Flavor Symmetry in the Lepton Sector

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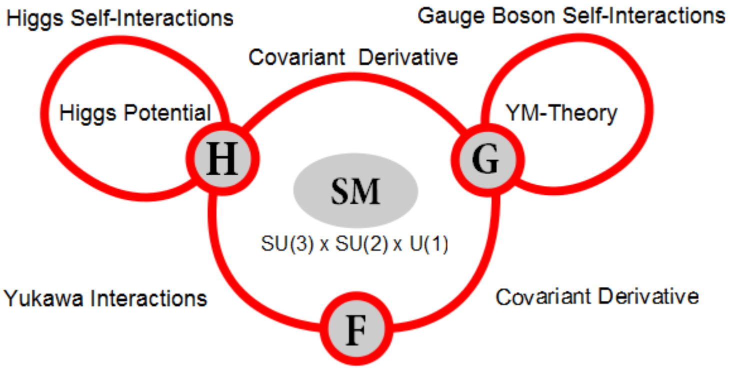
Outline

The Standard Model

The Lagrangian of the Standard Model (SM)

$$\mathcal{L}_{SM} = \mathcal{L}_F + \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_Y$$



The Flavor Puzzle

- **Family Structure:**
  Three generations of quarks and leptons.
- **Mass Hierarchies.**
- **Quark Mixing:**
  Origin of the CKM-structure.
- **Massive Neutrinos:**
  Not included in the SM
  $\Rightarrow$ Extension needed.
- **Neutrino-Nature:**
  Dirac or Majorana.
- **Lepton Mixing:**
  Origin of the PMNS-structure.
- **Neutrino-Spectrum.**
- **Differences**
  between quarks and leptons.

Continuous Flavor Symmetry in the Lepton Sector

Global $U(3)_L \otimes U(3)_R$ – Symmetry (Rotation in Generation Space)

$$F \quad \rightarrow \quad \mathcal{U}_F \, F, \quad \text{where} \quad F : L, \; e_R$$

FS explicitly broken by Yukawa Interactions

$$\mathcal{L}_{Y_E} = \sum_{ij} Y_{E\,ij} \, \bar{e}_{R_i} \, \Phi^\dagger \, L_j + h.c.$$

i,j: Generation index

Remaining Flavor Symmetries

$$G_F = U(3)_L \otimes U(3)_R \quad \xrightarrow{Y_E} \quad U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

$\Rightarrow$ 3 physical parameters: charged lepton masses m_e, m_μ and m_τ.

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(Solar) Neutrino Problem

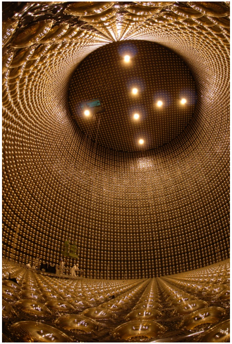


Figure: Inside Superkamiokande

- From 1970 on, there have been several neutrino experiments for:
  - **Solar Neutrinos:** Homestake, (S)Kamiokande, SNO, ...
  - **Atmospheric Neutrinos:** MACRO, (S)Kamiokande,
  - **Reactor Experiments:** KamLAND, CHOOZ, ...
  - **Accelerator Experiments:** K2K, MINOS, ...
- All Experiments detected a deficit of neutrino flavor coming from the source, and an excess of other flavors

  $\Rightarrow$ Neutrinos oscillate

  $\Rightarrow$ Neutrinos are massive.

Experimental Results from Neutrino Oscillations

Mass Differences (eV^2) and Angles

$$\Delta m^2_{\odot} \equiv \Delta m^2_{12} \approx 7.7 \cdot 10^{-5} \ll \Delta m^2_{atm} \equiv \Delta m^2_{23} \approx \Delta m^2_{13} \approx \pm 2.5 \cdot 10^{-3}$$

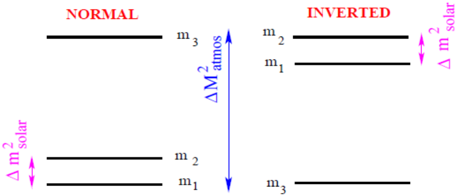
$$\theta_{12} \approx 35^\circ \qquad \theta_{23} \approx 45^\circ \qquad \theta_{13} \approx 0^\circ \qquad \delta_{CP} = ?$$

