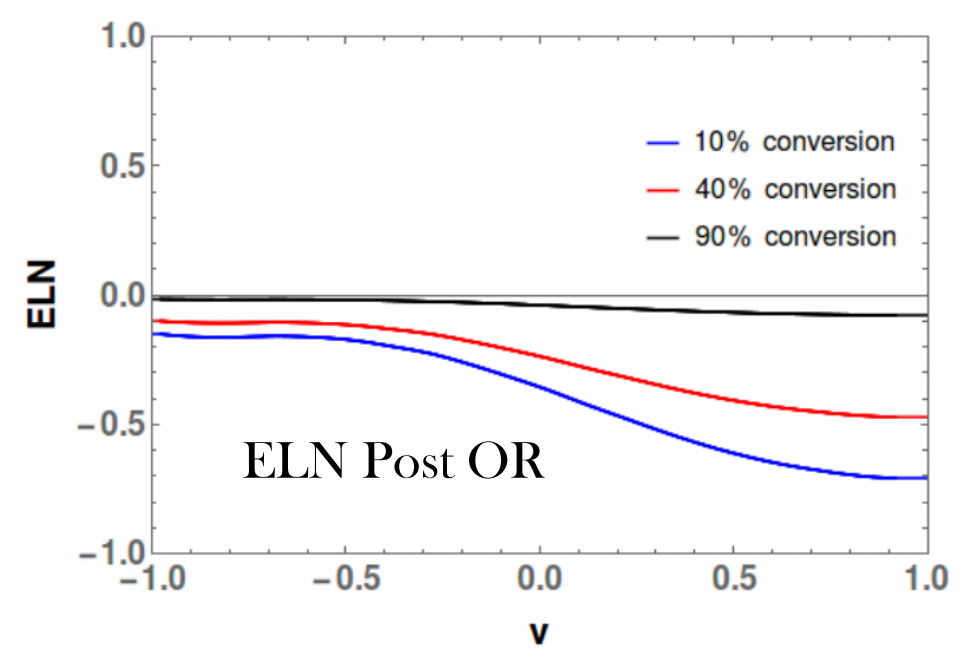
S.C & M. Chakraborty, in preparation







Beyond 3 Flavors: Sterile Neutrinos



S.C & M. Chakraborty, in preparation

Fast oscillations are suppressed for $R_{resonance} < R_{FO}$

Possible influence on nucleosynthesis or neutrino spectra at earth

Active-sterile MSW mostly looked into late phase for nucleosynthesis.

Interesting to see what happens in LESA simulations!!

Thank You, Georg!

Wish you a galactic SN!

Linearized System: 3 Flavor

Liouville like equations

$$v^\beta \partial_\beta \rho = -i[H_p, \rho_p]$$

$$v^\beta = (1, \mathbf{v})$$
$$\beta = 0, \ldots, 3$$

Matrix of densities & Hamiltonian

$$\rho_{\mathbf{p}} = \begin{pmatrix} \rho_{\mathbf{p}}^{ee} & \rho_{\mathbf{p}}^{e\mu} & \rho_{\mathbf{p}}^{e\tau} \\ \rho_{\mathbf{p}}^{\mu e} & \rho_{\mathbf{p}}^{\mu\mu} & \rho_{\mathbf{p}}^{\mu\tau} \\ \rho_{\mathbf{p}}^{\tau e} & \rho_{\mathbf{p}}^{\tau\mu} & \rho_{\mathbf{p}}^{\tau\tau} \end{pmatrix} \quad \& \quad H_{\mathbf{p}} = \frac{M^2}{2E} + H^{matter}$$

Taking into account the off-diagonals of ρ upto linear order,

$$i\, v^\beta \partial_\beta \rho_{\mathbf{p}}^{e\mu} = \left[\frac{m_1^2 - m_2^2}{2E} + v_\beta(\lambda_e^\beta - \lambda_\mu^\beta) \right] \rho_{\mathbf{p}}^{e\mu} - \sqrt{2}\, G_F\, (\rho_{\mathbf{p}}^{ee} - \rho_{\mathbf{p}}^{\mu\mu}) v^\beta \int d\mathbf{p}' v'_\beta\, (\rho_{\mathbf{p}'}^{e\mu} - \bar{\rho}_{\mathbf{p}'}^{e\mu})$$

