Introduction to Gauge/Gravity Duality

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A general Form of Gauge/Gravity duality

A Conformal Field Theory (CFT)
in d-dimensions

is dual to

Quantum Gravity in Anti-de Sitter (AdS) space
in (d+1)-dimensions

where the coupling constants behave as $g_{\mathrm{CFT}} = \frac{1}{g_{\mathrm{QG}}}$.

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Part 4: Different Applications

Conformal Symmetry

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- Conformal transformation leaves angles invariant

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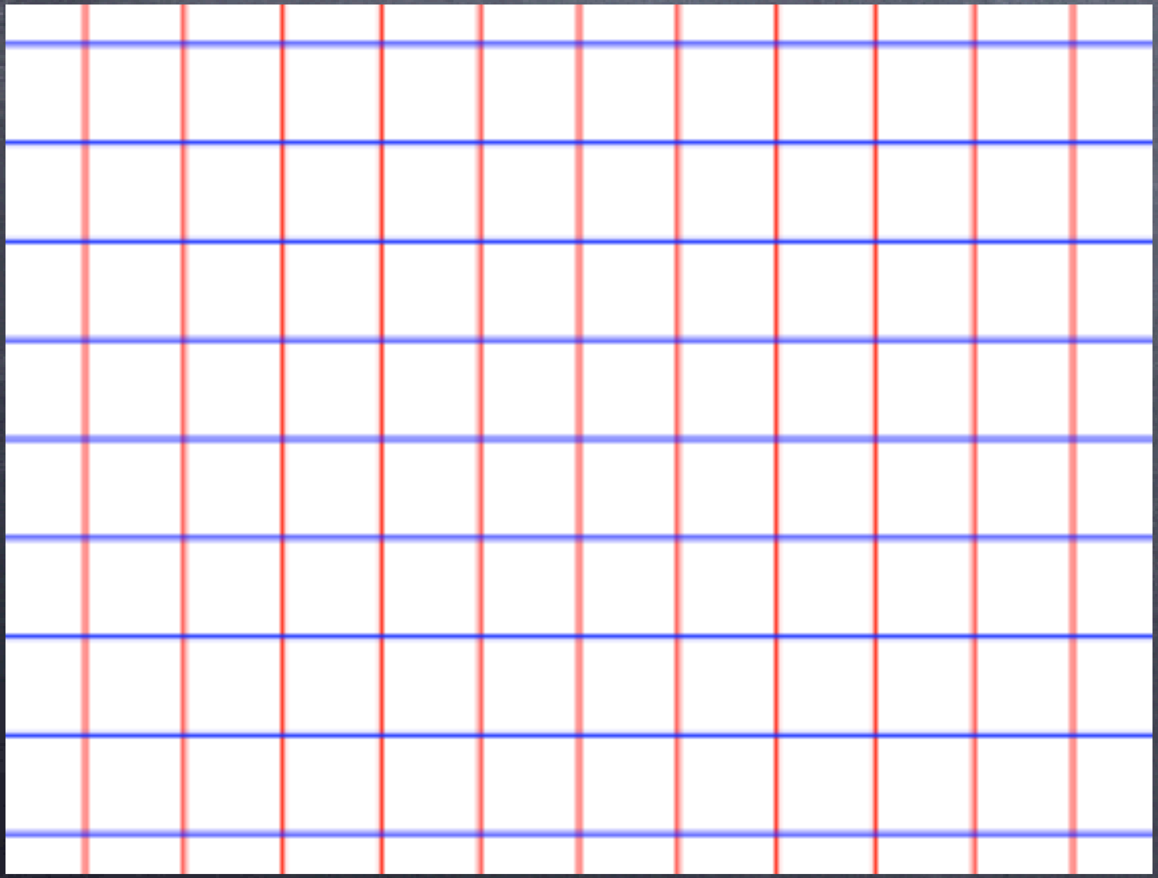

