Konzepte für Experimente an zukünftigen Hadroncollidern I

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Recapitulation of the previous lecture

Interaction of charged particles with matter

Two effects in the passage of charged particles through matter:

- Energy loss.
- Deflection from the original trajectory.

Processes causing energy loss and deflection

- Inelastic scattering off atomic electrons in the traversed material.
- Elastic scattering off the nuclei of the traversed material.
- Emission of Čerenkov radiation.
- Nuclear reactions.
- Bremsstrahlung.

The radiation field of an accelerated charge is proportional to its acceleration a_{charge}. The energy of the radiation is proportional to $|\vec{E}|^2$ which is proportional to $a_{charge}^2 = \left(\frac{F}{m}\right)^2 \propto \frac{1}{m^2}$. Hence bremsstrahlung is only important for electrons, but not for heavy charged particles.

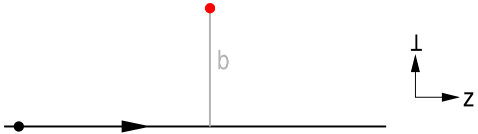
Recapitulation of the previous lecture

Interaction of heavy charged particles with matter

- Heavy charged particles: $\mu^{\pm}$, $\pi^{\pm}$, p, $\bar{p}$, α particles, light nuclei.
- Dominant processes for heavy charged particles:
  - Inelastic scattering off atomic electrons of the traversed material.
  - Elastic scattering off the nuclei of the material.

Inelastic scattering off atomic electrons

Semiclassical treatment



schweres geladenes
Teilchen mit der Masse M,
Ladung ze, Geschwindigkeit v

Momentum transferred to the electron:

$$\int_{-\infty}^{\infty} F_\perp dt = \int_{-\infty}^{\infty} e \cdot E_\perp dt = e \int_{-\infty}^{\infty} E_\perp \frac{dz}{v} = \frac{e}{v} \int_{-\infty}^{\infty} E_\perp dz \cdot \frac{2\pi b}{2\pi b} = \frac{e}{2\pi b v} \cdot 2\pi b \int_{-\infty}^{\infty} E_\perp dz$$

$$\Rightarrow \Delta p := \int_{-\infty}^{\infty} F_\perp dt = \frac{ze^2}{2\pi\epsilon_0 b v}$$

Recapitulation of the previous lecture

Inelastic scattering off atomic electrons

Energy gained by the electron:

$$\Delta E(b) = \frac{(\Delta p)^2}{2m_e} = \frac{z^2 e^4}{8\pi^2 \epsilon_0^2 m_e v^2 b^2}$$

N_e: Electron number density in the material.

$\Rightarrow$ Energy loss of electrons at distance between b and $b + db$ from the heavy particle in a thin layer dx:

$$-dE(b) = \Delta E(b) \cdot N_e \cdot 2\pi b \, db \, dx = \frac{z^2 e^4}{4\pi \epsilon_0^2 m_e v^2} \frac{1}{b} db \, dx$$

$$-\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} dE(b) = \frac{x^2 e^4}{4\pi \epsilon_0^2 m_e v^2} N_e \ln \frac{b_{max}}{b_{min}} = \frac{x^2 e^4}{8\pi \epsilon_0^2 m_e v^2} N_e \ln \frac{b_{max}^2}{b_{min}^2}$$

Inelastic scattering off atomic electrons

Energy loss of the heavy charged particle:

$$-\frac{dE}{dx} = \frac{x^2 e^4}{8\pi\epsilon_0^2 m_e v^2} N_e \ln \frac{b_{max}^2}{b_{min}^2}$$

- b_{min} can be computed from the largest possible energy transfer to the electron:

$$2\gamma^2 m_e v^2 = \frac{z^2 e^4}{8\pi^2\epsilon_0^2 m_e v^2 b_{min}^2} \Leftrightarrow b_{min}^2 = \frac{x^2 e^4}{16\pi^2\gamma^2 m_e^2 v^4 \epsilon_0^2}$$

