# The role of the Higgs radial mode as a UV completion of the non linear sigma model.

Master thesis project

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# Introduction

## NGB EFT Lagrangian: Weakly or strongly coupled?

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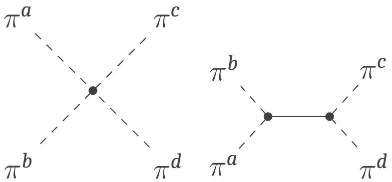

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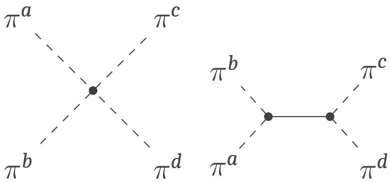
  - *Answer*: Adding by hand new degrees of freedom: NL$\Sigma$M$\to$L$\Sigma$M;



# Preliminary concepts

## Coset construction!

- Let's take for example the $N_f = 2$ QCD-like theory with SSB $SU(2)_L \times SU(2)_R / SU(2)_V$:

$$\Sigma = \exp\left(\frac{2i}{f}\pi^a X^a\right), \quad \Sigma \xrightarrow{\mathcal{G}} L\Sigma R^\dagger, \quad \Sigma\Sigma^\dagger = \mathbb{I}, \quad \det\Sigma = 1$$

$$\mathcal{L} = \frac{f^2}{4}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) \quad \Rightarrow \quad \mathcal{L} = \frac{1}{2}(\partial_\mu \pi^a)^2 + \frac{1}{f^2}\pi^a \partial_\mu \pi^b \pi^c \partial^\mu \pi^d f^{ab\gamma} f^{cd\gamma} + \dots$$

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- $\exists \Lambda^* \sim 4\pi f$ above which the EFT approach fails.
- $\Sigma$ has not enough d.o.f. to provide a linear irreducible representation of $\mathcal{G}$.

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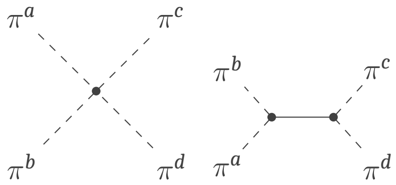
allow us to construct a linear irreps of $\mathcal{G}$ just adding a singlet extra radial mode, "curing" the energy growing behaviour of the scattering amplitudes and making the theory weakly coupled at high energies.
- **Notice the link:**

Linear irreps of $\mathcal{G} \Longleftrightarrow$ Renormalizability $\Longleftrightarrow$ Weakly coupled theory.

# Examples

## Standard Model and two flavours QCD-like theory.

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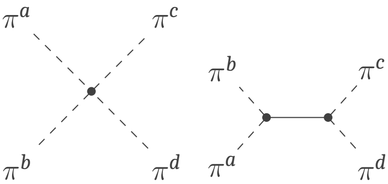


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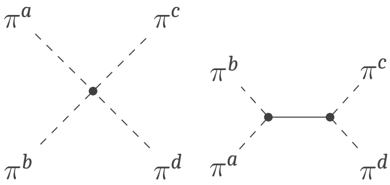


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- Two possible scenarios:
  - Non Linear Sigma Model just as an EFT (ex: QCD, no other resonances intervene to restore perturbative unitarity) $\Rightarrow$ The theory remain strongly coupled.
  - $SO(4)/SO(3)$ custodial symmetry for the EW sector in the SM, the Higgs boson radial mode *unitarize* the SM amplitudes as the WW-scattering $\Rightarrow$ The theory became weakly coupled.

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  - Finding the minimal number of degrees of freedom necessary to *unitarize* the theory.

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- $\tilde{\Sigma} \xrightarrow{\mathcal{G}} UL\tilde{\Sigma}R^\dagger, \quad U \in U(1), L \in SU(N)_L, R \in SU(N)_R$
- We perform the chiral perturbation theory of the Non Linear Sigma Model

$$\mathcal{L}_{nl\sigma m} = \frac{1}{2}(\partial_\mu \pi^a)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \frac{\tilde{C}_\pi}{6f^2}[(\pi^a \partial_\mu \pi^a)^2 - \pi^a \pi^a \partial_\mu \pi^b \partial_\mu \pi^b] + \dots$$

$$\mathcal{A}(ab \rightarrow cd) = i\tilde{C}_\pi \left( \delta^{ab}\delta^{cd}\frac{s}{f^2} + \delta^{ac}\delta^{bd}\frac{t}{f^2} + \delta^{ad}\delta^{bc}\frac{u}{f^2} \right). \qquad \boxed{\tilde{C}_\pi = \frac{N}{N^2 - 2}}$$

# Linear Sigma Model



- Without the pseudo-reality condition, $\mathscr{H}$ contains more then one singlet radial mode and the more general linear irreps could be found in $\mathscr{H} \sim \mathrm{Mat}(N, \mathbb{C}) \Rightarrow 2N^2_{\mathbb{R}}$ d.o.f.

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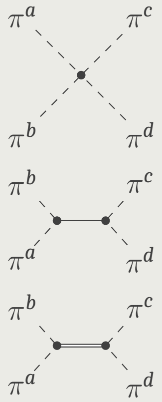
- Using polar decomposition one can find an useful parametrization of $\mathscr{H}$:

$$\mathscr{H} = \tilde{\Sigma}(x)\phi(x) = e^{\frac{2i}{f}\pi^a X^a} e^{\frac{i}{f}\eta} \left(1 + \frac{H^a(x)}{f} X^a\right)(f + h(x)).$$

# Linear sigma model framework



## Feynmann Diagrams




## Lagrangian

$$\mathcal{L} = \underbrace{\frac{1}{4} Tr[\partial_\mu \mathscr{H}^\dagger \partial^\mu \mathscr{H}]}_{(1)} - \underbrace{V(\mathscr{H}\mathscr{H}^\dagger)}_{(2)} + \underbrace{\mathrm{Pol}(\det \mathscr{H})}_{(3)}$$

$$(1) \Rightarrow \mathcal{L}_{4\pi} + \mathcal{L}_{h\pi} + \mathcal{L}_{H\pi}$$

$(2) \Rightarrow$ Gives information on $m_h^2$ and $m_H^2$.

$(3) \Rightarrow$ Explicitly breaking of the $U(1)$ abelian symmetry $\Rightarrow$ mass contribution to the $\eta$ field.

- Looking at the two to two $\pi\pi$ scattering we notice that

$$\mathcal{A}_{tot} = \mathcal{A}_{4\pi} + \mathcal{A}_h + \mathcal{A}_H$$

$$\mathcal{A}^{\text{tot}}(\pi^a\pi^b \to \pi^c\pi^d) = \frac{i}{f^2}(\tilde{C}_\pi - \tilde{C}_h - \tilde{C}_H)\left(\delta^{ab}\delta^{cd}s + \delta^{ac}\delta^{bd}t + \delta^{ad}\delta^{bc}u\right)$$

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- Finally we arrive to the main result:

$$\boxed{\tilde{C}_\pi + \tilde{C}_h + \tilde{C}_H = 0}$$

**ALL THE HIGGS RADIAL MODES ARE INVOLVED IN RESTORING PERTURBATIVE UNITARITY.**

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# Conclusions.



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  - *Warning*: Those theories manifest, as well as the Standard Model, the hierarchy problem.
- Our studies provides *new model building tools* to extend the UV completion throught the L$\Sigma$M procedure for more general SSB patterns.

# The role of the Higgs radial mode as a UV completion of the non linear sigma model.

*Thank you for listening!*

*Any questions?*