Aspects of the Inner Entanglement Inside Black Holes in the Presence of Memory Burden

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Overview

Introduction: Quantum N-portrait

Black holes are bound state of large N soft gravitons of wavelength $\lambda = R_S$ at a critical point where a large number of gapless(holographic) modes emerge [G.Dvali, C.Gomez '11 '12].

Properties of semiclassical black holes such as:

- Thermal(Hawking) Radiation: $T = \frac{1}{R}$
- Half decay time: $t_{\text{half}} = R^3 M_p^2$
- Bekenstein-Hawking entropy: $S_{\text{BH}} \sim \frac{R^2}{L_p^2}$

are very well understood in Quantum N-portrait. Quantum corrections are $\sim \dfrac{1}{N}$

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The Quantum N-portrait predicts new phenomena

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Goal: study these two phenomena and investigate how the first affects the second

Idea: study a simple prototype model, to gain knowledge about the black hole beyond semiclassical description.

Prototype model

Guidelines [G.Dvali '17]:

- Large number of hard quanta(memory modes) $\hat{a}_k$ with energy gap ϵ. QI can be stored in memory patterns $|n_1, n_2, .., n_k\rangle$. High entropy $\Rightarrow$ degenerecy in microstates $\rightarrow$ $\hat{a}_k$ gapless
- soft modes(master mode) $\hat{a}_0$, attractively coupled to memory burden and are highly occupied to a certain critical level N_c s.t. memory modes become gapless.
- allow the system to deplete via occupation numbers exchange of master modes with an external (soft)mode $\hat{b}_0$(Hawking radiation)
- coupling constants $C_0 \sim C_m \sim \frac{1}{N_c}$

$$
\begin{aligned}
\hat{H} = & \epsilon_0 \overbrace{\hat{n}_0}^{\hat{a}_0^\dagger \hat{a}_0} + \epsilon_0 \overbrace{\hat{m}_0}^{\hat{b}_0^\dagger \hat{b}_0} + C_0 \left(\hat{a}_0^\dagger \hat{b}_0 + h.c \right) + \overbrace{\left(1 - \frac{\hat{n}_0}{N_c}\right)\epsilon}^{\text{eff. gap } \tilde{\epsilon}} \sum_{k=1}^{K} \hat{n}_k \\
& + C_m \sum_{k=1}^{K} \sum_{\substack{l=1 \\ l>k}}^{K} \left(\hat{a}_k^\dagger \hat{a}_l + h.c \right)
\end{aligned}
\tag{1}
$$

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Effect shared by systems that have high capacity of memory storage.
Back reaction of the memory modes $\hat{a}_k$ on the master modes $\hat{a}_0$
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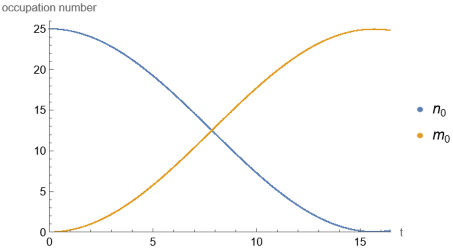

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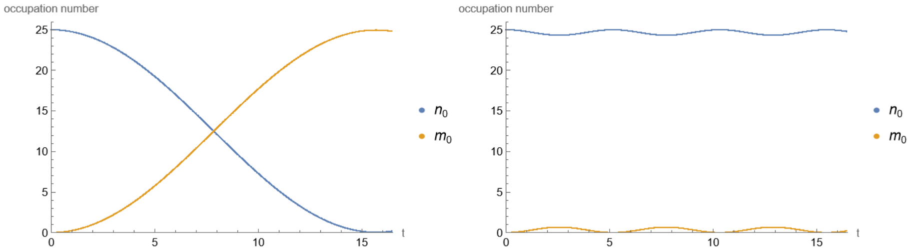

$\mu = 0$ $\qquad$ $\mu \neq 0$

$\Rightarrow$ large load of QI ties the system to its initial state

Weakening the memory burden effect

Can we weaken the memory burden effect without lowering the amount of QI? [G.Dvali, S.Zell, M.Michel, L.Eisemann '18]