Mixing Matrix

$$|U|_{90\%} = \begin{pmatrix} .80 \to .84 & .53 \to .60 & .00 \to .17 \\ .29 \to .53 & .51 \to .69 & .61 \to .76 \\ .26 \to .50 & .46 \to .66 & .64 \to .79 \end{pmatrix}$$

Gonzalez-Garcia, Maltoni '07

Possible Neutrino Mass Spectra

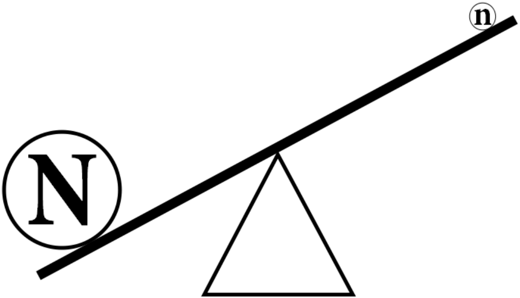


Summary of Oscillation Experiments

- Only masses^2 are accessible.
- The CP-phase is averaged out and thus cannot be determined.
- The current data still allow for the three scenarios of a normal, inverted and almost degenerate (i.e. maximum absolute mass scale) neutrino spectrum.

Concepts of See-Saw Model

- 3 heavy right-handed singlet neutrinos N_I added to the SM.
- At low scales the N_I get integrated out (effective theory).
- Mass scale of light neutrinos suppressed by the scale of the heavy neutrinos N_I.
- Natural and minimal explanation of tiny neutrino masses.



See-Saw Model Lagrangian (Type-I)

FS-Breaking Term at High Energies

$$\mathcal{L}_{SS} = -N_I^c \, Y_{\nu\, Ij} \, L_j \, H - \frac{1}{2} \, N_I^c \, M_{IJ} \, N_J^c = -\mathcal{N}_\mathcal{I} \, \mathcal{M}_{\mathcal{I}\mathcal{J}} \, \mathcal{N}_\mathcal{J}$$

$$\mathcal{N}_\mathcal{I} = (\nu_i, N_I) \qquad \mathcal{M}_{\mathcal{I}\mathcal{J}} = \begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix}$$

At low energies the effects of the heavy neutrinos N_I can be neglected
$\Rightarrow$ effective theory for light neutrinos:

FS-Breaking Term at Low Energies Λ_{LNV}

$$\mathcal{L}_{eff} = +\frac{1}{2} \, (L_i \, H)^T \, (\hat{g}_\nu)_{ij} \, (L_j \, H)$$

$$\hat{g}_\nu = \hat{g}_\nu^T = Y_\nu^T \, M^{-1} \, Y_\nu \equiv \frac{g_\nu}{M}$$

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Properties of $\mathcal{L}_{eff}$

- Lepton number violation (LNV) by 2 units, $\Lambda_{LNV} \sim M \Rightarrow$ LNV can lead to baryogenesis from leptogenesis in the early universe. (strictly speaking, the decay of the N_I plays the important role here.)
- The dimension-5 operator yields mass term for neutrinos after electro-weak symmetry breaking.

Neutrino Mass Matrix

$$m_\nu = \hat{g}_\nu\, v^2/2 \quad \propto \quad g_\nu \frac{v^2}{\Lambda_{LNV}}, \qquad v = \langle H \rangle \approx 250\,\mathrm{GeV} \ll \Lambda_{\mathrm{LNV}}$$

- Although neutrino masses are highly suppressed, g_ν can have generic entries of $\mathcal{O}(1)$.

Counting of Physical Parameters in $\mathcal{L}_{eff} + \mathcal{L}_{Y_E}$

$$Y_E \quad \rightarrow \quad \mathcal{U}_{e_R}^{\dagger}\, Y_E\, \mathcal{U}_L = diag\left(y_e, y_\mu, y_\tau\right) \propto diag\left(m_e, m_\mu, m_\tau\right)$$

$$+$$

$$m_\nu \quad \rightarrow \quad \mathcal{U}_L^T\, m_\nu\, \mathcal{U}_L \;=\; \left(U_{PMNS}^{-1}\right)^T diag\left(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\right)\, U_{PMNS}^{-1}$$

$$\Downarrow$$

$$G_F = U(3)_L \otimes U(3)_R \quad \xrightarrow{g_\nu,\, Y_E} \quad \text{nothing}$$

- $12 + 18$ parameters in $\mathcal{L}_{eff} + \mathcal{L}_{Y_E}$
- 18 broken symmetry generators
- 12 physical parameters:
  - 6 masses of charged leptons and light neutrinos.
  - 3 angles describing mixing among the generations.
  - 3 CP-violating phases: 1 Dirac phase and 2 Majorana phases.

