# Applications of AdS/CFT Methods to Quantum Criticality in Condensed Matter Physics

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# Outline

1. Quantum criticality in condensed matter systems

2. "Real Life" condensed matter systems

3. AdS/CFT application to condensed matter physics

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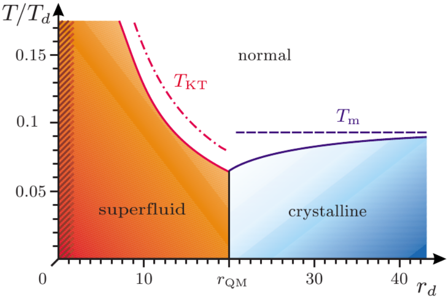
# Quantum phase transitions (QPT)

## Properties of QPTs

- driven by pure quantum fluctuations
- identified by non–analytical points in ground state energy
- accessible by varying a physical parameter $g$ at $T = 0$
- $1^{st}$ order or continuous phase transition




QPTs in the $T \neq 0$ regime

- thermal phase transitions with emerging topological order
  - Berezinskii–Kosterlitz–Thouless transition (BKT)
- No spontaneous symmetry breaking
- No emergence of spatially uniform order parameter

# Quantum phase transitions (QPT)

## Properties of BKT

- $T > T_{BKT}$ disordered phase possess exponentially decaying correlations
- $T < T_{\mathrm{BKT}}$ quasi–long–range order phase with algebraic decaying order parameter correlations
- higher order physical quantities are continuous $\longrightarrow$ phase transition of infinite order

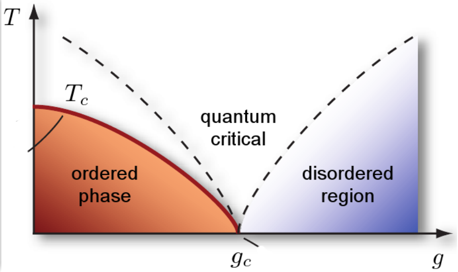
## Interesting quantum phase transitions

- order $\longleftrightarrow$ disorder (also known in classical systems)
- supersolid $\longleftrightarrow$ superfluid ($^4$He at ultra cold temperatures)
- High–$T_C$ superconductor $\longleftrightarrow$ insulator (thin disordered films)

# Quantum critical point (QCP)

## Properties of QCPs

- energy fluctuations about ground state vanishes (no mass gap)
- correlation length diverges (no coherence)
- system possess scale invariance in space and time




- scale transformations with dynamical scaling exponent $z$

$$t \longrightarrow \lambda^z t \qquad\qquad x \longrightarrow \lambda x$$

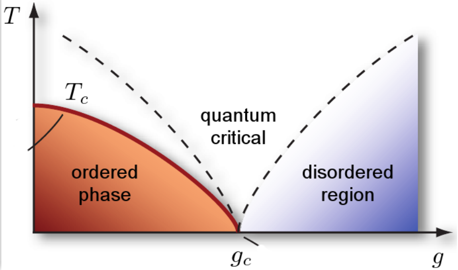
- Two scalings, energy and distance are related by $\Delta \propto \xi^{-z}$

$$\Delta \propto (g - g_c)^{\nu z} \qquad\qquad \xi^{-1} \propto (g - g_c)^{\nu}$$

# Quantum critical region (QCR)

## Properties of QCRs

- QCP are felt over thermal regions $\longrightarrow$ QCR
- growing with increasing temperature (non–classical)
- allows finite temperature crossovers (avoiding CMWH)




$\longrightarrow$ QCP/QCR are difficult to handle using "traditional" CMT tools

- In the QCR there are no weakly coupled quasi–particle
- No order parameter description for finite $T$ (see BKT)
  - Field theory works for $T \neq 0$ in imaginary time only
  - Failure in real–time domain for $t > \hbar/k_B T$
- AdS/CFT provides models of strongly coupled QC in $2+1$ dimensions

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# BCS superconductors

## Properties of BCS superconductors

- Cooper pair formation via interchange of virtual phonons
- Cooper paired dressed electrons lead to energy gap

$$E_{\text{gap}} = 3.52 k_B T_c \sqrt{1 - (T/T_c)}$$

- Isotope effect $T_c \propto 1/\sqrt{M}$ due to phonon exchange
- Infinte DC conductivity
- Ideal diamagnet due to Meißner–Ochsenfeld effect

## Non BCS superconductors

- No isotope effect
- Deviation from universal energy gap law
- Heavy fermions with extremely high "effective thermal mass"
- High temperature $T_c > 30\mathrm{K}$

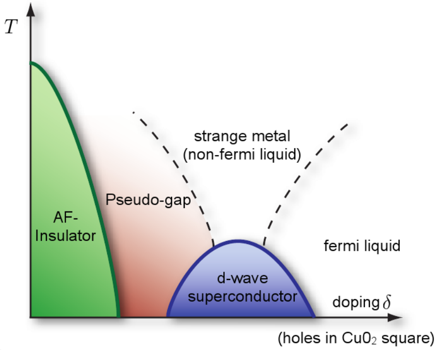
# High $T_c$ superconductors

## Properties of SC QPT

- $2^{\text{nd}}$ order phase transition
- mean field critical exponent $1/2$
- spontaneously broken local $U(1)$

