Edge states and the Luttinger liquid in M-theory

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Outline

- Motivation
- Introduction
  - What is M-theory?
  - What is the ABJM model?
- Model and Calculations
  - Reduction to Type IIA SUGRA
  - Brane embedding
  - Finite chemical potential
- Results
  - Spectral function
  - Phase diagram
- Condensed Matter

Lower dimensional CMT with String Theory?

Condensed matter systems

- Compute quantities in condensed matter systems with strong coupling

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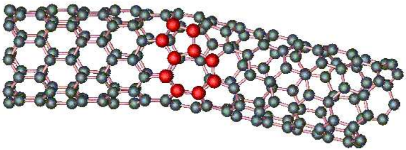
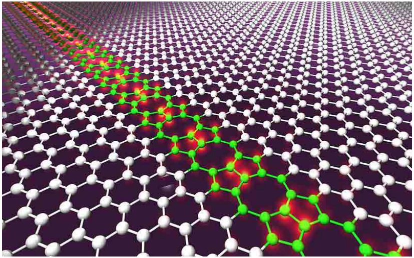


[V. H. Crespi]



[Y. Lin, USF]

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- UV-completion of SUGRA
- 11-dimensional theory
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From string theory we can uplift to M-theory

ABJM

- dubbed after Aharony, Bergman, Jafferis and Maldacena [0806.1218]
- holographic principle, gauge/gravity duality
- N_c M2-branes probe a $\mathbb{C}^4/\mathbb{Z}_k$ singularity

(Super-) Gravity side

limit for large N_c where 't Hooft coupling $\lambda = N_c/k$ is kept fixed:
$AdS_4 \times S^7/Z_k$ in SUGRA
limit for $k \longrightarrow \infty$ type IIA string theory on $AdS_4 \times \mathbb{CP}^3$

Gauge theory side

$1+2$ dimensional theory with $\mathcal{N} = 6$ SUSY
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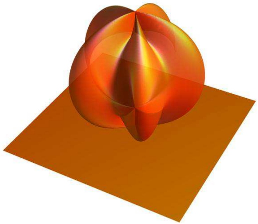
Chern-Simons terms $\rightarrow$ often occur in condensed matter systems!

M-theory content

Coincident Stack of N_c M2-branes
transverse space is $\mathbb{C}^4/\mathbb{Z}_k$ orbifold



[J. Davey]

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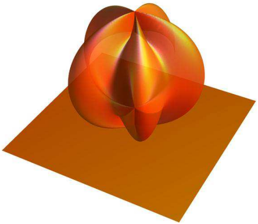

[J. Davey]

Dimensional reduction to type IIA

- Zoom to near-horizon $\Rightarrow AdS_4 \times S^7/\mathbb{Z}_k$ SUGRA
- Radius of compact direction $R/k\ell_p = 2^{5/6}\pi^{1/3}(N_c k)^{1/6}/k$
- sending the Chern-Simons level $k \to \infty$ gives type IIA string theory
- Branes become: M2->D2

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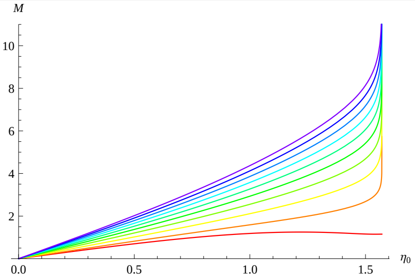

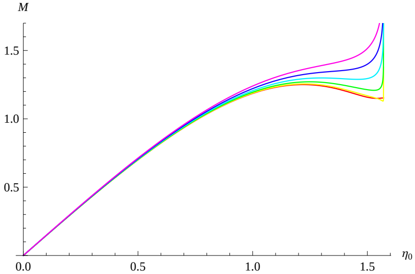

Brane embedding

Introduce $U(1)$ gauge field A_μ living on D6-brane

Gauge field A_μ gauged to have A_t the only nonvanishing component

Generalized action

action: Minimal area law

$$S \quad \propto \int \mathrm{d}\sigma^{1+6} \sqrt{-\det(\mathcal{P}[g] \qquad\qquad)}$$

