Neutrino clusters and neutrino stars

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Neutrino bound states and systems

M. Markov, Phys.Lett. 10, 122 (1964): "Neutrino superstars"

Massive neutrinos + gravity, analogy with neutron stars, $n \rightarrow \nu$

$$R = \sqrt{\frac{8\pi}{G_N}} \frac{1}{m_\nu^2}$$

$M \sim 1/m_\nu^2$

Oppenheimer - Volkoff limit

For $m_\nu = 0.5$ MeV: $M = 4\ 10^6\ M_{sun}$, $R = 4\ 10^{12}$ cm

Now $m_\nu = 0.05$ eV: $M = 4\ 10^{20}\ M_{sun}$, $R = 5\ 10^{26}$ cm

R. D.Viollier et al, Phys.Lett. B306, 79 (1993) ,....

Gravity, $m_\nu = (10 - 100)$ keV:
$M = (10^8 - 10^{10})\ M_{sun}$, $R = (10^{14} - 10^{16})$ cm
- essentially, warm DM

Heavy sterile neutrinos ?

Neutrino bound states and systems

G. J. Stephenson et al, Int. J. Mod. Phys. A13, 2765 (1998) ...

Long range scalar (Yukawa) forces with coupling y,
m_ν = 13 eV, motivated by ^{3}H exp. anomaly, negative m^2

$$G_N \rightarrow G_\nu = \frac{y^2}{4\pi m_\nu^2}$$

Analogy with hadrodynamics, Thomas - Fermi approximation ...

Equations for final configurations → density profiles,
Formation of clouds in the Universe.

$$R = 4\sqrt{2}\,\pi \frac{1}{y\, m_\nu}$$

$M = (10^8 - 10^{10})\, M_{sun}$, $R = 10^{13}$cm, central density: 10^{15} cm^{-3}

Equation for effective mass of neutrino m* in the scalar field

Dark Matter Nuggets

M.B. Wise and Y. Zhang, Phys. Rev. D 90, 055030 (2014), JHEP 02, 023 (2015)

Dirac fermions with $m_D \sim 100$ GeV and coupling constant with scalar $\alpha_\phi = 0.01 - 0.1$ → Applications to asymmetric dark matter

Description is similar to that by Stephenson et al:

System of equations for scalar field and Fermi momentum of DM

Solved numerically in *M.I. Gresham, H.K. Lou and K.M. Zurek, Phys. Rev. D96, 096012 (2017), Phys. Rev. D 98, 096001 (2018)*

Dependences of properties of nuggets on N and m_ϕ:

With increase of N radius R(N) first decreases, reaches minimum and then increases. The increase is in the relativistic regime

In relativistic case $R > 1/m_\phi$ is possible - "saturation" regime

Content

A.Y.S, and Xun-Jie Xu,
JHEP 08 (2022) 170,
2201.00939 [hep-ph]

Neutrino clusters:

Physics and Equations

Final configurations

Formation and applications

Physics and equations

Interaction. Equations for fields

$L = \quad .... y \bar{\nu}\nu \phi - \frac{1}{2} m_\phi^2 \phi^2 - m_\nu \bar{\nu}\nu + \ldots$ single neutrino type

where ϕ – scalar with mass m_ϕ
m_ν - neutrino mass
y - effective coupling pheno bound $y < 10^{-7}$

Equations of motion:

$$\boxed{\begin{array}{l} i\cancel{d}\,\nu - m^*\nu = 0 \\ (d^2 + m_\phi^2)\,\phi + y\,\bar{\nu}\nu = 0 \end{array}}$$

(*)

(**)

$m^* = m_\nu + y\phi$ - effective mass of neutrino in medium

$V = y\phi$ - potential

$m_\phi \ll m_\nu$ ϕ – classical field, ν - particle quanta

in eq. (**) should take expectation value

$\bar{\nu}\nu \rightarrow \langle \bar{\nu}\nu \rangle$ $\rightarrow$ source of ϕ field

ν - ϕ system

neutrino gas with density n and momentum distribution f(p, t, x), and long range attractive forces due to Yukawa interactions

