Integrability for ABJM theory: Quantum Spectral Curve and Regge trajectories

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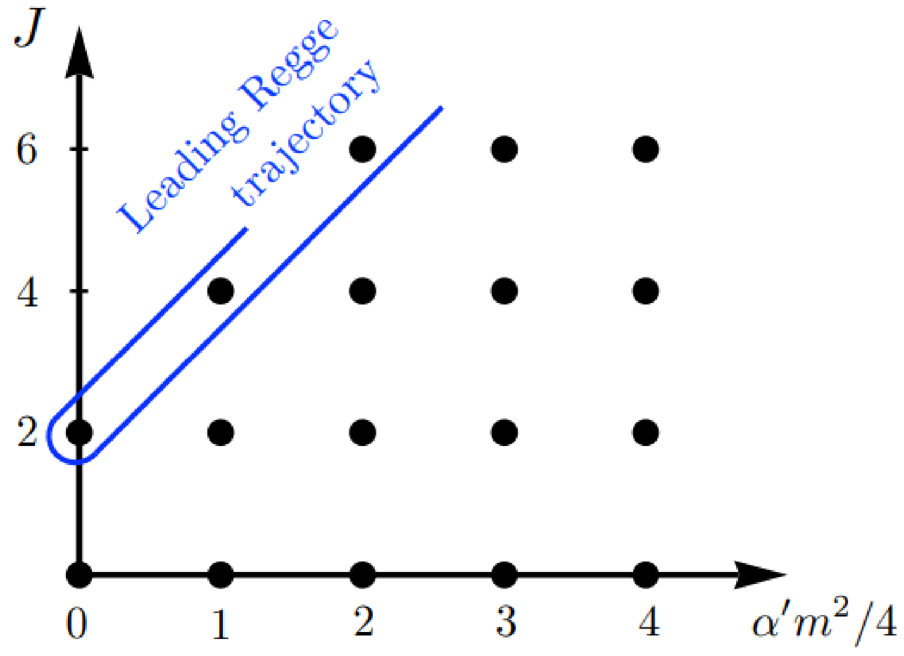
Regge trajectories

Regge theory: study of the dependence of the spectrum on the spin of exchanged particles

