25 minutes

Effective field theory for quantum fields in de Sitter space

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IMPRS Colloquium

MAX-PLANCK-INSTITUT
FÜR PHYSIK

Outline of the talk

1. Motivation: why an EFT in de Sitter space?
2. Soft de Sitter Effective Theory
3. Our work
4. Summary and outlook

Inflationary paradigm

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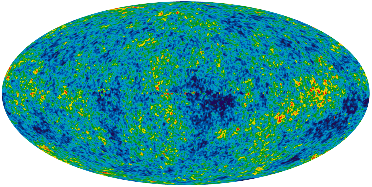

Source: NASA/WMAP Science Team

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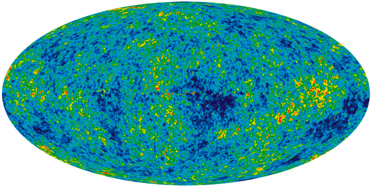
- Explains approximate spatial flatness and isotropy of the Universe we see today.
- Explains approximate isotropy of the CMB and offers a source for its temperature fluctuations as well.
- Offers a mechanism for the seeding of large-scale structure.



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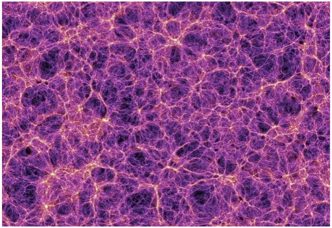

Source: Cherenkov Telescope Array

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with a very flat potential $V(\Phi)$. The fluctuation ϕ is well-described as a massless scalar field living in the inflationary spacetime. Background geometry is also allowed to fluctuate, have metric perturbations γ_{ij}. Main observables of interest are cosmological correlators of inflaton and metric fluctuations,

$$\langle \phi(t,\vec{x}_1)...\phi(t,\vec{x}_m)\gamma_{ij}(t,\vec{x}_{m+1})...\gamma_{ab}(t,\vec{x}_n)\rangle\,,$$

evaluated at equal and late time t, a long time after inflation ended (i.e. today).

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Real, minimally coupled, massless scalar field in dS is a good testing ground to start to understand the dynamics of quantum fields in an inflationary spacetime.

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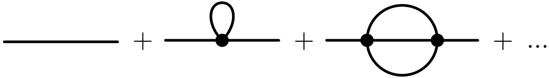

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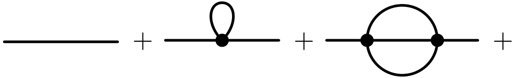
$\langle \phi(t,\vec{x})\phi(t,\vec{y}) \rangle =$  $+ \ldots$

- Massless scalar propagator is infrared-divergent $\Rightarrow$ cannot construct perturbative expansion:

 $\sim \int \frac{\mathrm{d}^3 k}{k^3} \to \infty\,, \quad k \equiv |\vec{k}|\,.$

$\Rightarrow$ Perturbation theory is ill-defined, need to go beyond it to obtain physically sensible results.

Theoretical challenges

- IR-regularized perturbation theory (e.g. with small mass $m^2 \ll H^2$ or IR-cutoff) develops secular logarithms:

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$$O(t \gg 1/H) = \sum_{n=0}^{\infty} \tilde{O}_n \underbrace{g^n \log^n(e^{Ht})}_{\geq 1},$$

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$\Rightarrow$ In both cases, perturbation theory fails due to field modes with

$$\frac{k}{a(t)H} \ll 1\,.$$

The IR-structure of this theory appears to be non-trivial and needs to be understood in a framework that goes beyond fixed-order perturbation theory.

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Idea: split the field up in a large- and small-momentum part,

$$\phi(t,\vec{k}) = \underset{k<a(t)H}{\underset{\uparrow}{\bar{\phi}(t,\vec{k})}} + \underset{k>a(t)H}{\underset{\uparrow}{\delta\phi(t,\vec{k})}}\,, \quad \delta\phi \ll \bar{\phi}\,,$$

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and treat $\bar{\phi}$ as a classical, stochastic random variable. Its properties are described by a probability distribution function (PDF) $\rho(t,\varphi)$, which satisfies a Fokker-Planck equation,

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The stochastic “noise” is generated by the short-wavelenegth modes $\delta\phi$.

