The Anomalous Magnetic Moment of the Muon

Providing an Explanation using a Visible QCD Axion

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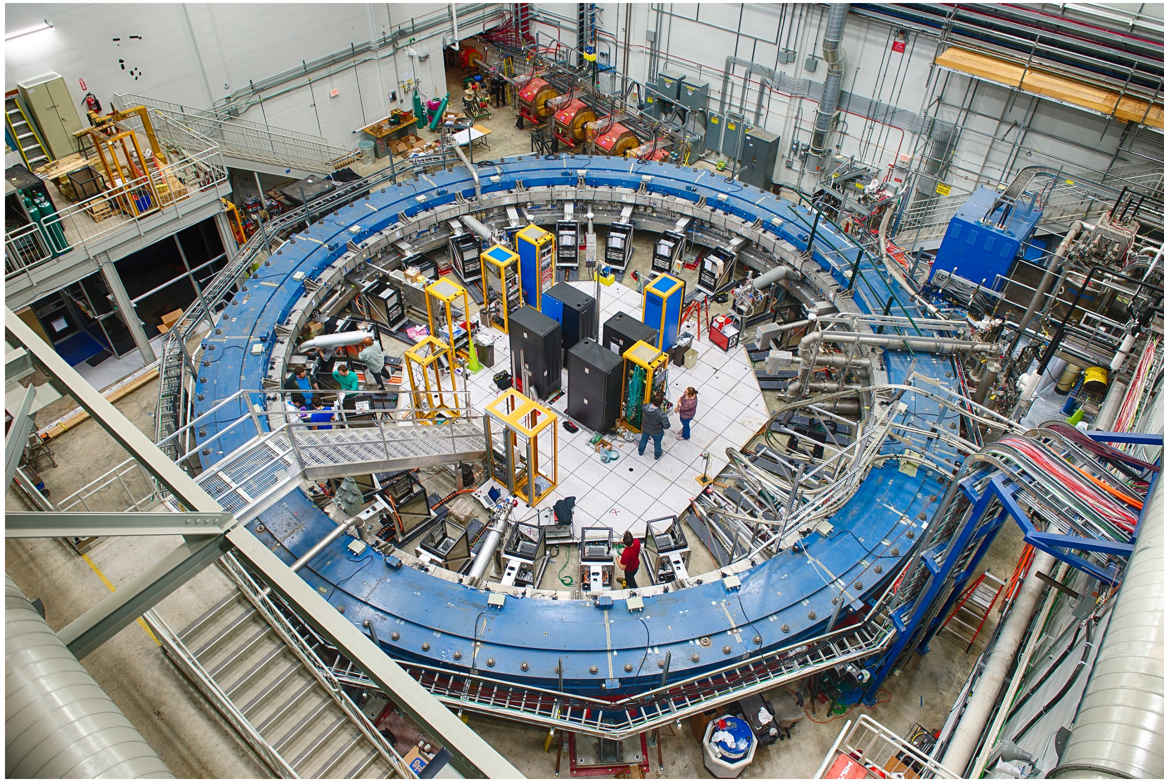
Master’s Thesis with U. Nierste, T. Schwetz-Mangold, R. Ziegler at KIT

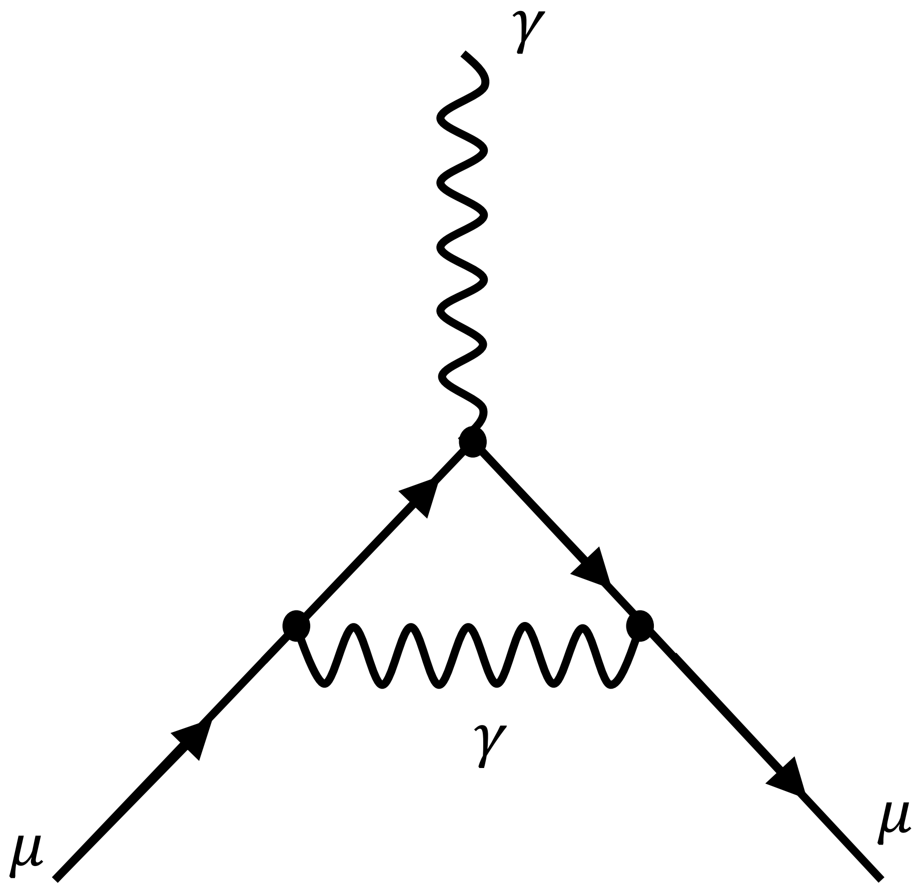
Content

(1) Experiments at BNL and FNAL

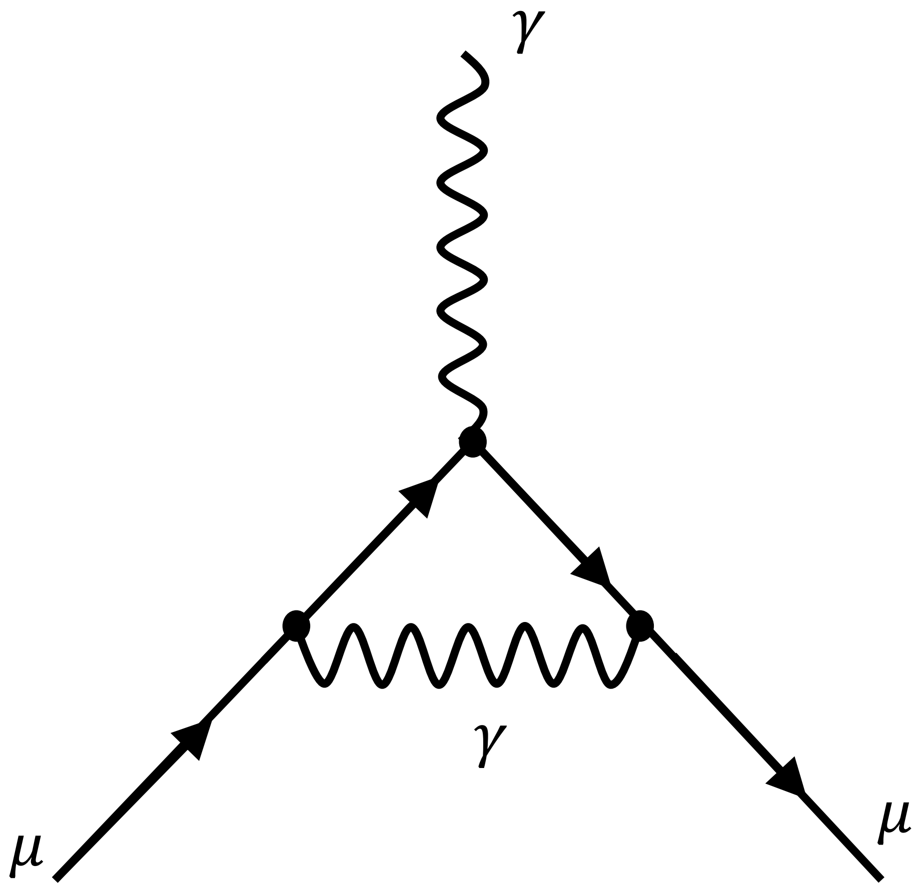
- Magnetic moment $\vec{\mu}$ of the muon
  - $\vec{\mu} = -g \frac{q}{2m} \vec{L}$
- Measured at Brookhaven National Laboratory (BNL) and Fermi National Accelerator Laboratoy (FNAL)
- $a_\mu = \left(\frac{g-2}{2}\right)_\mu$