Expectation value:

$$\boxed{\langle \bar{\nu}\nu \rangle = n^* = \frac{1}{2\pi^2}\int p^2 dp \, \frac{m^*}{E_p} f(p)}$$

$$E_p = \sqrt{p^2 + m^{*2}}$$

$$m^* = m_\nu + y\phi$$

effective neutrino mass in the field

Non-relativistic limit, $p \ll m^*, m_\nu$: $n^* \to n$ usual density

Relativistic case, $p \gg m^*$: chiral suppression: $n^* \ll n$

The field (potential) is suppressed, attractive force is suppressed

→ difference from gravity - no collapse

Equations for neutrino stars. Equilibrium condition

Ground state - degenerate neutrino gas. Static configuration

Attraction by Yukawa forces

equilibrium

Pressure of neutrino gas

$P_{yuk} = - P_{deg}$

In non-relativistic case: Hydrostatic equilibrium:

$F_{deg}(r) = - F_{yuk}(r)$ where $F_{deg}(r) = dP_{deg}(r)/dr$

In general, chemical equilibrium: $d\mu/dr = 0$

for degenerate gas: $dE_F/dr = 0$ E_F - Fermi energy

Non-relativistic case

In non-relativistic case: Hydrostatic equilibrium

$$F_{deg}(r) = - F_{yuk}(r)$$

$$F_{deg}(r) = \frac{(6\pi^2)^{2/3}}{5\, r\, m_\nu}\, n(r)^{5/3} \qquad F_{yuk}(r) = - \frac{y^2}{4\pi\, r^2}\, n(r)\, N(r)$$

Reduced to the Lane-Emden equation (after differentiation over r)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dn^{2/3}}{dr}\right) = - \kappa y^2 n$$

$$\kappa = \frac{2\, m_\nu}{(6\pi^2)^{2/3}} \qquad \gamma = 3/2 \text{ - solution with finite radius}$$

Boundary condition in the center:

$n(0) = n_0$ or $p_F(0) = p_{F0}$

Relativistic case

With increase of N the Fermi momentum increases as $p_{F0} \sim N^{2/3}$
Transition to relativistic case at $p_{F0} > m^*$

Chemical equilibrium condition

$$d\mu/dr = dE_F/dr = 0$$

or $dE_F^2/dr = 0$, $E_F^2 = p_F^2 + m^{*2}$

$$\Rightarrow \quad \underbrace{m^* \frac{dm^*}{dr}}_{F_{yuk}(r)} = \underbrace{- p_F \frac{dp_F}{dr}}_{F_{deg}(r)}$$

- generalization of the Hydrostatic equilibrium.

Indeed, $F_{yuk}(r) = dV/dr = y\, d\phi/dr = \dfrac{dm^*}{dr}$

Relativistic equations

Static case

$$(\nabla^2 - m_\phi^2)\, m^* = y\, n^*$$

$$m^* \frac{dm^*}{dr} = -\, p_F \frac{dp_F}{dr} \qquad (*)$$

$$n^* = \frac{1}{2\pi^2} \int_0^{p_F} \frac{m^*}{\sqrt{p^2 + m^{*2}}}\, p^2 dp$$

from chemical equilibrium condition

Equations for m^* (instead of ϕ) and p_F (neutrino density)

Boundary conditions:

$p_F(0) = p_{F0}$ - external (given) parameter

$m^*(0) = m^*_0$ m^*_0 is tuned in such a way that at $r \to$ infty $m^* \to m_\nu$

In non-relativistic case the system (*) can be reduced to the Lane-Emden equation

