

Quark mass hierarchies in heterotic orbifold GUTs

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Motivation

Bottom up meets top down

GUTs in extra dimensions

Heterotic orbifold compactifications

Yukawa couplings and quark masses

Summary

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up quark	$1.5 - 3.3 \text{ MeV}$
charm quark	$1.27 \left(\begin{smallmatrix} +0.07 \\ -0.11 \end{smallmatrix} \right) \text{ GeV}$
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Topic of this talk: Quark masses in heterotic orbifold GUTs

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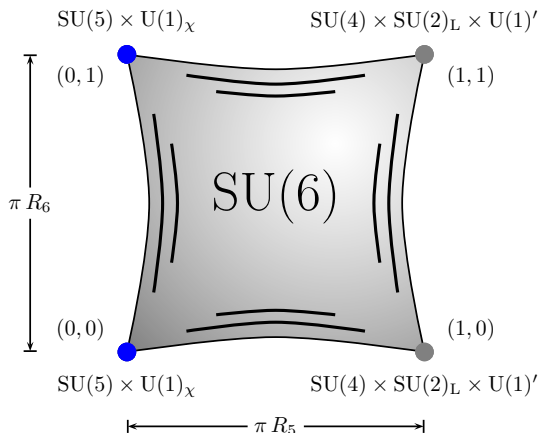
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- Orbifolds are the 'simplest' extra dimensional setup with chiral spectrum

Bottom up: A GUT in six dimensions

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- Higher dimensional GUT with $\mathcal{M}^4 \times \mathbb{T}^2/\mathbb{Z}_2$ geometry



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This setup is called heterotic mini-landscape

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Can we predict the quark masses in this setup?

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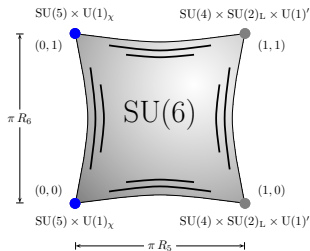
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- Strategy: Take the CFT results to calculate the ratios between the quark masses

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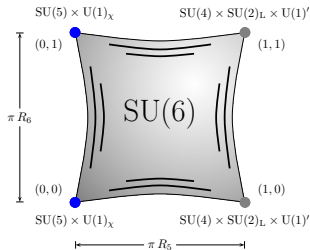
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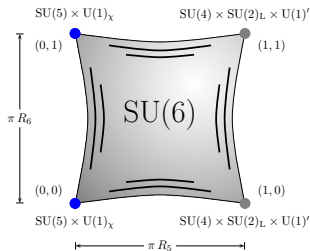


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- Localization of the particles is predicted by string theory
- Two families live at fixed points, the third family lives in the bulk

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- Let us focus on a specific model from the mini-landscape

$$Y_U = \begin{pmatrix} e^{-\frac{\pi}{4} R_6^2 A(\tilde{s})} & \frac{\langle s_{11}^0 \rangle}{\langle s_5^0 \rangle} e^{-\frac{\pi}{4} R_6^2 A(\tilde{s})} & \frac{\langle s_6^0 \rangle \langle s_{11}^0 \rangle}{\langle s_5^0 \rangle} e^{-\frac{\pi}{4} R_6^2 A(\tilde{s})} \\ \frac{\langle s_{11}^0 \rangle}{\langle s_5^0 \rangle} e^{-\frac{\pi}{4} R_6^2 A(\tilde{s})} & A(\tilde{s}) & \langle s_6^0 \rangle A(\tilde{s}) \\ \langle s_9^0 \rangle e^{-\frac{\pi}{4} R_6^2 A(\tilde{s})} & \frac{\langle s_9^0 \rangle \langle s_{11}^0 \rangle}{\langle s_5^0 \rangle} e^{-\frac{\pi}{4} R_6^2 A(\tilde{s})} & y_t \end{pmatrix}$$

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- The quantity $A(\tilde{s})$ is hard to compute but

$$\frac{m_u}{m_c} \approx e^{-\frac{\pi}{4}R_6^2}, \quad \frac{m_c}{m_t} \approx A(\tilde{s})$$

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- We get a prediction for R_6 !

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$$\frac{m_d}{m_s} \approx \frac{62}{63}, \quad \frac{m_c}{m_t} \approx e^{-\frac{\pi}{4} R_6^2} \frac{126}{125} \frac{D(\tilde{s})}{U(\tilde{s})}$$

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- Other models in the heterotic mini-landscape can also be tested

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Thank you for your attention!