Simillar equations for e-τ and μ-τ

S. Airen, F. Capozzi, S.C, B. Dasgupta, G. Raffelt & T. Stirner, JCAP 2018

Linearized System: 3 Flavor

Taking into account the off-diagonals of ρ upto linear order,

$$i\, v^\beta \partial_\beta \rho_{\mathbf{p}}^{e\mu} = \left[\frac{m_1^2 - m_2^2}{2E} + v_\beta(\lambda_e^\beta - \lambda_\mu^\beta)\right]\rho_{\mathbf{p}}^{e\mu} - \sqrt{2}\, G_F\, (\rho_{\mathbf{p}}^{ee} - \rho_{\mathbf{p}}^{\mu\mu})v^\beta \int d\mathbf{p}' v'_\beta\, (\rho_{\mathbf{p}'}^{e\mu} - \bar{\rho}_{\mathbf{p}'}^{e\mu})$$

Simillar equations for e-τ and μ-τ

Evolution of the off-diagonal 'S_p' holds all flavor coherence information,

$$\rho_{\mathbf{p}} = \frac{f_{\nu_e,\mathbf{p}} + f_{\nu_\mu,\mathbf{p}} + f_{\nu_\tau,\mathbf{p}}}{3} 1 + \frac{f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}}}{3}\begin{pmatrix} s_{\mathbf{p}} & S_{1\mathbf{p}} & 0 \\ S_{1\mathbf{p}}^* & -s_{\mathbf{p}} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Three S_p's

$$+ \frac{f_{\nu_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}}}{3}\begin{pmatrix} s_{\mathbf{p}} & 0 & S_{2\mathbf{p}} \\ 0 & 0 & 0 \\ S_{2\mathbf{p}}^* & 0 & -s_{\mathbf{p}} \end{pmatrix} + \frac{f_{\nu_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}}}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & s_{\mathbf{p}} & S_{3\mathbf{p}} \\ 0 & S_{3\mathbf{p}}^* & -s_{\mathbf{p}} \end{pmatrix}$$

M. Chakraborty & S.C, JCAP 2019

Linearized System: 3 Flavor

$$\int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\mathbf{v}}{4\pi},$$

In the flavor isospin picture,

$$i\, v^\beta \partial_\beta \underline{S_{1E,\mathbf{v}}} = (\omega_{12} + v^\beta \lambda_{1\beta}) \underline{S_{1E,\mathbf{v}}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} \underline{S_{1E',\mathbf{v}'}}$$

$$i\, v^\beta \partial_\beta S_{2E,\mathbf{v}} = (\omega_{13} + v^\beta \lambda_{2\beta}) S_{2E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{2E',\mathbf{v}'} S_{2E',\mathbf{v}'}$$

$$i\, v^\beta \partial_\beta S_{3E,\mathbf{v}} = (\omega_{13} - \omega_{12} + v^\beta \lambda_{3\beta}) S_{3E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{3E',\mathbf{v}'} S_{3E',\mathbf{v}'}$$

Spectra : $$g_{1E,\mathbf{v}} = \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

Simillar spectra for e-τ and μ-τ

M. Chakraborty & S.C, JCAP 2019

Linearized System: 3 Flavor

$$\int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\mathbf{v}}{4\pi},$$

In the flavor isospin picture,

$$i\, v^\beta \partial_\beta \underline{S_{1E,\mathbf{v}}} = (\omega_{12} + v^\beta \lambda_{1\beta}) \underline{S_{1E,\mathbf{v}}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} \underline{S_{1E',\mathbf{v}'}}$$

$$i\, v^\beta \partial_\beta S_{2E,\mathbf{v}} = (\omega_{13} + v^\beta \lambda_{2\beta}) S_{2E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{2E',\mathbf{v}'} S_{2E',\mathbf{v}'}$$