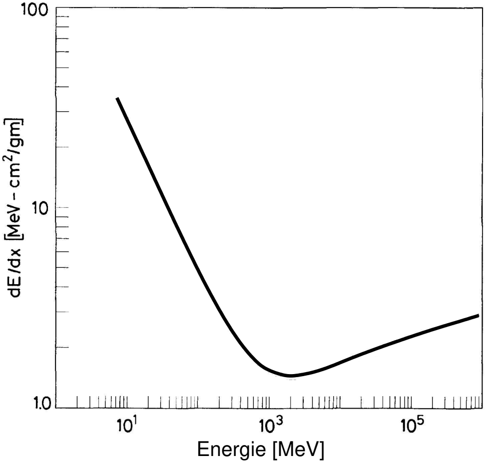
- b_{max} can be computed from the smallest allowed energy transfer following from the quantization of the electron's binding energy:

$$\Delta E_{min} = \frac{z^2 e^4}{8\pi^2\epsilon_0^2 m_e v^2 b_{max}^2} \Leftrightarrow b_{max}^2 = \frac{z^2 e^4}{8\pi^2\epsilon_0^2 m_e v^2} \frac{1}{\Delta E_{min}}$$

$\Rightarrow$ Bohr's approximation of the Bethe-Bloch formula:

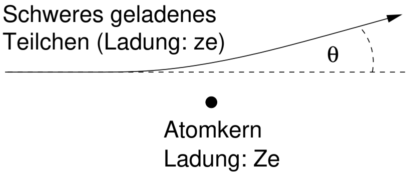
$$\boxed{-\frac{dE}{dx} = \frac{z^2 e^4}{8\pi\epsilon_0^2 m_e v^2} N_e \ln \frac{2 m_e \gamma^2 v^2}{\Delta E_{min}}}$$

Graphical illustration, minimum ionizing particles



- First rapid decrease of the energy loss with increasing energy of the charged particle.
- After a minimum weak, only logarthmic increase of the energy loss with increasing energy of the heavy charged particle.
- Particles with an energy for which the energy loss is minimum are called minimum ionizing particle.

Multiple scattering

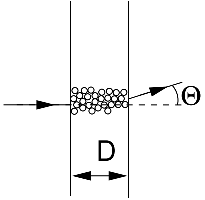


Scattering off a single nucleus:

$$\theta = \frac{\Delta p}{p} \propto \frac{z \cdot Z}{p}.$$

$$< \theta >= 0, 0 \neq \theta_0^2 := Var(\theta) \propto \frac{z^2 \cdot Z^2}{p^2}.$$

Scattering off many nuclei:



$$< \Theta > = 0$$

$$\Theta_0^2 := Var(\Theta) = \sum_{collisions} \theta_0^2 \propto D \cdot z^2 \cdot Z^2 p^{-2}.$$

Hence $\boxed{\Theta_0 \propto \frac{\sqrt{D}}{p}.}$

The exact treatment gives

$$\theta_0 := \frac{13,6 \text{ MeV}}{E} \sqrt{\frac{d}{X_0}},$$

with the radiation length X_0 of the material.

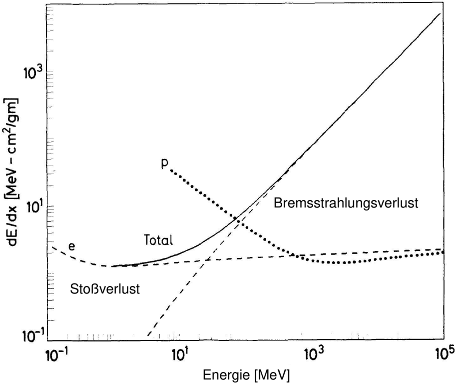
Energy loss of electrons (and positrons)

m_e is so small that the acceleration that an electron experiences in collisions with the atomic nuclei becomes so large that bremsquanta can be emitted.

