Review of event-shape measurements of α_s

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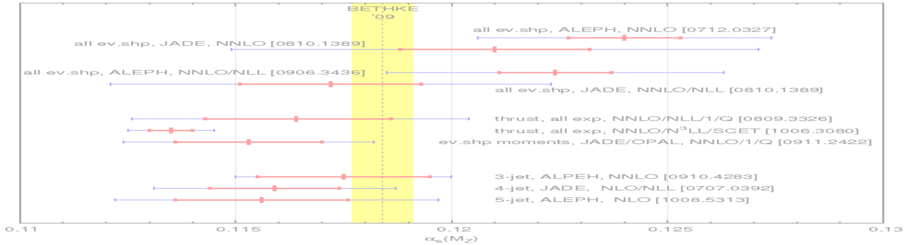
CERN, Princeton & LPTHE/CNRS (Paris)

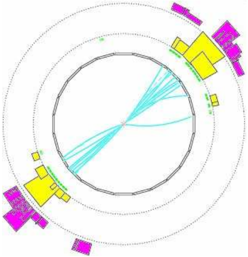
Workshop on precision measurements of α_s
Munich, Germany, 9–11 February 2011



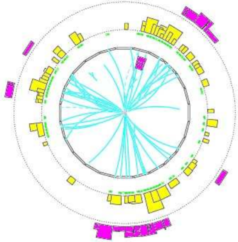
e^+e^- Event Shapes: Thrust

Various proposals to measure shape of events. Most famous example is **Thrust**:

$$T = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i.\vec{n}_T|}{\sum_i |\vec{p}_i|} \,,$$



2-jet event: $T \simeq 1$



3-jet event: $T \simeq 2/3$

Fraction of events with $T \simeq 2/3$ is $\propto \alpha_s$

Other widely-used event shapes

"Jet" masses: $M_i^2 = \left(\sum_{k\in H_i} p_k\right)^2, \; i = 1, 2;$

- Heavy-jet mass $\rho \equiv M_H^2 = \frac{\max(M_1^2, M_2^2)}{E_{vis}^2}$

Broadenings: $B_i = \frac{\sum_{k\in H_i} |\vec{p}_k \times \vec{n}_T|}{2\sum_j |\vec{p}_j|}, \; i = 1, 2;$

- Total broadening: $B_T = B_1 + B_2$
- Wide-jet broadening: $B_W = \max(B_1, B + 2)$

C-parameter: $C = \frac{3}{8}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$

Jet rates: most often defined using the k_t algorithm, as a function of jet resolution parameter $y_{\text{cut}} \sim k_t^2/Q^2$

NB: n-jet rate with $n > 3$ sensitive to α_s^{n-1}, i.e. enhanced sensitivity to α_s.

Data sources (e^+e^-)

4 LEP experiments (ALEPH, DELPHI, L3, OPAL):

- LEP 1: very high statistics at $Q = M_Z$
- LEP 2: modest statistics at energies from $133 \ldots 207$ GeV
- Events with radiated photon giving effective hadronic $Q < M_Z$

Use of these data as "pure QCD with lower Q" is of arguable legitimacy

SLD @ SLAC:

- very high statistics at $Q = M_Z$

JADE @ PETRA:

- good statistics from $14 \ldots 44$ GeV
- data reanalysed with modern tools and variables

TASSO, AMY, etc.:

- older data covering $14 \ldots 55$ GeV

Also DIS event shapes from HERA

Won't discuss here.

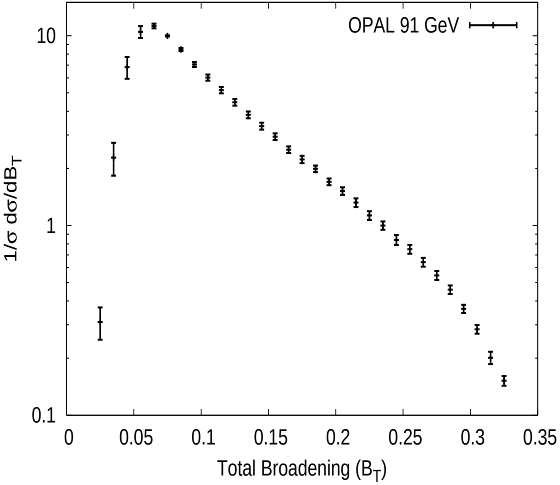
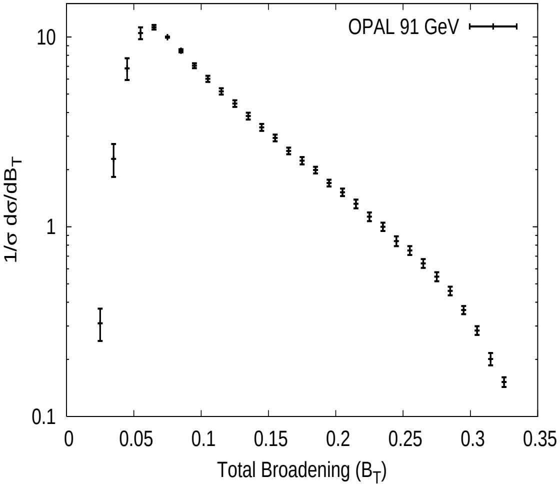


ILLUSTRATION OF QUALITY OF DATA

Extracting α_s from event shapes



Some typical kinds of ingredient

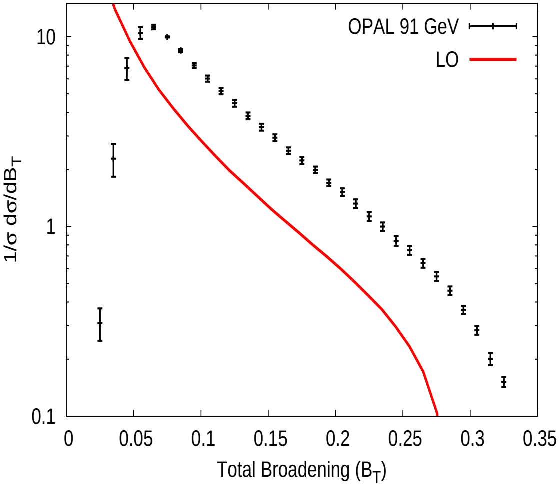
Fixed order calculation:

- LO: $\frac{d\sigma}{dB_T} = A_1(B_T)\alpha_s$
- NLO: add $A_2(B_T)\alpha_s^2$

Resummation of $\alpha_s^n \ln^{2n} \frac{1}{B_T}$

Hadronisation corrections

Extracting α_s from event shapes



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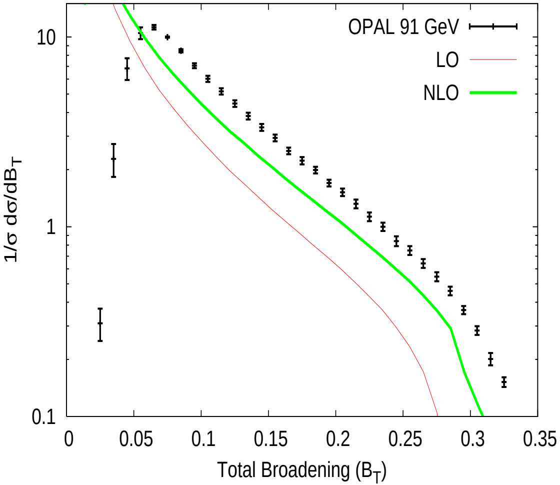

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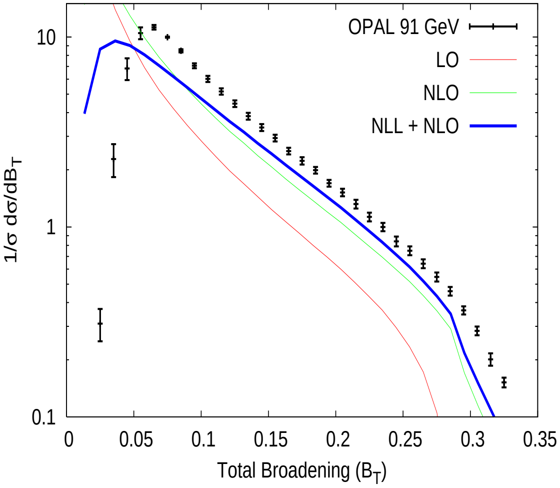

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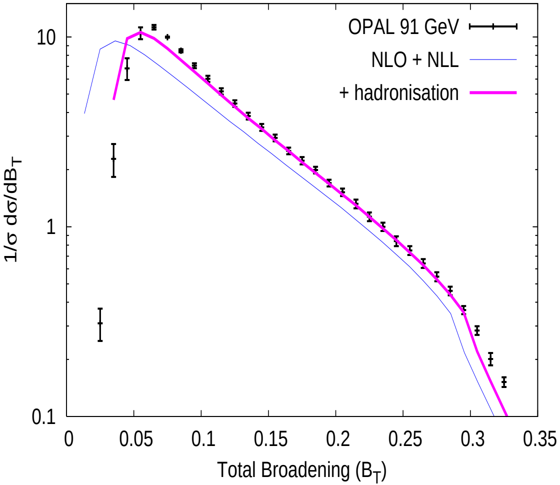

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Hadronisation corrections

What's happened recently that's relevant for α_s?

1. NNLO calculations for 3-jet type observables

Gehrmann–De Ridder et al '07; Weinzierl '08

2. More accurate resummation (N^3LL) for thrust and heavy-jet mass in SCET

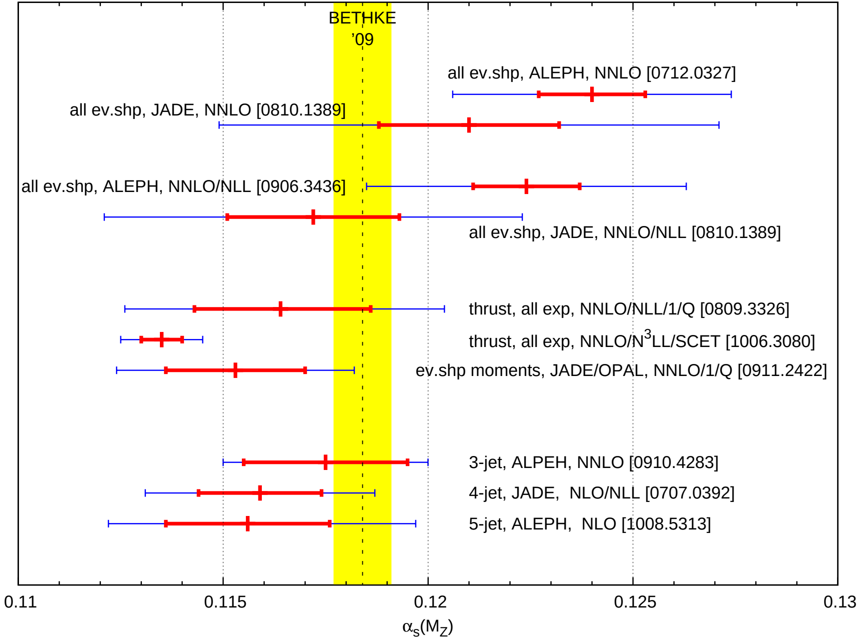
Becher & Schwartz '08; Chien & Schwartz '10

3. NLO calculation of 5-jet rate

Frederix et al '10

4. Lots of phenomenology

Overview of recent fits



What choices go into a fit?

Data choices

- One experiment (and one energy): simpler combination of systematics
- Many experiments / many energies: more data, lever-arm in $\sqrt{s}$, etc.
- One "excellent" observable? Or many, as cross check of systematics?

Resummation

- Traditional resummation, available for many observables, but only NLL?
- Or SCET N^3LL, available only for thrust and heavy-jet mass?
- Or no resummation at all?

Hadronisation

- Do you correct for it?
- Do you estimate it from Monte Carlos?
- Or from analytical models?

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Re

Often the choices are entangled with each other

- Or SCET N^{3}LL, available only for thrust and heavy-jet mass?
- Or no resummation at all?