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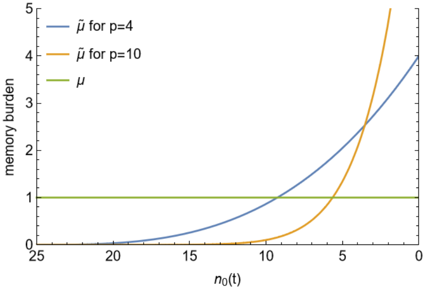

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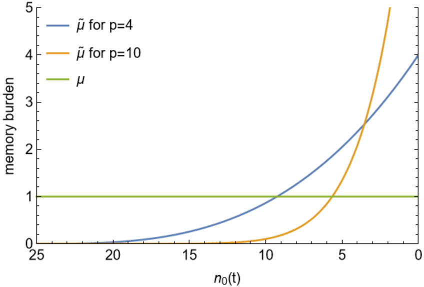

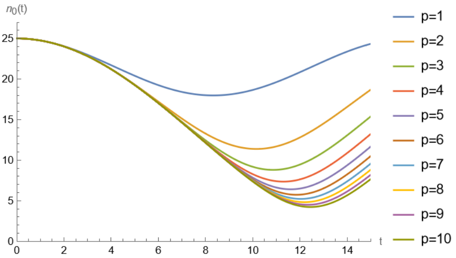

Conclusion: memory burden strength can be varied by varying the order of nonlinearity p

Inner entanglement

Two entanglements happening simultaneously:

- Standard: entanglement of black hole with outgoing radiation[Page '93]

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$$|\mathrm{in}\rangle = |n_1, n_2, ..., n_K\rangle \longrightarrow \sum_{\tilde{n}_j}^{K} c_{\tilde{n}_1,\tilde{n}_2,...,\tilde{n}_K} |\tilde{n}_1, \tilde{n}_2, ..., \tilde{n}_K\rangle$$

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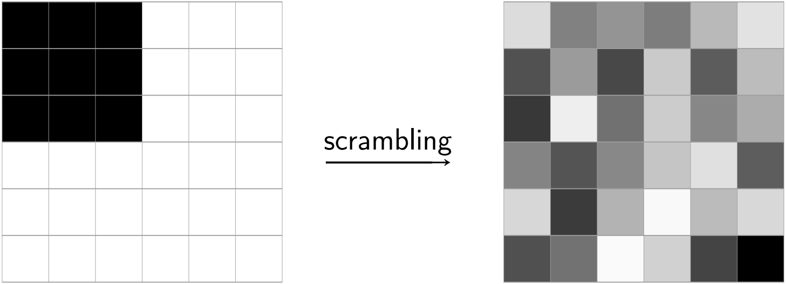

Measure for the inner entanglement

Inner entanglement can be measured by:

- define one-particle density matrix $\rho_1 = \mathrm{Tr}_{a_2,..,a_k}\, \rho_{\mathsf{mem}}$
- Von Neumann entropy $S_1 = -\frac{1}{n}\,\mathrm{Tr}\big(\rho_1 \ln \rho_1\big)$

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Time evolve the state:

$$|\text{in}\rangle = |\underbrace{N_c}_{n_0}, \underbrace{0}_{m_0}\rangle \otimes \underbrace{|n_1, ..., n_K\rangle}_{\text{memory pattern}} \longrightarrow |N_c - \Delta N, \Delta N\rangle \otimes |\tilde{n}_1, ..., \tilde{n}_K\rangle$$

One particle density matrix:

$$\rho \quad \xrightarrow{\text{trace out } a_0 b_0} \quad \overbrace{\rho_{\text{mem}}}^{\to S_{a_0 b_0/\text{mem}}} \quad \xrightarrow{\text{trace out } \{a_2..a_K\}} \quad \overbrace{\rho_1}^{\to S_1}$$

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But,

ρ_{mem} is a mixed state in general

S_1 = inner entanglemeent + (classical)statistical correlations($a_0 b_0$ effects)