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The Idea behind Spurion Parametrization

- Promotion of Y_E and g_ν to scalar "spurion" fields.
- Spurion fields transform under chiral FS as

$$g_\nu \sim (6,0)_{-2,0}\,, \qquad Y_E \sim (\bar{3},3)_{-1,1}$$

- Vacuum expectation values (VEVs) of the spurion fields break FS.

see also Cirigliano *et al.*'05

- Hierarchical structure of masses and mixing results from breaking FS in a stepwise fashion.
- The way of how FS gets broken depends on the properties of the (eff.) spurion potential.

Patterns of Symmetry Breaking

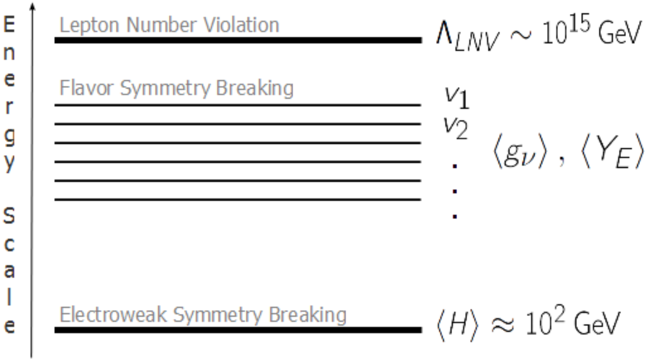

Figure: Pattern of symmetry breaking by VEVs of spurions $v_1, v_2, \ldots$ below the scale of lepton number violation Λ_{LNV}. At $\langle H \rangle$ electroweak gauge symmetry is broken.

Characteristics of Symmetry Breaking (SB)

- Specific example: Normal Hierarchy $m_{\nu_3} \gg m_{\nu_2}$, and $m_{\nu_2} \gg m_{\nu_1}$.

$$\Rightarrow \quad \Delta m_{13} \approx m_{\nu_3}^2 \quad \Rightarrow \quad m_{\nu_3} \approx 50\,\text{meV}$$
$$\Rightarrow \quad \Delta m_{12} \approx m_{\nu_2}^2 \quad \Rightarrow \quad m_{\nu_2} \approx 9\,\text{meV}$$

- See-Saw Scale: $\Lambda_{LNV} = v^2/m_{\nu_3} \approx 10^{15}\ GeV$.

- From $Y_{E_i} \sim \frac{m_{E_i}}{v}$ and $g_{\nu_i} \sim \frac{m_{\nu_i} \Lambda_{LNV}}{v^2}$ (diagonal basis) and assuming $g_{\nu_2}/g_{\nu_3} \approx g_{\nu_1}/g_{\nu_2}$, one finds the

Order of VEVs

$$1 \equiv g_{\nu_3} \quad \rightarrow \quad g_{\nu_2} \quad \rightarrow g_{\nu_1} \quad \approx \quad y_\tau \quad \rightarrow \quad y_\mu \quad \rightarrow \quad y_e$$

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Parameter Counting during SB

During each step of SB the following relations hold:

$$\# \text{ Spurions} + \# \text{ VEVs} - \# \text{ Symmetries} = \# \text{ Physical Parameters} = 12$$

$$\# \text{ Spurions} + \# \text{ VEVs} + \# \text{ Goldstones} = \# \text{ Spurion Parameters} = 30$$

To see how the mechanism of SB in the spurion picture works, let's consider an explicit example:

Go to basis where g_ν is diagonal (convenience)

$$\Rightarrow \quad Y_E \quad \rightarrow \quad diag\,(y_e, y_\mu, y_\tau)\; U_{PMNS} \qquad (\star).$$

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Bi-Tri-Maximal Mixing (TBM-Mixing)

Currently best fit of the observed neutrino-mixing structure from neutrino oscillations:

$$\theta_{23} \simeq \pi/4 \text{ (maximal)}, \ \sin\theta_{12} \simeq 1/\sqrt{3} \text{ (close to maximal)}, \ \theta_{13} \simeq 0.$$

