EFT for $\mu \to e$ Flavour Change(LFV)

Sacha Davidson (IN2P3/CNRS, FR)
+ Marco Ardu(applying for pd), B Echenard, M Gorbahn, Y Kuno,...

LFV is boring $\begin{cases} \text{no anomalies} \\ \text{uncountable models predict signals in upcoming expts} \end{cases}$

and EFT is tedious ... $\begin{cases} \text{no predictions} \\ \text{endless incomprehensible parameters} \end{cases}$

So how to entertain such an excellent phenomenologist as Georg for 20 minutes?

(eat chocolate?)

What to learn about m_ν-mechanism from $\mu \to e$ rates?

Sacha Davidson (IN2P3/CNRS, FR)
+ Marco Ardu, B Echenard, M Gorbahn, Y Kuno, M Yamanaka, U Uesaka,...

LFV is boring $\begin{cases} \text{no anomalies} \\ \text{uncountable models predict signals in upcoming expts} \end{cases}$

and EFT is tedious ... $\begin{cases} \text{no predictions} \\ \text{endless incomprehensible parameters} \end{cases}$

So how to entertain such an excellent phenomenologist as Georg for 20 minutes?

(eat chocolate?)

But LFV might be interesting later: *has to exist*, and many exptal searches....
...while we wait, ask "what can we learn about neutrinos from LFV?"

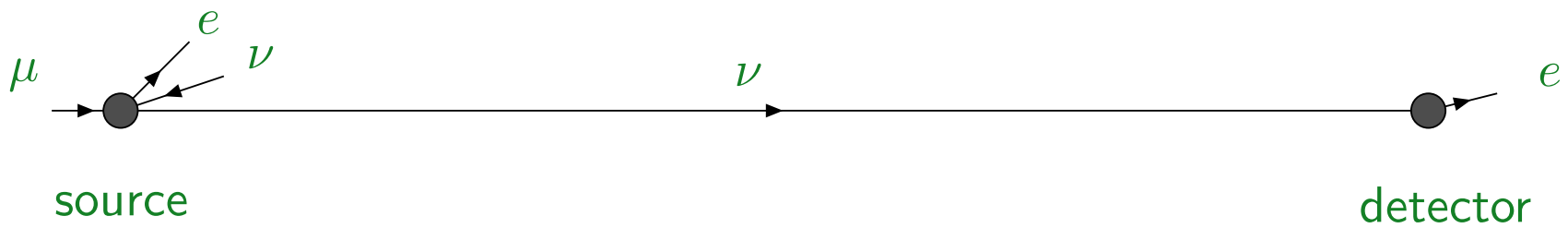
about the mass mechanism? magnetic moments? number of singlets? NSI? ...

A pheno question, so

1. start from data : bounds on LFV (... three $\mu \to e$ processes)
   parametrise with contact interactions
2. EFT to take data to models (peel off SM loops)
3.. what can it tell us about heavy leptonic NP?

Meet lepton flavour change...categories

- the only leptons we "see" are charged — and we see them change flavour:

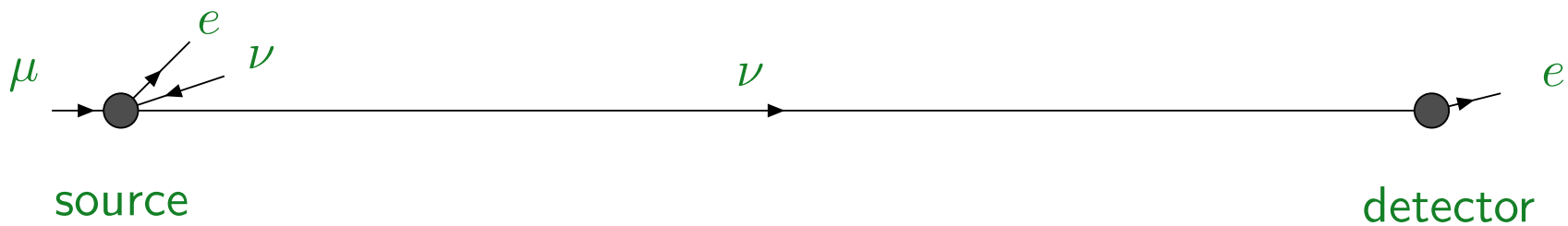


But 'tis not "LFV"!

LFV = charged lepton flavour change at a point, without missing energy. eg $\mu \to e\gamma$.

Meet lepton flavour change...categories

- the only leptons we "see" are charged — and we see them change flavour:



But 'tis not "LFV"!
LFV = charged lepton flavour change at a point, without missing energy. eg $\mu \to e\gamma$.

- Can make categories of LFV processes:

J Heeck

$$\boldsymbol{\Delta LF = 1, \Delta QF = 0}$$
$\mu A \to eA$, $\tau \to 3l$, $h \to \tau^{\pm} l^{\mp}$... $(l \in \{e, \mu\})$

$$\boldsymbol{\Delta LF = 2}$$
$\mu\bar{e} \to e\bar{\mu}$, $\tau \to ee\bar{\mu}$...

$$\boldsymbol{\Delta LF = \Delta QF = 1}$$
$K \to \mu\bar{e}$, $B \to K\tau\bar{\mu}$, ...