$\Rightarrow S_1$ is not a reliable measure for inner entanglement

Two memory sectors

Trick: Add a second memory sector $\{a'_1, .., a'_{K'}\}$ [G.Dvali '18]

$$\hat{H} = \epsilon_0 \hat{n}_0 + \epsilon_0 \hat{m}_0 + C_0\left(\hat{a}_0^\dagger \hat{b}_0 + h.c\right) + \left(1 - \frac{\hat{n}_0}{N_c}\right)\sum_{k=1}^{K} \epsilon \hat{n}_k$$
$$+ \left(1 - \frac{\hat{n}_0}{N_c - \Delta N_c}\right)\sum_{k'=1}^{K'} \epsilon \hat{n}'_{k'} + C_m\Big\{\sum_{k=1}^{K}\sum_{k'=1}^{K'} f_1(k,k')\left(\hat{a}_k^\dagger \hat{a}'_{k'} + h.c\right)$$
$$+ \sum_{k=1}^{K}\sum_{\substack{l=1\\ l>k}}^{K} f_2(l,k)\left(\hat{a}_k^\dagger \hat{a}_l + h.c\right) + \sum_{k'=1}^{K'}\sum_{\substack{l'=1\\ l'>k'}}^{K'} f_3(l',k')\left(\hat{a}_{k'}^\dagger \hat{a}'_{l'} + h.c\right)\Big\}$$

2nd sector becomes gapless for $n_0 = N'_c = N_c - \Delta N_c$

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2nd sector becomes gapless for $n_0 = N'_c = N_c - \Delta N_c$

<u>Time evolution:</u>

$$|\underbrace{N_c}_{n_0}, \underbrace{0}_{m_0}\rangle \otimes \underbrace{|n_1, ..., n_K\rangle}_{\text{1st sector}} \otimes \underbrace{|0, ..., 0\rangle}_{\text{2nd sector}} \rightarrow |N_c - \Delta N_c, \Delta N_c\rangle \otimes |\tilde{n}_1, ..., \tilde{n}_K\rangle \otimes \left|n'_1, .., n'_{K'}\right\rangle$$

At $n_0 = N_c - \Delta N_c$: Rewriting of QI from first to second sector

Two regimes

Idea: Try to isolate the classical correlations contribution in S_1 by considering two distinct regimes:

- 1st regime: $\Delta N_c \sim 1$: The system reaches gaplessness for the 2nd sector $n_0 \to N_c - \Delta N_c$
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- 1st regime: $\Delta N_c \sim 1$: The system reaches gaplessness for the 2nd sector $n_0 \to N_c - \Delta N_c$
  $\Rightarrow$ rewriting of QI ($n'_{k'} \neq 0$)
- 2nd regime: $\Delta N_c \gg 1$: The system does not reach gaplessness of the 2nd sector $n_0 \not\to N_c - \Delta N_c$ $\Rightarrow$
  2nd sector is not excited $n'_{k'} = 0$

In the second regime we expect inner entanglement between two sectors to vanish. only the contribution from statistical correlations is left.

First regime $\Delta N_c \sim 1$

1st regime: $n'_{k'} \neq 0$

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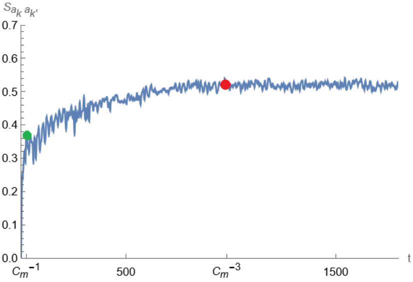
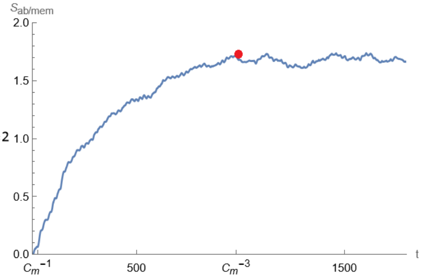


Entanglement between
memory sectors



Entanglement between $a_0 b_0$
and memory sectors

- $S_{a_k/a'_{k'}}$ has local maximum at $t \sim C_m^{-1}$ and global maximum at $t \sim C_m^{-3}$
- Only a global maximum at $t \sim C_m^{-3}$