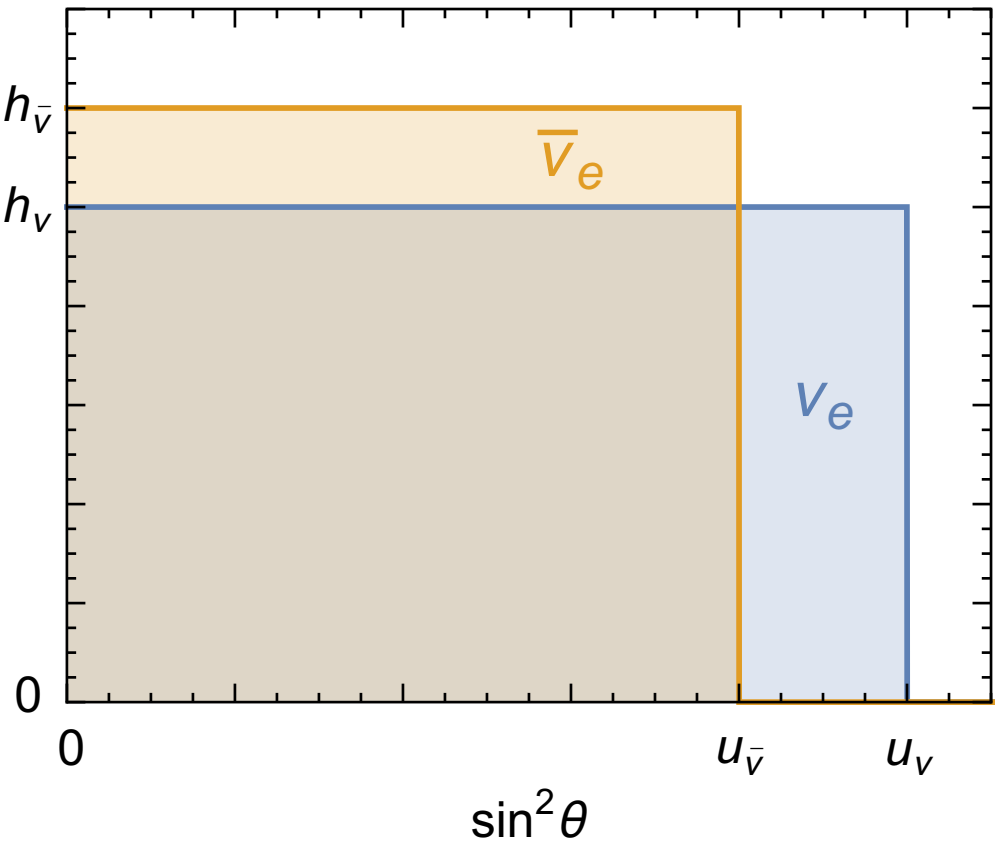
$$i\, v^\beta \partial_\beta S_{3E,\mathbf{v}} = (\omega_{13} - \omega_{12} + v^\beta \lambda_{3\beta}) S_{3E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{3E',\mathbf{v}'} S_{3E',\mathbf{v}'}$$

In the lineraized regime,

- Three off diagonal modes evolve independently!
- Three flavor evolution is equivalent to three two flavor evolution.

M. Chakraborty & S.C, JCAP 2019

FAST INSTABILITY



SN: *a > 0, b > 0*

$$h_{\nu_e}(u) = \int_0^\infty d\omega\, g(\omega, u)$$

$$h(u) = \frac{1 \pm a}{1 \pm b} \times \begin{cases} 1 & \text{for } 0 \le u \le 1 \pm b\,, \\ 0 & \text{otherwise}\,, \end{cases}$$

Uniform but different distribution
For neutrinos and antineutrinos,
width parameter (b)

$$-1 < b < +1$$

The distribution also connected
to the lepton asymmetry of the system,
asymmetry parameter (a)

