A unique $\mathbb{Z}_4^R$ for the MSSM

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in collaboration with
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Discrete Anomalies

$\mathbb{Z}_\mathbf{N}$ anomalies

The setup

- gauge theory with simple gauge group G_{gauge}
- a $\mathbb{Z}_N$ symmetry
- fields $\psi^{(i)}$ in irreducible representation $\mathbf{r}^{(i)}$ of G_{gauge}; $\mathbb{Z}_N$ charge $q^{(i)}$

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Under a $\mathbb{Z}_N$ transformation

$$\psi^{(i)} \to \exp\left(\frac{2\pi \mathrm{i}}{N} q^{(i)}\right) \psi^{(i)}$$

$$\mathcal{D}\Psi\mathcal{D}\bar{\Psi} \to \exp\left(\frac{2\pi \mathrm{i}}{N} n_\text{gauge} \sum_i q^{(i)}\, 2\ell(\mathbf{r}^{(i)})\right) \mathcal{D}\Psi\mathcal{D}\bar{\Psi}$$

T. Araki (2007)

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$$n_{\text{gauge}} \in \mathbb{Z}$$
$$n_{\text{gauge}} \sim \int \mathrm{d}^4x\, F\tilde{F}$$

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Dynkin index

factor 2 comes from normalization $\ell(\mathbf{M}) = \frac{1}{2}$ for $\mathrm{SU}(M)$

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Anomaly condition: $\sum_i q^{(i)} \ell(\mathbf{r}^{(i)}) = 0 \mod \eta$

where $\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$

GS anomaly cancellation

- One can absorb the change of the path integral measure in a change of Lagrangian

$$\Delta\mathscr{L}_{\text{anomaly}} = \sum_G \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a \, A_{G-G-\mathbb{Z}_N} - \frac{\alpha}{384\pi^2} \mathcal{R}\widetilde{\mathcal{R}} \, A_{\text{grav}-\text{grav}-\mathbb{Z}_N}$$

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anomaly coefficients

- Provided the Lagrangian also includes axion couplings

$$\mathscr{L} \supset -\frac{a}{8} F^a \widetilde{F}^a + \frac{a}{4} \mathcal{R}\widetilde{\mathcal{R}}$$

$\Delta\mathscr{L}_{\text{anomaly}}$ can be compensated by a shift of the axion a
if the anomaly coefficients are universal Green & Schwarz (1984)

$$A_i = \rho \mod \eta$$

R-symmetries

- *R*-symmetries do not commute with supersymmetry
- different component fields of a supermultiplet carry different *R*-charge
- in superfield notation: superspace coordinate θ carries *R*-charge 1

$$\Phi = \phi + \sqrt{2}\,\theta\psi + \theta\theta F\,,$$

$$V = -\theta\sigma^\mu\bar{\theta}\,A_\mu + \mathrm{i}\theta\bar{\theta}\bar{\theta}\bar{\lambda} - \mathrm{i}\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D\,.$$

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- also gauginos λ carry *R*-charge
- superpotential $\mathscr{W}$ carries *R*-charge 2 because

$$\mathscr{L} \supset \int \mathrm{d}^2\theta\,\mathscr{W}$$

bottom-line:

For *R*-symmetries also gauginos and superpotential carry *R*-charge.

Application to the MSSM

MSSM: good features and open questions

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- However:
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  - ☹ dimension four and five proton decay operators
  - ☹ CP and flavor problems
- Supersymmetry alone seems not to be enough

Problems of the MSSM

Superpotenial up to order 4

$$
\begin{aligned}
\mathscr{W} = {} & \mu\, h_u h_d + \mu_i'\, h_u \ell_i \\
& + (Y_u)^{ij}\, q_i h_u \bar{u}_j + (Y_d)^{ij}\, q_i h_d \bar{d}_j + (Y_e)^{ij}\, \ell_i h_d \bar{e}_j \\
& + \lambda^{(1)}_{ijk}\, \ell_i \ell_j \bar{e}_k + \lambda^{(2)}_{ijk}\, q_i \bar{d}_j \ell_k + \lambda^{(3)}_{ijk}\, \bar{u}_i \bar{d}_j \bar{d}_k \\
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μ-problem:
Why is μ around the weak scale?

$$m_Z \sim \mu$$

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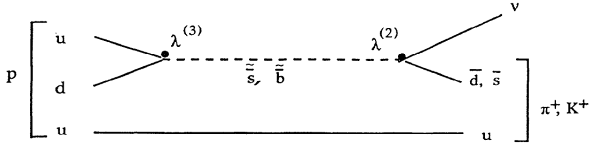
baryon- and lepton number violation:



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Usually assumed: matter parity

	q_i	$\bar{u}_i$	$\bar{d}_i$	ℓ_i	$\bar{e}_i$	h_u	h_d
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0

Farrar, Fayet (1978) and Dimopoulos, Raby, Wilczek (1982)

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proton hexality

	q_i	$\bar{u}_i$	$\bar{d}_i$	ℓ_i	$\bar{e}_i$	h_u	h_d
$\mathbb{Z}_6^{\mathcal{P}}$	0	1	5	4	1	5	1

Dreiner, Luhn, Thormeier (2006)

Comments on proton hexality

Appealing features

- forbids dimension four and five proton decay operators
- has matter parity as a subgroup
- allows Yukawa couplings, Weinberg operator neutrino mass operator $\ell\, h_u\, \ell\, h_u$ and the μ-term
- unique, anomaly-free symmetry with these features

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Drawbacks

- inconsistent with grand unification (universal charges for all matter fields)
- does not address the μ-problem

What are we looking for?