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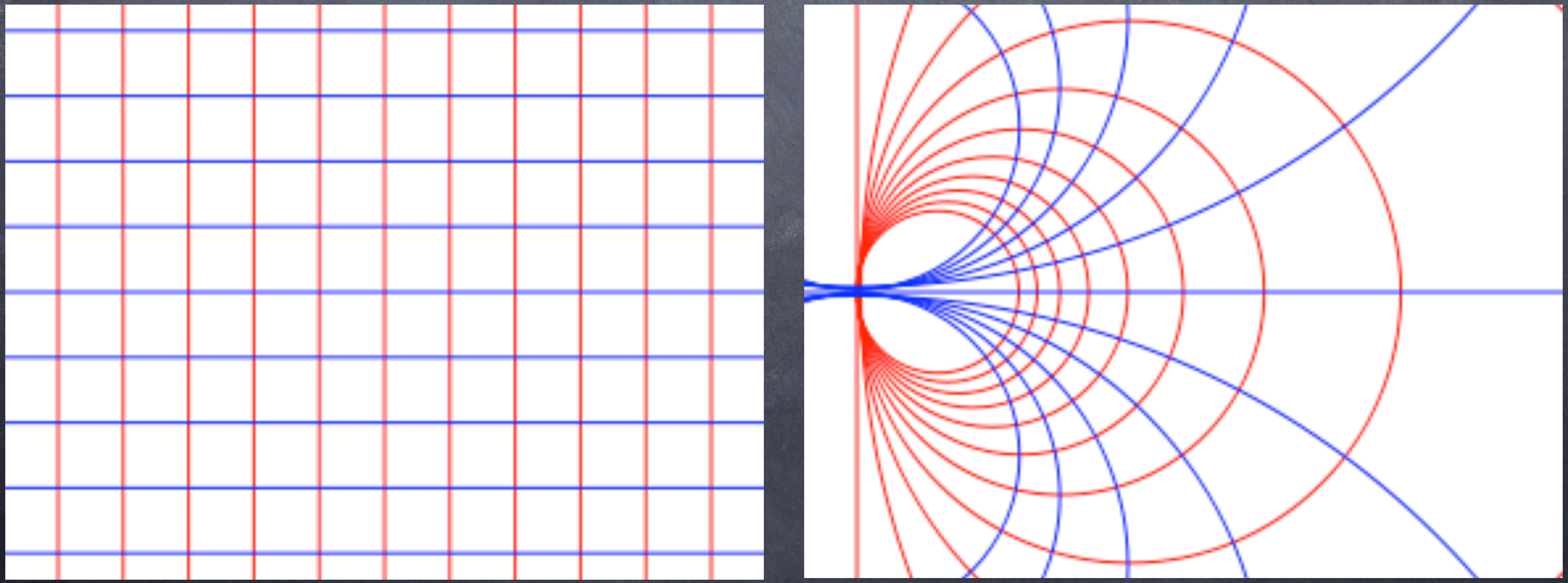

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- Very strong symmetry

$$\langle \phi_i(x)\phi_j(y)\rangle = \frac{c\,\delta_{ij}}{|x-y|^{2\Delta}}$$

$$\langle \phi_i(x_1)\phi_j(x_2)\phi_k(x_3)\rangle = \frac{c_{ijk}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3} + |x_{13}|^{\Delta_1+\Delta_3-\Delta_2}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}}$$