- Tri-maximal mixing among ν_e, ν_μ and ν_τ in ν_2.
- Bi-maxinmal mixing among ν_μ and ν_τ in ν_3.

$$|U|^2_{PMNS} \simeq |U|^2_{TBM} \equiv \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix} \end{array}$$

Harrison, Scott '04

1. VEV

$$g_\nu \xrightarrow{VEV} \begin{pmatrix} 0 & & \\ & 0 & \\ & & v_1 \end{pmatrix}, \qquad Y_E \xrightarrow{VEV} 0$$

FS-Breaking

$$G_F \rightarrow G_F' = SU(2)_L \otimes SU(3)_R \otimes U(1)_L \otimes U(1)_R \otimes \mathbb{Z}_2$$

Residual Spurions

$$\begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix} \sim (2,1)_{2,0}, \qquad \begin{pmatrix} y_{11} & y_{21} & y_{31} \\ y_{12} & y_{22} & y_{32} \end{pmatrix}^T \sim (2,3)_{-1,1}$$

$$\begin{pmatrix} y_{13} & y_{21} & y_{33} \end{pmatrix}^T \sim (1,3)_{0,1}$$

Parameter Counting: $(6+18)+1-13=12$

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4. VEV

$$g_\nu = \begin{pmatrix} v_3 & & \\ & v_2 & \\ & & v_1 \end{pmatrix}, \; Y_E \xrightarrow{VEV} \begin{pmatrix} 0 & & \\ & v_4 & \\ & & \tilde{v}_3 \end{pmatrix} U_{TBM} = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{v_4}{\sqrt{6}} & \frac{v_5}{\sqrt{3}} & -\frac{v_4}{\sqrt{2}} \\ -\frac{v_3}{\sqrt{6}} & \frac{v_3}{\sqrt{3}} & \frac{v_3}{\sqrt{2}} \end{pmatrix}$$

FS-Breaking

$$G_F^{(3)} \quad \to \quad G_F^{(4)} = U(1)_R$$

Residual Spurions

$$(y_{11}) \sim (1)_3 \,, \quad (y_{21}) \sim (1)_3 \,, \quad (y_{31}) \sim (1)_3 \,,$$

$$(y_{32}) \sim (1)_0 \quad \text{uncharged!}$$

Parameter Counting: $(0+8)+5-1=12$

Summary of FS-Breaking

VEV	FS
–	$U(3)_L \otimes U(3)_R$
v_1	$SU(2)_L \otimes SU(3)_R \otimes U(1)_L \otimes U(1)_R \otimes \mathbb{Z}_2$
v_2	$SU(3)_R \otimes U(1)_L \otimes U(1)_R \otimes \mathbb{Z}_2$
$v_3,\ \tilde{v}_3$	$SU(2)_R \otimes U(1)_R$
v_4	$U(1)_R$
v_5	nothing

Table: Residual FS during each step of symmetry breaking.

Results from SB

- 1 uncharged complex spurion (y_{32}) left
  $\Rightarrow$ 1 phase.
- 3 charged complex spurions + 1 residual symmetry
  $\Rightarrow$ 2 phases.
- Angles describe the relative orientation of mass eigenvectors given by the patterns of SB.
- Deviations from TBM-mixing, i.e. deviations from ($\star$), generally enter at later stages of SB.
- In strict TBM-mixing, (y_{11}) would be the next spurion to acquire its VEV (others would be a deviation of TBM) ...

Summary

- Latest experimental data confirm that neutrinos are massive and mix.
- See-Saw Models are an efficient way to introduce neutrino masses and mixing to the SM.
- Spurion parametrization and spontaneous symmetry breaking are an elegant way to dynamically fix masses and mixing parameters.

  for quarks see also Feldmann, Mannel, Jung '08,'09
- Starting point for model-independent approaches to the flavor puzzle.

Future Outlook

- Understanding the (effective) potential responsible for SB:
  - How does it look like?
  - How it comes about?
- Implications for specific processes like $\mu \rightarrow e\,\gamma$.

  see also Cirigliano *et al.'05*
- Possible implementation in a more fundamental theory, for instance through gauged flavor symmetries.

  for quarks see also Albrecht, Feldmann, Mannel '10
- The role of discrete symmetries.

Thanks