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Action for D6-brane

$$S_{\mathrm{D6}} \propto \int \mathrm{d}\chi\, \chi^{-4} \left[2\cos\eta \sqrt{1 + \eta'^2\, \chi^2 (1-\chi^3) - \chi^4 (\partial_\chi A_t)^2} - \sin^2\eta \right]$$

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Before embedding can be computed: Solve for $\partial_\chi A_t$

Chemical potential

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Asymptotics of gauge field

At the boundary of AdS ($\chi \to 0$)

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EOM for gauge field

Solving the equation of motion for the gauge field

$$\partial_\chi A_t = d \sqrt{\frac{1 + \eta'^2\, \chi^2(1-\chi^3)}{4\cos^2\eta + d^2\chi^4}} \qquad \mu = -d \int_0^1 \mathrm{d}\chi \sqrt{\frac{1 + \eta'^2\, \chi^2(1-\chi^3)}{4\cos^2\eta + d^2\chi^4}}$$

Scalar excitations

Scalar fluctuations

Fluctuations of the brane in one component $\eta(\chi)$

scalar mode $\eta(\chi) \rightarrow \eta(\chi) + \delta\eta(\sigma^i)$

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Equations of motion for fluctuations

Plug into DBI-action and expand to 2nd order in the fluctuations

Equations of Motion are partial differential equations

Fourier trafo $\delta\eta(\chi, \vec{x}) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i}\vec{k}\vec{x}} \delta\eta(\chi, \vec{k})$ with $\vec{k} = (\omega, 0, q)$

Linear eom $\partial_\chi^2(\delta\eta) + \mathfrak{C}_1 \partial_\chi(\delta\eta) + \mathfrak{C}_2(\delta\eta) = 0$

The spectral function

Definition

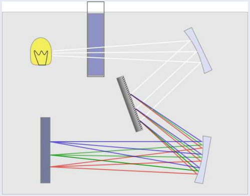
The spectral function is related to the retarded correlator G^R by

$$\mathfrak{R} = -2\mathrm{Im}\, G^R \quad \text{where}$$

$$G^R(k) = -\mathrm{i} \int \mathrm{d}x^4\, \mathrm{e}^{ikx} \theta(x^0) \langle [J(x), J(0)] \rangle$$

$$\text{with} \quad G^R = \left. \frac{\delta^2 S_{\mathrm{SUGRA}}}{\delta \tilde{A}^2} \right|_{\mathrm{boundary}}$$

[Son, Starinets, '02]



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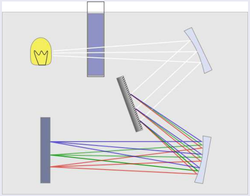
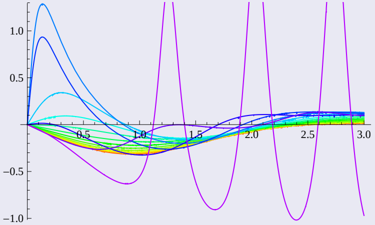


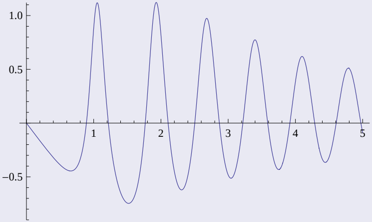
In our coordinates

$$\mathfrak{R}(\omega, \vec{q} = 0) \propto \mathrm{Im}\, \frac{\partial_\chi \eta}{\eta}\Big|_{\mathrm{boundary}}$$

Numerical Calculation

Spectral function for vanishing density





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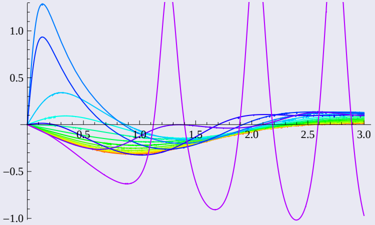