categories $\approx$ independent below $\boldsymbol{\Lambda}_{\mathrm{LFV}}$

bounds/upcoming reach to $\Delta LF = 1, \Delta QF = 0$

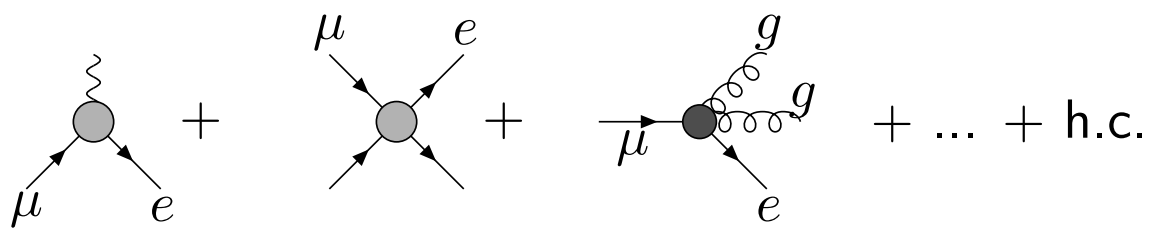
some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUMII)	$10^{-(16\to?)}$ (Mu2e,COMET)
		$10^{-(18\to?)}$ (PRISM/PRIME/ENIGMA)
$\tau \to \{e, \mu\}\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to e\bar{e}e, \mu\bar{\mu}\mu, e\bar{\mu}\mu ...$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$(Belle-II, LHCb?)
$\tau \to \begin{Bmatrix} e \\ \mu \end{Bmatrix} \{\pi, \rho, \phi, ...\}$	$\lesssim$ few $\times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^{\pm}\ell^{\mp}$	$< 1.5, 2.2 \times 10^{-3}$(ATLAS/CMS)	$< 2.4 \times 10^{-4}$ (ILC)
$h \to \mu^{\pm}e^{\mp}$	$< 6.1 \times 10^{-5}$ (ATLAS/CMS)	2.1×10^{-5} (ILC)
$Z \to e^{\pm}\mu^{\mp}$	$< 7.5 \times 10^{-7}$(ATLAS)	
$Z \to l^{\pm}\tau^{\mp}$	$< \times 10^{-7}$(ATLAS)	

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

parametrising the data — EFT part 1

parametrise LFV processes via contact interactions

(at low E)



suppose $\{C_O^\zeta\}$ momentum-indep (no form factors) $\Leftrightarrow$ New Particles are heavy

($\zeta =$ flavour, $O =$ Lorentz structure of operators $\times$chirality$=X$)

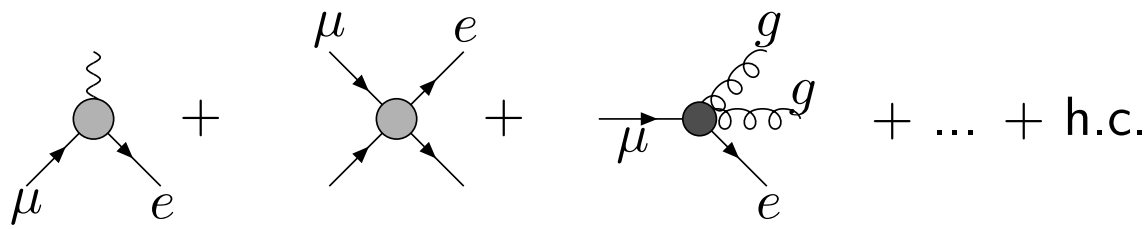
$$\delta\mathcal{L} = \sum_\zeta \sum_O \left(\frac{m_\mu C_{D,X}^{e\mu}}{v^2} \overline{e}\sigma \cdot F P_X \mu + \frac{C_O^\zeta}{v^2} O^\zeta + \frac{C_O^\zeta}{v^3} O^\zeta + ... + h.c. \right)$$

($v = 174$ GeV)

parametrising the data — EFT part 1

parametrise LFV processes via contact interactions

(at low E)



suppose $\{C_O^\zeta\}$ momentum-indep (no form factors) $\Leftrightarrow$ New Particles are heavy

(ζ = flavour, O = Lorentz structure of operators $\times$chirality$=X$)

$$\delta\mathcal{L} = \sum_\zeta \sum_O \left(\frac{m_\mu C_{D,X}^{e\mu}}{v^2} \overline{e}\sigma \cdot F P_X \mu + \frac{C_O^\zeta}{v^2} O^\zeta + \frac{C_O^\zeta}{v^3} O^\zeta + ... + h.c. \right)$$

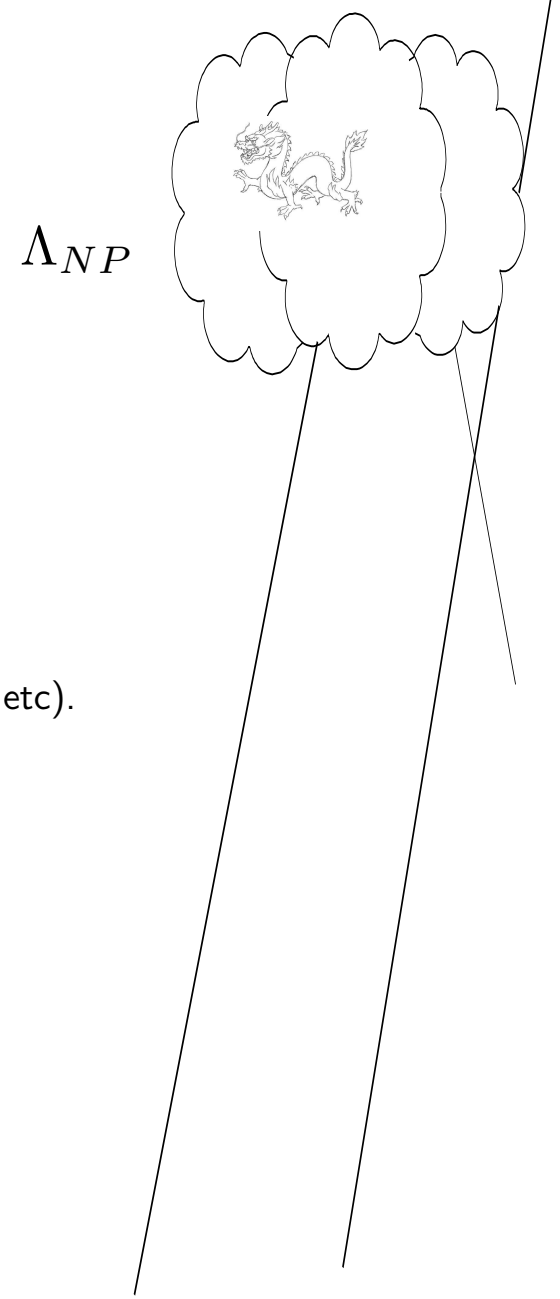
($v = 174$ GeV)