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$\Rightarrow$ Interpretation: the massless, interacting field develops a dynamical mass

$$m_{\mathrm{dyn}}^2 \sim H^2 \sqrt{g} \, ,$$

non-perturbatively. The IR-divergence self-regularizes.

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$\Rightarrow$ This motivates the construction of an effective framework in the context of QFT to understand how to systematically incorporate subleading corrections.

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- Exhibit the property of "factorization" of hard and soft physics, schematically:

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Breakthrough in this direction: Soft de Sitter Effective Theory (SdSET) [Cohen, Green 2020; Cohen, Green, Premkumar, Ridgway 2021; Cohen, Green, Premkumar 2021].

Structure of SdSET

The SdSET is an EFT built to compute the late-time/small-wavenumber limit of equal-time correlation functions,

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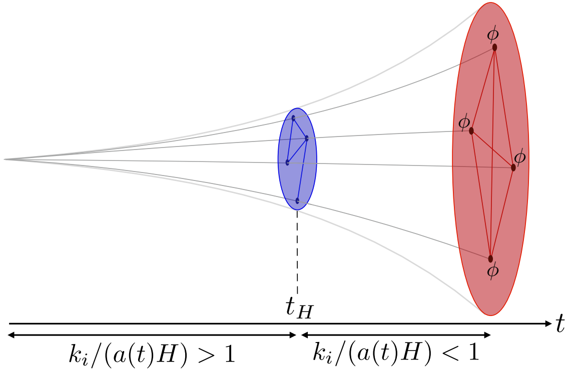

Time of horizon crossing t_H defined by

$$\frac{k_i}{a(t_H)H} \sim 1\,.$$

Structure of SdSET

Full-theory field is split up as

$$\phi(t,\vec{k}) = \underbrace{H\left[\varphi_+(t,\vec{k}) + [a(t)H]^{-3}\varphi_-(t,\vec{k})\right]}_{\text{EFT fields, } k/\Lambda(t)<1} + \underset{\substack{\uparrow \\ k/\Lambda(t)>1}}{\phi_{\text{UV}}(t,\vec{k})}\,,$$

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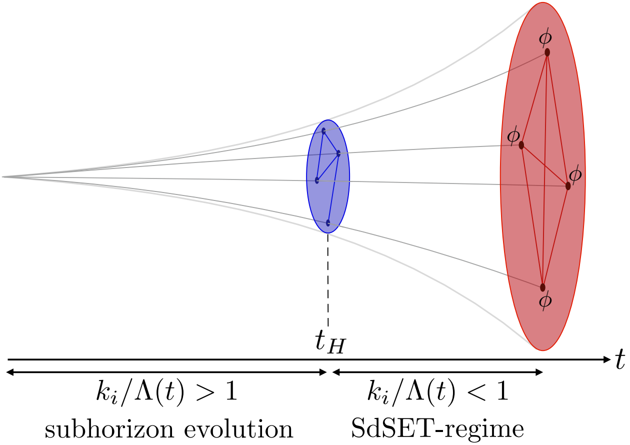
$\phi_{\rm UV}$ is not described as a dynamical field in the EFT (it is "integrated out"). Its effects are captured by Wilson coefficients and non-Gaussian initial conditions (IC's).

Structure of SdSET

Physical origin of the IC's:



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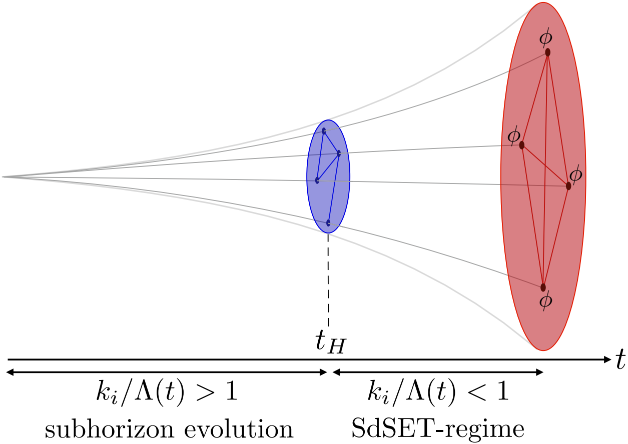

- SdSET describes the time evolution from t_H on, field modes already had time to evolve and interact $\Rightarrow$ leads to non-Gaussian correlations at t_H, which need to be specified as input for the EFT via the IC's.