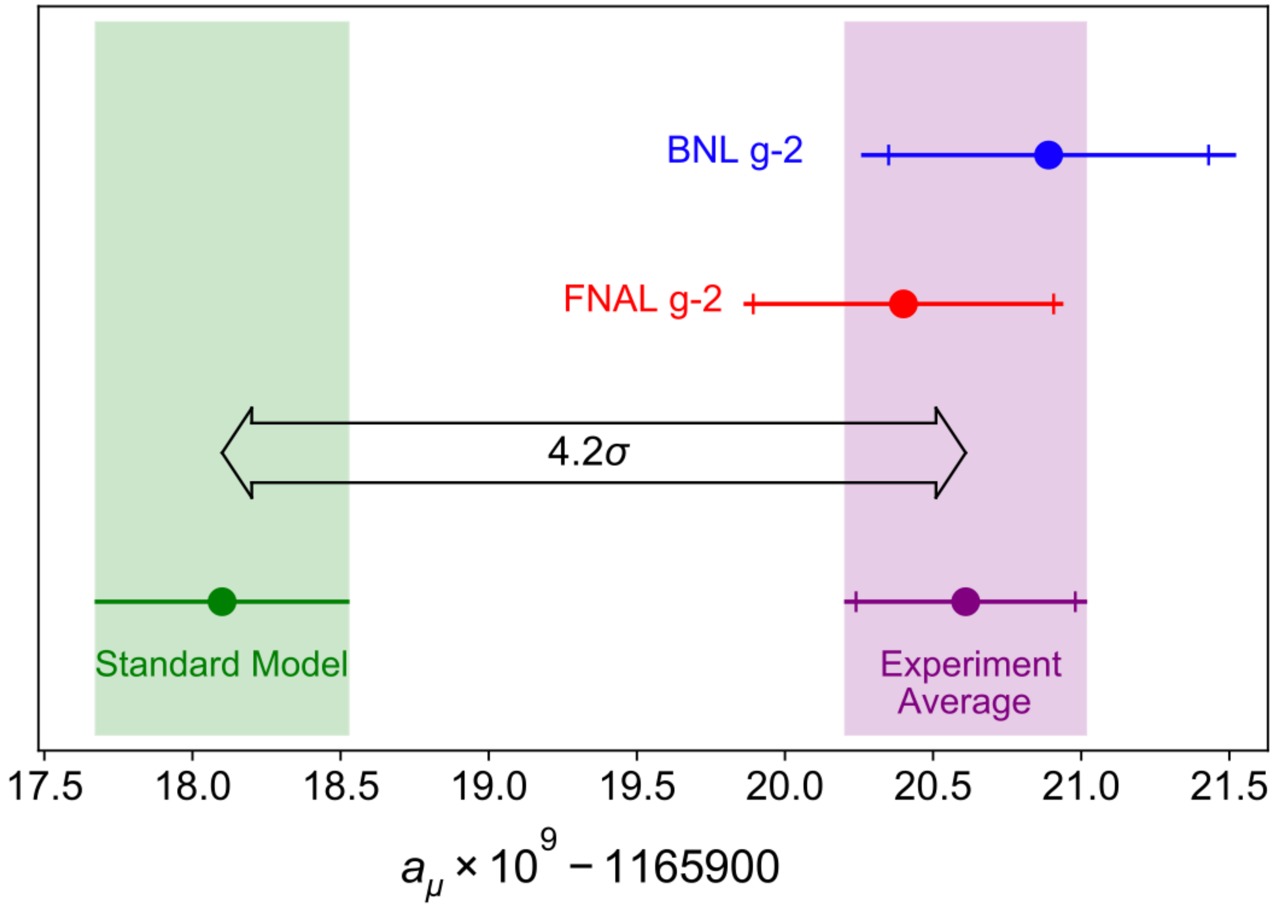
(2) Theoretical Calculation of $(g-2)_\mu$



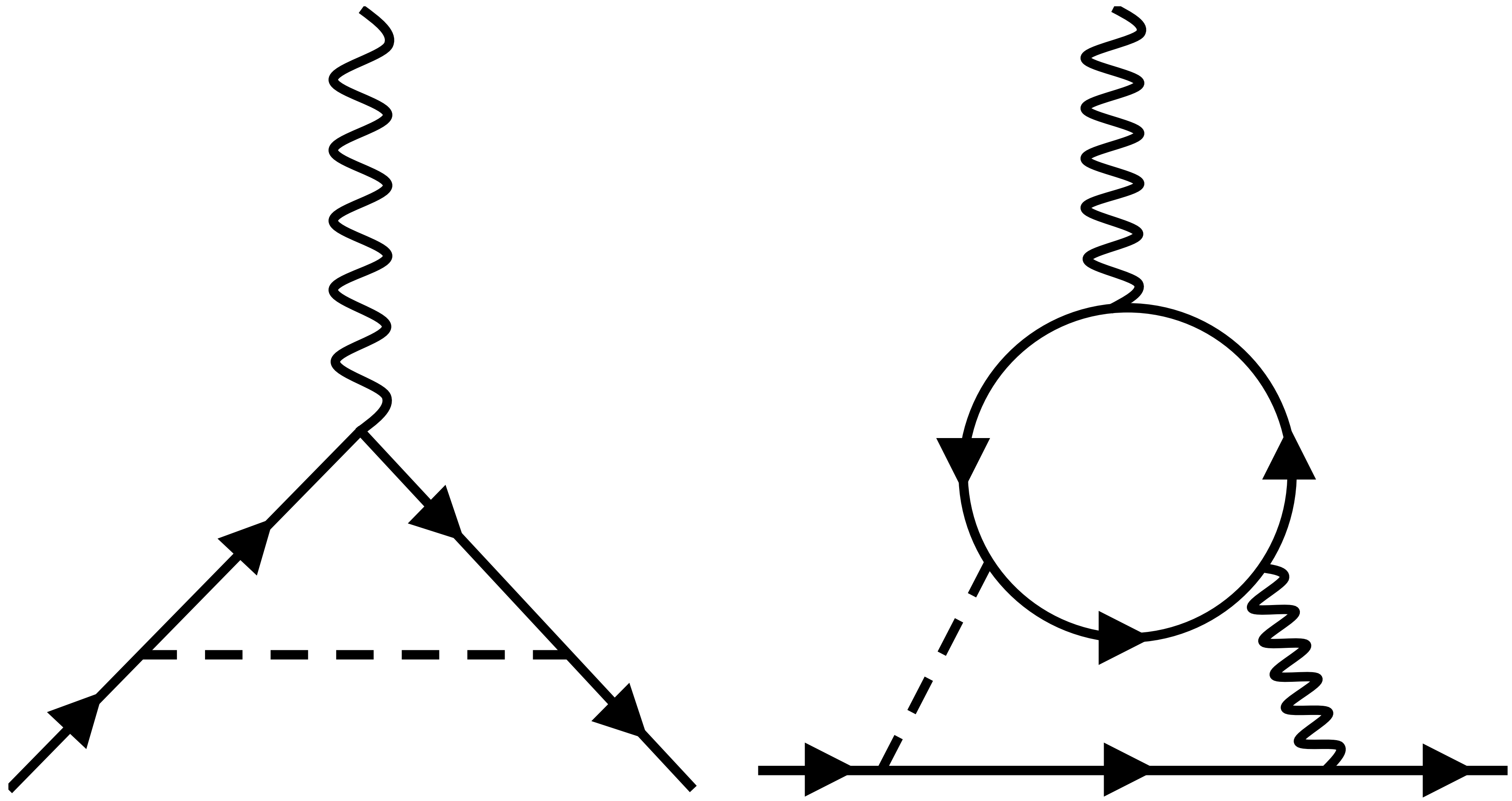
(2) Theoretical Calculation of $(g-2)_\mu$