A discrete abelian symmetry which

- is anomaly-free (possibly with Green-Schwarz mechanism)
- allows Yukawa couplings, Weinberg operator neutrino mass operator $\ell\, h_u\, \ell\, h_u$
- forbids the μ-term (at the perturbative level)

We will show:
There is a unique $\mathbb{Z}_4^R$ with these features.

Claim 1: It has to be an R-symmetry

- Notation of $\mathbb{Z}_N$ charge: quarks and leptons q, Higgses q_{h_u}, q_{h_d}
- Anomaly coefficients

$$\mathrm{SU}(3) - \mathrm{SU}(3) - \mathbb{Z}_N : \quad A_3 = 6q$$

$$\mathrm{SU}(2) - \mathrm{SU}(2) - \mathbb{Z}_N : \quad A_2 = 6q + \frac{1}{2}(q_{h_u} + q_{h_d})$$

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bottom-line:

The μ-term is always allowed for anomaly-free, non-R symmetries!

Claim 2: Higgs discrete charges have to vanish

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- u- and d-type Yukawas allowed requires that

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Claim 3: The order has to be 4

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$$N = 4$$

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	q_i	$\bar{u}_i$	$\bar{d}_i$	ℓ_i	$\bar{e}_i$	h_u	h_d
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0

A unique $\mathbb{Z}_4^R$ for the MSSM

- We know: $\mathbb{Z}_4^R$ where Higgses have charge zero
- To allow Yukawa couplings: quarks and leptons have charge one

	q_i	$\bar{u}_i$	$\bar{d}_i$	ℓ_i	$\bar{e}_i$	h_u	h_d
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0

Superpotenial up to order 4

$$\begin{aligned}\mathscr{W} = {} & \mu\, h_u h_d + \mu_i'\, h_u \ell_i \\ & + (Y_u)^{ij}\, q_i h_u \bar{u}_j + (Y_d)^{ij}\, q_i h_d \bar{d}_j + (Y_e)^{ij}\, \ell_i h_d \bar{e}_j \\ & + \lambda^{(1)}_{ijk}\, \ell_i \ell_j \bar{e}_k + \lambda^{(2)}_{ijk}\, q_i \bar{d}_j \ell_k + \lambda^{(3)}_{ijk}\, \bar{u}_i \bar{d}_j \bar{d}_k \\ & + \kappa^{(1)}_{ijkl}\, q_i q_j q_k \ell_l + \kappa^{(2)}_{ijkl}\, \bar{u}_i \bar{u}_j \bar{d}_k \bar{e}_l + \kappa^{(3)}_{ijk}\, q_i q_j q_k h_d + \kappa^{(4)}_{ijk}\, q_i \bar{u}_j \bar{e}_k h_d \\ & + \kappa^{(5)}_{ij} \ell_i \ell_j h_u h_u\end{aligned}$$

Non-perturbative violation of $\mathbb{Z}_4^R$

- In orbifold GUTs $h_u\, h_d$ is proportional to $\langle \mathscr{W} \rangle$ we will get a holomorphic contribution to the μ term of the right order

$$\mu \sim \frac{\langle \mathscr{W} \rangle}{M_{\mathrm{P}}^2} \simeq m_{3/2}$$

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- Whatever gives us $\langle \mathscr{W} \rangle$ will be a measure for $\mathbb{Z}_4^R$ breaking
- for instance, one may replace/describe hidden sector superpotential by gaugino condensate Nilles (1982)

$$\langle \mathscr{W} \rangle \simeq \langle \lambda\lambda \rangle \simeq \Lambda^3$$

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- Dimension 5 proton decay operators will have highly suppressed coefficients

$$\mathscr{W}_{QQQL}^{\mathrm{np}} \;\sim\; \frac{\langle\mathscr{W}\rangle}{M_\mathrm{P}^4} Q\,Q\,Q\,L \;\sim\; \frac{m_{3/2}}{M_\mathrm{P}}\,\frac{1}{M_\mathrm{P}} Q\,Q\,Q\,L \;\sim\; 10^{-15}\,\frac{1}{M_\mathrm{P}} Q\,Q\,Q\,L$$

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- No R parity violation because $\mathbb{Z}_4^R$ has a non-anomalous subgroup which is equivalent to matter parity

Summary

- Green-Schwarz mechanism works also for discrete symmetries
- We found a unique $\mathbb{Z}_4^R$ in the MSSM which
  - is anomaly-free (with Green-Schwarz)
  - forbids all dimension four nucleon decay operators
  - is compatible with grand unification
  - has matter parity as an anomaly-free subgroup
  - suppresses dimension five nucleon decay operators
  - provides a solution to the μ problem

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Thank You!