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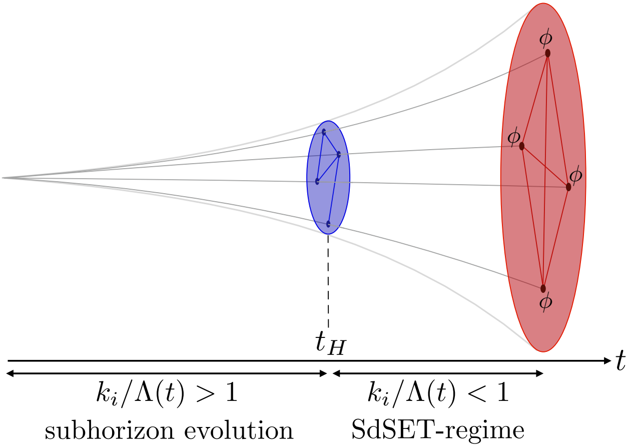
Physical origin of the Wilson coefficients:



- While the fields evolve in time after t_H, they can still interact via modes with momenta $\vec{l}$ which satisfy $l/\Lambda(t) > 1$.

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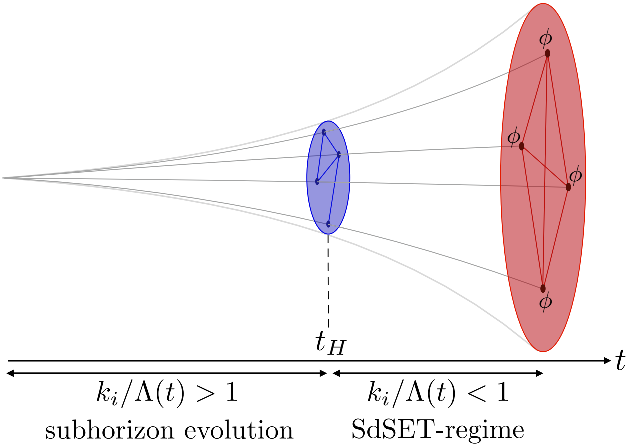

- While the fields evolve in time after t_H, they can still interact via modes with momenta $\vec{l}$ which satisfy $l/\Lambda(t) > 1$. These contributions cannot be described by the dynamical d.o.f. of the EFT and are reproduced by the Wilson coefficients.

Structure of SdSET

In a bit more detail: SdSET is defined at leading power in λ by the action

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Both the Ξ_n and c_n are determined by matching computations: compute quantity in the full theory, compute analogous quantity in the EFT, the difference determines Ξ_n and c_n.

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First steps:

- Since in dS there is no consensus on a good regulator for divergent loop integrals, need to define regularization and renormalization schemes both in the full theory and EFT. Introduced an analytic regulator via

$$\phi(t,\vec{x}) \equiv \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left(\frac{k}{a(t)\mu}\right)^{-\varepsilon} \phi(t,\vec{k})\,,$$

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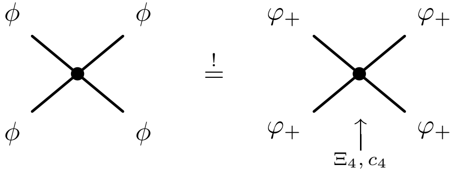
  to regularize both UV- and IR-divergences, analogously for EFT fields.
- Opportunity to apply tools from flat-space computations to cosmological correlators. In particular: applied the method of regions [Beneke, Smirnov 1997] to simplify computation of correlators in the late-time limit and elucidate the origin of the contributions to the Ξ_n and c_n.