B. Abi et al.: arXiv:2104.03281



(3) Explaining the Anomaly



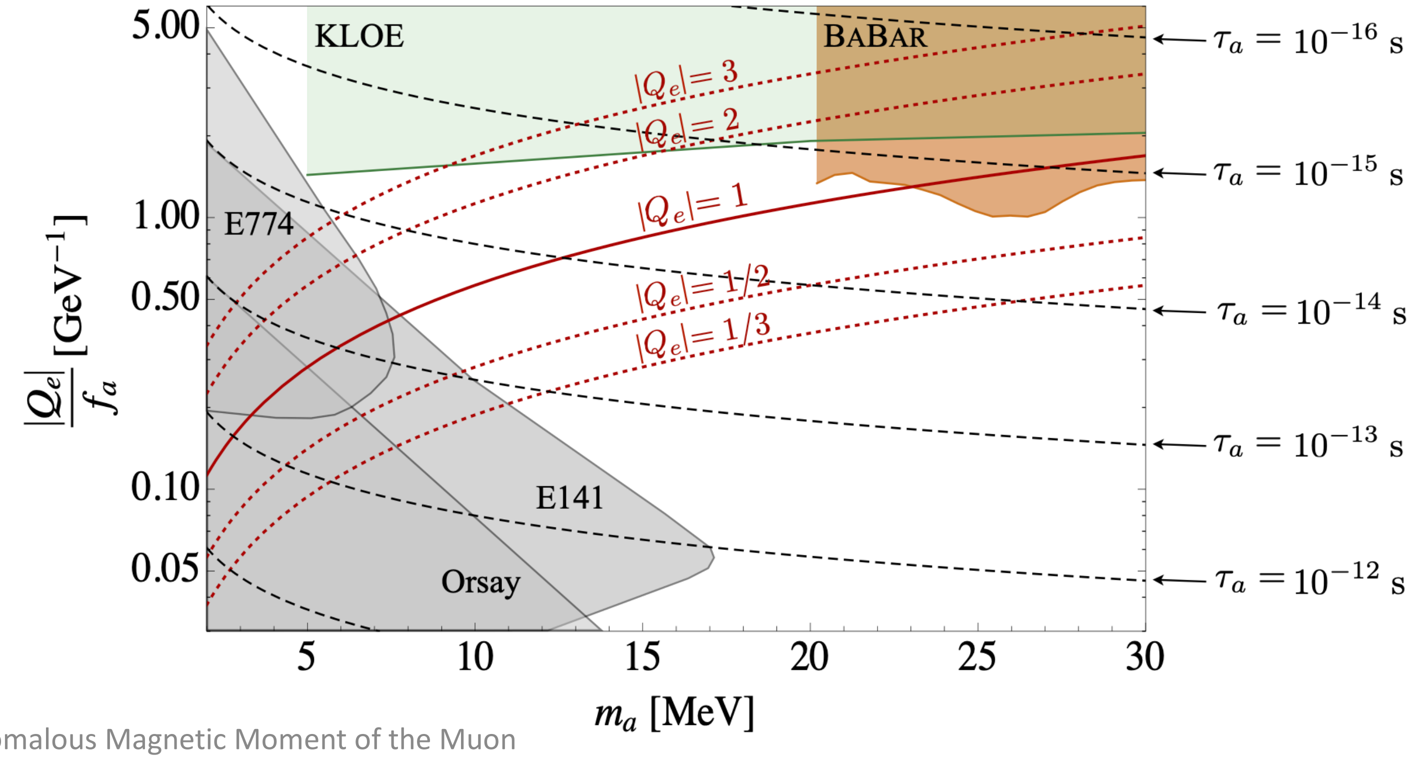
(4) The QCD Axion

- Pseudoscalar particle
- Coupling to fermions, effective coupling to photons
- Broken $U(1)$ symmetry to solve the “strong CP puzzle” (R. Peccei, H. Quinn)

$$\mathcal{L}_{QCD} \supset \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a,\mu\nu} \, , \bar{\theta} < 10^{-10}$$

(4) The Visible QCD Axion

- Decays (promptly) into SM fermions ("visible")
- D. Alves, N. Weiner (arXiv:1710.03764):
  - $m_a \sim \mathcal{O}(\mathrm{MeV})$
  - Couples to first generation
    - "Pionphobic"
    - "Nucleophobic"
  - Avoids constraints
    - Beam-dump experiments
    - Collider experiments



(4) Goals of the new Model

- Address $(g-2)_\mu$
  - $\Delta a_\mu = (25.1 \pm 5.9) \cdot 10^{-10}$ (4,2σ)
  - Use 1-loop and 2-loop contributions
  - Fulfil constraints by beam-dump and collider experiments
- Address strong CP puzzle
  - Identify pseudoscalar mediator a as visible QCD axion
  - Pionphobic, nucleophobic, promptly decaying, …

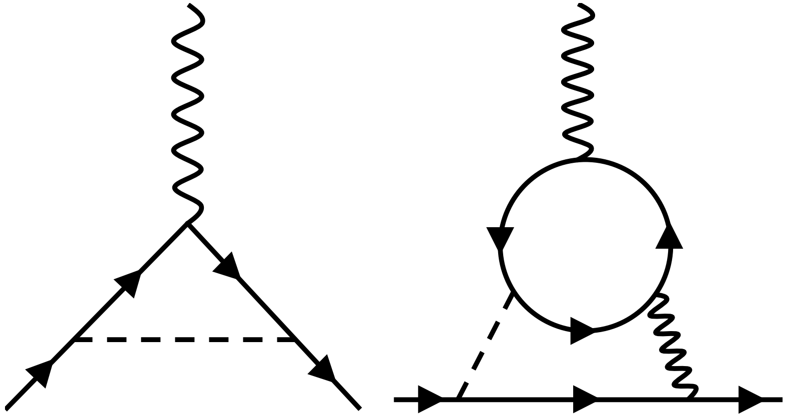
(5) The Model

$$\mathcal{L}_{\text{eff}} = \frac{\partial^\mu a}{2f_a} c_i \bar{\psi}_i \gamma_\mu \gamma_5 \psi_i$$

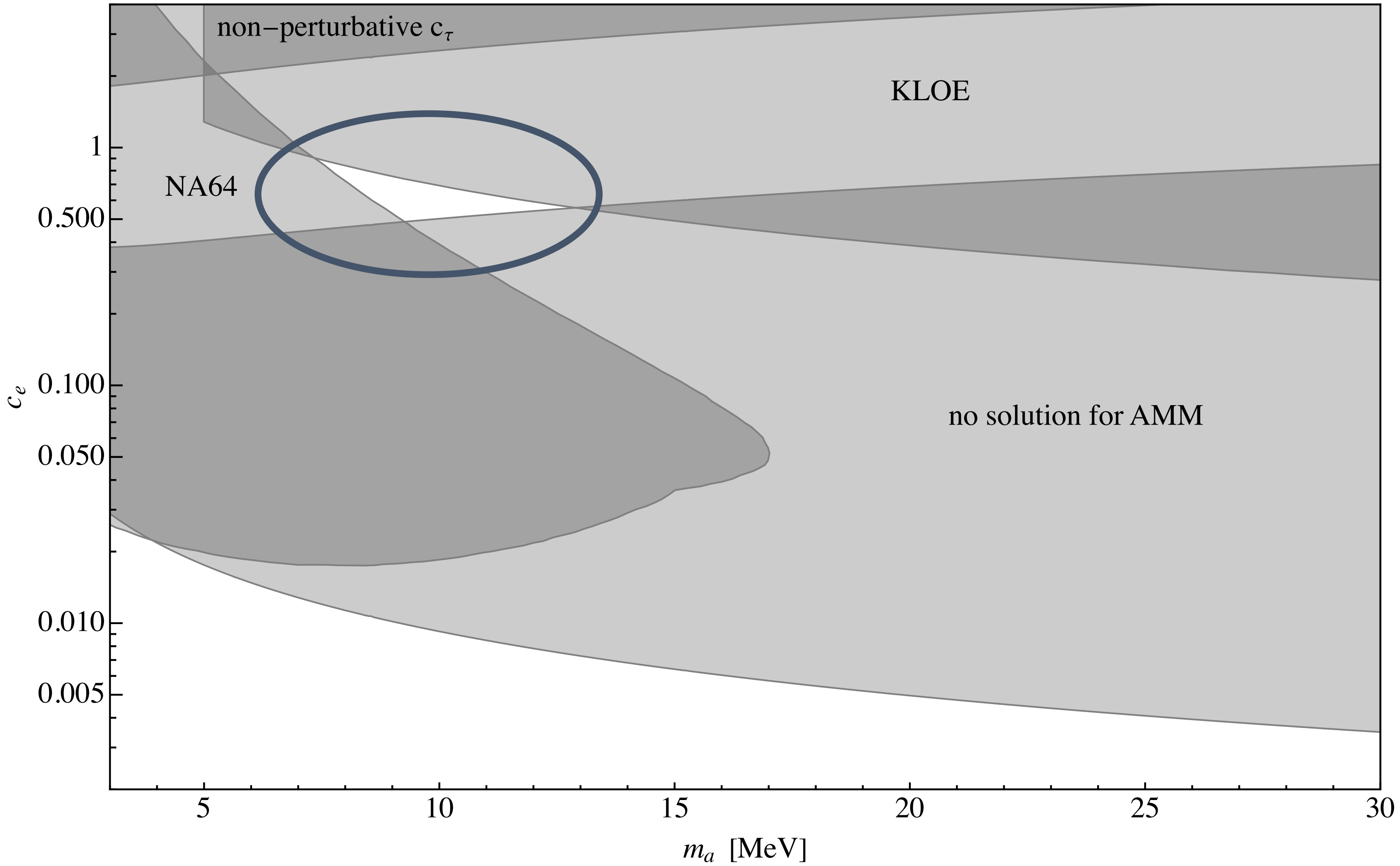
- $c_u = \frac{2}{3}$
- $c_d = \frac{1}{3}$

} pionphobic, nucleophobic

- $c_s = c_c = c_b = c_t = 0$
- Hierarchy: $c_e \simeq c_\tau \gg c_\mu$



(5) The Model



non−perturbative c_τ
KLOE
NA64
no solution for AMM
1
0.500
0.100
0.050
0.010
0.005
c_e
5
10
15
20
25
30
m_a [MeV]