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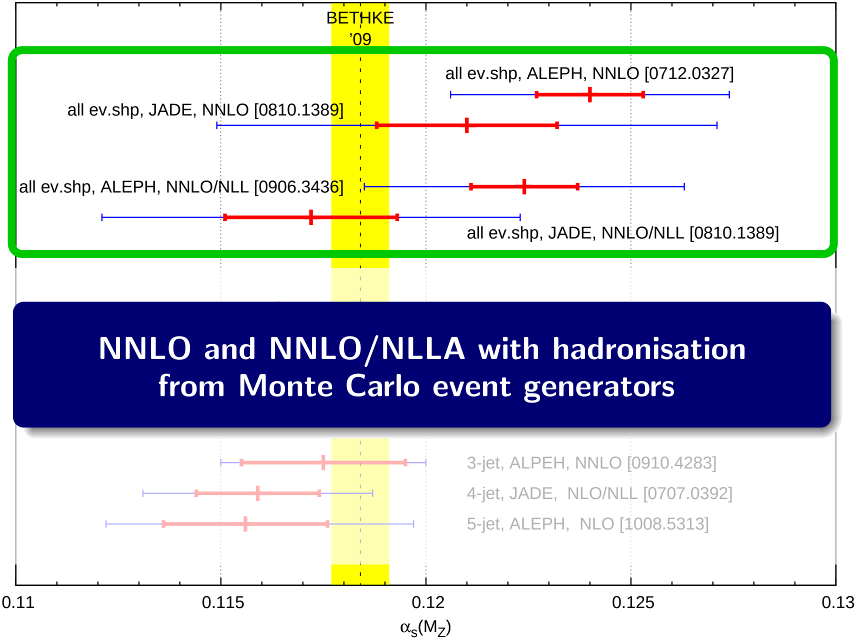
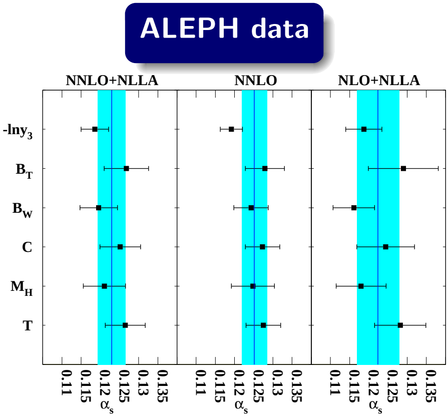
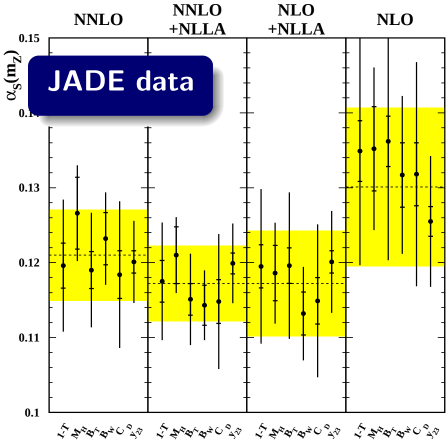


NNLO and NNLO/NLLA with hadronisation from Monte Carlo event generators

NNLO,	ALEPH	$0.1240 \pm .0008_{\rm stat} \pm .0010_{\rm exp} \pm .0011_{\rm had} \pm .0029_{\rm theo}$
NNLO	JADE	$0.1210 \pm .0007_{\rm stat} \pm .0021_{\rm exp} \pm .0044_{\rm had} \pm .0036_{\rm theo}$
NNLO/NLL,	ALEPH	$0.1224 \pm .0009_{\rm stat} \pm .0009_{\rm exp} \pm .0012_{\rm had} \pm .0035_{\rm theo}$
NNLO/NLL,	JADE	$0.1172 \pm .0006_{\rm stat} \pm .0020_{\rm exp} \pm .0035_{\rm had} \pm .0030_{\rm theo}$



Dissertori et al '07, '08



Bethke et al '08

In these results, an $\mathcal{O}(5 - 10\%)$ contribution to α_s comes from **hadronisation corrections**.

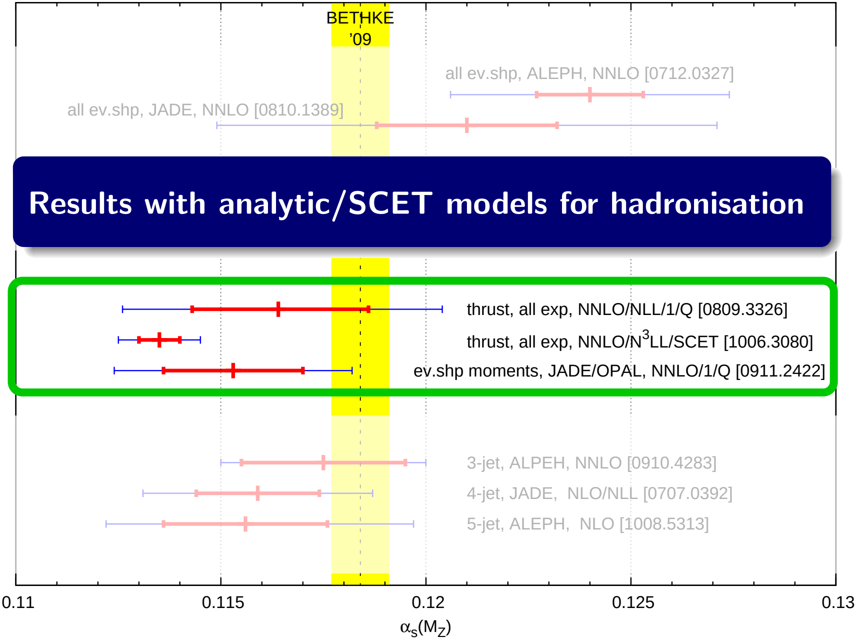
Determined by taking Pythia/Herwig/Ariadne MCs and looking at ratio of parton-level event-shape distribution to hadron-level distribution.

But who says MC parton level is in any way related to NNLO/NNLA parton level?