Non-degenerate case *

$$f(p) = \frac{1}{\exp(p/T) + 1}$$

$$n = \frac{I_3}{2\pi^2} T^3$$

Results from degenerate case (upto numerical coefficients) by substitution

$$p_{F0} \rightarrow T$$

Radius is bigger: $R_T = 1.9\, R_{deg}$

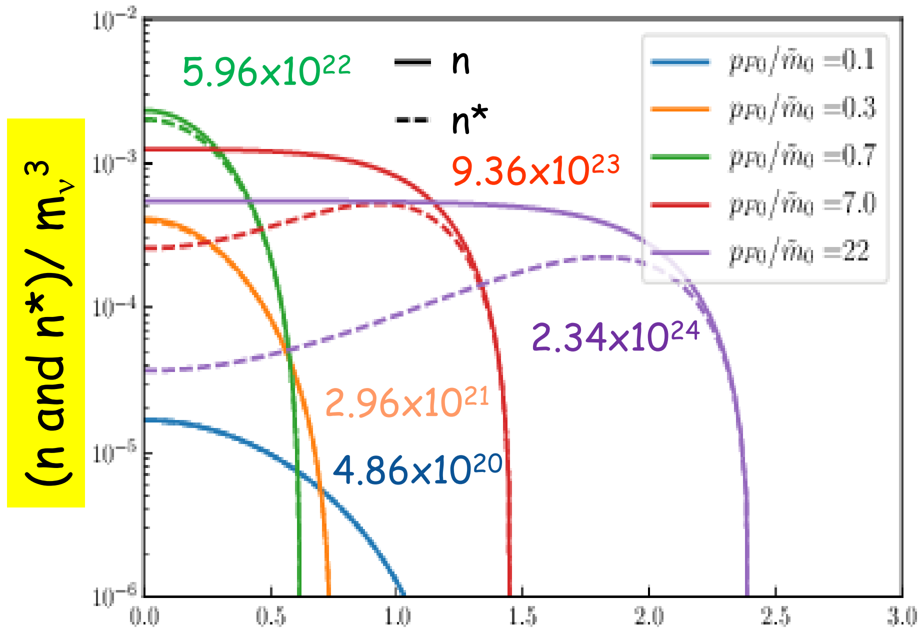
Final stable configuration

Density distribution

Density and effective density distributions for different values of p_{F0}/m_ν (corresponding values of N indicated)

$1.2\ 10^{11}\ cm^{-3}$

$y = 10^{-7}$, $m_\nu = 0.1$ eV, $m_\phi = 0$



$R \sim 1/N^{1/3}$ r, km R

With increase of N

for $N < 6\ 10^{22}$ (non-rel)
R decreases, n_0 increases

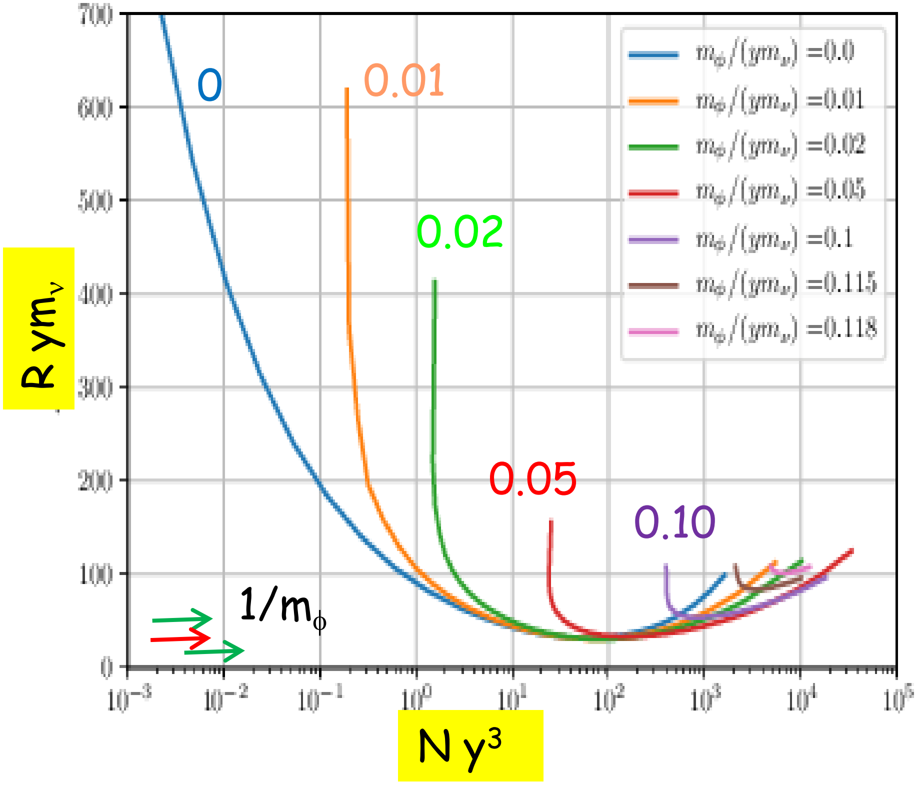
for $N > 6\ 10^{22}$ (rel)
R increases, n_0 decreases