$$\left.\frac{dE}{dx}\right|_e = \left.\frac{dE}{dx}\right|_{collision} + \left.\frac{dE}{dx}\right|_{bremsstrahlung}.$$

- $\left.\frac{dE}{dx}\right|_{Collision}$ denotes the energy loss due to excitation and ionization of atoms. The corresponding formula is similar to the Bethe-Bloch formula, but differs in details because
  - the electrons are deflected when scattering off atomic electrons,
  - and the impinging electron is indistinguashable from the atomic electron.
- $\left.\frac{dE}{dx}\right|_{Bremsstrahlung}$ denotes the energy loss via bremsstrahlung.

Critical energy and radiation length



Critical energy E_k

$$\left.\frac{dE}{dx}\right|_{collisions}(E_k) = \left.\frac{dE}{dx}\right|_{Bremsstrahlung}(E_k).$$

$E_k \approx \frac{800\ \text{MeV}}{Z+1/2}$ so bremsstrahlung is the dominant process for $E_{e^\pm} > 1$ GeV.

Radiation length X_0

$$-\left.\frac{dE}{dx}\right|_{bremsstrahlung} = N \cdot E_e \cdot \Phi_{radiation},$$

hence

$$E_e(x) = E_e(0)e^{\frac{-x}{X_0}}.$$

Interaction of photons with matter

Dominant processes

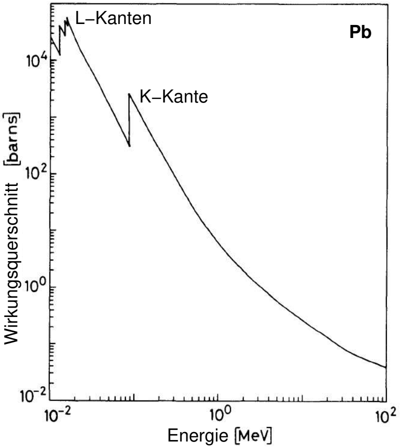
1. Photoelectric effect.
2. Compton scattering
3. e^+e^- pair production

$\Rightarrow$ A beams of photons does not lose energy when passing through matter, but intensity because all three processes remove photons from the beam.

Photoelectric effect

Absorption of a photon by an atomic electron

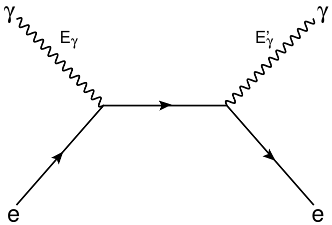
$$E_e = \hbar\omega_\gamma - \text{Bindungsenergie des Elektrons}$$

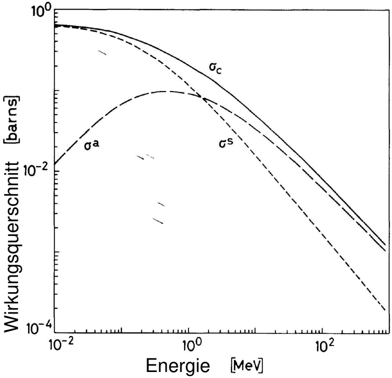


- Cross section decreasing with increasing photon energy.
- Peaks in the cross section when the photon energy reaches the binding energy of the electrons in an atomic shell.
- Process important for $E_\gamma \sim 10 - 100$ keV.

The process is forbidden for free electrons due to energy-momentum conservation. Consider a free electron at rest:

$$m_e^2 + 2E_\gamma m_e = (p_\gamma + p_{e,A})^2 = p_{e,E}^2 = m_e^2 \;\Rightarrow\; E_\gamma = 0.$$



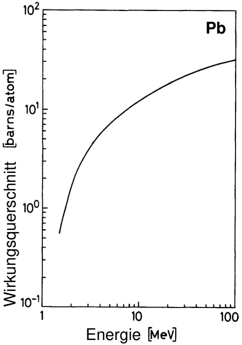


- Scattering of a photon off an electron.