$\Rightarrow$ express LFV rates in terms of $\{C_O^\zeta\}$, $X \in \{L, R\}$

$$\begin{aligned} BR(\mu \to e\gamma) &\Rightarrow C_{D,L}, C_{D,R} \leq 10^{-8} \\ BR(\mu A \to eA)_{SI} &\Rightarrow |\sum r_{O,X} C_{O,X}| \leq 10^{-7} \qquad \text{(also SD)} \\ BR(\mu \to e\bar{e}e) &\Rightarrow C_{D,X}, C_{V,RX}, C_{V,LX} C_{S,XX} \leq 10^{-6} \end{aligned}$$

12 constraints.

Next...how to get to models?



- can calculate LFV rates in BSM neutrino models

...but is backwards = want to start from data

- ? try EFT?

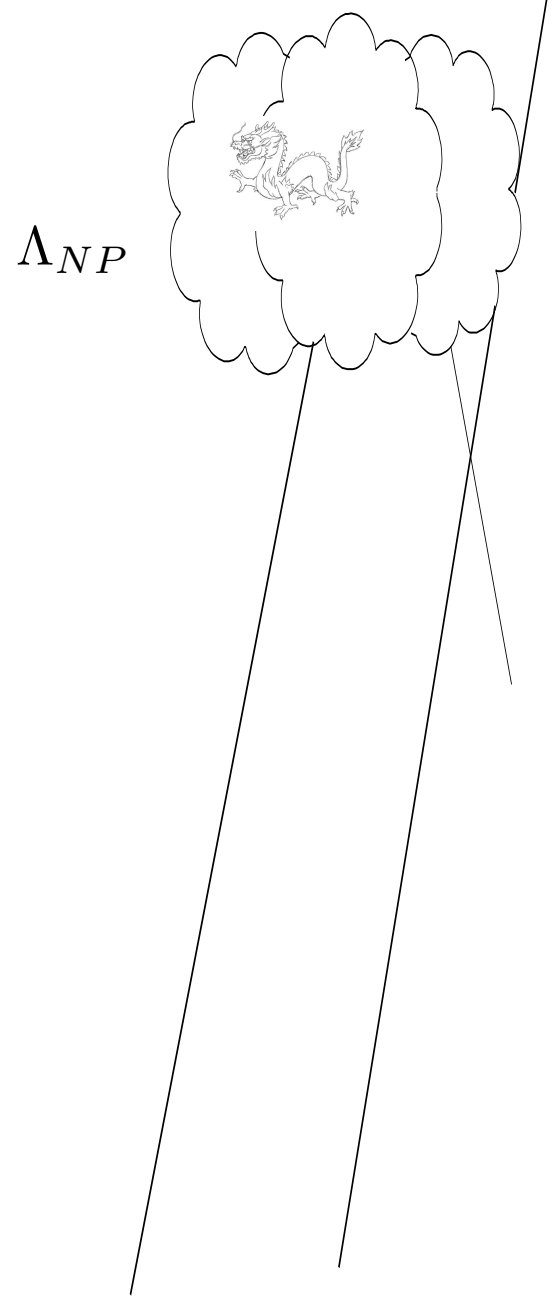
  assumes heavy New Physics

  ($\Rightarrow$ neglects effects of new light steriles $\Leftrightarrow$ not link LFV and neutrino mag mos, etc).

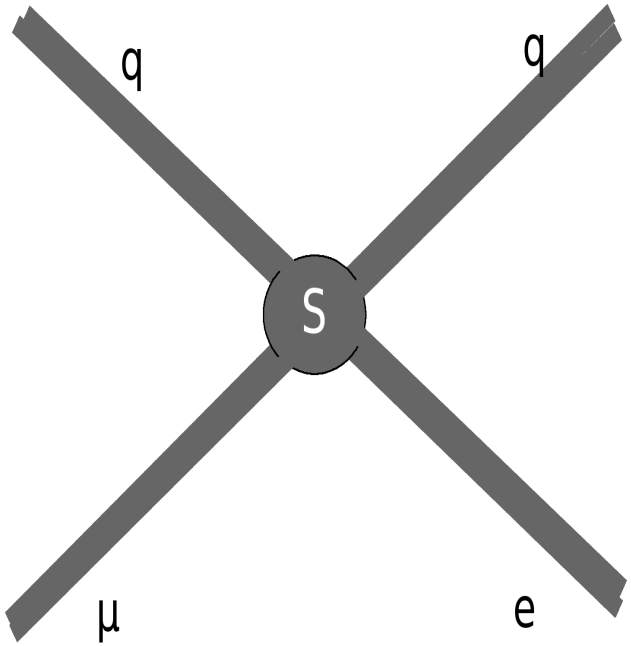
  $\{C_O^\zeta\}$ depend on (energy) scale, due to SM loops, described by RGEs

$$\boxed{\text{data } (\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA)}$$