$n^*/n \sim \langle m^*/E \rangle \sim \langle m^*/p_{F0} \rangle$

Dependence on coupling – scaling: $N \sim 1/y^3$ $R \sim 1/y$

p_{F0}/m_ν - gives the central density

Dependence on mass of a mediator



Radius (Rym_ν) as function of number of neutrinos for different values of m_ϕ/ym_ν

Radius of interaction $r_\phi = 1/m_\phi$

★ Lover bound on N, for non-zero m_ϕ which increases with m_ϕ

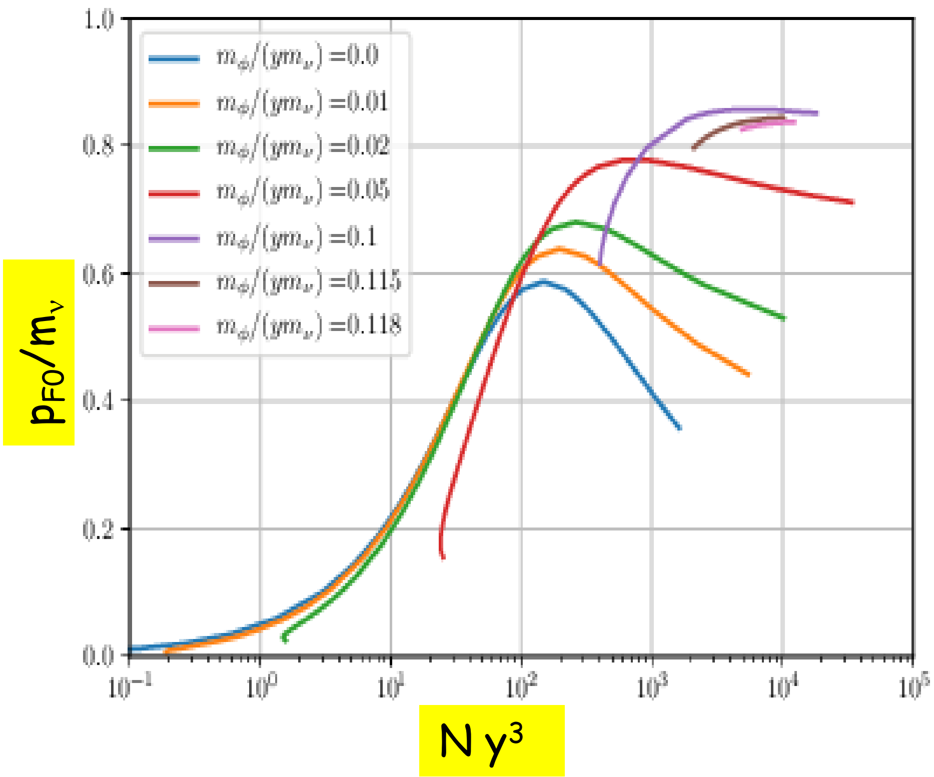
★ Minimal radius: increases with m_ϕ and shifts to larger N

With increase of m_ϕ:

in non- relativistic range binding effect becomes weaker, R increases

in relativistic range – R decreases as a result of shift of minimum

Properties of neutrino clusters



Fermi momentum in center of star as function of number of neutrinos for different values of m_ϕ/m_ν

With increase of m_ϕ

Maximum of p_{F0} increases and shifts to larger N

Absolute maximum of p_{F0} and therefore central density are determined by value of neutrino mass

$$n_\nu^{max} = 4\ 10^8\ \text{cm}^{-3} \left(\frac{m_\nu}{0.1\ \text{eV}}\right)$$

Strength of interaction. Bounds. Radius

Definition $$S_\phi = \frac{y^2 m_\nu^2}{m_\phi^2}$$

Numerically: stable solutions exist for

$S_\phi^{-1/2} < 0.12$ ⇒ $S_\phi > 70$

Radius of cluster R can be smaller, comparable or bigger than radius of interactions $R_\phi = 1/m_\phi$

$$\frac{R}{R_\phi} = Y S_\phi^{-1/2}$$

$Y = Rym_\nu$ - vertical axis value in Fig. for R

E.g. for $S_\phi^{-1/2} = 0.1$, $Y = (55 - 100)$ ⇒ $R/R_\phi = (5 - 10)$

(in relativistic range)

Formation of clusters and applications

Maximal density and fragmentation

$n_{\nu 0}^{max}$ depends on neutrino mass only

General picture:

Expansion of uniform cosmic neutrino background
→ decrease of density and temperature

When $n_\nu(T) \sim n_{\nu 0}^{max}$ fragmentation may start

After separation the pre-clusters may shrink
and their central density – increase. Therefore

Fragmentation density

$n_{frag} < n_{\nu 0}^{max} \sim 10^9$ cm^{-3} for $m_\nu = 0.1$ eV

This density is realized in the epoch

$$z_f + 1 = \left(\frac{n_{frag}}{n_\nu(0)} \right)^{1/3} \sim 200$$

→ fragmentation may start when $p_\nu \sim m_\nu$

Evolution of the effective mass

$m^* = m_\nu + y\phi$

For uniform static configuration equation of motion gives

$$\phi = -\frac{y \langle \bar{\nu}\nu \rangle}{m_\phi^2} = -\frac{y\, n^*}{m_\phi^2}$$

$$m^* = m_\nu - \frac{y^2\, n^*(m^*)}{m_\phi^2}$$

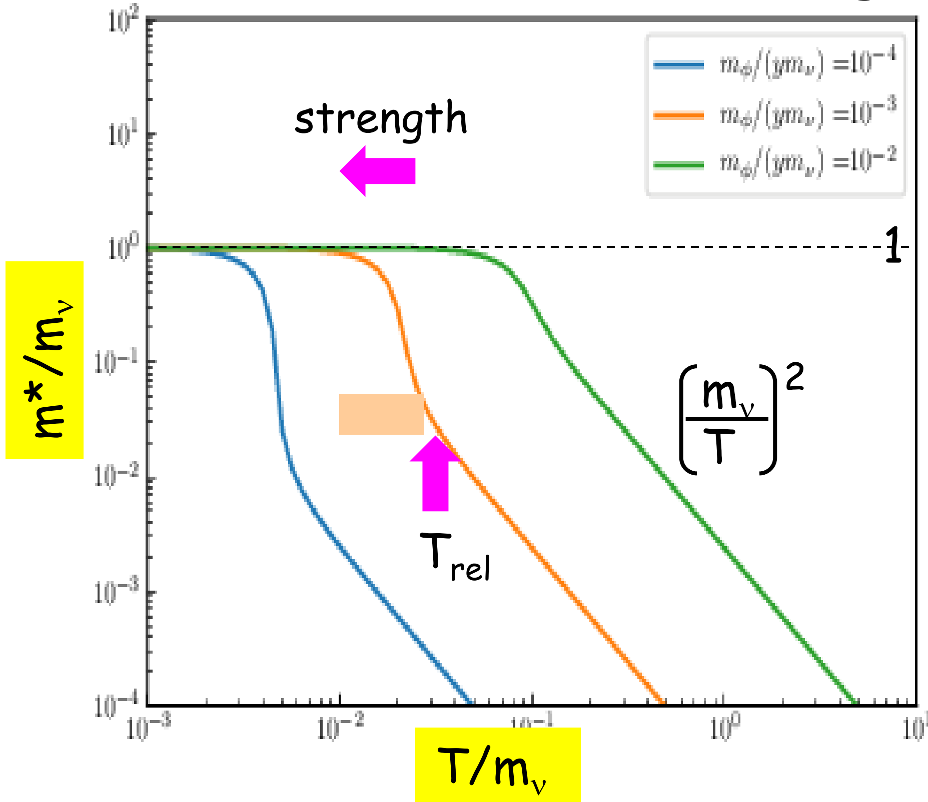
For thermal distribution

$$m^* = m_\nu - m^* \frac{y^2\, T^2}{24\, m_\phi^2}$$

transition region

$T_{rel}/m_\nu = (24/S_\phi)^{1/3}$

For different values of strength



non-relativistic

$m^* \to m_\nu$

relativistic

$m^* \to 0$

Energy of the system per neutrino

$\varepsilon_\alpha = \rho_\alpha / n_\nu$ $\quad \alpha = \text{tot}, \nu, \phi$

ρ_α - energy density in α component

in the uniform static medium
(all derivatives are zero)