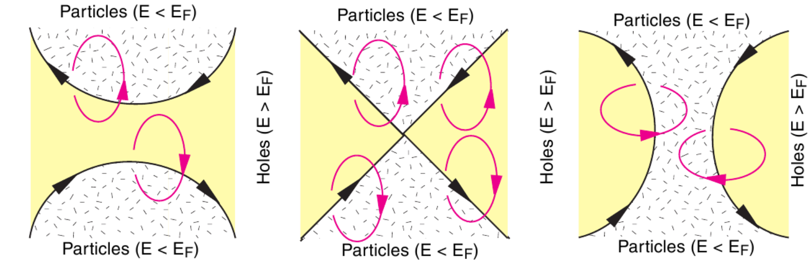
## Mysteries

- Origin / physics of pseudo gap
- Pairing mechanism of electrons
- Strange metal $\stackrel{?}{\equiv}$ QCR
- QCP under superconducting dome




- Attractive strong electron–electron interaction due to spin interactions
- Normal state already strongly coupled $\longrightarrow$ QCP with QCR

# Graphene




**Fermi surface before Lifshitz transition**
$\mu < 0$

**Fermi surface at Lifshitz point of quantum phase transition**
$\mu = 0$

**Fermi surface after Lifshitz transition**
$\mu > 0$

- Change Fermi surface as a function of charge carrier density $\propto \mu$
- "True" topological QPT without order parameter or broken symmetry

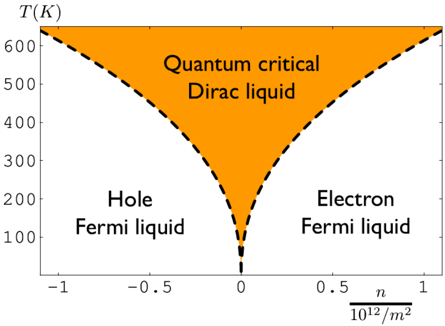
# Graphene

## Properties of Graphene

- Semi–metal / zero gap semiconductor
- Zero effective mass $\longrightarrow$ Two Dirac points (point like Fermi surface)
- Conical dispersion relation

$$E = \hbar v_F \left|\mathbf{k}\right|$$

- Relativistic spin 1/2 particles $\longrightarrow$ Dirac equation




- Mermin–Wagner–Hohenberg theorem forbids phase transition for $T \neq 0 \longrightarrow$ Crossover in QCR

# Outline

# Gravity duals used for condensed matter physics

## Correspondence

| Gravity | Condensed Matter |
|---|---|
| Einstein–Maxwell theory on AdS | effective theory |
| black hole with Hawking temperature | temperature $T$ |
| electric charged black hole | chemical potential $\mu$ |
| magnetic charged black hole | applied magnetic field |

- Solution to Einstein–Maxwell equation leads to AdS
- Black hole solution to EM equation $\longrightarrow$ AdS–Schwarzschild
- Dyonic (electrically & magnetically charged) black hole solution to EM equation $\longrightarrow$ Reissner–Nordstrom–AdS black hole

# Holographic superconductors

## Minimal Lagrangian

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F^2 - |\nabla\phi - iqA\phi|^2 - m^2|\phi|^2$$

## Normal phase $T > T_c$

- Gravity dual to normal state $\longrightarrow$ Reissner–Nordstrom–AdS black hole
- Charged condensate $\sim$ charged operator $\langle Q\rangle \neq 0$ dual to scalar field $\phi$

$$\langle Q\rangle = \frac{2\Delta - d}{L}\phi$$

## Superconducting phase $T < T_c$ and $\phi \neq 0$

- Below critical temperature different spacetime background needed

$$\mathrm{d}s^2 = \frac{L^2}{r^2}\left(-f(r)\,\mathrm{e}^{-\chi(r)}\,\mathrm{d}t^2 + f(r)^{-1}\,\mathrm{d}r^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}\right)$$

# Holographic superconductors

## Solution

- Plugging metric in Einstein–Maxwell equation (EOM of Lagrangian) $\longrightarrow$ Yields solution $\phi(r)$
- Expectation value of charged condensate $\langle Q \rangle$ can be read off
- Showing 2$^{\text{nd}}$ order phase transition with critical exponent $1/2$

## Further properties

- infinite DC conductivity
- gaped AC conductivity
- extreme type II superconductors since photon is not dynamical
  - critical field $B_c$ calculated via free energy difference
  - critical field $B_{c2}$ calculated from above

# Quantum critical transport

## Transport coefficients

- "Traditional" identifications as low–lying excitations of ground state
- described by quasi particles or non–linear waves

But: In QCR these concepts are not well defined

- In $2+1$ dim we can write transport coefficients as

$$\sigma = \frac{\tilde{e}^2}{h} \times \text{ universal dimensionales constant}$$

- AdS/CFT allows analytical and structural insights
- For graphene $\tilde{e} = e$ the transport coefficients are shown to obey the above relations with Fermi level at Dirac point