Using EFT, part 2: changing scale

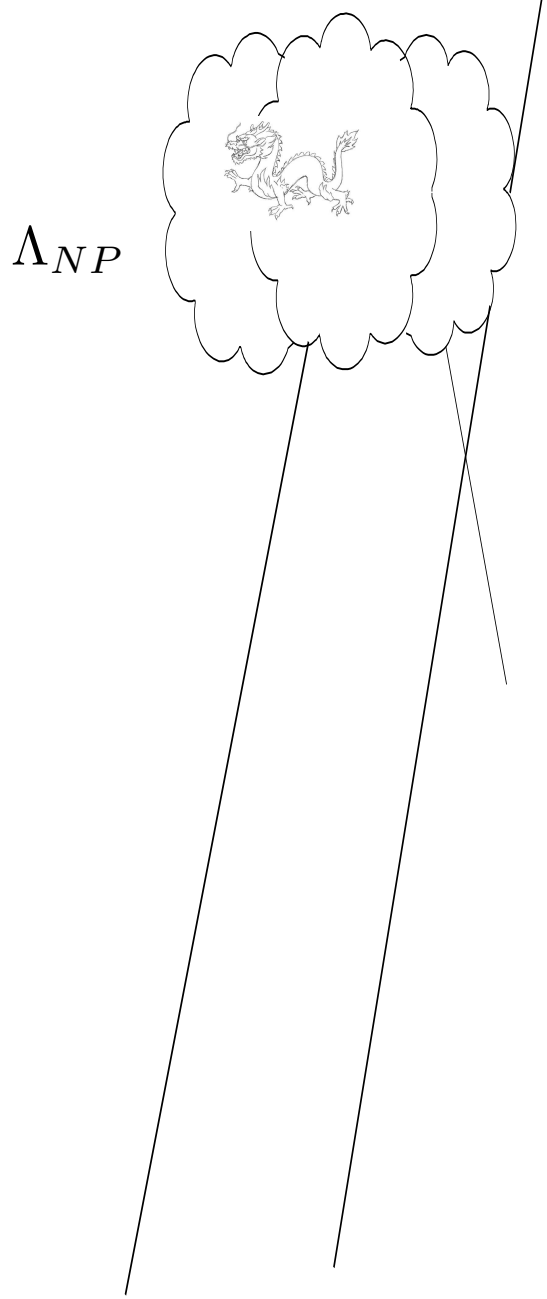


Renormalisation Group Eqns allow to change scale within an EFT$\Leftrightarrow$ add/peel off loops

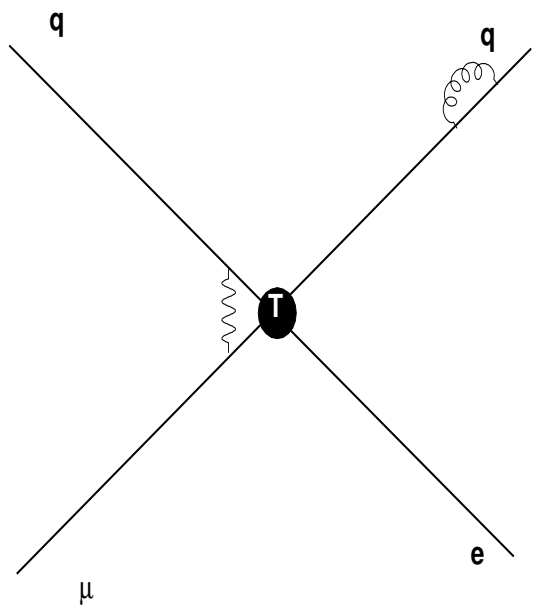


$$\boxed{\text{data } (\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA)}$$

Using EFT, part 2: changing scale



Renormalisation Group Eqns allow to change scale within an EFT$\Leftrightarrow$ add/peel off loops



worth to include loops, because few constraints; models might not generate exptal processes at tree

$$\boxed{\text{data } (\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA)}$$

(operators + RGEs: *everything to which data could be sensitive*)

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs **$\approx$100 ops.**
add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$
(not dim6: bottom-up perspective/ operator dim. not preserved in matching)
above m_W: dim 6 + selected dim 8 (guess by powercounting)
ArduDavidson

RGEs+matching: at "leading order" $\equiv$ largest contribution of each operator
to each observable. (2GeV$\rightarrow m_W$:resum LL QCD, $\alpha_e \log$, some $\alpha_e^2 \log^2$, $\alpha_e^2 \log$)
not just 1-loop RGEs; two-loop sometimes relevant
(1-loop vanishes/suppressed)

... but $\mu \to e$ rates only constrain $\sim 12\{C_O^\zeta\}$.

many operators+few constraints=using inconvenient basis

Have 6 (+6) constraints on e_L (e_R) operator coefficients. Focus on e_L.
Want to change basis to $scale\ \text{-}dependent$ **basis of constrained 6-d subspace.**

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$
Have RGEs for coefficients (arranged in row vector)

$$\mu \frac{\partial}{\partial \mu} \vec{C}(\mu) = \vec{C}(\mu) \mathbf{\Gamma}(\mu, g_s(\mu), ...) \quad \Rightarrow \quad \vec{C}(m_\mu) = \vec{C}(m_W) \boldsymbol{G}(m_\mu, m_W)$$

solved as scale-ordered exponential (resummed QCD, $\alpha \log$, some $\alpha^2 \log^2, \alpha^2 \log$)
$\Rightarrow$ define scale-dep $\vec{v}_{\mu \to e\gamma}(\Lambda)$, column of **G** such that: $C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu \to e\gamma}(\Lambda)$
$\boldsymbol{\vec{v}_{\mu \to e\gamma}(\Lambda)}$ **is scale-dep basis vector for constrainable subspace**

2-6. repeat for other independent constraints. So obtain scale-dep basis vectors for the subspace, defined from the observables.
The "flat directions" (experimentally inaccessible) are orthogonal, and therefore irrelevant.