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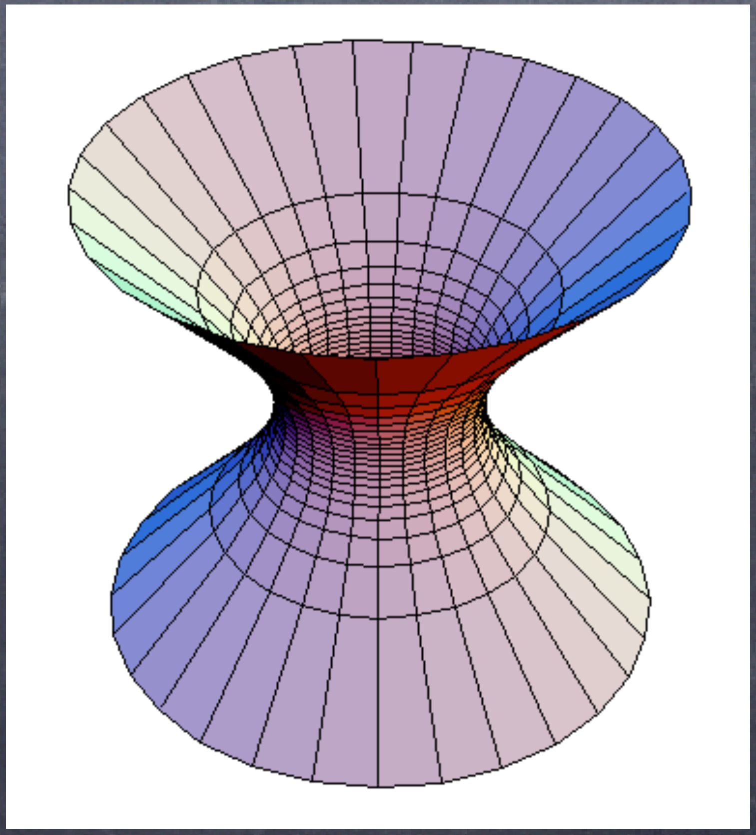

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- In addition: Dictionary between gravity fields and field theory operators needed.

Massive scalar in AdS_{d+1}

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- Equation of Motion (Klein-Gordon Equation)

$$(\nabla^2 - m^2)\phi = 0$$

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- Solutions close to the boundary $z = 0$

$$\phi(z) = A z^{d-\Delta} + B z^{\Delta} \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + R^2 m^2}$$

=> A, B have correct dimension to interpret them as source and vev of an operator $\mathcal{O}$ with scaling dimension Δ.

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- Identify A as source $S_{\mathrm{def}} = S + \int \mathrm{d}^d x\, A\mathcal{O}$ and B as vev $\langle \mathcal{O} \rangle = B$

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$$\langle \mathrm{e}^{\int \mathrm{d}^d x \phi_{\mathrm{bdy}}(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{\mathrm{CFT}} = Z_{\mathrm{QC}} \left[\phi(\vec{x}, z)\Big|_{z=0} = \phi_{\mathrm{bdy}}(\vec{x}) \right]$$

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- Correlation functions (Observables):

$$\langle \mathcal{O}_1(\vec{x}_1) \cdots \mathcal{O}_n(\vec{x}_n) \rangle_{\mathrm{CFT}} = \frac{\delta^n \log Z_{\mathrm{CFT}}}{\delta \phi^1_{\mathrm{bdy}}(\vec{x}_1) \cdots \delta \phi^n_{\mathrm{bdy}}(\vec{x}_n)}$$

$$= \frac{\delta^n \log Z_{\mathrm{QC}} \left[\phi^i(\vec{x}_i, z=0) = \phi^i_{\mathrm{bdy}} \right]}{\delta \phi^1_{\mathrm{bdy}}(\vec{x}_1) \cdots \delta \phi^n_{\mathrm{bdy}}(\vec{x}_n)}$$

Best understood example

$\mathcal{N}=4$ SYM Theory with gauge group $SU(N)$ in 4-dim at large N and large 't Hoft coupling $\lambda = N g_{\mathrm{YM}}^2$.

is dual to

Type IIB Supergravity on $AdS_5 \times S^5$.

Parameters of the theories are related by

$$g_{\mathrm{YM}}^2 = 2\pi g_s \qquad \frac{R^4}{(\alpha')^2} = 2\lambda$$

Large N Field Theory

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- For E propagators, V vertices, F loops

$$N^{E+F-V}\lambda^{E-V} = N^{\chi}\lambda^{F-\chi} \quad \chi = 2 - 2g$$

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- Closed strings (Gravitons) are emitted from stack => Curved spacetime $AdS_5 \times S^5$
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Constantin's talk

- Formal developments: Hidden Symmetries in scattering amplitudes, Integrability of $\mathcal{N} = 4$ SYM Theory

Conclusions

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- Example with String Theory origin

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