Flux Compactification In String Theory

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WHY?

HOW?

WHO?

What's the issue?

Example: Domain Wall SUSY Breaking In Heterotic String Theory


Why?

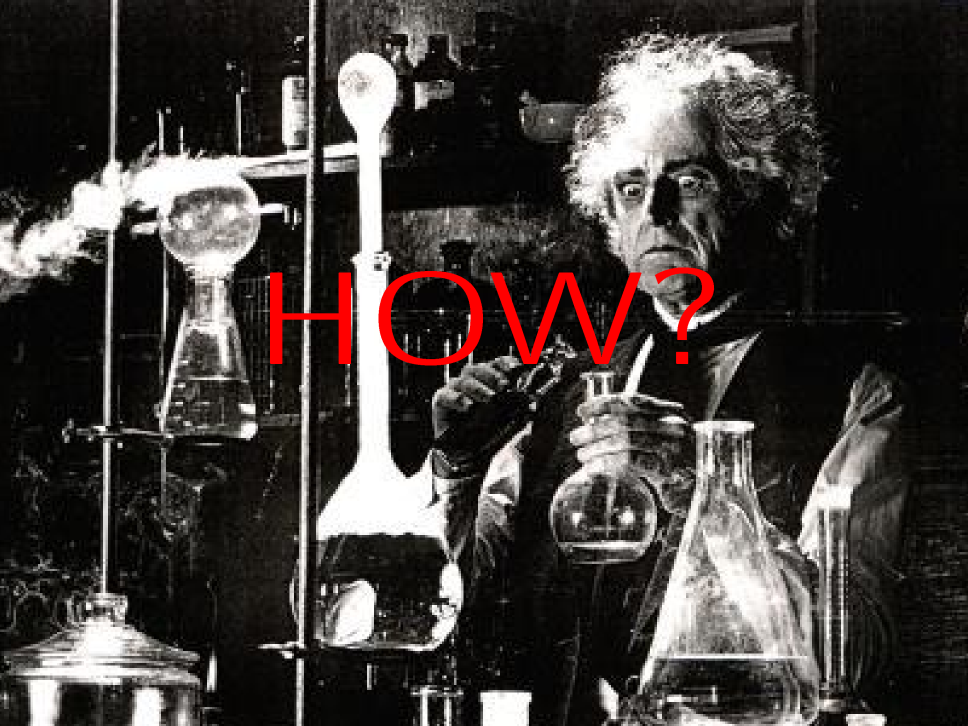
Why should one study string theory?

- Scribbling formulas on a piece of paper is entertaining
- Calculations done at home or in the office keep you out of the sun
- There is a clear distinction: experiment - theory - string theory
- Be one of the top-nerds at your institute
- Don't panic: experiment will never get you!

Why should one study string theory?

- The idea of unification is the guiding principle of high energy physics
- Particle physics is well described by quantum field theories
- A quantum theory of gravity is ill defined due to UV-divergences
- These divergences arise because of local interactions
- One has to introduce 'non-locality': quantum loop gravity, non-commutative space time, string theory

$\Rightarrow$ So maybe string theory is the right thing!


HOW?

How to study string theory?

- String theory is a theory of little strings. Natural ways to describe it
  1. 2d worldsheet embedded in 10 dimensions $\rightarrow$ CFT description
  2. low energy limit $\rightarrow$ 10d SUGRA theories
- Incorporate non-perturbative corrections: e.g. D-branes, instantons
- This can lead to even more fancy theories: F-Theory, M-Theory

In this talk the focus is on SUGRA

A. PICCARD E. HENRIOT P. EHRENFEST Ed. HERZEN Th. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H. FOWLER L. BRILLOUIN

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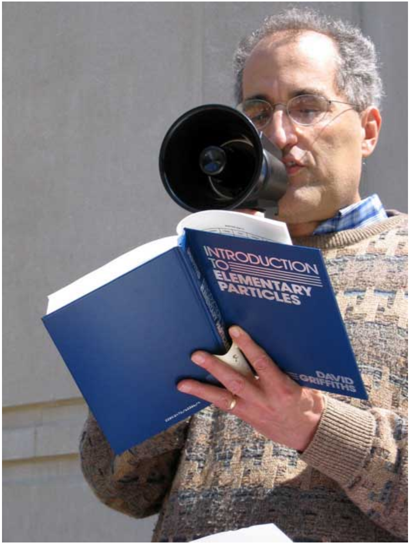
Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

Who is studying string theory?



Edward Witten et al.

What's the issue?