E.g. in MC's $\propto$ radiation $\propto \alpha_s(p_t)$ with a 1 GeV cutoff

In NNLO, radiation $\propto \alpha_s(\mu) + b_0 \alpha_s^2(\mu) \ln \mu / p_t + \dots$ with no cutoff

Results with analytic/SCET models for hadronisation



Effective coupling hadronisation

What matters for thrust (etc.) is how hadronsiation modifies the soft energy flow at large angles. Parametrise with effective coupling

$$\alpha_0 = \frac{1}{\mu_I}\int_0^{\mu_I} d\mu\, \alpha_{\text{eff}}(\mu)$$

Argue thrust shifted by amount proportional to α_0

$$\delta T = -\frac{8C_F\mathcal{M}}{\pi^2}\frac{\mu_I}{Q}\Bigg[\alpha_0 - \underbrace{\alpha_s(\mu_R) - \alpha_s^2(\mu_R^2)\left(\ln\frac{\mu_R}{\mu_I} + \ldots\right) - \alpha_s^3(\mu_R^2)\,(\ldots)}_{\text{subtract "perturbative" NNLO infrared piece}}\Bigg]$$

i.e.

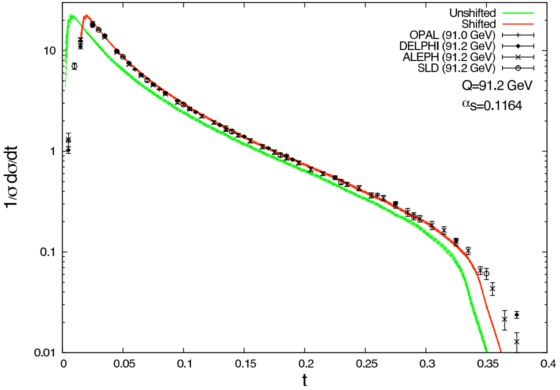
$$\frac{d\sigma}{dT}(T) \to \frac{d\sigma}{dT}(T - \delta T)$$

Dokshitzer & Webber ’95, ’97 + numerous related contributions
[Possibly] Valid in 2-jet limit; but seems to work also near 3-jet limit

NLO/NLL + power-corrections for thrust

$$\alpha_s(M_Z) = 0.1164 \pm 0.0022_{\mathrm{exp}} \pm 0.0031_{\mathrm{theo}}$$

Davison & Webber 0809.3326



Results based on all LEP and low-energy data

$\chi^2/d.o.f. = 1.09$

Experimental error corresponds to $\delta\chi^2 \simeq 14$ on grounds that fit not perfect.

SCET results

N^3LL resummation recently performed in context of Soft Collinear Effective Theory (SCET) for thrust and heavy-jet mass

Becher & Schwartz '08; Chien & Schwartz '10

SCET also argued to provide operator definition of hadronisation and rigorous factorisation (including subtraction terms shown for effective coupling) Choice of which operators to keep/neglect requires skill?

SCET provides event-shape α_s extraction with by far the smallest errors:

$$\alpha_s(M_Z) = 0.1135 \pm 0.0002_{\text{expt}} \pm 0.0005_{\text{hadr}} \pm 0.0009_{\text{pert}}$$

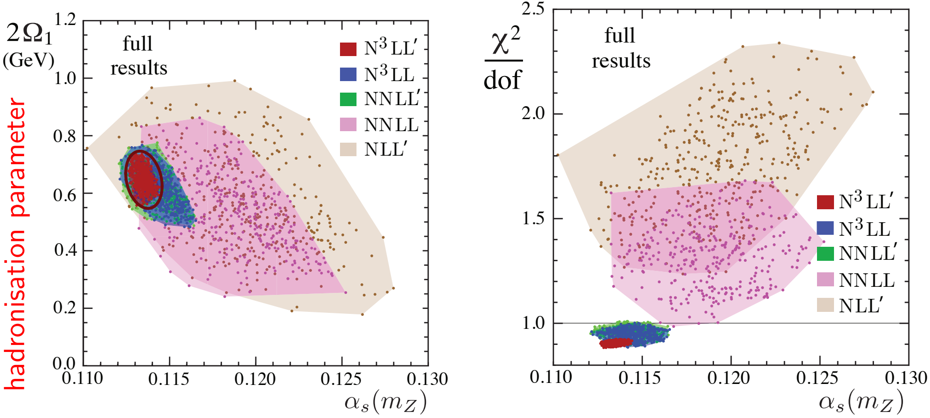
Abbate et al, arXiv:1006.3080

Davison & Webber had larger exp./hadr. err. (similar dataset); related to $\delta\chi^2$?

Not compatible with world average. Similar to *some* DIS extractions Is it corrrect?

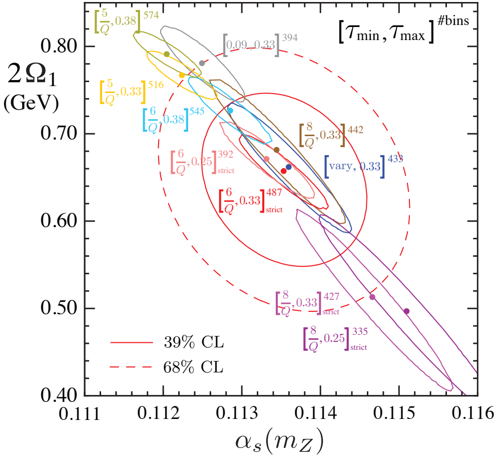
In favour of SCET fit

Good converge and fit quality



Questionnable aspects of SCET fit? [1]

Dependence on fit range



Suggestion of systematic dependence $\gtrsim$ quoted theory error.

Span of results is 0.003

Particular sensitivity to lower edge

What is basis for specific central choice?

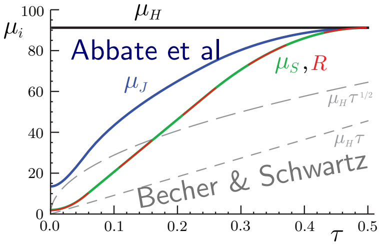
Questionnable aspects of SCET fit? [2]

Choice of "profile" function

Experiment	Energy	BS results [20]	our BS profile	default profile
ALEPH	91.2 GeV	0.1168(1)	0.1170	0.1223
ALEPH	133 GeV	0.1183(37)	0.1187	0.1235
ALEPH	161 GeV	0.1263(70)	0.1270	0.1328
• • •				
average		0.1172(10)	0.1180	0.1221
global fit (stat)	all Q		0.1188	0.1242
global fit (stat+syst)	all Q		0.1192	0.1245

TABLE VIII: Comparison of the results for $\alpha_s(m_Z)$ quoted by Becher and Schwartz in Ref. [20] with results we obtain from our adapted code [...] and employ their profile functions for the nonsingular, hard, jet and soft scales, with results shown in the column labeled "our BS profile". In the last column we show results with this same code, but using our default profile functions.