$$-1 < a < +1$$

S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

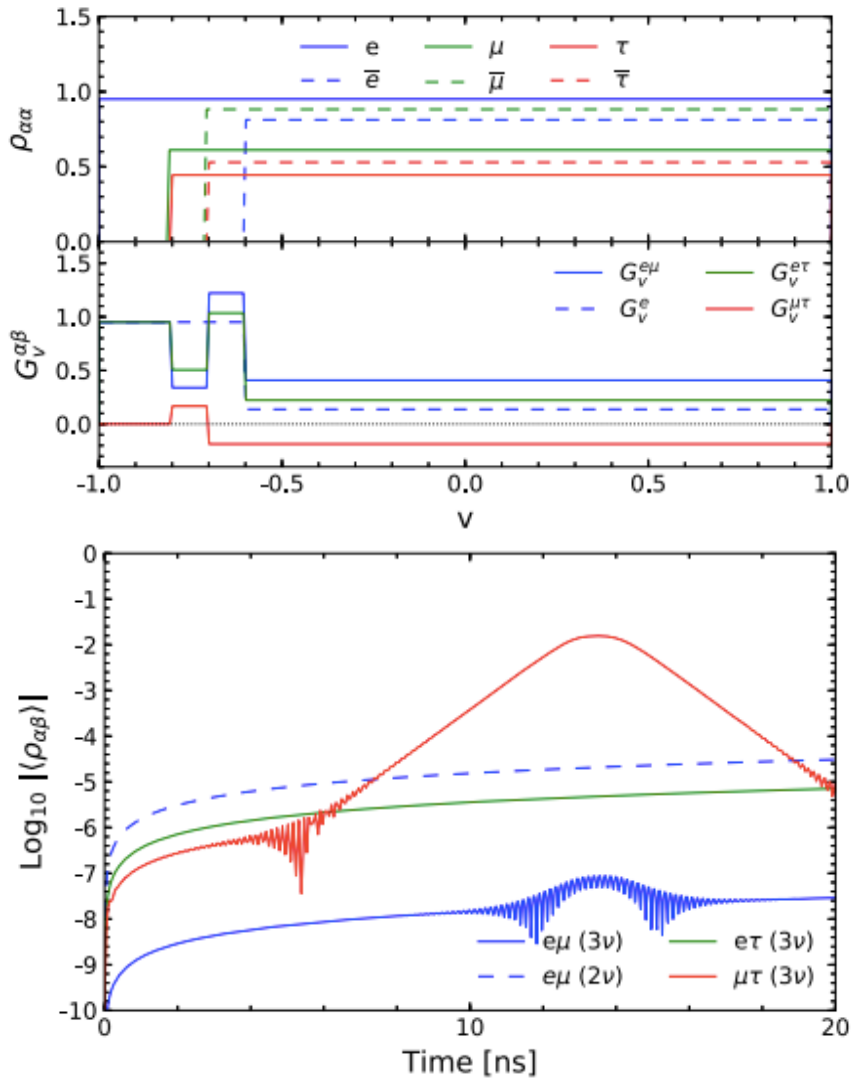
S.C, Hansen, Izaguirre & Raffelt, PLB 2016

Full flavor Test: Results

Case III:
No Fast Conversion
In 2 flavor

No Fast Conversion
in e-τ and e-μ.

μ-τ shows
instability



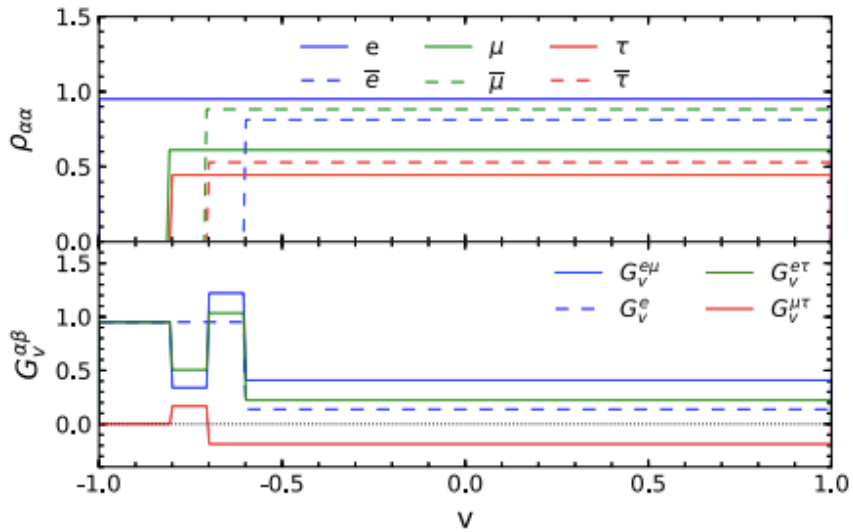
F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRL 2020