Scalar field

$$\rho_\phi = \tfrac{1}{2} m_\phi^2 \phi^2 = (m^* - m_\nu)^2 \frac{m_\phi^2}{2\, y^2}$$

$$\varepsilon_\phi = m_\nu \begin{cases} 1/\chi & \text{relativistic limit} \\ \chi/4 & \text{non-relativistic limit} \end{cases}$$

$$\chi \equiv \frac{S_\nu I_3}{\pi^2} \left(\frac{T}{m_\nu} \right)^3$$

$I_3 = 1.80$

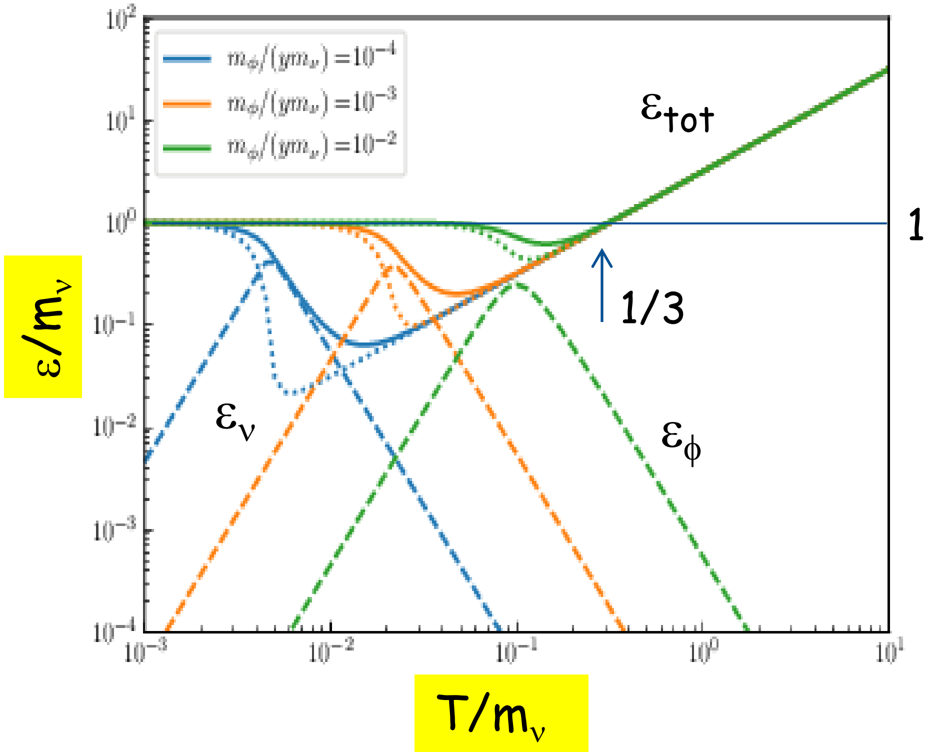
Neutrinos

$$\varepsilon_\nu = \langle E_\nu \rangle = \langle \sqrt{p^2 + m^{*2}} \rangle$$

$$\varepsilon_\nu = \begin{cases} 3.15T & \text{relativistic limit} \\ m_\nu & \text{non-relativistic limit} \end{cases}$$

Total energy $\quad \varepsilon_{tot} = \varepsilon_\nu + \varepsilon_\phi$

Evolution of energies. Dip



Dependence of energy per neutrino on T/m_ν for different values of m_ϕ/ym_ν
ε_ϕ - dashed, ε_ν – dotted, ε_{tot} - solid