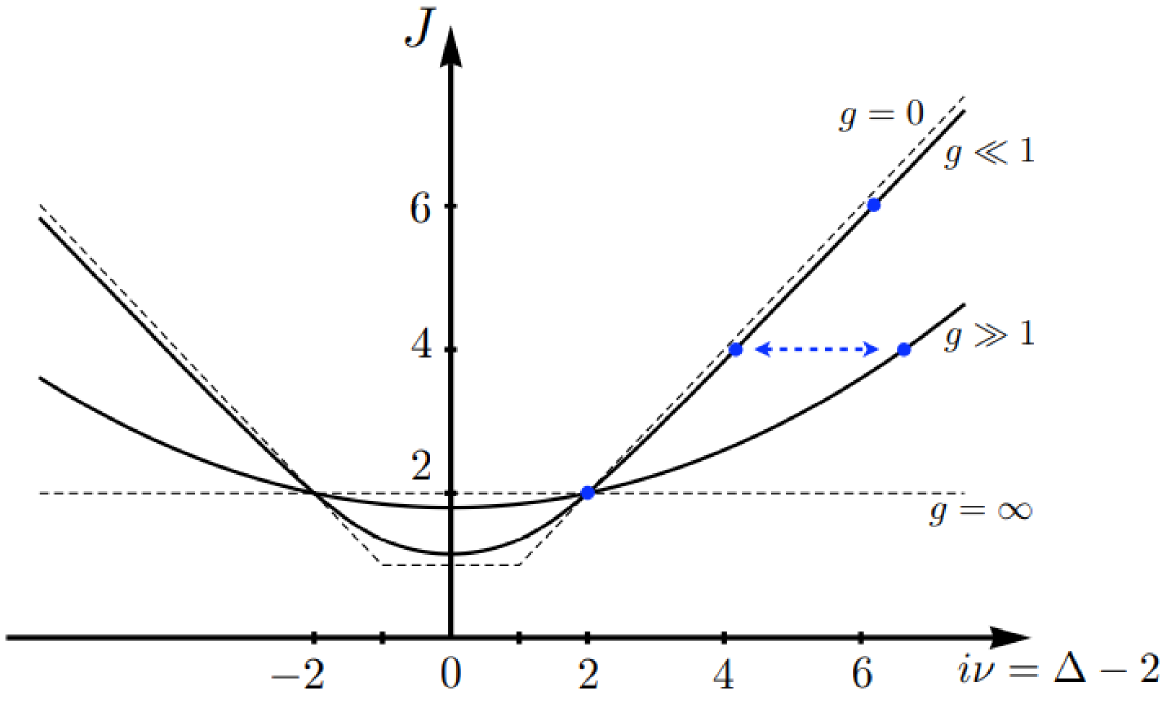
Regge theory in QFT

Scattering amplitudes:
$T(s,t) \sim s^{j(t)}, \quad s \to \infty, t$ fixed



Conformal Regge theory

Mellin amplitudes=integral transform of the n-point correlators in CFT



$\mathcal{N} = 4$ SYM vs ABJM theory

- Supersymmetry + Conformal symmetry $\Rightarrow$ Superconformal gauge theory
- These theories appear in AdS/CFT duality: string theories $\Leftrightarrow$ gauge theories

$\mathcal{N} = 4$ **Super Yang-Mills**	**ABJM theory**
4d Yang-Mills: $SU(N)$	3d Chern-Simons: $U(N) \times U(M)$
Conformal group: $SO(2,4) \simeq SU(2,2)$	Conformal group: $SO(2,3) \simeq Sp(4)$
Supercharges $\mathcal{N} = 4$	Supercharges $\mathcal{N} = 6$
Supergroup: $PSU(2,2\|4)$	Supergroup: $OSp(6\|4)$
Regge trajectories for generic spin: **understood**	Regge trajectories for generic spin: **NOT understood**

Spectrum of a CFT

- **Conformal dimension** Δ of an operator $O(x)$ $\rightarrow$ eigenvalue of dilatation generator acting on $O(x)$

- **Spectral problem** $\rightarrow$ compute the *conformal dimensions* of the operators in the theory

2-point correlator

$$\langle \mathcal{O}(x)\bar{\mathcal{O}}(y)\rangle \approx \frac{1}{|x-y|^{2\Delta}} \overset{\gamma<<\Delta_0}{\rightarrow} \frac{1}{|x-y|^{2\Delta_0}}\left(1-\gamma \ln\left(\Lambda^2|x-y|^2\right)\right)$$

$$\boxed{\Delta = \Delta_0 + \gamma}$$

Bare dimension Δ_0 (tree level)

Anomalous dimension γ: quantum corrections to Δ_0

Single trace operators

- **Single trace operators**: fields in adjoint representation → trace to build *gauge invariant* quantities

$$\boxed{\mathcal{O}_{i_1,\ldots,i_L}(x) \propto \mathrm{Tr}(\phi_{i_1}(x)\cdots\phi_{i_L}(x))}$$

- **Loop corrections** mix the flavour indices of the fields inside the trace

- Anomalous dimensions can be seen as the **spectrum of these mixing operators**

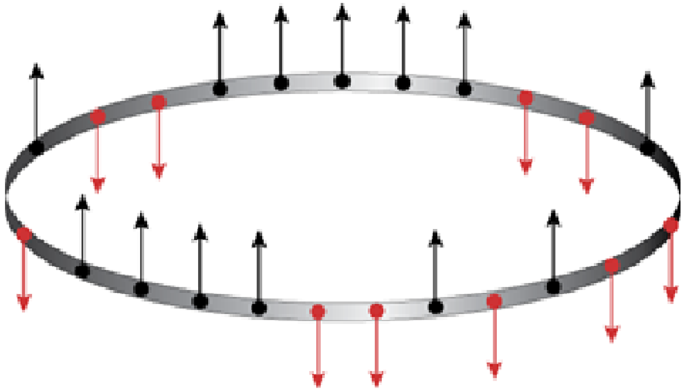
$$\langle O_{i_1\ldots i_L}(x)\bar{O}^{j_1\ldots j_L}(y)\rangle = \frac{1}{|x-y|^{2L}}\left(1-\frac{\lambda}{8\pi^2}\sum_{l=1}^{L}\underbrace{(1-\hat{P}_{l,l+1})}_{\text{Anomalous dimension}}\right)\left(\delta_{i_1}^{j_1}\ldots\delta_{i_L}^{j_L}+\text{cycles}\right)$$

Spin chain models

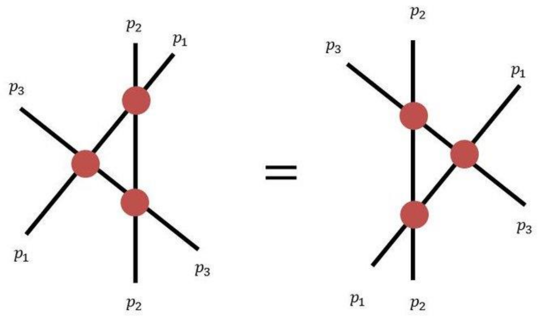
GAUGE THEORY	⟷	SPIN CHAIN MODEL
fields in the trace	⟷	sites in the spin chain
operator $Tr(\phi_{i_1} \ldots \phi_{i_L})$	⟷	states in $\mathcal{H}_1 \otimes \ldots \otimes \mathcal{H}_L$
cyclicity of the trace	⟷	periodic conditions (closed chain)
mixing operators	⟷	**integrable** hamiltonian on the spin chain
anomalous dimension	⟷	eigenvalues of integrable hamiltonian

