Supersymmetry and the W-Boson mass

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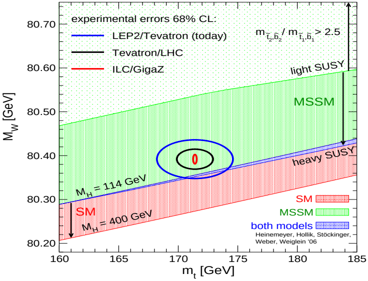
Aim of this talk:

Understand this plot!



Outline

- Introduction to Supersymmetry.
- On-shell renormalization of masses and couplings.
- m_W as dependent quantity.
- m_W: SM vs SUSY.

Standard Model and symmetries

Standard Model: gauge theory

Poincaré group (spacetime symmetry)	$\otimes$	internal symmetries (independent of momentum and spin)
$\downarrow$		$\downarrow$
translation, boost, rotation		$SU(3) \times SU(2)_L \times U(1)$

Why symmetries?

Emmy Noether

symmetries	$\longrightarrow$	conservation laws
time translation	$\longrightarrow$	energy conservation
rotation	$\longrightarrow$	angular momentum conservation
$U(1)_{QED}$	$\longrightarrow$	charge conservation

Why Supersymmetry?

Coleman-Mandula theorem

The most general Lie algebra of symmetry operators that commute with the S-matrix [...] consists of the generators P_μ and $J_{\mu\nu}$ of the Poincaré group, plus internal symmetry generators.

The theorem can be overcome by considering "graded Lie algebras":
$\rightarrow$ Instead of Bosonic symmetry generators one has Fermionic generators!

Supersymmetry

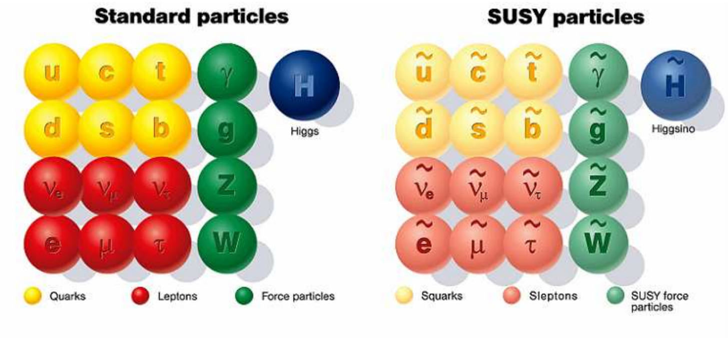
Pro

- Only (non-trivial) extension of the Poincaré group in 4D
- Relates particles of different spin
- Nice phenomenology
  gauge coupling unification, solution to hierarchy problem, EW precision data

Contra

- Many new particles, none of them observed.
- Supersymmetry $\rightarrow$ superpartners have same mass.
  $\Rightarrow$ has to be broken.

Supersymmetry: Particle spectrum



- Each SM field, gets a superpartner that differs by spin 1/2.
- Matter superfields: (spin 0, spin 1/2) doublets.
- Vector superfields: (spin 1, spin 1/2) doublets.
- Supersymmetry requires two Higgs fields → Ananda's talk.

Mass parameter

sfermion Lagrangian $(\tilde{f} = \{u, d\})$

$$\mathcal{L}_{\tilde{f}} = \left(\tilde{f}_L^\dagger, \tilde{f}_R^\dagger\right)\left(\partial_\mu \partial^\mu - \mathcal{M}_{\tilde{f}}\right)\begin{pmatrix}\tilde{f}_L \\ \tilde{f}_R\end{pmatrix}$$

$$\mathcal{M}_{\tilde{f}} = \begin{pmatrix} m_f^2 + M_L^2 + m_Z^2 \cos(2\beta)(T_f^3 - Q_f \sin^2\theta_W) & m_f(A_f^* - \mu\kappa) \\ m_f(A_f - \mu^*\kappa) & m_f^2 + M_{\tilde{f}_R}^2 + m_Z^2 \cos(2\beta) Q_f \sin^2\theta_W \end{pmatrix}$$

- Four fermions for each generation: $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$, $\tilde{d}_R$
- 1st generation: $m_{u,d} \approx 0$
  Only three parameters in this sector: M_L, $M_{\tilde{u}_R}$, $M_{\tilde{d}_R}$

3 free mass parameters (fixed by experiment)
1 dependent mass $\rightarrow$ prediction of the theory.