For large enough strength

$$S_\phi > S_\phi^{min} \sim 600$$

the dip develops in $\varepsilon^{tot}(T)$ dependence with

$$\varepsilon_{tot} < m_\nu \qquad (*)$$

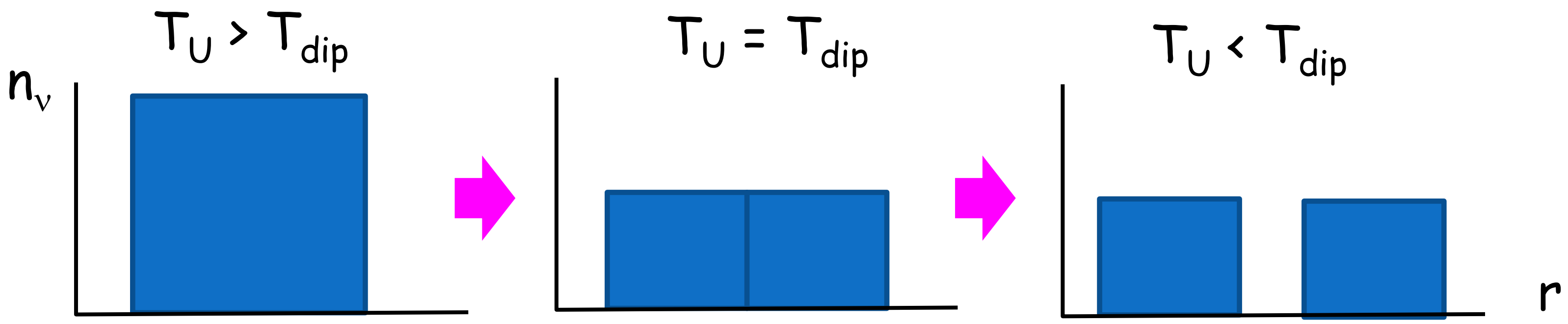
at $T \sim m_\nu/3$, when neutrinos become non-relativistic

(*) implies existence of bound state with $(m_\nu - \varepsilon_{tot})$ being the binding energy

With increase of strength the minimum of dip shifts to lower T and becomes lower

Instability and Fragmentation

Below T_{dip} further expansion and cooling require increase of energy of the system → fragmentation without decrease of T and density



T_U - temperature in the Universe

Fragmentation stars at z_f ~ 200: corresponds to maximal density

The size of the Universe that epoch D_U (200) = 20 Mpc

The radius of the biggest structures: $R_f \sim D_U(200)/4 = 5$ Mpc
Distance between structures: $d_f(200) \sim D_U(200)/2 = 10$ Mpc
$N_f = 1.2\ 10^{85}$, $M_f = 4\ 10^{17}\ M_{sun}$
Present size of voids $d(0) \sim z_f d_f = 2000$ Mpc

Parameters of clusters

The biggest possible structures which would satisfy the energy conditions correspond to $m_\phi \sim 3\ 10^{-32}$ eV

If $m_\phi \gg 3\ 10^{-32}$ eV such structures are not stable $\rightarrow$ further fragmentation occurs down to $R \sim 1/\ m_\phi$

For $m_\phi\ /\ y m_\nu = 10^{-2}$

R	y	m_ϕ , eV
10 kpc	$1.4\ 10^{-26}$	$1.4\ 10^{-30}$
1 pc	$1.4\ 10^{-22}$	$1.4\ 10^{-26}$
10 km	$4\ 10^{-8}$	$4\ 10^{-11}$

If formation starts at z =200, voids are 200 bigger than clusters

Observational consequences

Further disintegrations : other perturbations, DM halos, gravity,

Ratio of distances between clusters d and radiuses of clusters

$d/R = 10^{-2}\, d_0\, m_\nu\, y^2\, N^{2/3}$

d_0 - distance between neutrinos without clustering

$d/R \sim 100$ does not depend on y for stable configuration

Affects detection of relic neutrinos depending on sizes of stars

Influence Early structure formation?

Strength of interactions and formation of clusters

$$S_\phi = \frac{y^2\, m_\nu^{\,2}}{m_\phi^{\,2}}$$

70 — 600 — S_ϕ

No stable configuration

Stable final configurations
No dip

Dip in total energy dependence on T

Formation of structures via growth of local density perturbations, virialization

Formation via phase transitions

Similar to formations of DM halos

In conclusion

Neutrino interaction with light or massless scalar boson with $y < 10^{-7}$ and $m_\phi < 10^{-10}$ eV can lead to formation of stable bound systems of neutrinos: stars, clusters

Due to chiral suppression of attraction, existence of relativistic regime in which dependence of characteristics, is opposite to the non-relativistic case. Absence of collapse.