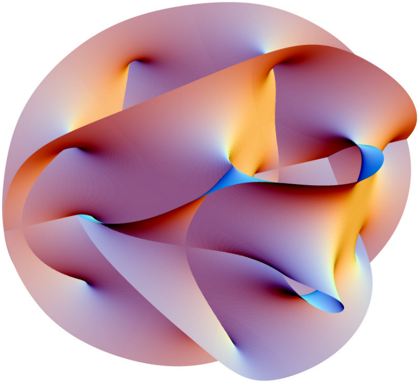
Compactification In String Theory

- String theory comes with
  6 dimensions to much
  $\Rightarrow$ these have to be compact

- A priori many consistent ways to compactify in string theory

- But compactification influences the 4d theory
  (spectrum, supersymmetry, etc.)
  $\Rightarrow$ reduction of possibilities

- For the simplest case
  (no background fields),
  Calabi–Yau–manifolds have to be used



Fluxes

- What happens for non-zero fluxes (i.e. background fields)?

  1. A potential for scalar fields (moduli) is generated $\Rightarrow$ Good!
  2. The geometry of the compact manifold is deformed $\Rightarrow$ Not so good!

- To study this and other implications of flux compactification one analyzes the G-structure of the compact manifold

What is a Manifold with G-structure?

On a G-structure manifold the transition functions between different coordinate patches are elements of the structure group G

What is a Manifold with G-structure?

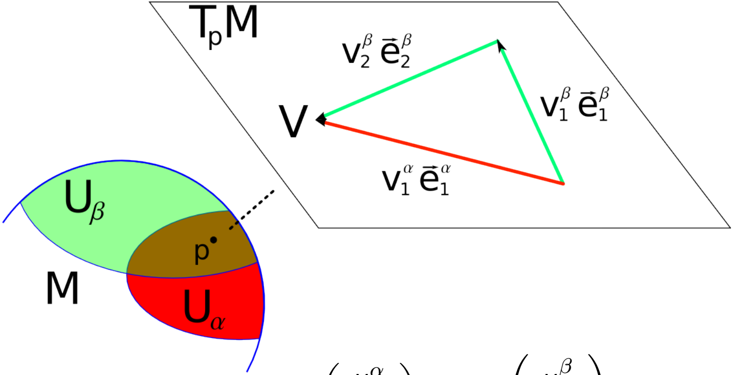
Let M be a compact manifold, U_α, U_β two coordinate patches of M with $U_\alpha \cap U_\beta \neq \varnothing$, $p \in U_\alpha \cap U_\beta$ and $V \in T_p M$.

Then there are two coordinate representations of V, V_a^α and V_b^β that are related by

$$V_a^\alpha = t_{ab}^{\alpha\beta} V_b^\beta \quad \text{with} \quad t^{\alpha\beta} \in G$$

If the group G is the same for all overlapping patches, G is called the structure group of M and M is a manifold with G-structure

What is a Manifold with G-structure?



$$\begin{pmatrix} v_1^\alpha \\ 0 \end{pmatrix} = t^{\alpha\beta} \begin{pmatrix} v_1^\beta \\ v_2^\beta \end{pmatrix}, \quad t^{\alpha\beta} \in G$$

What is it good for?

- In general this is $GL(d, \mathbb{R})$ for a d-dimensional manifold
- Globally defined tensors reduce the structure group
- Using these tensors gives new insight

Example: Domain Wall SUSY Breaking In Heterotic String Theory

based on arXiv:1004.0867
by J.H., D. Lüst, F. Marchesano, L. Martucci

Compactification ansatz and scalar potential

- Consider $X_{10} = X_4 \times M$: $\quad \mathrm{d}s_{10}^2 = e^{2A}\mathrm{d}s_4^2 + \mathrm{d}s_6^2$

  where M admits an SU(3)-structure

- The bosonic $d = 10$ heterotic SUGRA action up to $\mathcal{O}(\alpha')$

  Bergshoeff and de Roo '89

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{-\det g}\, e^{-2\phi}\left[\mathcal{R}_{10}+4(\mathrm{d}\phi)^2-\frac{1}{2}H^2+\frac{\alpha'}{4}(\mathrm{Tr}R_+^2-\mathrm{Tr}F^2)\right]$$

  can be written as $\qquad S = -\int_{X_4}\mathrm{d}^4x\ V$

- The scalar potential V is a function of $g_{(6)}$, H, ϕ, and A

SUSY conditions

- The SUSY conditions are $\delta\psi_\mu = \delta\psi_m = \delta\lambda = \delta\chi = 0$ Strominger '86, Hull '86
- They can equally be written in terms of the SU(3)-structure forms J and Ω Gauntlett,Martelli,Waldram '04

$$\mathrm{d}A = 0, \quad \Lambda_{4d} = 0, \quad F^{2,0} = 0, \quad J \cdot F^{1,1} = 0, \quad R_+^{2,0} = 0, \quad J \cdot R_+^{1,1} = 0$$

$$\mathrm{d}(e^{-2\phi} J \wedge J) = 0, \quad e^{2\phi}\mathrm{d}(e^{-2\phi} J) = *H, \quad \mathrm{d}(e^{-2\phi}\Omega) = 0$$