Basis should span the finite-eigenvalue subspace of the correlation matrix.

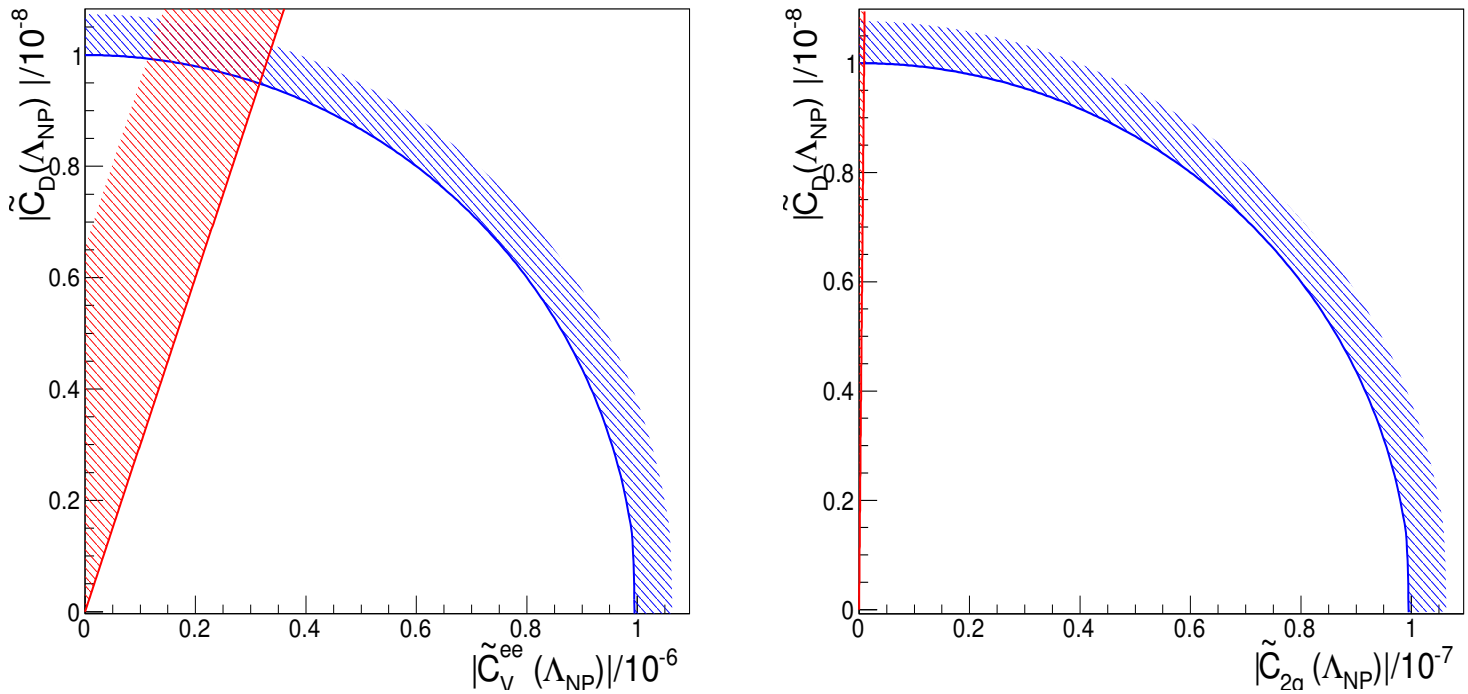
match to models, and explore what we can learn

match to models, and explore what we can learn

Ex: Type II seesaw (add triplet scalar $\vec{T}$, $[m_\nu] \propto [Y]\lambda_H$)

$$\mathcal{L} \supset \left([Y]_{\alpha\beta}\, \overline{\ell^c_\alpha} \varepsilon \vec{\tau} \cdot \vec{T} \ell_\beta + M_T \lambda_H \;\; H \varepsilon \vec{\tau} \cdot \vec{T}^* H + \text{h.c.} \right) + \ldots$$

Are some experimentally accessible regions inaccessible to some models?



veery prelim

model prediction = red hashed current expt exclusion = blue hashed

Summary: maybe charged leptons can help us learn about νs

$\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$ have exceptional sensitivity ($\Lambda_{\rm LFV} \lesssim 10^2 \to 10^3$ now, $\Lambda_{\rm LFV} \lesssim 10^3 \to 10^4$ upcoming), to only a few operators at low energy, so:

interesting to include RGEs(at leading order), because ensure that almost every $\mu \to e$ operator (in chiral basis) with ≤ 4 legs contributes at $\gtrsim \mathcal{O}(10^{-3})$ to $\mu \to e\gamma$ and/or $\mu \to e\bar{e}e$ and/or $\mu A \to eA$

Can even have interesting sensitivity to products of some $(\mu \to \tau) \times (\tau \to e)$ interactions!

But most directions in coefficient space are untestable (*(not an EFT-problem, its a consequence of searching for NP under the lamppost.)* Can circumvent this by changing operator basis: a convenient basis for comparing models to $\mu \to e$ flavour-changing observables can be constructed from the observables.