Bethe ansatz equations

Bethe ansatz for the eigenstates → **Integrable Scattering Matrix:**



- *Unitarity* → $SS^{\dagger} = 1$
- *Yang-Baxter* →



Solving the spectrum for closed chains

↕

Bethe ansatz equations

$$\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right)^L = \prod_{\substack{j=1 \\ j \neq k}}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$$

Quantum Spectral Curve

- Set of **auxiliary functions** dependent on a **spectral parameter** $\boldsymbol{u} \in \mathbb{C}$ → generalization of the Bethe ansatz equations

- **Algebraic constraints** → Non-linear system of finite difference functional equations

$$Q_1(u) = \prod_{i=1}^{N_1}(u - u_i)\,, \quad \frac{Q_1^{[++]}(u_i)}{Q_1^{[--]}(u_i)} = \left(\frac{u_i+\frac{i}{2}}{u_i-\frac{i}{2}}\right)^L \quad \Longrightarrow \quad \text{Bethe ansatz equations}$$

Spin chain

↓

Polynomial Q-functions

vs

Supersymmetric theory

↓

Non-trivial monodromy

Gluing conditions

1) Analytic and algebraic properties
+
2) Ansatz on asymptotic behaviour

$\Rightarrow$ Constraints on Δ and S

1) Symmetries of the QSC equations: **analytic continuation**, **conjugation** and **parity** $(u \to -u)$

2) Asymptotic behaviour $\leftrightarrow$ charges like conformal dimension and spin

- **Power-like** asymptotics $\longrightarrow$ Spin is **quantized**: $S \in \mathbb{Z}$
- **Exponential behaviour** $\longrightarrow$ Allows to study $\Delta(S)$ for CONTINUOUS SPIN

New results

Analytic results

- ✓ Study of the **parity symmetry** for the QSC equations for ABJM theory → **new constraints on the gluing conditions** (expressed as a matrix equation)

- ✓ **New ansatz** for the asymptotic behaviour → search for a relation between conformal dimension and **spin as a continuous parameter**

Numerical studies:

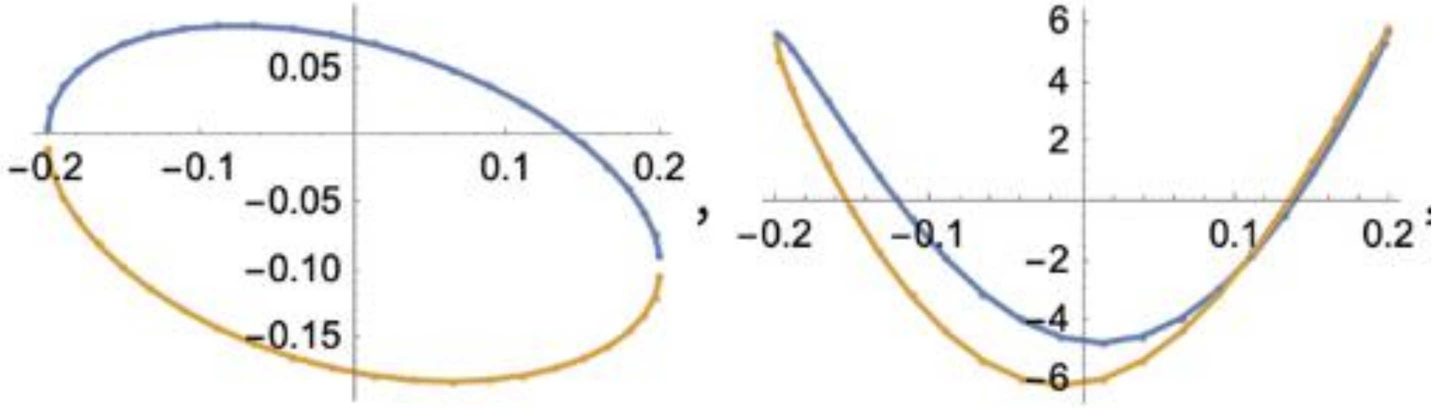
- ➢ **Numerical implementation** of the ansatz on the gluing constraints