(6) Conclusion

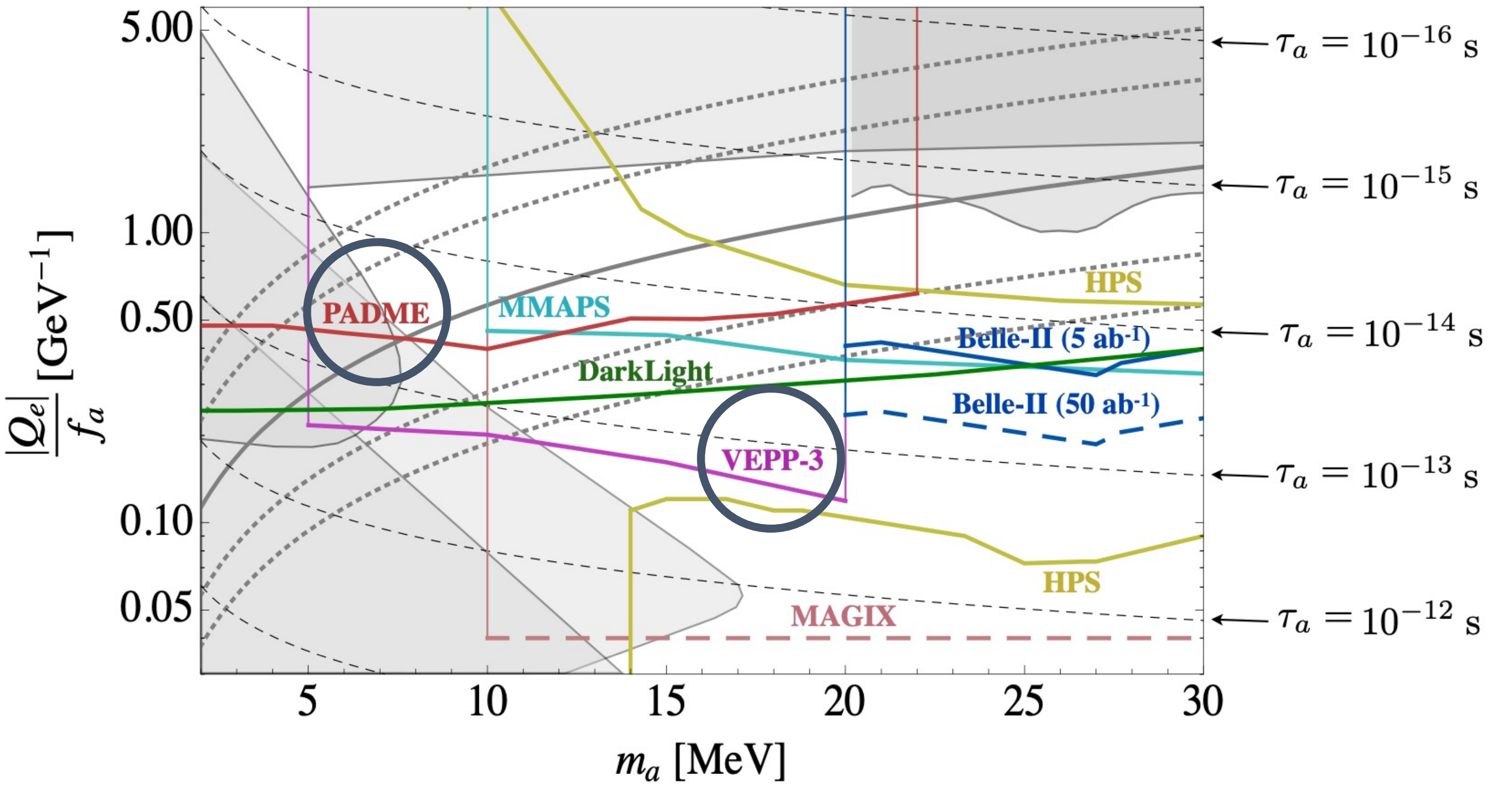
- Complete model addressing the strong CP puzzle and $(g-2)_\mu$
- Able to address further issues (XENON1T excess, $(g-2)_e$)
- Fulfils given constraints (pionphobic, beam-dump experiments, …)
- Future experiments will investigate interesting region (PADME, VEPP-3, …)

- Proof of concept

Backup

Backup: Outlook

D. Alves et al.: arXiv:1710.03764



Backup: Benchmark Model

$$m_a = 5.7\,\mu\text{eV}\left(\frac{10^{12}\text{ GeV}}{f_a}\right)$$

$$f_a = \frac{v_{\text{PQ}}}{2N} \in [0.66, 0.75] \text{ GeV}$$
$$c_c = c_s = c_b = c_t \approx 2.5 \cdot 10^{-5}$$

Prompt decay to e^+e^- : $\tau \approx 7 \cdot 10^{-15}$ s

BR$(a \to \gamma\gamma) \approx 4 \cdot 10^{-5}$

Decay of SM Higgs boson:

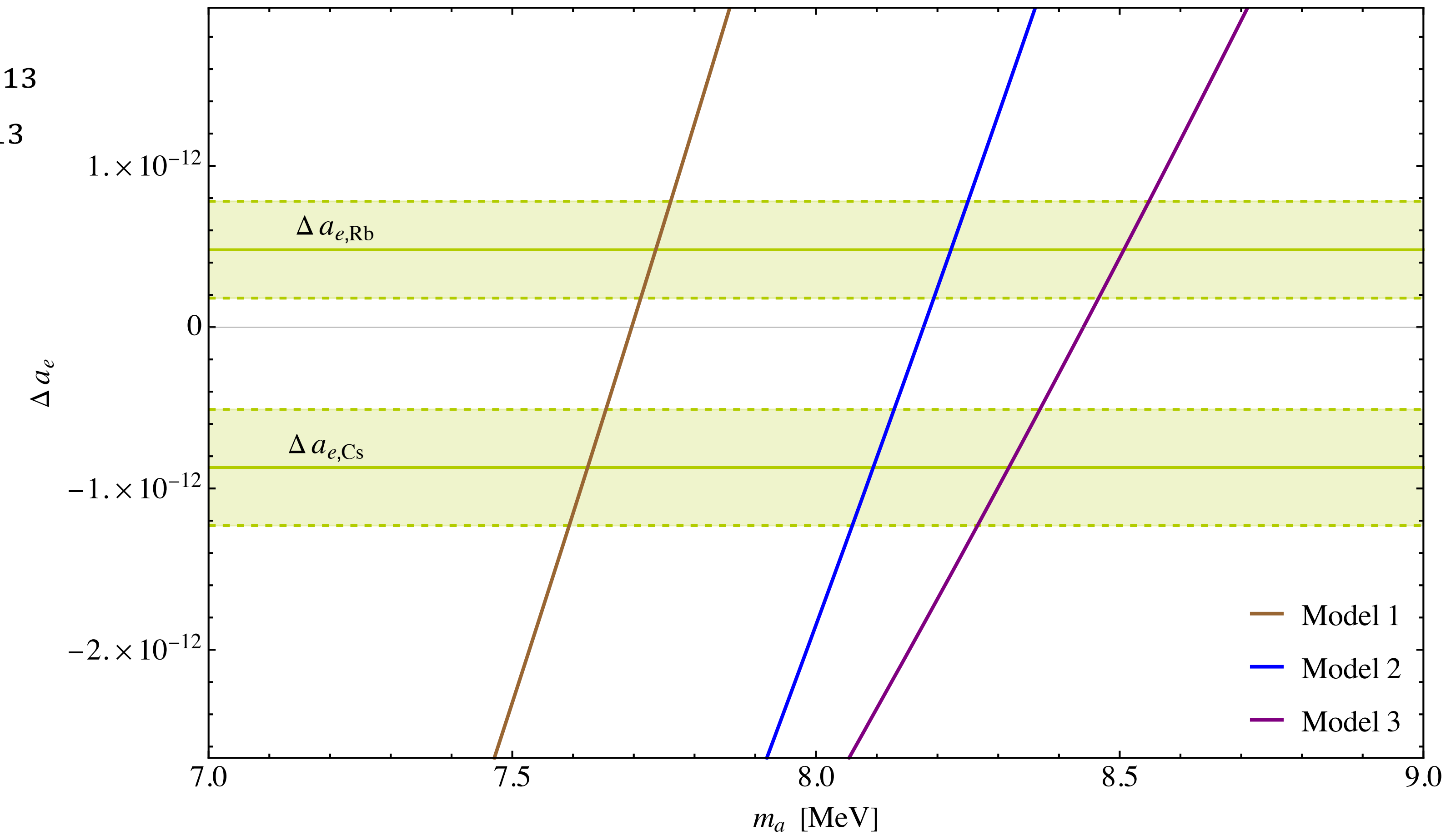
BR$(h_{\text{SM}} \to aa) \approx 10^{-2}$