Thanks Georg

BackUp

LFV categories $\approx$ independent below $\Lambda_{\rm LFV}$

$\boldsymbol{\Delta LF = 1, \Delta QF = 0}$

$\mu A \to eA,\ \tau \to 3l,\ h \to \tau^{\pm} l^{\mp} ...$ $(l \in \{e, \mu\})$

$\boldsymbol{\Delta LF = 2}$

$\mu\bar{e} \to e\bar{\mu},\ \tau \to ee\bar{\mu} ...$

$\boldsymbol{\Delta LF = \Delta QF = 1}$

$K \to \mu\bar{e},\ B \to K\tau\bar{\mu}, ...$

categories $\approx$ independent below $\Lambda_{\rm LFV}$

- SM loops corrections to $\Delta LF = 2$ cannot give $\Delta LF = 1$ (LFV is at $\Lambda_{\rm LFV}$)
- $(\Delta LF = 1)^2 \to \Delta LF = 2$, but better exptal bds on $\Delta LF = 1$.
- $\Delta LF = \Delta QF = 1$ mixes with $\Delta LF = 1$ in SMEFT. But quark FCNC small, so effect $<$ "forseeable" exptal reach on $\Delta LF = 1$. (for $\Lambda_{\rm LFV} > 4$ TeV). ArduDavidson

But to reconstruct $\mu \to e$ bottom-up, need all data?

$eg\ BR(\pi^0 \to e^\pm \mu^\mp) < 3.6 \times 10^{-10}$, or $BR(\Upsilon \to l_1 \bar{l}_2) \lesssim 10^{-6}$?

Ummm: μ decays weakly $\Leftrightarrow \tau_\mu \sim 10^{-6}$ sec.
vs $\tau_{\pi^0} \sim 10^{-16}$ sec (loop-suppressed QED), or $\tau_\Upsilon \sim 10^{-20}$ sec (tree QED/QCD)

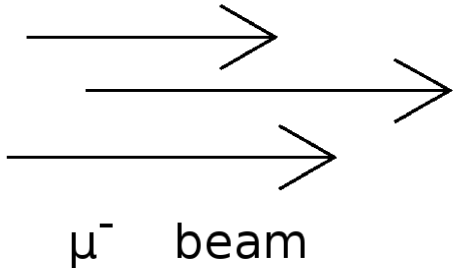
Compare *weak* μ decays to *anomalous QED* π_0 decay
(write $\delta\mathcal{L} \sim \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\mu)(\bar{q}q) + \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\gamma\mu)(\bar{e}\gamma e))$:

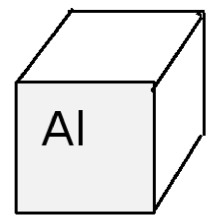
$$BR(\mu \to e\bar{e}e) = \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \sim \left|\frac{m_\mu^2/\Lambda^2_{\rm LFV}}{m_\mu^2 G_F}\right|^2 \sim \frac{v^4}{\Lambda^4_{\rm LFV}} \lesssim 10^{-12} \Rightarrow \Lambda_{\rm LFV} \gtrsim 10^5 {\rm GeV}$$

$$BR(\pi_0 \to \bar{e}\mu) = \frac{\Gamma(\pi_0 \to \bar{e}\mu)}{\Gamma(\pi_0 \to \gamma\gamma)} \sim \left|\frac{m_\pi^2/\Lambda^2_{LFV}}{\alpha/4\pi}\right|^2 \sim \left(\sqrt{\frac{4\pi}{\alpha}}\frac{m_\pi}{\Lambda_{LFV}}\right)^4 \Rightarrow \Lambda_{\rm LFV} \gtrsim {\rm TeV}$$

... rare μ processes have exceptional *sensitivity*, because μ decay weak.
Other $\mu \to e$ processes constrain "orthogonal" operator coefficients, less well.

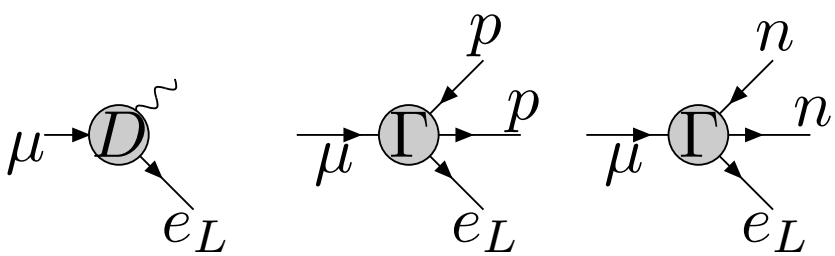
$\mu A \to eA$: most sensitive process, expt + th





target
(Z=13,A=27, J=5/2)

- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV:μ interacts with $\vec{E}$, nucleons (via $\widetilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e



$(E_e \approx m_\mu$ so $e_L/e_R)$

$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

KitanoKoikeOkada

$$BR_{SI} \sim \quad Z^2|\sum ...\tilde{C}_{SI}|^2 \quad , \quad \tilde{C}_{SI} \in \{\tilde{C}^p_V, \tilde{C}^p_S, \tilde{C}^n_V, \tilde{C}^n_S, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over $N \propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim ...|\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

CiriglianoDavidsonKuno
HoferichterEtal

Can't we do without RGEs, etc?

in discovery mode for LFV+electroweak loops are small...include later?

counterex: $\mu A \to eA$ in model giving tensor $2\sqrt{2}G_F C_T^{uu}(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u)$ at weak scale