Potential in squares

$$
\begin{aligned}
V_0 =& \frac{1}{4\kappa^2}\int \mathrm{vol}_M\ e^{4A-2\phi}\left[e^{-4A+2\phi}\mathrm{d}(e^{4A-2\phi}J) - *H\right]^2 \\
&+\frac{1}{4\kappa^2}\int \mathrm{vol}_M\ e^{4A-2\phi}\left\{\frac{1}{4}\left[e^{-2A+2\phi}\mathrm{d}\left(e^{2A-2\phi}J\wedge J\right)\right]^2 + 4(\mathrm{d}A)^2\right\} \\
&+\frac{1}{4\kappa^2}\int \mathrm{vol}_M\, e^{-2A+2\phi}\left[|\mathrm{d}(e^{3A-2\phi}\Omega)|^2 - |J\wedge \mathrm{d}(e^{3A-2\phi}\Omega)|^2\right] \\
&-\frac{1}{4\kappa^2}\int \mathrm{vol}_M\, e^{4A-2\phi}\Big\{2\mathrm{d}A + \frac{1}{4}e^{-2A+2\phi}(J\wedge J)\lrcorner\mathrm{d}(e^{2A-2\phi}J\wedge J) \\
&\qquad\qquad + \frac{1}{2}e^{-3A+2\phi}\mathrm{Re}[\bar{\Omega}\lrcorner\mathrm{d}(e^{3A-2\phi}\Omega)]\Big\}^2 \\
V_1 =& \frac{\alpha'}{8\kappa^2}\int \mathrm{vol}_M e^{4A-2\phi}\left[2\mathrm{Tr}|F^{2,0}|^2 + \mathrm{Tr}|J\cdot F^{1,1}|^2\right] - \left[2\mathrm{Tr}|R_+^{2,0}|^2 + \mathrm{Tr}|J\cdot R_+^{1,1}|^2\right] \\
&-\frac{\alpha'}{2\kappa^2}\int_M \mathrm{vol}_M\ e^{4A-2\phi}\Big[2\,e^{-2A}(\nabla^m\nabla^n e^A)(\nabla_m\nabla_n e^A) \\
&\qquad\qquad - (\iota_m H\cdot\iota_n H)\,\nabla^m A\nabla^n A - 6(\mathrm{d}A\cdot\mathrm{d}A)^2\Big]
\end{aligned}
$$

$\Rightarrow$ SUSY + BI imply $V = 0$ and EOM

DWSB conditions

- We want to break SUSY in a controlled way
- From the external Einstein equations in 10d one finds that $\mathrm{d}A = \Lambda_{4d} = 0$.

$$\Rightarrow V = 0 \text{ also for broken SUSY}$$

- We adopt the following conditions Lüst,Marchesano,Martucci,Tsimpis '08

$$\mathrm{d}(e^{-2\phi} J \wedge J) = 0, \quad e^{2\phi}\mathrm{d}(e^{-2\phi} J) = *H, \quad \bar{\Omega} \lrcorner \mathrm{d}(e^{-2\phi}\Omega) = 0$$

- We allow that $\mathrm{d}(e^{-2\phi}\Omega) \neq 0$
$\Rightarrow$ SUSY breaking implies non-complex manifold
- The geometry gets further constrained by the conditions $V = 0$ and $\delta V = 0$

A simple example: $T^2 \hookrightarrow M \rightarrow K3$

- M has to split into a two and a four dimensional part (locally)
- As a toy model we consider an elliptically fibered $K3$-manifold

  $$ds_6^2 = e^{-2D} ds_{K3}^2 + \frac{L_T^2}{\mathrm{Im}\tau} \theta \otimes \overline{\theta}$$

  e.g. Dasgupta,Rajesh,Sethi '99, Yau et al. '01-'09, Becker,Bertinato,Chung,Guo '09

  with $D \sim \phi$ and τ constant
- Satisfying the Bianchi identity while keeping $M_{SB} \ll M_{KK}$ yields
  - Poisson equation for ϕ
  - $L_T^2 \simeq 6(2\pi)^2 \alpha'$
  - $\tau = (1 + i\sqrt{35})/6$
  - $m_{3/2} \simeq \frac{1}{6} \times \frac{g_s M_P}{M_s^4 L_{K3}^4}$

$\Rightarrow$ Consistent SUSY breaking together with moduli stabilization

Conclusion

What is sweet and swings through the refrigerator?

Conclusion

The Tarzipan