2-group Symmetries in 6 dimensions

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Outline

- 2-group Symmetries
  - Symmetries in QFT
  - Higher Form Symmetries
  - 2-group Symmetries
- 2-group Symmetries in 6D QFTs
- Conclusions

Symmetries in QFT

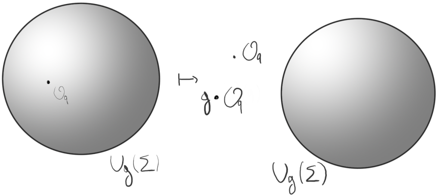
- Transformations that leave the theory invariant
- They give information about the QFT:
  - Noether
  - Ward identities
  - Anomalies
  - ...
- Global Symmetries vs Gauge Redundancies
- Example: complex scalar $\phi \mapsto \mathrm{e}^{i\alpha}\phi$

$$\mathcal{S} = \int \mathrm{d}\phi \wedge \star \mathrm{d}\phi \tag{1}$$

Mathematical Description

- Symmetry structure: group, acting by unitary transformations.
- Implemented by **extended topological operators** (codim-1) $U_g(\Sigma_{d-1})$ wrapping **charged local operators** $\mathcal{O}_q(x)$

$$\mathcal{O}_q(x) \mapsto g \cdot q\mathcal{O}_q(x) \tag{2}$$



Mathematical Description

- Noether current is a conserved 1-form $\mathrm{d} \star J = 0$, charge

$$Q(\Sigma_{d-1}) = \int_{\Sigma_{d-1}} \star J \tag{3}$$

- Background fields A "1-forms" (locally) $(A \mapsto A + \mathrm{d}\lambda)$
  - Continuous case: couple to Noether current $\int A \wedge \star J$, e.g. complex scalar $U(1)$ global symmetry

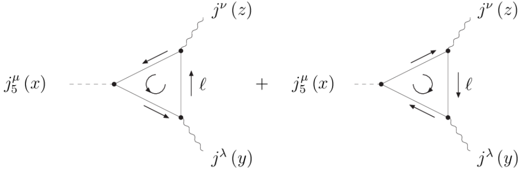
$$J \sim \phi^\star \mathrm{d}\phi \implies \mathcal{L} \sim \phi^\star D_A \phi \tag{4}$$

  - Discrete case: implemented topologically (topological couplings or networks of symmetry defects)

In general: coupling to a G-bundle $(A : X \to BG)$

Anomalies

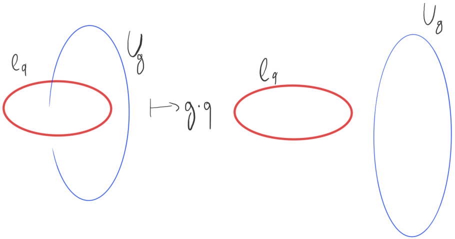
- Non-invariance of the partition function.
- Measured (perturbatively) by $\left(\frac{d}{2}+1\right)$-gon loop diagrams, e.g. ABJ



- Measured (perturbatively) by **anomaly polynomial** I^{d+2}, $(d+2)$-form constructed from characteristic classes, e.g. ABJ in 4d $\kappa_{(GAUGE)^2 GLOBAL} F_{gauge} \wedge F_{gauge} \wedge F_{global}$
- Non-perturbative treatment is more subtle.

Higher Form Symmetries

- p-form symmetries charge **extended** p-dimensional operators, implemented by unitary codim-$(p+1)$ topological operators.



- Noether current is a $(p+1)$ differential form, charge

$$Q(\Sigma_{d-p-1}) = \int_{\Sigma_{(d-p-1)}} \star J \tag{5}$$

Higher Form Symmetries

LMU

- For $p \geq 1$ the underlying group is **abelian**
- Background gauge fields B are $(p+1)$-forms, gauge invariance $B^{(p+1)} \mapsto B^{(p+1)} + \mathrm{d}\Lambda^{(p)}$
  - Continuous case: $\int B \wedge \star J$
  - Discrete case: implemented by network of symmetry defects

In general couple to higher generalization of bundles $(B : X \to B^p G)$

Higher Form Symmetries: examples

- 4d Maxwell: electric $U(1)$ symmetry $A \mapsto A + \alpha$, $\mathrm{d}\alpha = 0$

$$S = \int \frac{1}{2e^2} \mathrm{d}A \wedge \star \mathrm{d}A \tag{6}$$

  with current $J_e = \frac{2}{e^2} F$ charging Wilson Lines $W_q(\ell)$; magnetic symmetry charging 't Hooft lines $H_q(\ell)$, with current $J_m = \frac{1}{2\pi} \star F$.