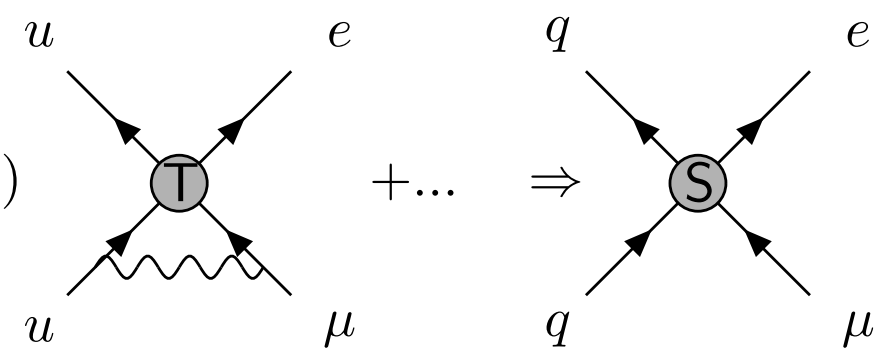
1: forget loops quark tensor matches to nucleon spin $\bar{N}\gamma\gamma_5 N$: $(N \in \{n,p\})$

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \tfrac{1}{2}|C_T^{uu}|^2}$$

(CiriglianoDKuno)
Hoferichter etal

2: include QED loops $m_W \to 2$ GeV:

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$



$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\, C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$$

$$\Delta C_S^{uu}(m_\tau) \sim \tfrac{1}{7}C_T^{uu}(m_W)$$

Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained. Important for $\mu \to e$. (?not $\tau \to l$?)

3 processes, many ops: if $\Delta QF = 0$, $\mu \to e$ occurs, will it contribute to $\mu \to e\gamma$, $\mu \to e\bar{e}e$ or $\mu A \to eA$?

2010.00317

Probably yes: SM loops ensure almost every $\Delta QF = 0$, $\mu \to e$ interaction with ≤ 4 legs, contributes $\gtrsim \mathcal{O}(10^{-3})$ to amplitudes $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and/or $\mu A \to eA$

(not $\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)

coefficient	$\mu \to e\gamma$	$\mu \to e\bar{e}e$	$\mu A \to eA$
$\lvert C_{D,X}\rvert$	1.12×10^{-8}	4.30×10^{-7}	2.35×10^{-7}
$\lvert C^{ee}_{V,XX}\rvert$	1.10×10^{-4}	7.80×10^{-7}	1.86×10^{-5}
$\lvert C^{ee}_{V,XY}\rvert$	2.55×10^{-4}	9.34×10^{-7}	3.77×10^{-5}
$\lvert C^{ee}_{S,XX}\rvert$	1.73×10^{-4}	2.8×10^{-6}	(3.64×10^{-3})
$\lvert C^{\mu\mu}_{V,XX}\rvert$	1.10×10^{-4}	5.60×10^{-5}	1.85×10^{-5}
$\lvert C^{\mu\mu}_{V,XY}\rvert$	2.56×10^{-4}	1.12×10^{-4}	3.77×10^{-5}
$\lvert C^{\mu\mu}_{S,XX}\rvert$	8.24×10^{-7}	(1.58×10^{-5})	(1.73×10^{-5})
$\lvert C^{\tau\tau}_{V,XX}\rvert$	3.80×10^{-4}	1.95×10^{-4}	1.24×10^{-5}
$\lvert C^{\tau\tau}_{V,XY}\rvert$	4.40×10^{-4}	1.91×10^{-4}	1.25×10^{-5}
$\lvert C^{\tau\tau}_{S,XX}\rvert$	5.33×10^{-6}	1.02×10^{-4}	1.12×10^{-4}
$\lvert C^{\tau\tau}_{S,XY}\rvert$	—	—	—
$\lvert C^{\tau\tau}_{T,XX}\rvert$	1.10×10^{-8}	(4.20×10^{-7})	(2.30×10^{-7})

sensitivities/1-at-a-time bds for $\delta\mathcal{L} = 2\sqrt{2}G_F C_i \mathcal{O}_i$; if model gives smaller coefficients, it is consistent with data. If it generates larger coefficients, need to arrange a cancellation...

$\Leftrightarrow$ **modulo cancellations, probably find $\mu \leftrightarrow e$**

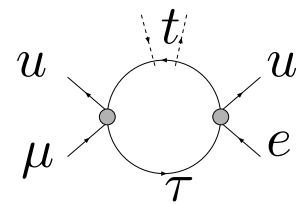
$[\mu \to \tau] \times [\tau \to e] = [\mu \to e] \Rightarrow$?

recall exptal reach: BR$(\mu \to e) \to 10^{-(18\to 20)} \sim [\mathrm{BR}(\tau \to l) \to 10^{-9}]^2$
? learn about $\tau \to l$ from $\mu \to e$?