Matching massless ϕ^4-theory onto SdSET

Start with tree-level matching: match tree-level four-point function onto SdSET



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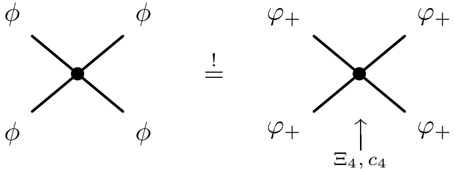

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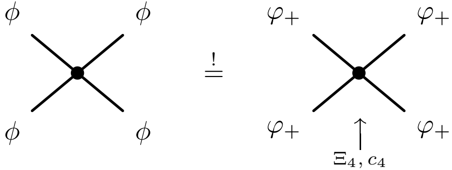

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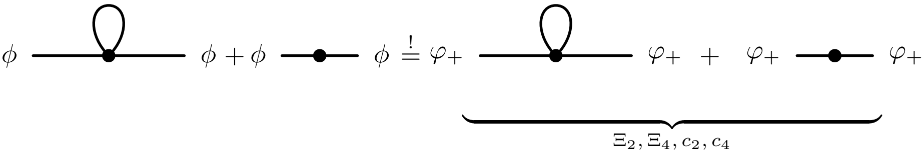

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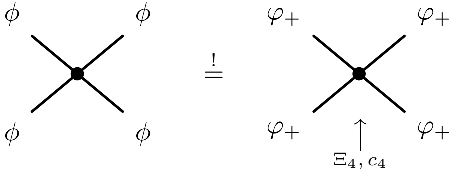

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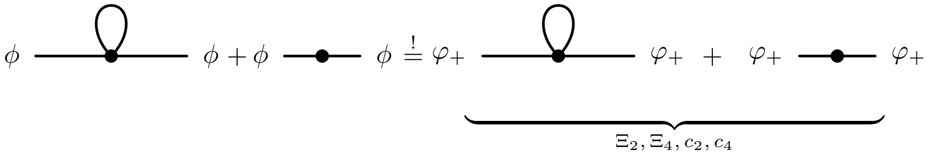

Ξ_4, c_4 already determined, need to be able to reproduce full result only using Ξ_2, c_2 $\Rightarrow$ first consistency check ✓.

Matching massless ϕ^4-theory onto SdSET

Finally, matching of composite-operator correlation function

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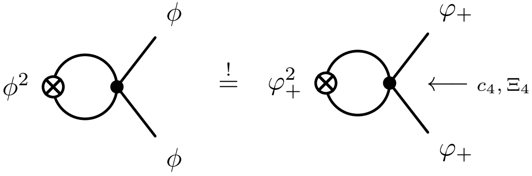

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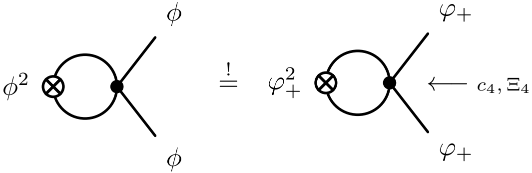

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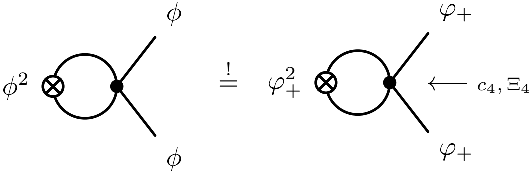

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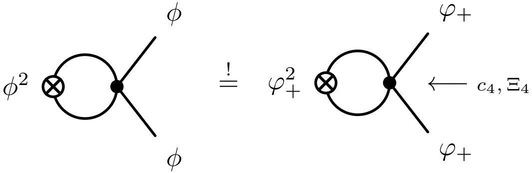

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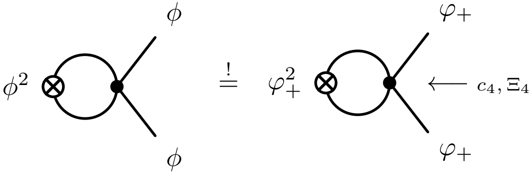

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$\Rightarrow$ Would like to derive this rigorously from an RGE analysis and put the framework on a solid QFT footing.

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- We carried out one-loop matching computations of massless ϕ^4-theory onto SdSET as a consistency check on the framework. This necessitated the introduction of a regularization and renormalization scheme both for the full and effective theories.
- To simplify full-theory calculations and elucidate the origin of the contributions to Wilson coefficients and IC's we were able to use the method of regions.

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Thank you for your attention!

Backup

Approximately exponential inflation of spacetime occurs if

$$V(\phi_0) \gg \left(\frac{\partial \phi_0}{\partial t}\right)^2 .$$

$\Rightarrow \phi_0$ "slowly rolls down" a flat potential V.