- Coupling to background

$$S = \int \frac{1}{2e^2} (\mathrm{d}A - B_e) \wedge \star(\mathrm{d}A - B) + B_m \wedge F \tag{7}$$

- Breaking gives interpretation to photons as Goldstone modes

Higher Form Symmetries: examples

- Center Symmetry YM $SU(N)$, shift A by flat $\mathbb{Z}_N$ connection

$$S = -\int \frac{1}{2g^2} \mathrm{tr}(F \wedge \star F) \tag{8}$$

Charges Wilson lines $W_n(\ell)$.

- Center symmetry $\mathcal{Z}(G) \iff$ Global form of $G = {}^{SU(N)}\!/_{Z}$, with $Z \subset \mathbb{Z}_N$, and allowed electric/magnetic lines $\iff$ choice of θ-angle

- Coupling to background

$$"S = -\int \frac{1}{2g^2} \mathrm{tr}((F - B_Z) \wedge \star(F - B_Z))" \tag{9}$$

2-group Symmetries

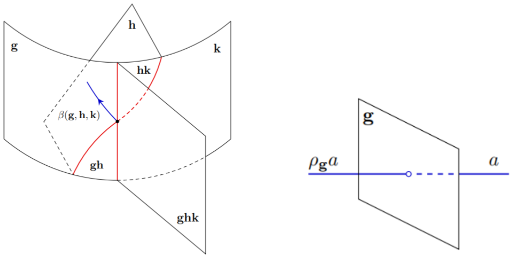
- Consider a QFT with a 0-form symmetry G and 1-form symmetry $\mathcal{A}$.
- G and $\mathcal{A}$ can mix in a non-trivial way $G^{(0)} \times_{\hat{\kappa}} \mathcal{A}^{(1)}$:
  - Background gauge transformations: $A^{(1)} \mapsto A^{(1)} + \mathrm{d}\lambda^{(0)}$, $B^{(2)} \mapsto B^{(2)} + \mathrm{d}\Lambda^{(1)} + \frac{\hat{\kappa}}{2\pi}\lambda^{(0)}\mathrm{d}A^{(1)}$
  - 2-group non-conservation (*fusion*):

$$\mathrm{d} \star J_B = 0, \quad \mathrm{d} \star j_A = \frac{\hat{\kappa}}{2\pi} F_A \wedge \star J_B \tag{10}$$

  - Ward-identities

2-group defects

- Modified fusion of defects



- Measured by topological data $\beta \in H^3_\rho(BG, \mathcal{A})$ (related to $\hat{\kappa}$), $\rho : BG \to Aut(\mathcal{A})$

Applications

- Classify extended objects
- Selection rules on amplitudes
- Symmetry breaking and Symmetry enhancement in RG flow
- Gauging $\sim$ discrete θ angles
- Characterizing phases:
  - Spontaneously broken discrete 1-form symmetry $\implies$ deconfined lines
  - unbroken $\implies$ confinement of charged line operators
- Gauging $\implies$ dualities
- Anomalies

- In a gauge theory with gauge group $U(1)$ (gauge field c) and a global $U(1)$ 0-form symmetry (background A), with mixed anomaly polynomial (4d)

$$I^6_{mixed} = \frac{k_{mixed}}{(2\pi)^3 3!} F_A \wedge F_A \wedge f_c \tag{11}$$

Resulting in 4d anomaly (measuring charge non-conservation)

$$\mathcal{A} = \frac{ik_{mixed}}{8\pi^2} \int F_A \wedge f_c \tag{12}$$

- **Operator valued** $\implies$ broken theory?