- ➢ *Work in progress*…numerical evaluation of the Regge trajectory at non-integer values of the spin

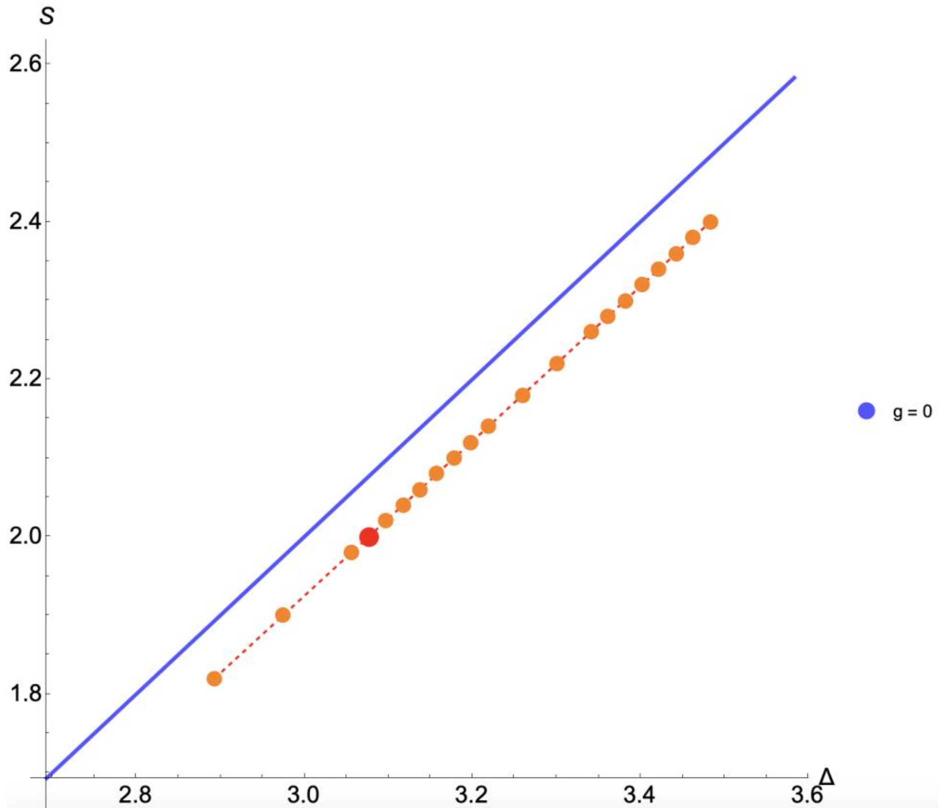
New results

Recently with the doctoral student **Nicolò Brizio**

Numerical evaluation of
Regge trajectories and
Q-functions at generic spin



Q-functions



Regge trajectory

References

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2. *Studies on Wilson loops, correlators and localization in supersymmetric quantum field theories*, Michelangelo Preti
3. *Quantum integrability and functional equations*, Dmytro Volin
4. *A review of the AdS/CFT Quantum Spectral Curve*, Fedor Levkovich-Maslyuk
5. *Introduction to Integrability*, N. Beisert
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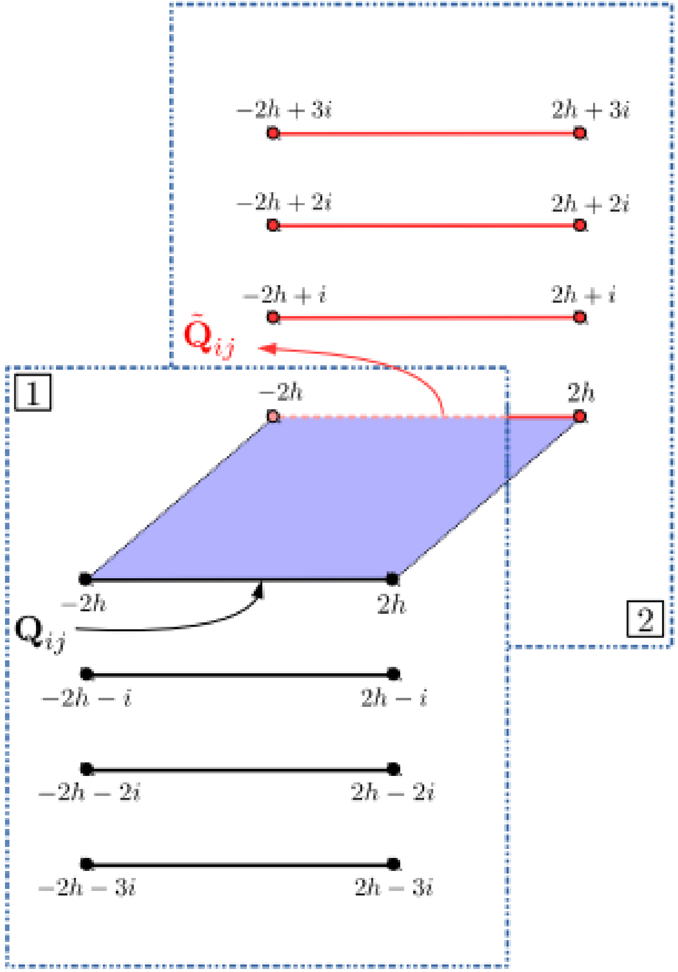
7. *BFKL Spectrum of N = 4 SYM: non-Zero Conformal Spin*, 1802.06908v4 [hep-th]
8. *Supersymmetric Gauge Theories and the AdS/CFT Correspondence*, Eric D'Hoker, Daniel Z. Freedman
9. *Lectures on Supersymmetry*, M. Bertolini
10. *Conformal Regge Theory*, V. D. Fonseca Gonçalves