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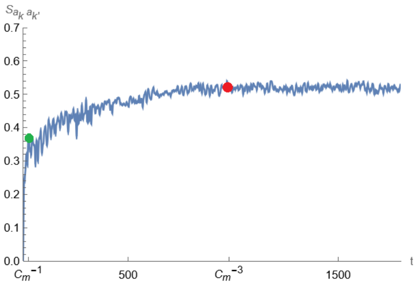

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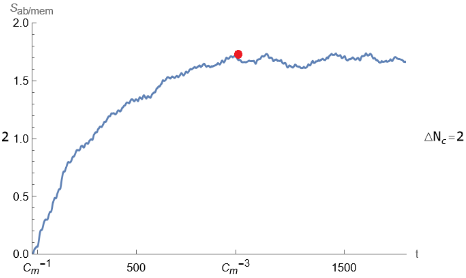


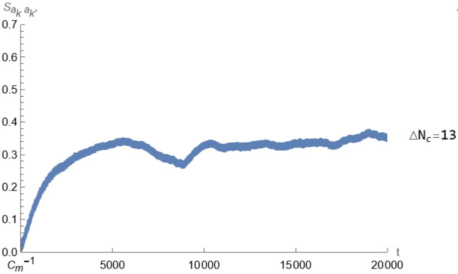
Entanglement between $a_0 b_0$ and memory sectors

- $S_{a_k/a'_{k'}}$ has local maximum at $t \sim C_m^{-1}$ and global maximum at $t \sim C_m^{-3}$
- Only a global maximum at $t \sim C_m^{-3}$

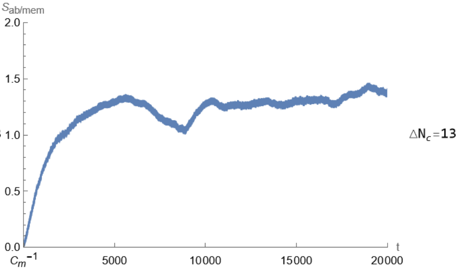
Claim: global maximum at C_m^{-3} is coming from the statistical correlations i.e tracing out $a_0 b_0$

second regime $\Delta N_c \gg 1$

2nd regime: $n'_{k'} = 0$



Entanglement between memory sectors



Entanglement between $a_0 b_0$ and memory sectors

- only classical correlations contribution is left in $S_{a_k/a'_{k'}}$
  $\Rightarrow$ both curves are the same up to a scaling factor
- for $t \sim C_m^{-1}$, $S_{ab/mem} \approx 0$

Conclusion: For time scales $t \sim C_m^{-1}$, ρ_{mem} is almost pure even after tracing out $a_0 b_0$

Effect of the memory burden on the inner entanglement

The one-particle entropy S_1 is then a reliable measure for inner entanglement for $t \sim C_m^{-1}$

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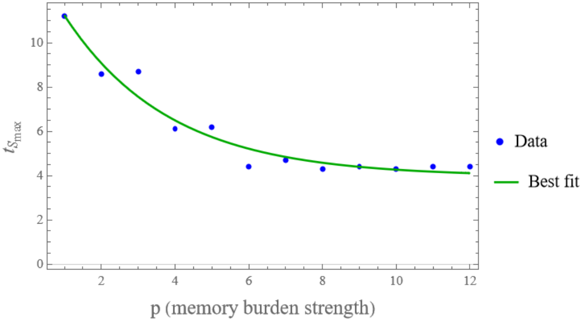
Effect of memory burden on the inner entanglement:



Conclusion: As the memory burden strength decreases, the memory modes take less time to fully entangle

Summary and outlook

- Increasing the nonlinearity of the interaction decreases the strength of the memory burden
- Soft/hard modes entanglement takes time order of magnitude higher than the inner entanglement. Since $C_m \sim \frac{1}{N}$, this difference in timescales becomes more dramatic for large N
- The stronger the memory burden effect the longer takes the system to fully entangle