Example: 2-group from Mixed Anomaly

- $U(1)$ gauge theory has $U(1)$ magnetic 1-form symmetry with coupling

$$\int B \wedge \star J_m = \frac{i}{2\pi} \int B \wedge f_c \tag{13}$$

- **Fixes** anomaly if $B \mapsto B + \frac{\hat{\kappa}}{2\pi} \lambda_A F_A$ with $\hat{\kappa} = -\frac{k_{mixed}}{2}$
- We have a 2-group $U(1)^{(0)}_{mixed} \times_{\hat{\kappa}} U(1)^{(1)}_{m}$. Non-trivial flavor background $\implies$ B_m background **cannot** be flat.
- This is easily generalized to $U(1)_{gauge} \times G_{flavor}$ and $U(1)_{gauge} \times \mathcal{P}$ mixed anomalies.
- Truncating flavor anomalies via GS counterterms

$$in \int B \wedge F_A \tag{14}$$

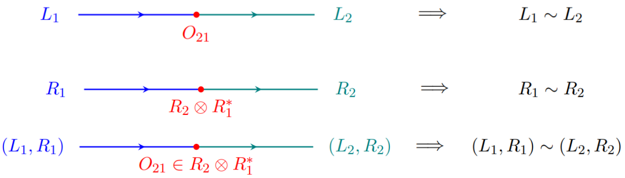
Example: 2-group from Mixed Anomaly

- Similar in 6d: from $I^8_{A^2c^2} \sim k\,\mathrm{tr}(F_A \wedge F_A) \wedge \mathrm{tr}(f_c \wedge f_c)$
- $J^{(2)} \sim \star(f_c \wedge f_c)$ generates $U(1)^{(1)}$ instanton 1-form symmetry
- Gauge theory is well defined if $B^{(2)}_{inst}$ undergoes 2-group shift $B \mapsto B + \frac{\hat{\kappa}}{2\pi}\,\mathrm{tr}(\lambda \mathrm{d}A)$
- Similar for local Lorentz invariance.
- Truncating flavor anomalies via GS counterterms (more counterterms)

Bockstein 2-groups

- Non-trivial topology for flavor group $F \implies$ Bockstein 2-groups
- $F = {}^{\mathcal{F}}\!/_{\mathcal{Z}}$, $\Gamma^{(1)} \subset Z(G)$, $\mathcal{E}$

$$1 \to \Gamma^{(1)} \to \mathcal{E} \to \mathcal{Z} \to 1 \tag{15}$$



- 2-group measured by $\beta = Bock(w_2)$

2-groups in 6D QFTs

- Continuous 2-groups:
  - SCFTs: no continuous 1-form symmetries $\implies$ no continuous 2-groups $\implies$ **mixed gauge/flavor anomalies** vanish
  - LSTs: instanton $U(1)$ 1-form symmetry often gives 2-groups with flavors, R and Poincaré: invariants under T-duality. Truncates big parts of 't Hooft anomalies.
- Bockstein 2-groups:
  - SCFTs: exhibit Bockstein 2-groups of this type (*mainly* in "exotic theories").
  - LSTs: do **not** exhibit Bockstein 2-groups.
  - Anomaly computation is more subtle (discrete structures).
- Defect analysis via inflow.

Conclusions: Possible directions

- More general 2-group structures.
- Higher groups.
- Geometric engineering of these symmetries from M/F-theory.
- 2-group anomaly inflow, symmetry TFTs and global anomalies.
- Swampland conjectures.
- Non-invertible symmetries.
- Particle Physics applications.

Questions?

Thank you for your attention.

Appendix 1: 2-groups in Mathematics

- (Weak) 2-category with 1-object:
  - 1-morphisms form a group G
  - 2-morphisms form a group with a crossed module structure. 2-morphisms $a : 1 \implies 1$ $(1 \in G)$ form an abelian group $\mathcal{A}$
  - $\rho : G \to Aut(\mathcal{A}) \sim$ conjugation.
  - $\beta \in H^3_\rho(BG, \mathcal{A})$ *associator.*
- Equivalent definitions: weak monoidal categories, crossed module, a group object in ***Cat***, a category object in ***Grp***

Appendix 2: Higher Symmetry and Confine

- A spontaneously broken discrete 1-form symmetry leads to deconfined line defects (i.e. defects obeying a perimeter law) at long distances.
- Unbroken 1-form symmetry implies confinement of the line operators charged under it.
- A discrete broken symmetry leads to a TQFT in the IR – long range topological order