- Compton scattering cross section described by the Klein-Nishina formula.
- σ_C: Compton scattering cross section.
- $\sigma_a := \sigma_C \frac{E'_\gamma}{E_\gamma}$, $\sigma_s := \sigma_C - \sigma_A$.
- Large energy transfer to the electron at $E_\gamma \sim 1$ MeV.

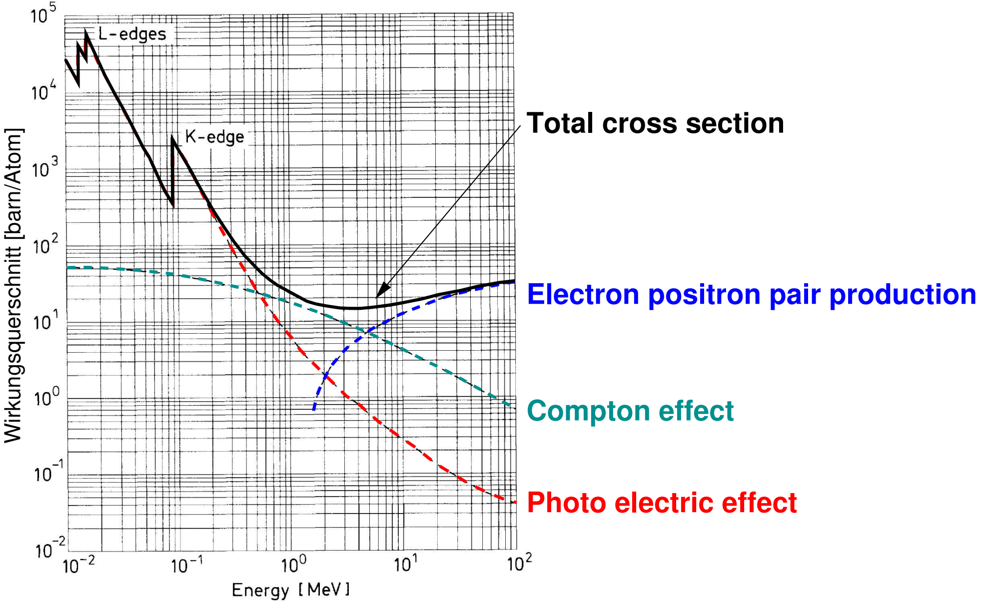
e^+e^- pair production

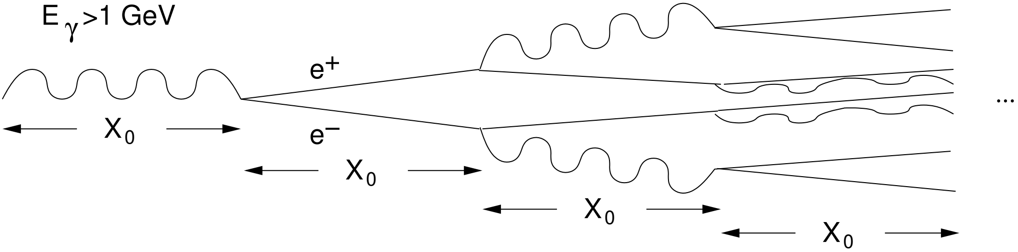
- $\gamma \rightarrow e^+e^-$ only possible when a third body, i.e. a nucleus participate in the process due to energy-momentum conservation ($0 = p_\gamma^2 \neq (p_{e^+} + p_{e^-})^2 > 0$).
- Cross section for pair production $\propto Z^2$ (Z: atomic number of the material).
- $E_{\gamma,min} = 2m_e$
- Probability for pair production after a distance x is proportional to $\exp(-\frac{x}{\lambda_P})$ with $\lambda_P \approx \frac{9}{7}X_0$.