QSC for ABJM

Q_{ij} → analytic continuation: $\tilde{Q}_{ij}$



branch points and cuts

Zhukovski's variable

$$x(u) = \frac{u+\sqrt{u^2-4h^2}}{2h}$$

MASTER equation: $Q_{a|i}^{[+]} = P_{ab}P^{[-2]bc}Q_{c|i}^{[-3]}$

Asymptotic behaviour

charges of the theory ↔ exponents in the power-like asymptotics

Gluing conditions for integer spin

Idea: analytic properties + asymptotic behaviour ⇒ constraints on Δ and S

Conjugation

$$\boxed{\overline{Q}_{a|i}(u) = Q_{a|j}(u)\Omega_i^{j[+]}(u)}$$

symmetry of the MASTER equation

⇓

Gluing matrix $\mathcal{L}(u)$

relation between $\tilde{Q}_{ij} \leftrightarrow \bar{Q}_{ij}$

$$\mathcal{L}(u) \equiv \begin{pmatrix} \frac{e^{i\pi\Delta}}{\cos(\pi\Delta)} & 0 & 0 & \delta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{tan^2(\pi\Delta)}{\delta} & 0 & 0 & \frac{e^{-i\pi\Delta}}{\cos(\pi\Delta)} \end{pmatrix}$$

ansatz: simple
asymptotic behaviour

⇓

$\mathcal{L}(u)$ **CONSTANT** and $S \in \mathbb{Z}$ QUANTIZED

Gluing conditions for continuous spin

Parity $\boldsymbol{u \to -u}$

$$\boxed{\mathbb{R}_a{}^b Q_{b|i}(-u) = Q_{a|j}(u)\Theta_i{}^j(u)}$$

symmetry of the MASTER equation

Gluing matrix $\mathrm{K}(u)$

$$\mathcal{K}(-u) = -k\mathcal{K}^T(u)k^{-1}$$

relation between $\tilde{Q}_{ij}(u) \leftrightarrow Q_{ij}(-u)$

Modified ansatz on the asymptotic behaviour

$K(u)$ analytic, but not bounded at infinity $\Rightarrow$ **NOT CONSTANT**

New ansatz for continuous spin

Ansatz for **exact** $K(u)$ → entries are *even or odd* polynomials in X or X^{-1}:

$$\mathcal{K}_{\text{exact}}(X) = \begin{pmatrix} P_1^{(4)}(X) & P_2^{(3)}(X) & P_3^{(3)}(X) & P_4^{(4)}(X) \\ P_5^{(3)}(X) & P_6^{(2)}(X) & P_7^{(2)}(X) & P_8^{(3)}(X) \\ P_9^{(3)}(X) & P_{10}^{(2)}(X) & P_{11}^{(2)}(X) & P_{12}^{(2)}(X) \\ P_{13}^{(4)}(X) & P_{14}^{(3)}(X) & P_{15}^{(3)}(X) & P_{16}^{(4)}(X) \end{pmatrix}$$

$$P^{(n)}(X) := \begin{cases} \sum_{\substack{i=-n \\ i \text{ even}}}^{n} a_i X^i, & \text{if } n \text{ even} \\ \sum_{\substack{i=-n \\ i \text{ odd}}}^{n} a_i X^i, & \text{if } n \text{ odd} \end{cases}$$

Imposing the constraint on this guess and comparing the **leading order** at $u \to +\infty$ and $u \to -\infty$

numerical implementation

conditions on the parameters of the ansatz for τ_i's and relation $\Delta \leftrightarrow S$