Final stable configurations: degenerate (close to degenerate) Fermi gas with the following features

- Existence of minimal radius determined by $1/y\, m_\nu$
- in relativistic regime R can be bigger than radius of interactions $R_\phi = 1/m_\phi$
- the lower bound on N , for a given strength
- upper bound on central density

Formation: via development of instabilities and fragmentation of the uniform relic neutrino background at $z < 200$

Affects programs of detection of relic neutrinos

Backup

*Global characteristics of stars

R N p_{F0}

$m_\phi = 0$

Considering uniform sphere $N = \frac{4\pi}{3} R^3 \frac{p_{F0}^3}{6\pi^2}$ V n

Radius $R = \frac{20}{y\sqrt{m_\nu p_{F0}}}$ from hydrostatic equilibrium

At $p_{F0} \sim m_\nu$ (transition between non-relativistic and relativistic cases):

$R \sim \frac{20}{y\, m_\nu} \sim R_{min}$ (as in the gravitational case)

In terms of N: $R = \frac{90.4}{y^2\, m_\nu} \frac{1}{N^{1/3}}$ radius decreases with increase of N

Fermi momentum and density in the center

$p_{F0} = 0.0485\, m_\nu\, y^2 N^{2/3}$

$n_{\nu 0} = 2\; 10^{-6}\, m_\nu^{\,3} y^6 N^2$

$\rightarrow$ fast increase with y and m_ν

Ground state

Degenerate Fermi gas distribution of neutrinos over p

$$f(p) = \begin{cases} 1, & p < p_F \\ 0, & p > p_F \end{cases}$$

p_F - Fermi momentum

$$n = \frac{p_F{}^3}{6\pi^2}$$

$$P_{deg} = \frac{p_F{}^5}{30\pi^2\, m_\nu}$$

Equation for m*

static case **

$$(\nabla^2 - m_\phi^2)(m^* - m_\nu) = y\, n^*(m^*) \qquad n^* = \frac{1}{2\pi^2}\int_0^{p_F} \frac{m^*}{\sqrt{p^2 + m^{*2}}}\, p^2 dp$$

Equations for m* (instead of ϕ)

G. J. Stephenson, et al

Boundary conditions:

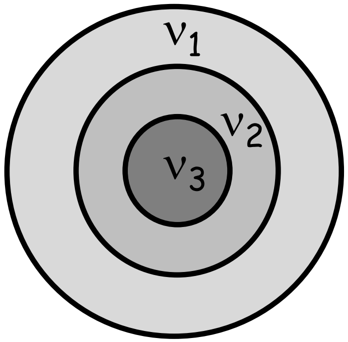
$m^*(0) = m^*_0$

$r \rightarrow$ infty $m^* \rightarrow m_\nu$

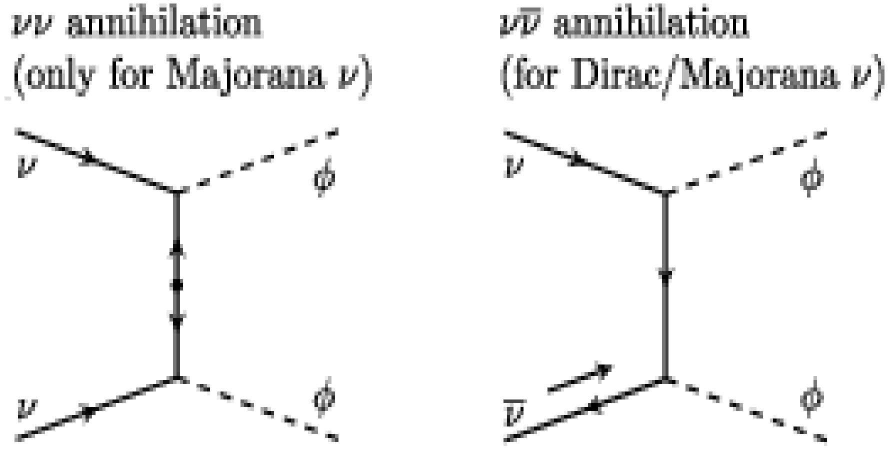
Characteristics of nu clusters

For fixed N

$$R \sim \frac{1}{y^2 m_\nu}$$



Radiation, cooling



ϕ - bremstrahlung

Formation of neutrino stars

From the cosmological neutrino background

For $y < 10^{-7}$ cooling mechanisms: ϕ –emission (bremstrachlung), annihilation $\nu\nu \rightarrow \phi\phi$, are negligible

Formation of ν-stars in analogy to formation of DM halos?

In terms of effective neutrino mass $m^* = m_\nu + y\phi$

At early epoch (large n) $m^* \ll m_\nu$

G. J. Stephenson ,et al.

With decrease of density $m^* \rightarrow m_\nu$ due to decrease of kinetic energy $\rightarrow$ formation of degenerate neutrino gas