Backup: Axion Lagrangian

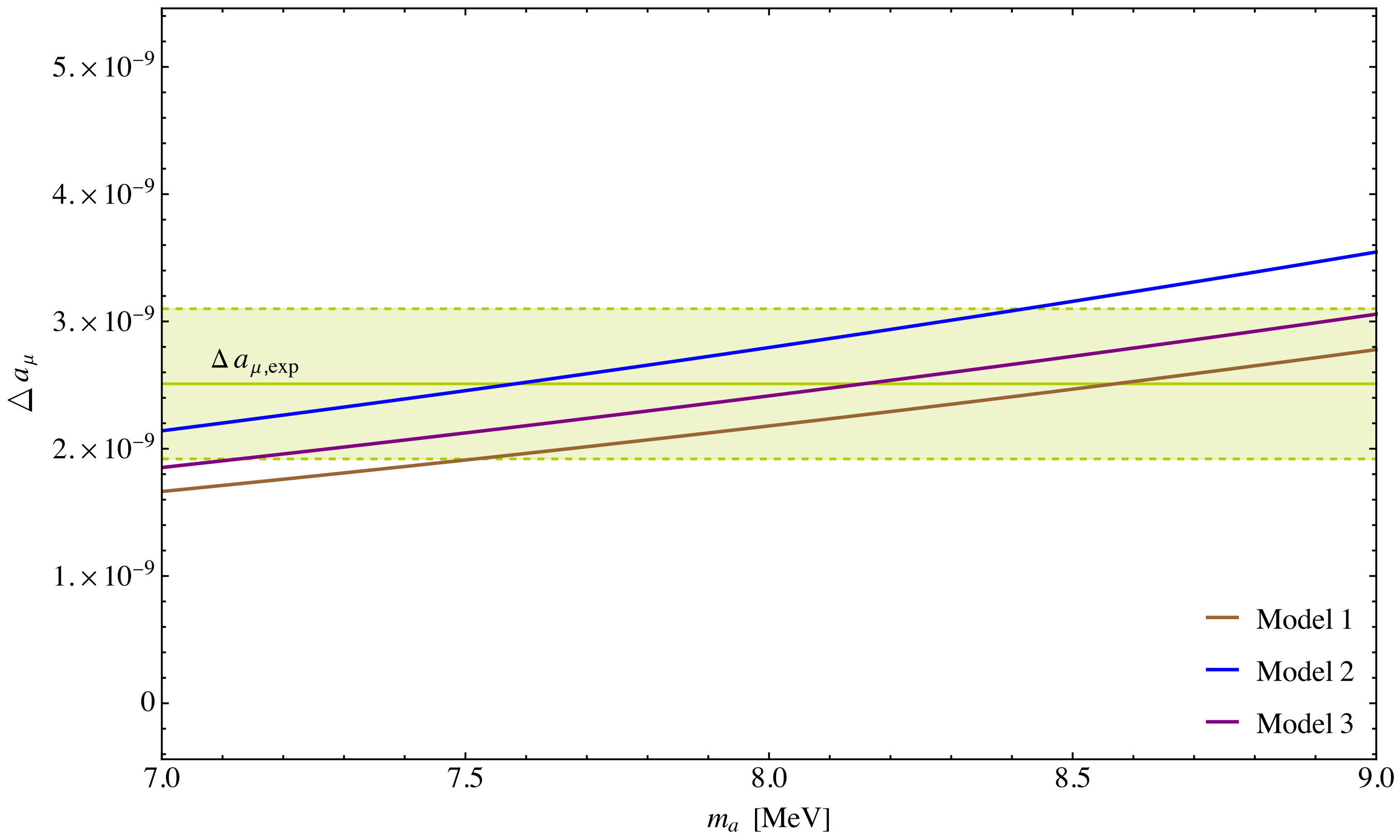
$$\mathcal{L} \supset \frac{a}{f_a}\frac{g_S^2}{32\pi^2}\mathrm{G}_{\mu\nu}^A \tilde{G}^{A,\mu\nu} + \frac{a}{f_a}\frac{e^2}{32\pi^2}\frac{E}{N}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a}c_i\overline{\psi_i}\gamma^\mu\gamma_5\psi_i$$

Backup: Reproducing Δa_e

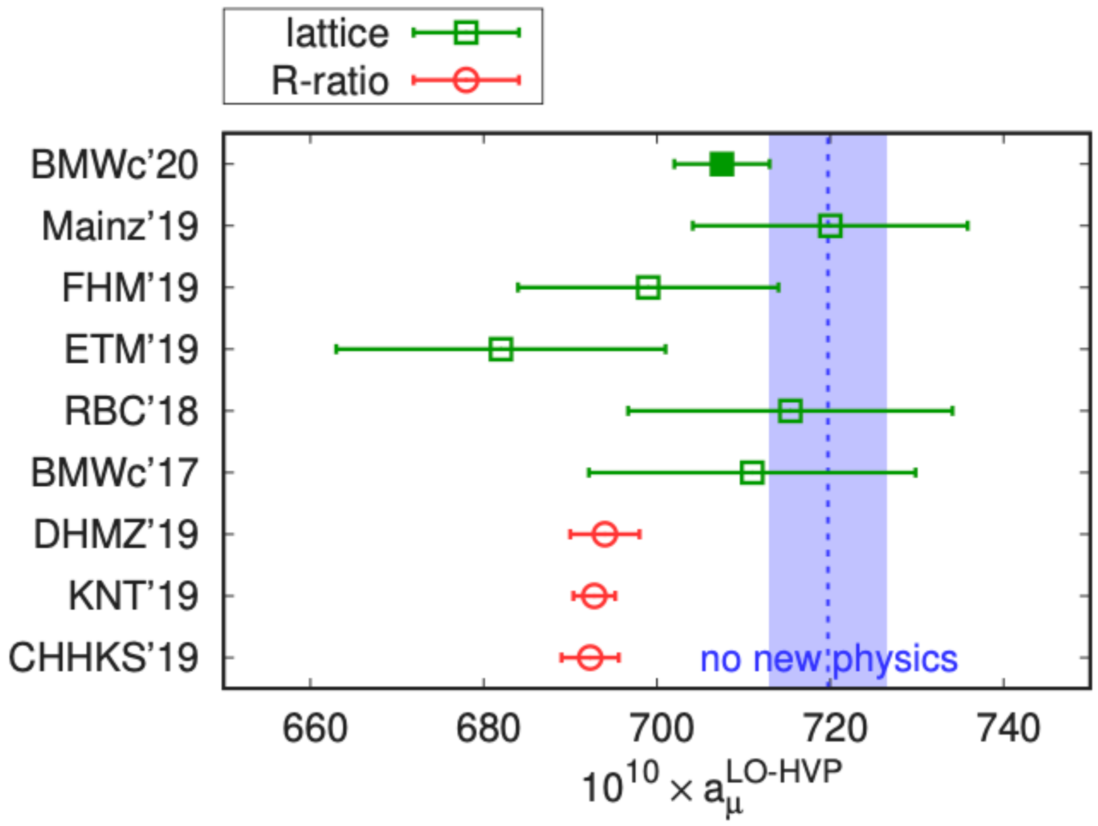
$$\Delta a_{e,\mathrm{Cs}} = (-8.8 \pm 3.6) \cdot 10^{-13}$$
$$\Delta a_{e,\mathrm{Rb}} = (4.8 \pm 3.0) \cdot 10^{-13}$$