0.005 difference ≫ theory error



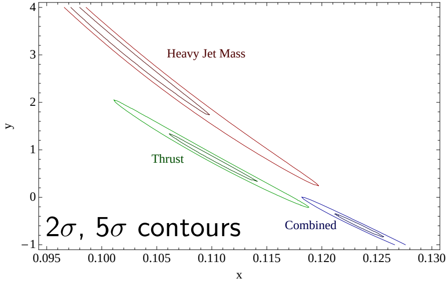
Profile function determines

- Weight of fixed-order v. resummation
- Scales in resummation [∼ modified logs of CTTW]

Schwartz agrees his orig. choice not right.
But is Abbate choice sufficiently broad?

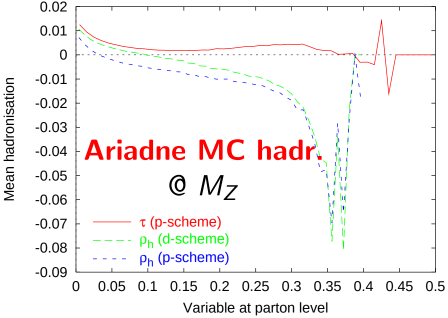
Questionnable aspects of SCET fit [3]? ρ v. T

hadronisation systematics?



Chien & Schwartz arXiv:1005.1644
But also Gardi & Rathsman '03

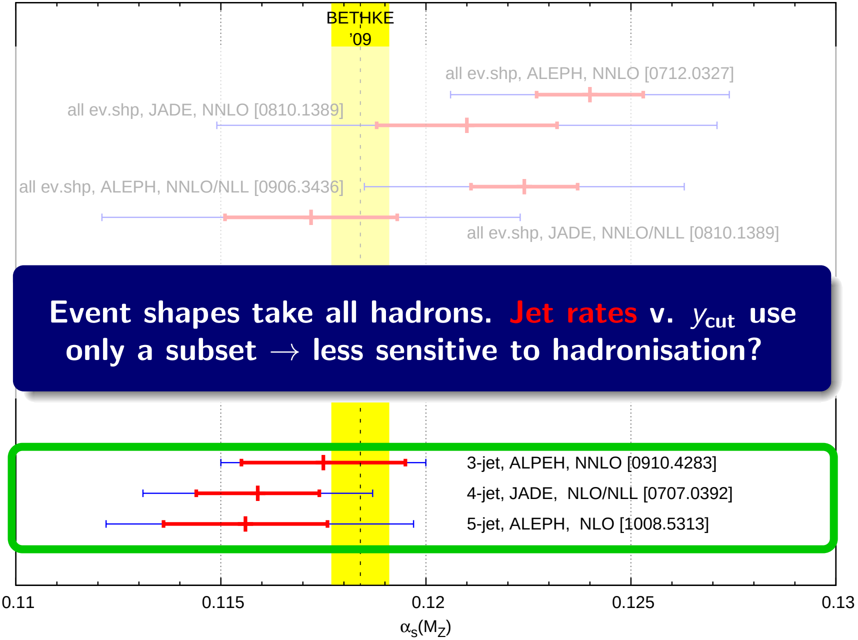
hadronisation ⟨shift⟩ v. τ, ρ



GPS & Wicke '01

- Jet masses' hadronisation very sensitive to π, K and p/n masses
- Heavy/wide observables are tricky (esp. for dists, higher moments).
- Do related systematics propage also into thrust?

Monte Carlo hadronisation gets these things "right"



Event shapes take all hadrons. Jet rates v. y_{cut} use only a subset → less sensitive to hadronisation?

α_s from 5-jet rate (pure NLO)

	LEP1, hadr. $\sigma_{\rm tot}^{-1} d\sigma/dy_{45}$, R_5	LEP1, no hadr. $\sigma_{\rm tot}^{-1} d\sigma/dy_{45}$, R_5
stat.	$^{+0.0002}_{-0.0002}$	$^{+0.0002}_{-0.0002}$
syst.	$^{+0.0027}_{-0.0029}$	$^{+0.0027}_{-0.0029}$
pert.	$^{+0.0062}_{-0.0043}$	$^{+0.0068}_{-0.0047}$
fit range	$^{+0.0014}_{-0.0014}$	$^{+0.0005}_{-0.0005}$
hadr.	$^{+0.0012}_{-0.0012}$	–
$\alpha_s(M_Z)$	$0.1159\,^{+0.0070}_{-0.0055}$	$0.1163\,^{+0.0073}_{-0.0055}$

Hadronisation with Sherpa (multijet-matched) is small effect.

➥**do not correct for it**

NB: depends on MC

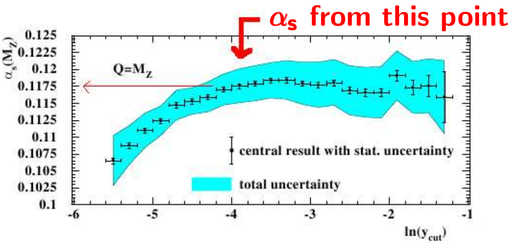
Final result with LEP1 & LEP2 data:

$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$$

Frederix et al arXiv:1008.5312

Table 2: Values of the strong coupling constant $\alpha_s(M_Z)$ obtained from fits to ALEPH LEP1 data for $\sigma_{\rm tot}^{-1} d\sigma/dy_{45}$ and R_5. NLO QCD predictions are used. Hadronization corrections are estimated with SHERPA. Default fit ranges are $3.8 \le -\ln y_{45} \le 5.2$, and $4.0 \le -\ln y_{\rm cut} \le 5.6$. See the text for details.

3-jet rate @ NNLO



3-jet rate as a function of y_{cut}

Measure α_s from point where

- determination stable
- systematics small

From ALEPH LEP 1 data:

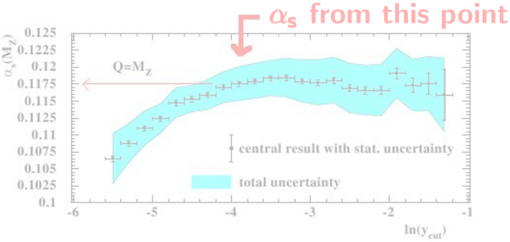
$$\alpha_s(M_Z) = 0.1175 \pm 0.0020_{exp} \pm 0.0015_{theo}$$

Dissertori et al., arXiv:0910.4283

- Aside from SCET analysis, this is measurement with smallest theory error
- Can exp. error be reduced by combining other LEP experiments' data?
- Total effect of hadronisation is 0.0015. Adding this (conservatively) as hadroniation error, get $\alpha_s = 0.1175 \pm 0.0029$

Would other jet definitions (e.g. Cambridge) have yet smaller hadronisation?