Full flavor Test: Results

Case III:
No Fast Conversion
In 2 flavor

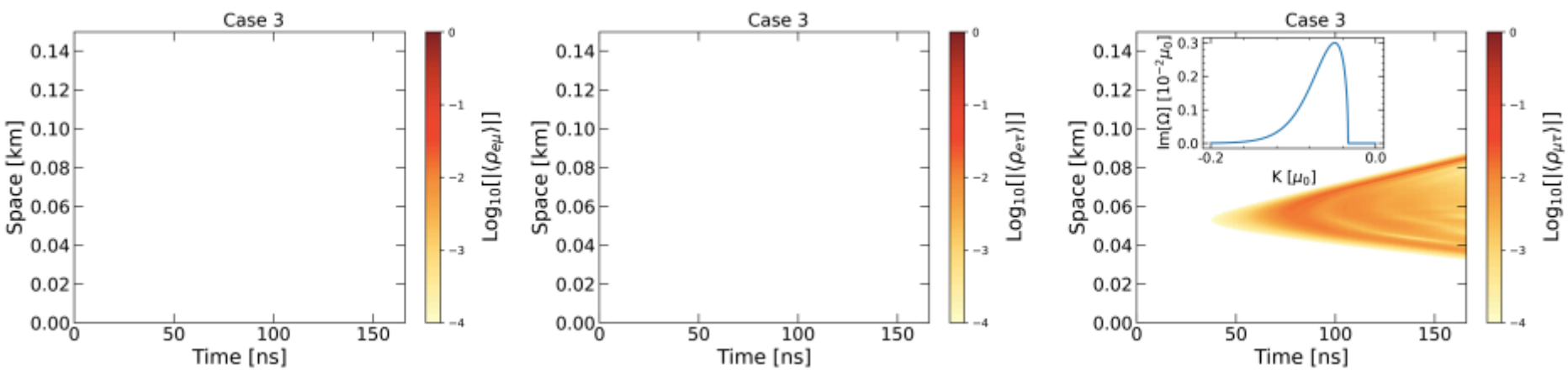
No Fast Conversion
in e-τ and e-μ

μ-τ shows
instability



1 + 1D results

F. Capozzi, M. Chakraborty, **S.C** and M. Sen, PRD 2022



BREAKING OF STATIONARITY (1+3+3)

$$i(\partial_t + \mathbf{v}\cdot\boldsymbol{\nabla}_{\mathbf{r}})\varrho = [\mathsf{H},\varrho], \quad \varrho = \varrho(t,\mathbf{r},E,\mathbf{v}),$$

Abbar & Duan, PLB 2015
Dasgupta & Mirizzi, PRD 2015

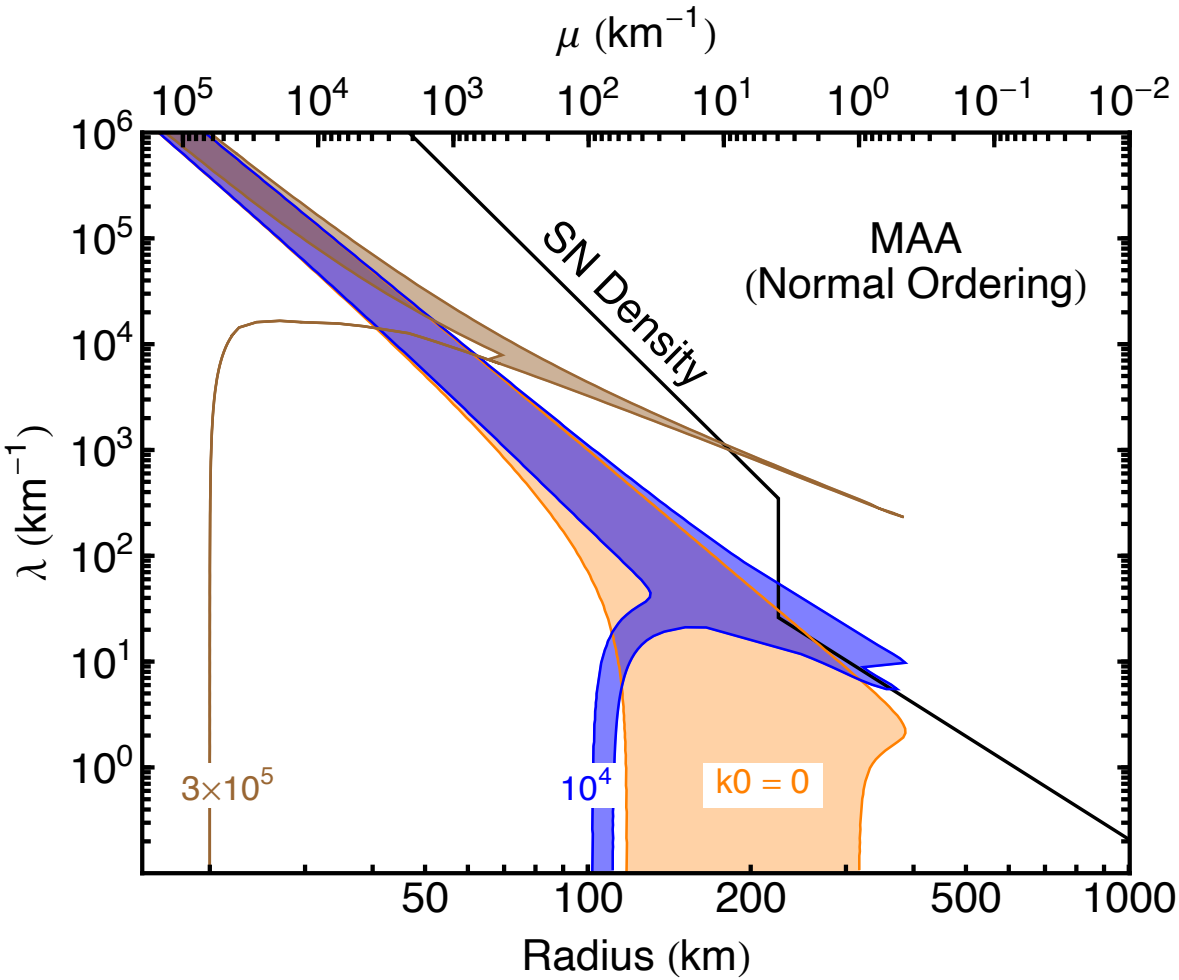
$$\left(\boxed{\bar{\lambda}_r \mathbf{v}^2 + k_0 \frac{R^2}{2r^2}\mathbf{v}^2} + \mathbf{k}\cdot\mathbf{v} + \omega - \Omega_r\right) Q_{\Omega,k_0,k,\omega,v} =$$