Backup: Reproducing Δa_μ

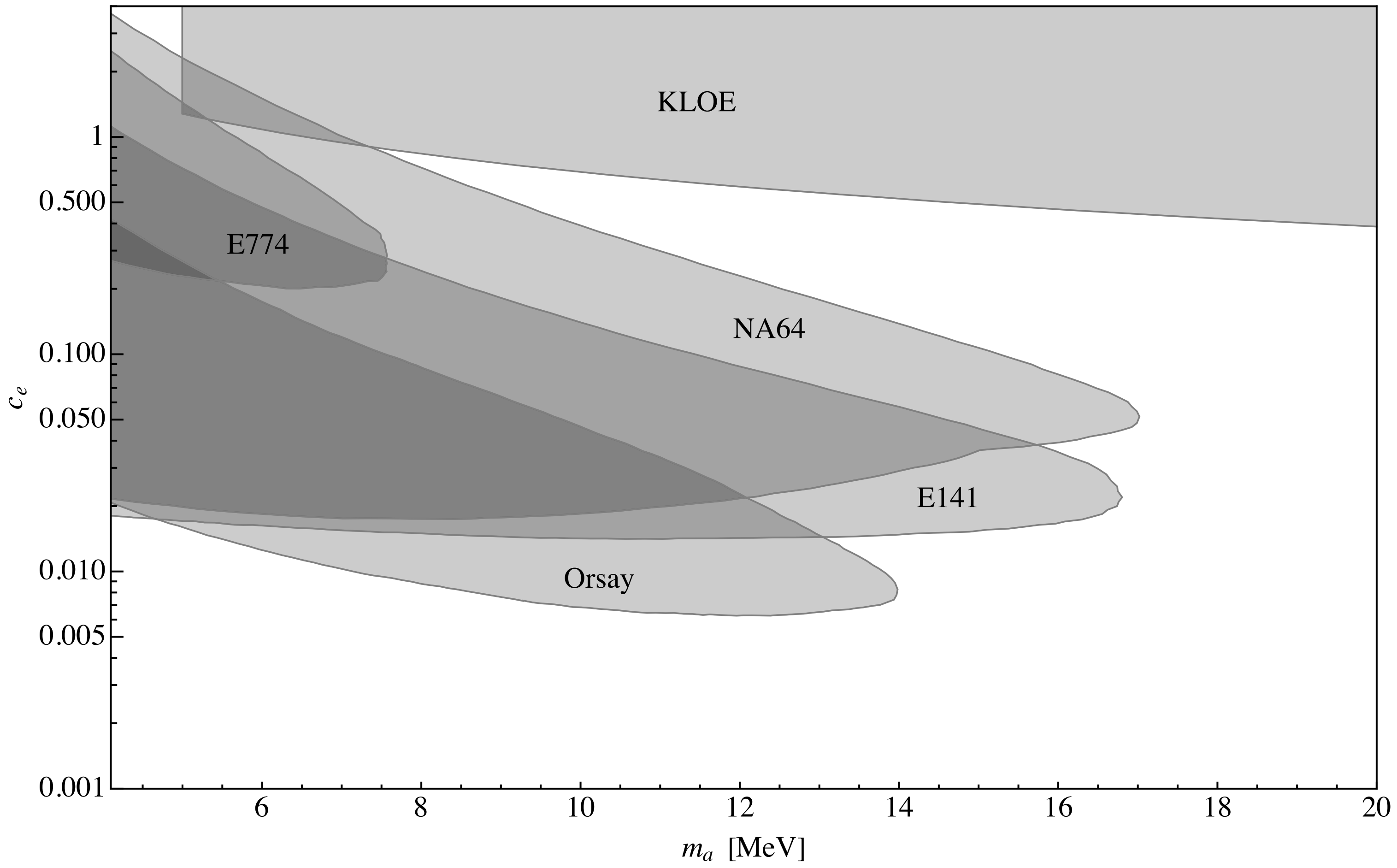


Backup: Lattice-QCD

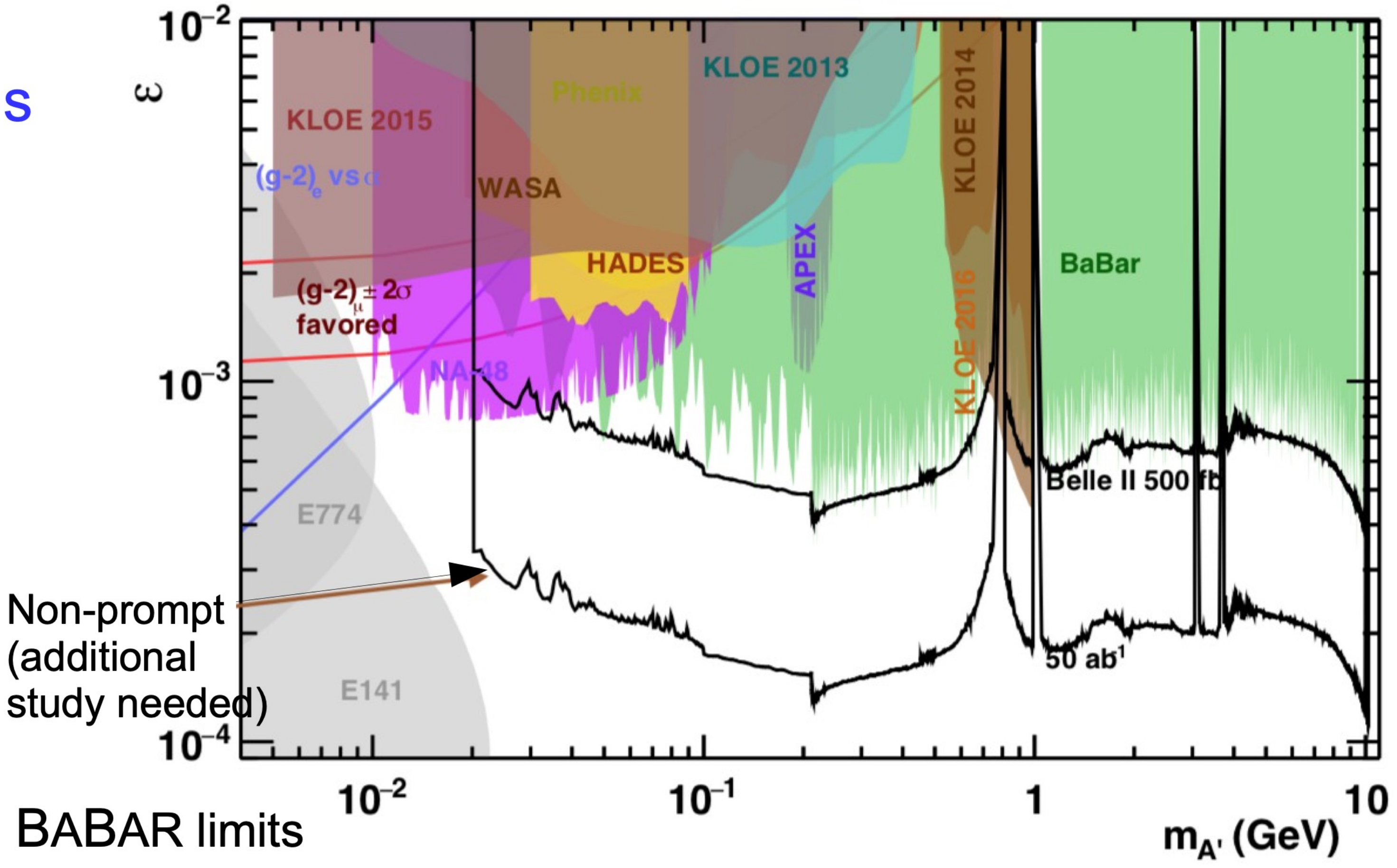


arXiv:2002.12347

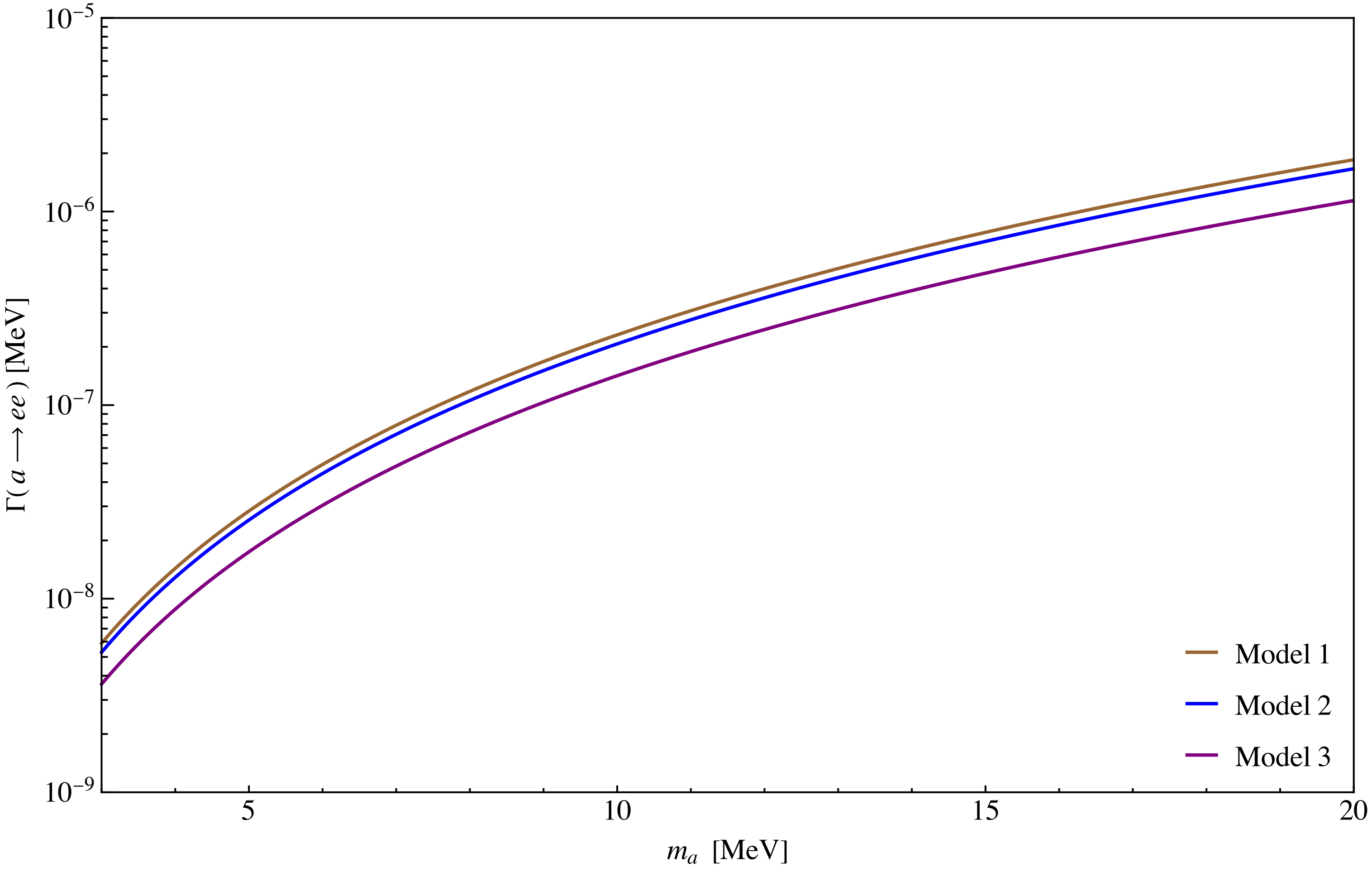
Backup: Experimental constraints



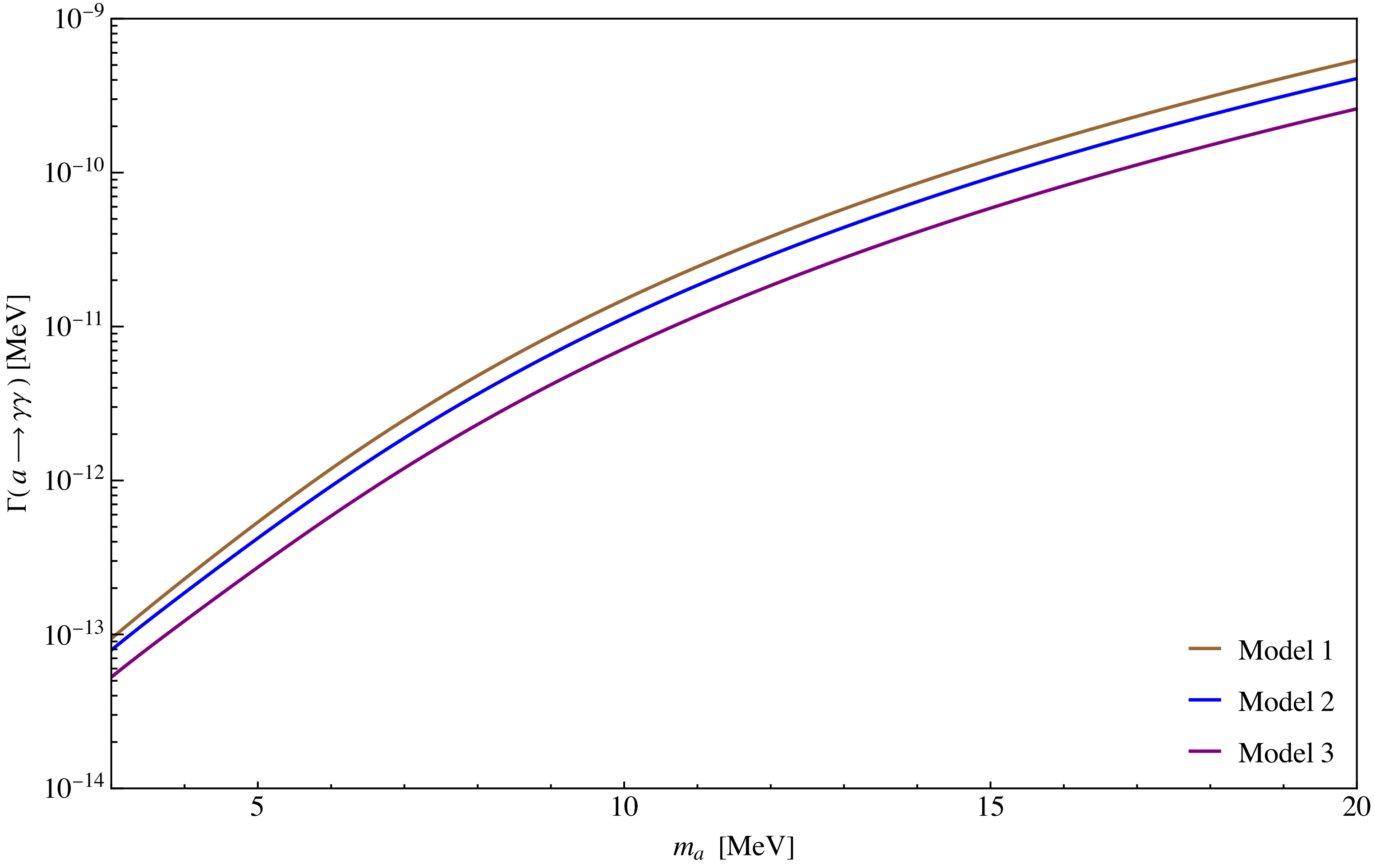
Backup: BaBar, Belle2



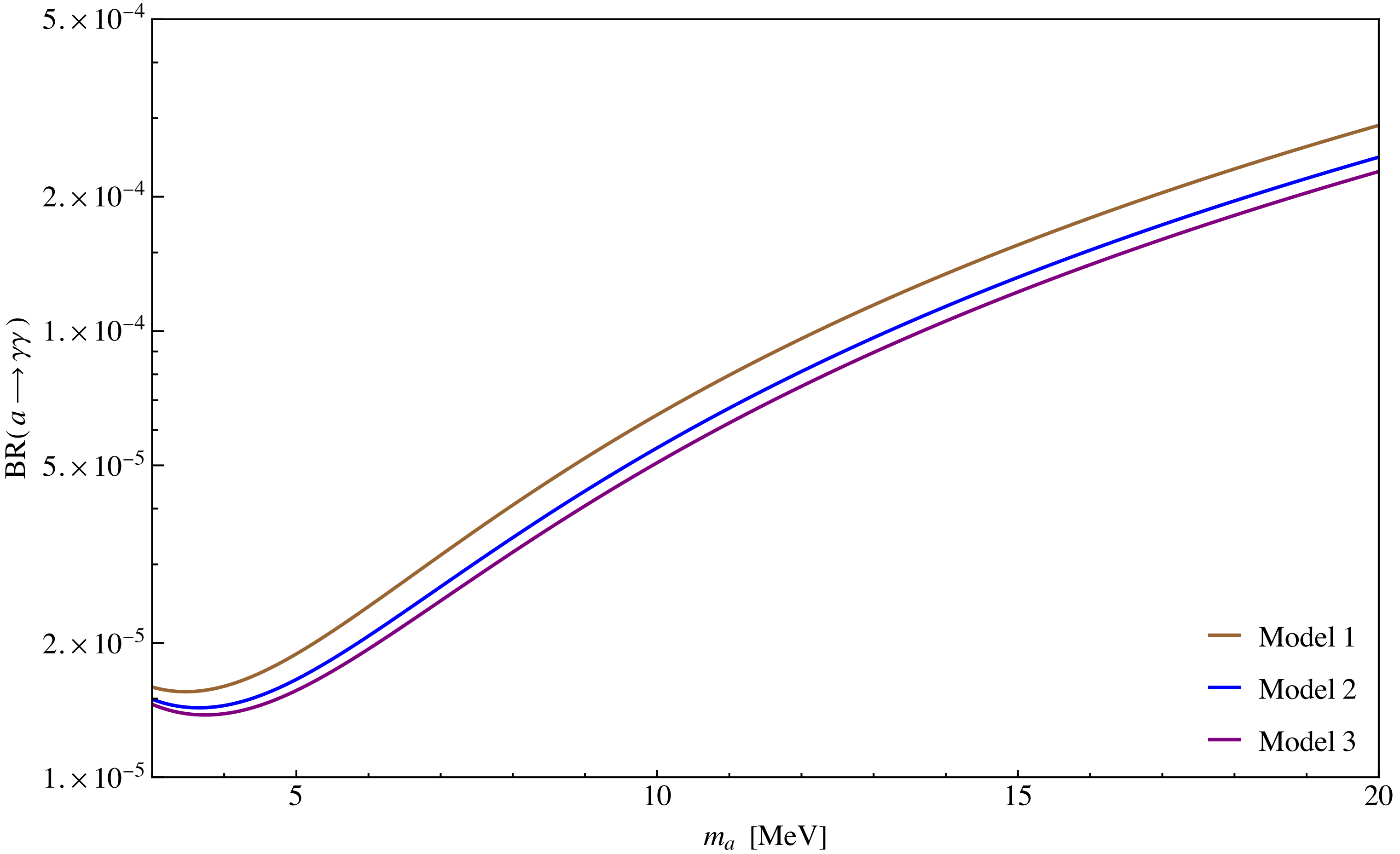
Backup: Axion decay into electrons



Backup: Axion decay into photons



Backup: Branching ratio axion into photons



Backup: Calculation

1-loop contribution:

$$\Delta a_i^1 = -\frac{m_i^2}{16\pi^2 f_a^2} |c_i|^2 f_1\left(\frac{m_a^2}{m_i^2}\right)$$

2-loop contribution (exact, $j \in \{s, c, b, t, e, \mu, \tau\}$):

R. Ziegler et al: arXiv:2011.08919

$$\Delta a_i^{2,j} = N_j \frac{m_i^2 \alpha}{8\pi^3 f_a^2} c_i c_j f_3\left(\frac{m_a^2}{m_i^2}, \frac{m_a^2}{m_j^2}\right)$$

2-loop contribution (effective, $\{u, d\}$):

M. Neubert et al.: arXiv:1708.00443

$$\Delta a_i^{2,u/d} = \frac{2\alpha m_i^2}{\pi f_a^2} c_i C_{\gamma\gamma}^{\text{eff}} \left(f_2\left(\frac{m_a^2}{m_i^2}\right) - \ln\left(\frac{16\pi^2 f_\pi^2}{m_i^2}\right)\right)$$

Backup: UV Completion/Mass Spectra

Table 5.1.: Fermionic part of the particle content. Shown are the PQ charges and hypercharges of all SM fermions.

Particle	$q_{L,i}$	$u_{R,1}$	$u_{R,2}$	$u_{R,3}$	$d_{R,1}$	$d_{R,2}$	$d_{R,3}$	$l_{L,i}$	$e_{R,1}$	$e_{R,2}$	$e_{R,3}$
PQ charge	0	X_u	X_c	X_t	X_d	X_s	X_b	0	X_e	X_μ	X_τ
Hypercharge	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-1	-1

Table 5.2.: Scalar doublet part of the particle content. Shown are their PQ charges, hypercharges and VEV.