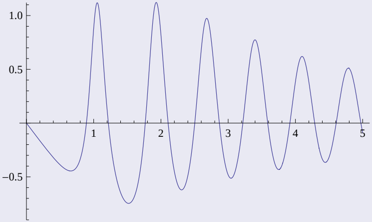

- higher quark mass gives very pronounced peaks
- expected from earlier studies in D3/D7

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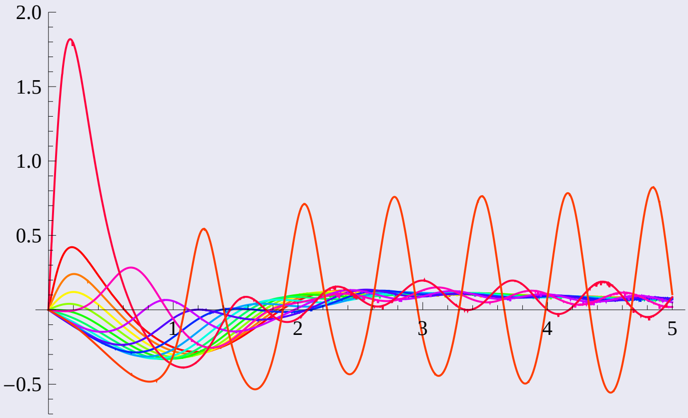
Spectral function for non–vanishing density



Phase diagram

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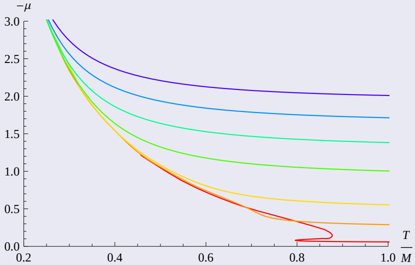

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loops indicate an instability

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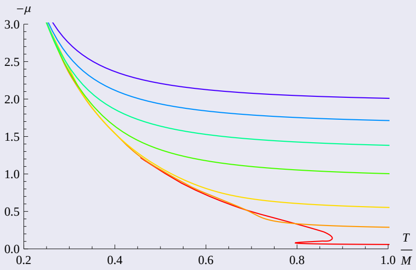
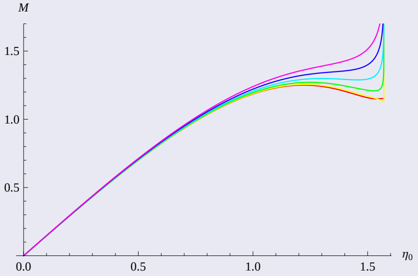


Phase Transition



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SOLUTION:

comparison with the embeddings indicates that instability lies at the phase transition

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- Materials with defects, lower dimensional i. e. graphene
- Electrons confined to a lower-dimensional subspace
- Luttinger-Liquid: 1d electron gas, behaves as a quantum liquid
  But no description with Fermi-Landau possible!
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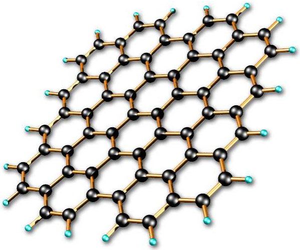

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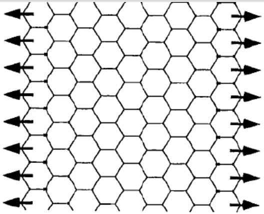

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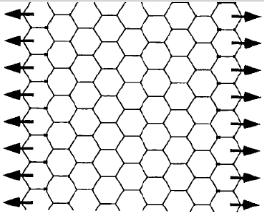

- AdS_4: codimension 0,1 or 2 defect
- here: codimension 0 defect
- calculated in [Erdmenger et al., 0909.3845]

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Outlook

- Connection to condensed matter systems
- Compute further observables: quark condensate,. . .
- Build in chirality

Conclusion

Thank you for your attention!