- Cross section increasing with increasing E_γ.
- Dominant process for $E_\gamma \gtrsim 10$ MeV.

Total photon absorption cross section for Pb





- After a distance $n \cdot X_0$: 2^n particles with energy $E_n \approx \frac{E_\gamma}{2^n}$.
- End of the cascade (shower), if $E_n = E_k$: $n = \frac{\ln \frac{E_\gamma}{E_k}}{\ln 2}$.
- Shower length: $n \cdot X_0 = X_0 \cdot \frac{\ln \frac{E_\gamma}{E_k}}{\ln 2}$.

Example

- $E_\gamma = 100$ GeV.
- Material: iron, d.h. $X_0 \approx 2$ cm, $E_k \approx 20$ MeV.

$\Rightarrow$ $n = 12$, d.h. ~ 4000 particles.
Shower length: $L_{longitudinal} \approx 24$ cm.

Kinematics in the approximation of massless particles

Initial state

$$p_i = (E_i, \underbrace{0, 0, E_i}_{\vec{p}_i})$$

Final state

$$p_1/2 = (E_1/2, p_{1/2,\perp}, 0, p_{1/2,\parallel})$$

$$p_{1/2}^2 = 0 \;\Rightarrow\; E_{1/2} = \sqrt{p_{1/2,\perp}^2 + p_{1/2,\parallel}^2}$$

$$p_i = p_1 + p_2$$

Transverse size of an electron photon shower

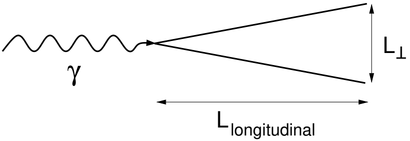
Kinematics in the approximation of massless particles

Hence

$$
\begin{aligned}
p_{1,\perp} + p_{2,\perp} = 0 \quad &\Leftrightarrow \quad p_{2,\perp} = -p_{1,\perp} \\
E_i = p_{1,\parallel} + p_{2,\parallel} \quad &\Leftrightarrow \quad p_{2,\parallel} = E_i - p_{1,\parallel} \\
E_i = E_1 + E_2 \quad &\Leftrightarrow \quad E_i - E_1 = E_2 = \sqrt{p_{2,\parallel}^2 + p_{2,\perp}^2} = \sqrt{(E_i - p_{1,\parallel})^2 + p_{1,\parallel}^2} \\
(E_i - E_1)^2 \quad &= \quad (E_i - p_{1,\parallel})^2 + p_{1,\perp}^2 \\
E_i^2 - 2E_iE_1 + E_1^2 \quad &= \quad E_i^2 - 2E_ip_{1,\parallel} + p_{1,\parallel}^2 + p_{1,\perp}^2 = E_i^2 - 2E_ip_{1,\parallel} + E_1^2 \\
E_1 = p_{1,\parallel} \quad &\Rightarrow \quad p_{1,\perp} = 0 = p_{2,\perp}
\end{aligned}
$$

The transverse size of the shower is 0 independently of $_{\gamma/e^{\pm}}$ in the limiting case of massless particles.

The full treatment with massive electrons and positrons leads to the following result.



$$L_{\perp} \approx 4R_M = 4X_0 \frac{21,2 \text{ MeV}}{E_k}$$

R_M: Molière radius

- The transvers size of the shower $L_{\perp}$ is independent of $E_{\gamma/e^{\pm}}$.
- $L_{T,Fe} = 4 \cdot 1,8$ cm $\cdot \frac{21{,}2\text{MeV}}{30{,}2\text{MeV}} \approx 5$ cm.
- Characteristic for electromagnetic showers: small transverse size which is independent of $E_{\gamma,e^{\pm}}$.
- The number of generated particles is a measure for $E_{\gamma,e^{\pm}}$ and proportional to $E_{\gamma,e^{\pm}}$.