Example: Scalar QED

Consider a scalar field with $U(1)$ interaction (electrodynamic):

$$
\begin{aligned}
\mathcal{L} = & \left(D_\mu \Phi^* D^\mu \Phi - m_0^2\right) + F^{\mu\nu} F_{\mu\nu} \\
= & \, \partial_\mu \Phi^* \partial^\mu \Phi - m_0^2 + F^{\mu\nu} F_{\mu\nu} + ieA^\mu (\partial_\mu \Phi^*) \Phi - ieA^\mu \Phi^* (\partial_\mu \Phi) + e^2 A_\mu A^\mu \Phi^* \Phi \\
= & \, \mathcal{L}_{0,\Phi} + \mathcal{L}_{0,\gamma} + \mathcal{L}_{int}
\end{aligned}
$$

$D^\mu = \partial^\mu + ieA^\mu, \; F^{\mu\nu} = \frac{1}{ie}[D^\mu, D^\nu]$

Free Φ field with mass m_0:	$\mathcal{L}_{0,\Phi} = \partial_\mu \Phi^* \partial^\mu \Phi - m_0^2$
Free photon field:	$\mathcal{L}_{0,\gamma} = F^{\mu\nu} F_{\mu\nu}$
Scalar–photon interaction:	$\mathcal{L}_{int} = ieA^\mu (\partial_\mu \Phi^*) \Phi - ieA^\mu \Phi^* (\partial_\mu \Phi) + e^2 A_\mu A^\mu \Phi^* \Phi$

How does the interaction term affect the free Lagrangian?
What is the meaning of m_0?

Interaction as a perturbation

- Find the field solutions for the free Lagrangian $\mathcal{L}_0$

$$\Phi(t_0, \mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p e^{i\mathbf{p}\cdot\mathbf{x}} + a_p^\dagger e^{-i\mathbf{p}\cdot\mathbf{x}} \right)$$

- Switch to the interaction picture.

$$\Phi_{\mathrm{I}}(t, \mathbf{x}) = e^{iH_0(t-t_0)}\ \Phi(t_0, \mathbf{x})\ e^{-iH_0(t-t_0)}$$
$$\Phi(t, \mathbf{x}) = U^\dagger(t, t_0)\ \Phi_{\mathrm{I}}(t, \mathbf{x})\ U(t, t_0)$$

- Compute time evolution $U(t, t_0)$ of the interaction picture fields

$$i\frac{\partial}{\partial t} U(t, t_0) = H_{\mathrm{I}} U(t, t_0)$$

$$U(t, t_0) = 1 + (-i)\int_{t_0}^{t} dt_1 H_{\mathrm{I}}(t_1) + (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 H_{\mathrm{I}}(t_1) H_{\mathrm{I}}(t_2) + \ldots$$

- The two point correlation function is now given by:

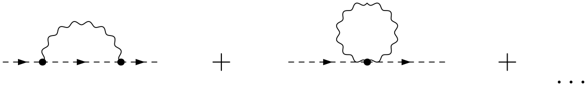
$$\langle \Omega | \mathrm{T}\{\Phi(x)\Phi(y)\} | \Omega \rangle = \lim_{\mathrm{T}\to\infty(1-i\epsilon)} \frac{\langle 0 | \mathrm{T}\left\{\Phi_{\mathrm{I}}(x)\Phi_{\mathrm{I}}(y)\ U(\mathrm{T}, -\mathrm{T})\right\} | 0 \rangle}{\langle 0 | \mathrm{T}\ U(\mathrm{T}, -\mathrm{T}) | 0 \rangle}$$

Physical mass

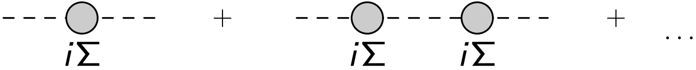
Feynman diagrammatic expansion of the perturbation

The propagator for the free field is given by: 

But there is interaction with the $U(1)$ gauge field:



To get the full (dressed) propagator, one has to resum all the contributions:



$$G(p^2) = \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2} i\Sigma \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2} i\Sigma \frac{i}{p^2 - m_0^2} i\Sigma \frac{i}{p^2 - m_0^2} + \dots$$

$$= \frac{i}{p^2 - m_0^2} \sum_{n=0}^{\text{inf}} \left(\frac{-\Sigma(p^2)}{p^2 - m_0^2} \right)^n = \frac{i}{p^2 - (m_0^2 - \Sigma(p^2))}$$

Physical mass and on-shell renormalization

The physical mass is given by the pole of the propagator:

$$0 = p^2 - (m_0^2 - \Sigma(p^2))$$
$$\Rightarrow m^2 = m_0^2 - \Sigma(m^2)$$

Case 1: m_0 is a free parameter

The value of m has to be matched to observables!

Case 2: m_0 is a dependent quantity (e.q. in the squark sector)

The value of m is predicted by the underlying theory!

e.g.:
$m_{\tilde{d}_L} = m_{\tilde{d}_L}^{tree}(m_{\tilde{u}_L}, m_{\tilde{u}_R}, m_{\tilde{d}_R}) + \Sigma(\text{all parameters off the theory})$

Corrections for dependent quantities is often mild but there are exceptions.
SUSY Higgs sector $\rightarrow$ Ananda's talk.

Electroweak sector in the SM

In the standard model, all masses that are free parameters in the SM Lagrangian can be renormalized on-shell, i.e the tree level value can be set to the experimental measured value.

Consider following parameters of the SM:

- W-boson mass: m_W
- Z-boson mass: m_Z
- Weak mixing angle: $\sin\theta_W$

One of these parameters is a dependent quantity, since there is the tree-level relation:

$$\cos\theta_W = \frac{m_W}{m_Z}$$

$\Rightarrow$ only two of these parameters can be renormalized on-shell!

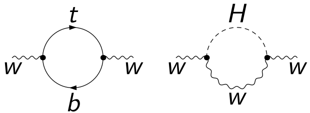
The dependent parameter is determined by the underlying theory!

$m_Z, \sin\theta_W \longrightarrow m_W$

$m_Z,\ m_W$	$\longrightarrow$	direct mass measurements (LEP)
$\sin\theta_W$	$\longrightarrow$	Kaon decay

PDG:

- $m_Z = 91.1876 \pm 0.0021$ GeV
- $m_W = 80.403 \pm 0.029$ GeV
- $\sin^2\theta_W = 0.23153 \pm 0.00016$



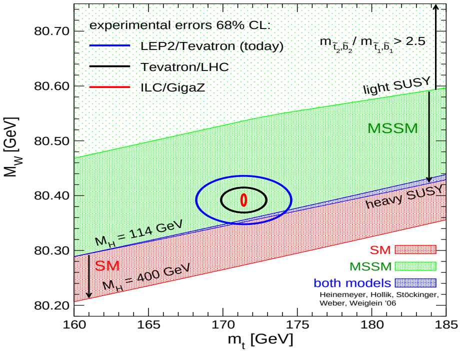
But $m_W^{tree} = m_z\sqrt{1-\sin^2\theta_W} = 79.937$ GeV

Not surprising, since m_W gets radiative corrections:

$$m_W^2 = m_W^{2\ tree} + \Sigma(m_Z, m_W, m_t, m_H, \dots)$$

- Σ leads to a finite shift of the W-mass.
- The precise value of the shift depends on the particle spectrum of the theory.

m_W: SM vs. SUSY



SUSY spectrum

- squark, slepton:
  $M_L,\ M_R = 100 \ldots 2000$ GeV
  $A^{t,b} = -2000 \ldots 2000$ GeV
- gauginos:
  $M_{1,2} = 100 \ldots 2000$ GeV
  $m_{\tilde{g}} = 195 \ldots 1500$ GeV
  $\mu = -2000 \ldots 2000$ GeV
- Higgs:
  $m_A = 90 \ldots 1000$ GeV
  $\tan_\beta = 1.2 \ldots 60$

Summary

- Supersymmetry $\rightarrow$ Unique extension of spacetime.
- Parameters in the Lagrangian have to be fixed by experiment (renormalized).
- Mass as dependent quantity: Value depends on the underlying theory.
- Prediction for m_W depends on the particle spectrum.
  $\Rightarrow$ SUSY slightly preferred.