Particle	h_0	h_1	h_2	h_3	h_4	h_5
PQ charge	X_{h0}	X_{h1}	X_{h2}	X_{h3}	X_{h4}	X_{h5}
Hypercharge	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
VEV	v_{h0}	v_{h1}	v_{h2}	v_{h3}	v_{h4}	v_{h5}

Table 5.3.: Scalar singlet part of the particle content. Shown are their PQ charges, hypercharges and VEV.

Particle	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7
PQ charge	1	$X_{\phi1}$	$X_{\phi2}$	$X_{\phi3}$	$X_{\phi4}$	$X_{\phi5}$	$X_{\phi6}$	$X_{\phi7}$
Hypercharge	0	0	0	0	0	0	0	0
VEV	$v_{\phi0}$	$v_{\phi1}$	$v_{\phi2}$	$v_{\phi3}$	$v_{\phi4}$	$v_{\phi5}$	$v_{\phi6}$	$v_{\phi7}$

Table F.2.: The masses of all CP-even scalars m_{S} and CP-odd pseudoscalars m_{PS} of each model are displayed, sorted numerically. The CP-even mass of 125 GeV corresponds to the SM Higgs boson H_{SM} and the two massless CP-odd particles represent the two Goldstone bosons of the unbroken symmetries. For every model, the individual estimates of the required parameters are used.

Model 1		Model 2		Model 3	
m_{S} in GeV	m_{PS} in GeV	m_{S} in GeV	m_{PS} in GeV	m_{S} in GeV	m_{PS} in GeV
260	260	180	180	130	130
260	260	180	180	130	130
260	260	180	180	130	130
125	3.8	125	16	130	130
4.4	3.6	16	2.8	125	2.2
3.8	1.7	4.7	1.7	5.9	1.8
3.5	1.5	2.8	1.6	2.3	1.6
1.8	1.4	1.9	1.2	1.8	1.3
1.6	1.1	1.7	0.97	1.6	1.1
1.6	0.79	1.4	0.79	1.5	1.0
1.5	0.56	1.3	0.54	1.4	0.76
1.3	0.32	1.3	0.36	1.2	0.15
0.90	0	1.1	0	0.98	0
0.57	0	0.60	0	0.64	0

Backup: UV Lagrangian

$$\mathcal{L}_{\text{kin}} = \sum_{i=0}^{5} (\partial_\mu h_i)^\dagger \, \partial^\mu h_i + \sum_{i=0}^{7} (\partial_\mu \phi_i)^* \, \partial^\mu \phi_i \quad .$$

$$h_i = \frac{(v_{hi} + H_i)}{\sqrt{2}} e^{i \frac{X_{hi}}{V_{\text{PQ}}} a} \begin{pmatrix} 0 \\ 1 \end{pmatrix} ,$$

$$\phi_i = \frac{(v_{\phi i} + \Phi_i)}{\sqrt{2}} e^{i \frac{X_{\phi i}}{V_{\text{PQ}}} a} .$$

$$\begin{aligned} V_1 = \sum_{i=1}^{3} \Big(& \bar{q}_i Y^u_{i1} \tilde{h}_4 u_{R,1} + \bar{q}_i Y^u_{i2} \tilde{h}_0 u_{R,2} + \bar{q}_i Y^u_{i3} \tilde{h}_0 u_{R,3} \\ & + \bar{q}_i Y^d_{i1} h_2 d_{R,1} + \bar{q}_i Y^d_{i2} h_0 d_{R,2} + \bar{q}_i Y^d_{i3} h_0 d_{R,3} \\ & + \bar{l}_i Y^e_{i1} h_5 e_{R,1} + \bar{l}_i Y^e_{i2} h_1 e_{R,2} + \bar{l}_i Y^e_{i3} h_3 e_{R,3} \Big) + h.c. \end{aligned}$$

$$V_2 = \sum_{i=0}^{5} \left(-\mu^2_{hi} h_i^\dagger h_i + \lambda_{hi} \left(h_i^\dagger h_i \right)^2 \right) + \sum_{i=0}^{7} \left(-\mu^2_{\phi i} \phi_i^* \phi_i + \lambda_{\phi i} \left(\phi_i^* \phi_i \right)^2 \right) .$$

$$\begin{aligned} V_3^1 = - \Big(& A_0 \phi_1^* \phi_0^2 + A_1 \phi_2^* \phi_1^2 + A_2 \phi_3^* \phi_2 \phi_0 + A_3 \phi_4^* \phi_3^2 + A_4 \phi_5^* \phi_4^2 + A_5 \phi_6^* \phi_5^2 \\ & + A_6 \phi_7^* \phi_4 \phi_6 \Big) + h.c. \ , \end{aligned}$$

$$V_4^1 = - \left(B_0 \phi_0^* h_0^\dagger h_1 + B_1 \phi_5^* h_0^\dagger h_2 + B_2 \phi_3^* h_2^\dagger h_3 + B_3 \phi_6 h_0^\dagger h_4 + B_4 \phi_7^* h_1^\dagger h_5 \right) + h.c.$$

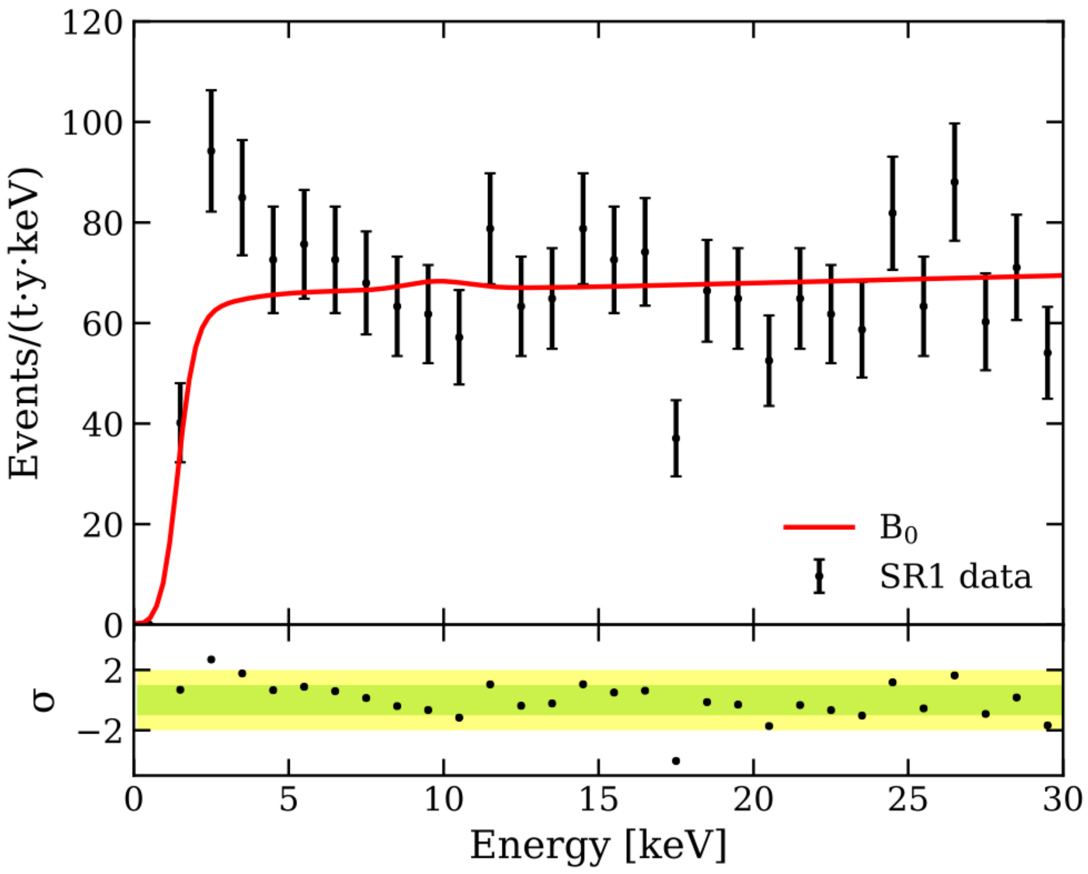
Backup: SM Higgs decay into axions

$$\mathcal{L}_{haa} = \frac{\gamma_1}{\sqrt{2}V_{PQ}^2 m_H^2} \left(\partial_\mu a\right)^2 H_{\rm SM} \,,$$

$$\begin{aligned}
\gamma_1^{\rm exp} &= \left(X_{\phi 0}^2 + X_{h1}^2\right) B_0 v_{\phi 0} v_{h1} + \left(X_{\phi 5}^2 + X_{h2}^2\right) B_1 v_{\phi 5} v_{h2} \\
&\quad + \left(X_{\phi 6}^2 + X_{h4}^2\right) B_3 v_{\phi 6} v_{h4} + \alpha_0^2 \sqrt{2} m_H^2 X_{h0}^2 v_{h0} \,, \\
&\approx 2\left(X_{\phi 0}^2 + X_{\phi 5}^2 + X_{\phi 6}^2\right) B v_{h1} v_{\phi} + \sqrt{2} m_H^2 X_{h0}^2 v \,.
\end{aligned}$$

Backup: Addressing the XENON1T excess

E. Aprile et al.: arXiv:2006.09721



Backup: Addressing the XENON1T excess