Summary and outlook

- Increasing the nonlinearity of the interaction decreases the strength of the memory burden
- Soft/hard modes entanglement takes time order of magnitude higher than the inner entanglement. Since $C_m \sim \frac{1}{N}$, this difference in timescales becomes more dramatic for large N
- The stronger the memory burden effect the longer takes the system to fully entangle
- extend the model to include species and investigate their effect
- Similarities with De Sitter case: Applications in inflationary cosmology: $t_{S\text{max}}$ provides an internal quantum clock$\Rightarrow$ gives an upper bound on the graceful exist from De Sitter(end of inflation)[G.Dvali S.Zell M.Michel L.Eisemann '18]
  $\Rightarrow$ investigate how bound changes due to memory burden

Thank you
for your attention

Backup slides

Numerical parameters

Values of the parameters are motivated by the quantum N-portrait:

$$N_c = 20, \quad K = K' = 4, \quad \epsilon_0 = R_S^{-1} = 1, \quad \epsilon = \epsilon_0 \sqrt{K} \ \left(\sim \frac{1}{\sqrt{G_N}}\right)$$

$$C_0 = C_m = 0.1$$

$$|f_1|, |f_2|, |f_3| \in [0.5, 1]$$

$$\Delta N_c \in [0, N_c]$$

The initial state has $N_m = \frac{K}{2}$ occupied memory modes.

$$N_m = \sum_{k=1}^{K} \hat{n}_k + \sum_{k'=1}^{K'} \hat{n}_{k'} = 2$$

Bounds on Couplings

The coupling must be $\sim \frac{1}{N_c}$ so that gaplessness is not spoiled
At gaplessness:

$$\hat{H} \quad \supset \quad \begin{array}{cc} & \begin{array}{cc} \hat{a}_k & \hat{a}_l \end{array} \\ \begin{array}{c} \hat{a}_k \\ \hat{a}_l \end{array} & \begin{pmatrix} 0 & \sim \frac{1}{N_c} \\ \sim \frac{1}{N_c} & 0 \end{pmatrix} \end{array}$$

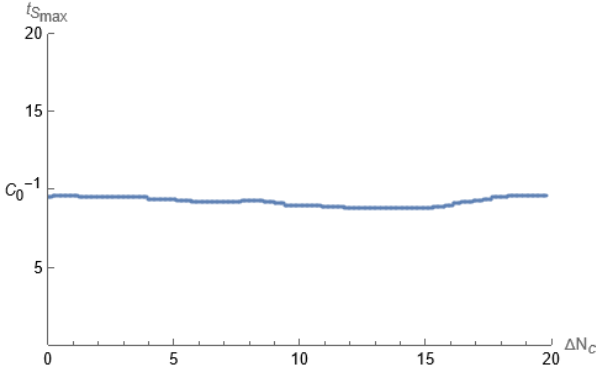
One can diagonlize the Hamiltonian via Bogolyubov transformation[G.Dvali, M.Panchenko '16] and the memory modes acquire an effective gap. This effective gap is $\frac{1}{N_c}$ suppressed.

It has been shown in [G.Dvali S.Zell M.Michel L.Eisemann '21] that (using our numerical values):

- $C_m \leq \frac{1}{2}$
- rewriting of information from 1st to 2nd sector happens at the ideal value $C_m = 0.1$

Consistency check: Page's time

Page's time: maximal entanglement of B.H($\hat{a}_0$ & $\{a_k, a'_{k'}\}$) and outgoing radiation($\hat{b}_0$) at half decay.



$t_{S_{max}} = t_{half} = t_{Page} \Rightarrow$ consistent with semiclassical picture

N.b: This does not mean that we can retrieve information after half decay since the information is controlled exclusively by the memory modes

3rd regime: $\Delta N_c = 0$

For the regime where both sectors are gapless at the same critical value: $N_c = N_c'$. Analytic expression for $n_0(t)$:

$$n_0(t) \sim \frac{4C_0^2}{4C_0^2 + \epsilon^2 \frac{(\sum n_k + \sum n'_{k'})^2}{N_c}} \sin^2\left(\frac{\sqrt{4C_0^2 + \epsilon^2 \frac{(\sum n_k + \sum n'_{k'})^2}{N_c}}}{2} . t \right)$$

$\Rightarrow$ No entanglement between soft and memory modes