3-jet rate @ NNLO



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- ... or
- ...
- ... hadronisation error, get $\alpha_s = 0.1175 \pm 0.0029$

Interesting combination of modest errors and simplicity of analysis

Would other jet definitions (e.g. Cambridge) have yet smaller hadronisation?

Conclusions

Situation with event shapes is not ideal

Benefits of NNLO somewhat counteracted by large hadronisation effects and limited perturbative convergence
Best "simple" analysis might be from jet rates?

SCET with N^3LL has small errors, but disturbing α_s.
How well controlled are the systematics?

EXTRAS

0910.4283: Dissertori et al; NNLO 3-jet rate

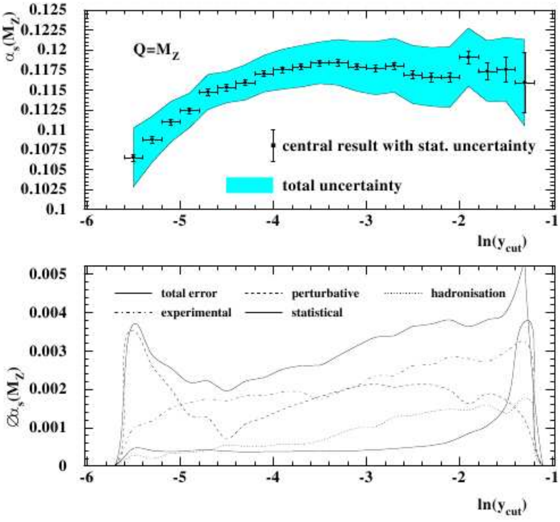


FIG. 1: Determinations of $\alpha_s(M_Z)$ from the three-jet rate, measured by ALEPH at the Z peak, for several values of the jet-resolution parameter $y_{\rm cut}$. The error bars show the

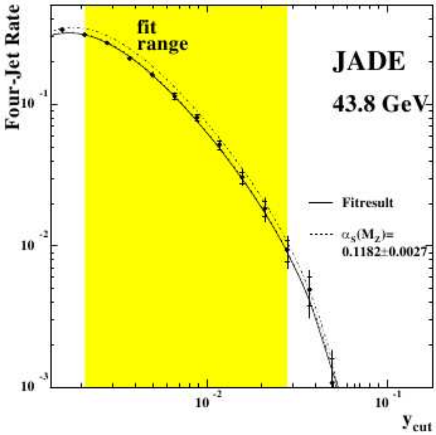
From LEP 1:

$$\alpha_s(M_Z) = 0.1175$$
$$\pm 0.0020_{\rm exp} \pm 0.0015_{\rm theo}$$

- Hadronisation is $\sim$ 5% correction
- α_s extracted at $y = 0.02$
- LEP2 consistent

0707.0392: Schieck et al/JADE; NLO+NLL 4-jet rate

$$\alpha_s(M_Z) = 0.1159 \pm 0.0004_{\text{stat}} \pm 0.0012_{\text{exp}} \pm 0.0024_{\text{had}} \pm 0.0007_{\text{theo}}$$



- Uses R_4 (exclusive?)
- fit ranges chosen by "requiring that hadronization corrections be less than 50% and the detector corrections be less than 50% in the fit range"

1008.5312: Frederix et al; NLO 5-jet rate

$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$$

	LEP1, hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$, R_5	LEP1, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$, R_5
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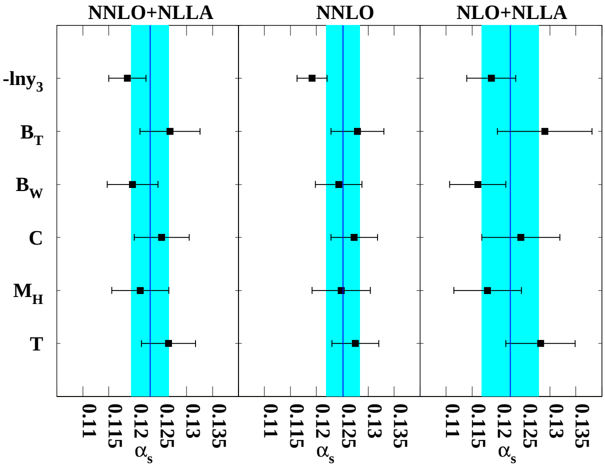
Table 2: Values of the strong coupling constant $\alpha_s(M_Z)$ obtained from fits to ALEPH LEP1 data for $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$ and R_5. NLO QCD predictions are used. Hadronization corrections are estimated with SHERPA. Default fit ranges are $3.8 \le -\ln y_{45} \le 5.2$, and $4.0 \le -\ln y_{\rm cut} \le 5.6$. See the text for details.

	LEP2, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$	LEP2, no hadr. R_5	LEP2, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$, R_5
stat.	$^{+0.0020}_{-0.0022}$	$^{+0.0022}_{-0.0025}$	$^{+0.0015}_{-0.0016}$
syst.	$^{+0.0008}_{-0.0009}$	$^{+0.0012}_{-0.0012}$	$^{+0.0008}_{-0.0008}$
pert.	$^{+0.0049}_{-0.0034}$	$^{+0.0029}_{-0.0020}$	$^{+0.0029}_{-0.0020}$
fit range	$^{+0.0038}_{-0.0038}$	$^{+0.0030}_{-0.0030}$	$^{+0.0028}_{-0.0028}$
$\alpha_s(M_Z)$	$0.1189^{+0.0066}_{-0.0057}$	$0.1120^{+0.0050}_{-0.0047}$	$0.1155^{+0.0044}_{-0.0039}$

Table 3: Values of the strong coupling constant $\alpha_s(M_Z)$ obtained from fits to ALEPH LEP2 data with $E_{\rm cm} \ge 183$ GeV for $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$ and R_5. NLO QCD predictions are used. Hadronization corrections are not included. Default fit ranges are $4.8 \le -\ln y_{45} \le 6.4$, and $2.1 \le -\log_{10} y_{\rm cut} \le 2.9$. See the text for details.