1. if model has $(\mu \to \tau)$,$(\tau \to e)$, then no conserved flavour, so "expect" $\mu \to e$

2. can one calculate anything model-independent? In SMEFT, (dim6)$^2 \to$ dim8, eg $\overline{\ell} e \varepsilon \overline{q} u \times (\overline{\ell}\gamma\ell)(\overline{q}\gamma q) \to \overline{\ell} e \varepsilon \overline{q} u H^\dagger H$

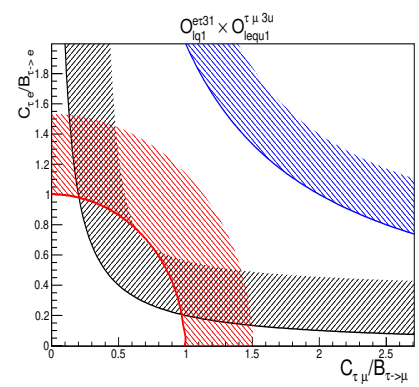
$$\frac{\Delta\ {}^{(8)}C^{e\mu uu}}{\Lambda^4_{\mathrm{LFV}}} \simeq \frac{\{y_t^2, g^2\}}{16\pi^2} \frac{C^{e\tau ut}_{LQ}}{\Lambda^2_{\mathrm{LFV}}} \frac{C^{\tau\mu tu}_{LEQU}}{\Lambda^2_{\mathrm{LFV}}}$$



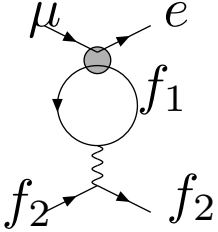
so effective low-energy 4-fermion interaction $2\sqrt{2}G_F C_S$

$$\Delta\ {}^{(6)}C^{e\mu uu}_S \propto \frac{v^4}{16\pi^2\Lambda^4_{\mathrm{LFV}}} C^{e\tau ut} C^{\tau\mu tu}$$

3. find eg, $\mu A \to eA$ sensitivity complementary to $B^- \to \{e, \mu\}\nu$ decays for some operators:

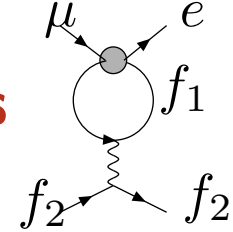


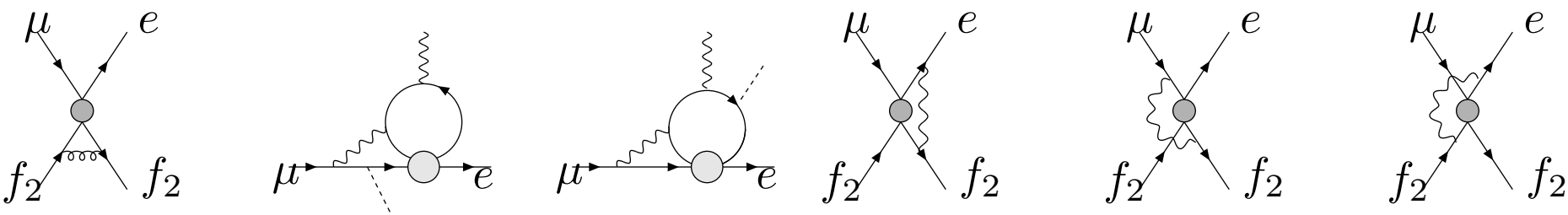
.



Including SM loop corrections to operators

ex: 1-loop QED + QCD (+2-loop QED V→D)





$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\boldsymbol{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\boldsymbol{\Gamma}$$

solve (analytically/numerically):

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV})\boldsymbol{G} \quad , \quad \boldsymbol{G} = \text{fn of SM parameters}, \log(\Lambda_{\rm LFV}/\Lambda_{exp})$$

For ex: $BR(\mu\to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.1\times 10^{-13} \Rightarrow C_{D,X} \lesssim 10^{-8}$

$$C_{D,X}(m_\mu) = C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) - \frac{\alpha_e}{4\pi e}\left(C^{\mu\mu}_{S,XX} - 8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C_{2loop}\right)\ln\frac{m_W}{m_\mu}$$

$$+16\frac{\alpha_e^2}{2e(4\pi)^2}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right)\ln^2\frac{m_W}{m_\mu} - 8\lambda^{a_T} f_{TD}\frac{\alpha_e}{4\pi e}\left(\frac{2m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_s}{m_\mu}C^{ss}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)\ln\frac{m_W}{2\text{GeV}}$$

$$+16\frac{\alpha_e^2}{3e(4\pi)^2}\left(\sum_{u,c} 4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)\ln^2\frac{m_W}{2\text{GeV}}$$

$C^{\zeta}_{Lor}(m_W)$ on right. $\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon +BowmanChengLiMatis

Operator basis $m_\tau \to m_W$: ~ 90 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad dim\ 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad dim\ 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad dim\ 7$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

...zzz...but ~ 90 coeffs!

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

But 3 processes, $\sim$100 operators $\Rightarrow$ zoo of flat directions?

DKunoYamanaka

Count constraints: (write $\delta\mathcal{L} = C^{flavour}_{Lorentz,XY}/v^n\ \mathcal{O}^{flav}_{Lorentz,XY}\ \ , X, Y \in \{L, R\}$)

$\boldsymbol{\mu \to e\gamma}$ **:** $\qquad BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ $\quad \Rightarrow$ **2 constraints**

$\boldsymbol{\mu \to e\bar{e}e}$ **:** (e relativistic $\approx$ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2$$
$$+ |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\} \qquad \Rightarrow \textbf{6 more constraints}$$

$\boldsymbol{\mu A \to eA}$ **:**(S^N_A,V^N_A=integral over nucleus A of N distribution$\times$lepton wavefns, **different** for diff. A)

$$BR_{SI} \sim Z^2|V^p_A\tilde{C}^p_{V,L} + S^p_A\tilde{C}^p_{S,R} + V^n_A\tilde{C}^n_{V,L} + S^b_A\tilde{C}^n_{S,R} + D_AC_{D,R}|^2 + |L \leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) $\qquad \Rightarrow$ **4 + 2 more constraints**

future: improved theory, 3SI+2SD targets $\qquad \Rightarrow$ **6+4** constraints

is 12-20 constraints on ~ 100 operators a problem?