$$\mu_r \int_{-\infty}^{+\infty} d\omega' \int d\mathbf{v}' \, (\mathbf{v}-\mathbf{v}')^2 \, Q_{\Omega,k_0,k,\omega',v'}$$

- k_0 from Fourier transform to the time part, i.e, frequency
- k_0 can be both +ve and –ve, thus can nullify matter effect

BREAKING OF STATIONARITY (1+1+3)

- **For simplicity assume spatial homogeneity, k = 0, $K_0 \neq 0$**



Cascading between different temporal modes would change the picture

However, that depends on the Duration of instability

Capozzi, Dasgupta & Mirizzi, JCAP 2016

S.C, Hansen, Izaguirre & Raffelt, PLB 2015.

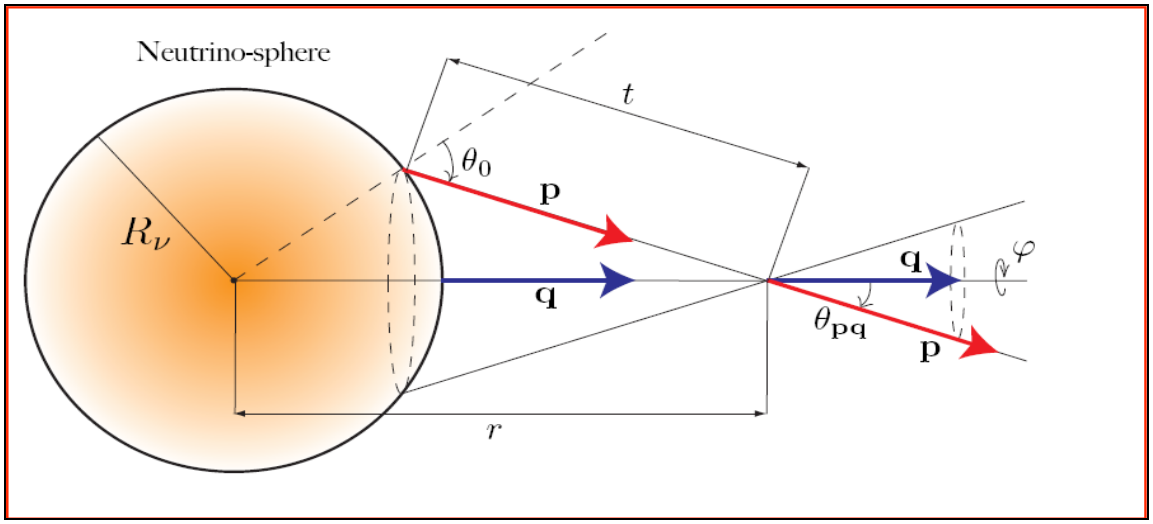
MULTI ANGLE PROBLEM (0+1+2):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\,[H(r, E, \theta), \rho(r, E, \theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}\,(E)$,
azimuthal symmetry in momentum: no ϕ
'v_r' Radial velocity depends on θ, leads to multi-angle matter effect

Ignore matter: Matter induced resonance happens far away from collective, however.....



Esteban-Pretel, Mirizzi, Pastor, Tomas, Raffelt, Serpico & Sigl, PRD 2008