0906.3436: Dissertori et al; NNLO+NLL ev. shapes

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009_{\text{stat}} \pm 0.0009_{\text{exp}} \pm 0.0012_{\text{had}} \pm 0.0035_{\text{theo}}$$



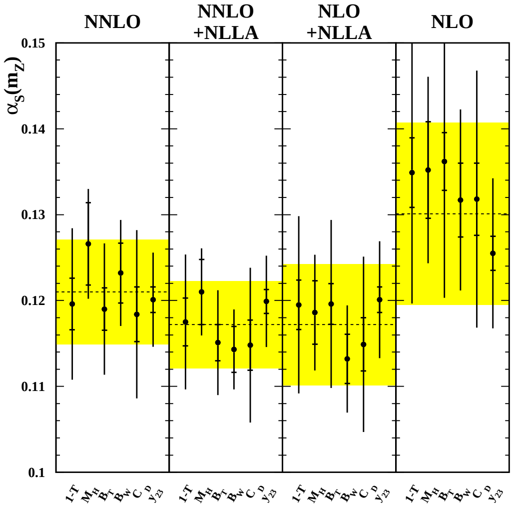
Results using Herwig++ + POWHEG for hadr. are 3% lower

0810.1389: Bethke et al/JADE; NNLO(+NLL)

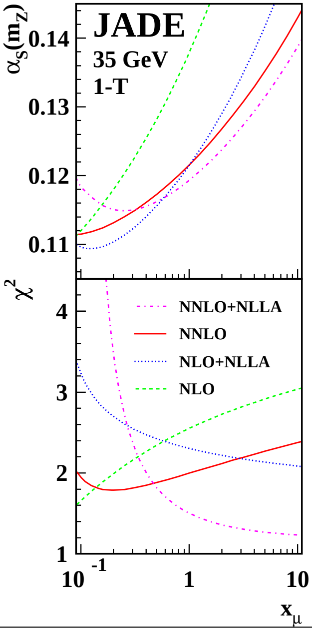
$$\text{NNLO} \quad : \alpha_s(M_Z) = 0.1210 \pm 0.0007_{\text{stat}} \pm 0.0021_{\text{exp}} \pm 0.0044_{\text{had}} \pm 0.0036_{\text{theo}}$$

$$\text{NNLO+NLL} : \alpha_s(M_Z) = 0.1172 \pm 0.0006_{\text{stat}} \pm 0.0020_{\text{exp}} \pm 0.0035_{\text{had}} \pm 0.0030_{\text{theo}}$$

- Fit range chosen such that LL terms less than unity; same range used for NNLO & NNLO+NLL
-



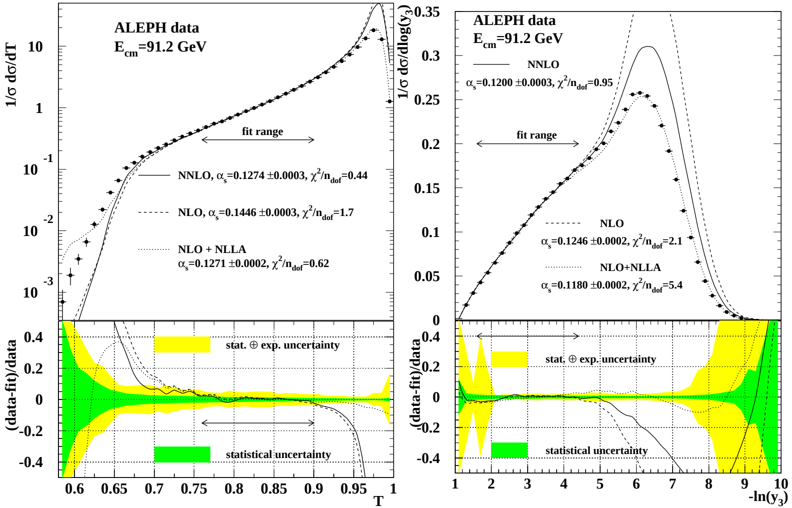
0810.1389 cont.



Illustrates clear improvement coming from inclusion of NNLO/NLL information

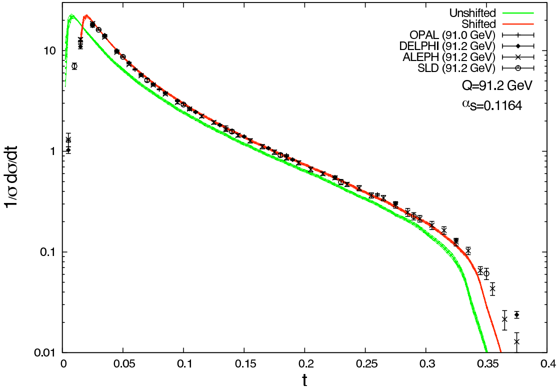
0712.0327: Dissertori et al; NNLO ev. shapes

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008_{\text{stat}} \pm 0.0010_{\text{exp}} \pm 0.0011_{\text{had}} \pm 0.0029_{\text{theo}}$$



0809.3326: Davison&Webber; NNLO/NLL/PC $\frac{d\sigma}{dT}$

$$\alpha_s(M_Z) = 0.1164 \pm 0.0022_{\text{exp}} \pm 0.00157_{\text{theo}}$$

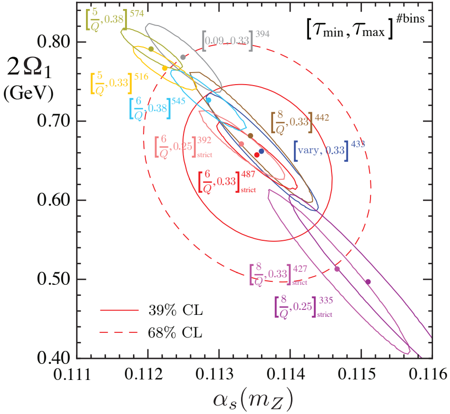


Note, scales varied in range $\sqrt{1/2} < x_\mu < \sqrt{2}$

With a conventional $\frac{1}{2} < x_\mu < 2$ variation, expect theory error to double roughly?

1006.3080: Abbate et al; SCET T NNLO+N^{3}LL

$$\alpha_s(M_Z) = 0.1135 \pm 0.0002_{\text{expt}} \pm 0.0005_{\text{hadr}} \pm 0.0009_{\text{pert}}$$

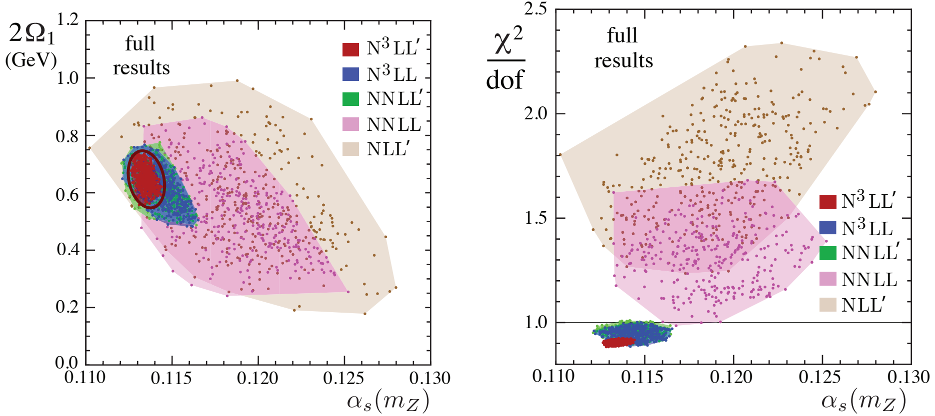


Experiment	Energy	BS results [20]	our BS profile	default profile
ALEPH	91.2 GeV	0.1168(1)	0.1170	0.1223
ALEPH	133 GeV	0.1183(37)	0.1187	0.1235
ALEPH	161 GeV	0.1263(70)	0.1270	0.1328
• • •	• •	• • •	• • •	• •
average		0.1172(10)	0.1180	0.1221
global fit (stat)	all Q		0.1188	0.1242
global fit (stat+syst)	all Q		0.1192	0.1245

TABLE VIII: Comparison of the results for $\alpha_s(m_Z)$ quoted by Becher and Schwartz in Ref. [20] with results we obtain from our adapted code [...] and employ their profile functions for the nonsingular, hard, jet and soft scales, with results shown in the column labeled "our BS profile". In the last column we show results with this same code, but using our default profile functions.

profile function matters at $\sim 4\%$

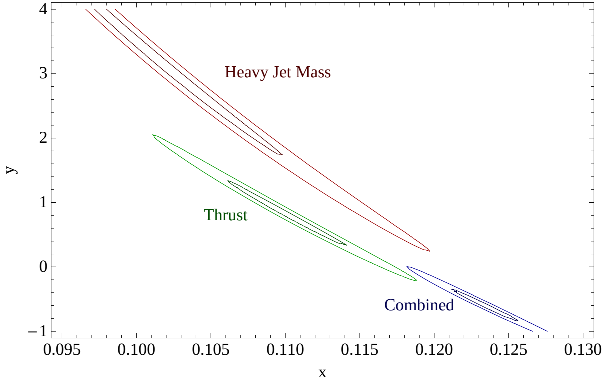
1006.3080 cont.



1005.1644: Chien & Schwartz; SCET ρ (& T)

Assuming no hadronisation (ρ): $\alpha_s(M_Z) = 0.1220 \pm 0.0031$
Assuming no hadronisation (T): $\alpha_s(M_Z) = 0.1193 \pm 0.0027$
Combined:
$\alpha_s(M_Z) = 0.1193 \pm 0.0011_{stat} \pm 0.0012_{syst} \pm 0.0017_{had} \pm 0.0013_{pert} \pm 0.0005_{soft}$

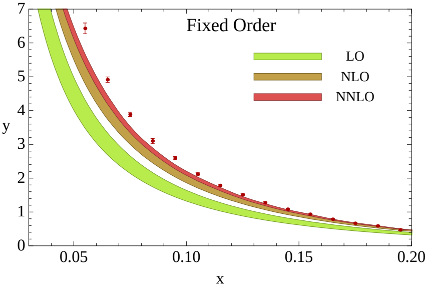
With hadronisation as shift (ρ): $\alpha_s(M_Z) = 0.1017$
With hadronisation as shift (T): $\alpha_s(M_Z) = 0.1101$

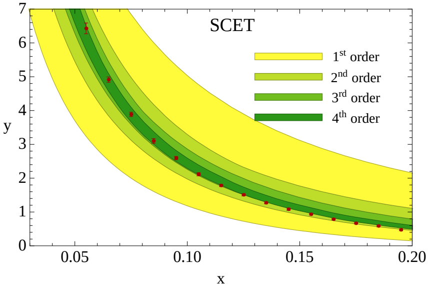


Is fit without hadronisation degrees of freedom reasonable?

Discrepancy between ρ and Thrust in power-correction fits has a long history.

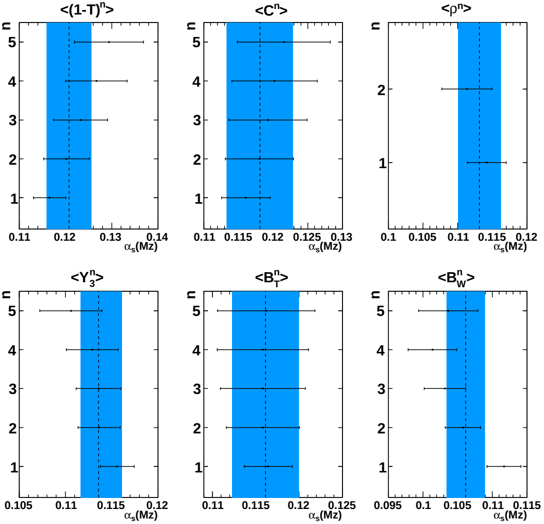
1005.1644 continued





0911.2422: Gehrmann et al; NNLO/PC moments

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017_{\text{exp}} \pm 0.0023_{\text{th}}$$



- PC formulae validity arguable for higher moments
- hadron-mass effects for the heavy jet mass
- Why not extract results based on first moments?