Thrust distributions at N^{3}LL with power corrections and precision determination of $\alpha_s(m_Z)$

LHCphenOnet

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α_s workshop, Munich

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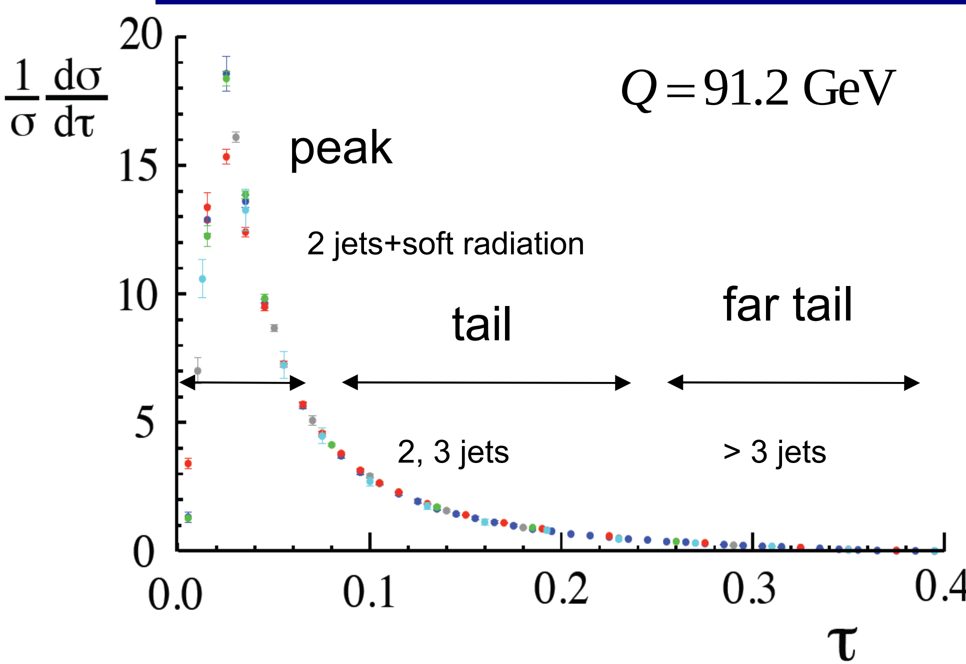
R. Abbate, M. Fickinger, A. Hoang, VM & I. Stewart – arXiv: 1006.3080 [hep-ph]

R. Abbate, A. Hoang, VM, M. Schwartz & I. Stewart – work in progress for HJM

Builds on work by Gehrmann et al & Weinzierl $O(\alpha_s^3)$ and Becher & Schwartz at N^{3}LL

	Experiment data	Values of Q	$e^+e^- \xrightarrow{Q} \text{jets}$
LEP	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}	
	DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}	
	OPAL	{91.0, 133.0, 177.0, 197.0}	
	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}	
SLAC	SLD	{91.2}	
DESY	TASSO	{14.0, 22.0, 35.0, 44.0}	
	JADE	{35.0, 44.0}	
KEK	AMY	{55.2}	



"standard" data set:

$Q \geq 35\,\text{GeV}$

$\frac{6\,\text{GeV}}{Q} \leq \tau \leq 0.33$

487 bins

Correlations treated in minimal overlap model

Factorization theorem

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

$$\frac{\Delta\alpha_s}{\alpha_s} \sim 0.5\%$$

b mass corrections

nonperturbative soft function , $\left(\frac{\Omega_1}{Q\tau} \right)$

$$\frac{d\hat{\sigma}_s}{d\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l \tau}{\tau} \right]_+$$

$$= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S)$$

Singular partonic for massless quarks QCD+QED final states

$$\frac{d\hat{\sigma}_{ns}}{d\tau} = \sum_{n,l} \alpha_s^n \ln^l \tau + \sum_n \alpha_s^n f_n(\tau)$$

Nonsingular partonic

Resummation for singular partonic

$$\ln\left(\int_0^\tau d\tau \frac{1}{\sigma} \frac{d\hat{\sigma}}{d\tau} \right) \sim (\ln\tau) \sum_{k=0} (\alpha_s \ln\tau)^{k+1} + \sum_{k=0} (\alpha_s \ln\tau)^{k+1} + \alpha_s \sum_{k=0} (\alpha_s \ln\tau)^k + \alpha_s^2 \sum_{k=0} (\alpha_s \ln\tau)^k + \ldots$$

LL NLL NNLL N^3LL

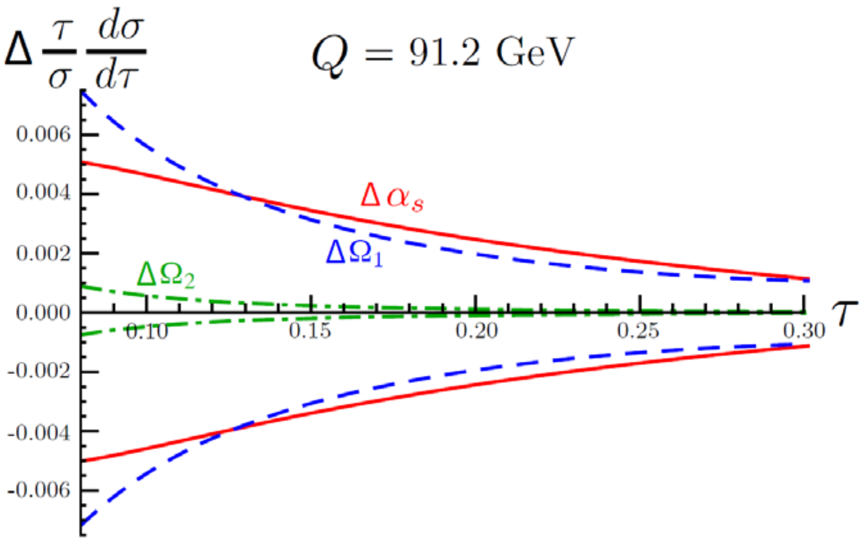
Factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}=\int \mathrm{d}k\left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau}+\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau}+\frac{\mathrm{d}\hat{\sigma}_{b}}{\mathrm{d}\tau}\right)\left(\tau-\frac{k}{Q}\right)S_{\tau}^{\mathrm{mod}}(k-2\bar{\Delta})+O\left(\sigma_0\frac{\alpha_s\Lambda_{\mathrm{QCD}}}{Q}\right)$$

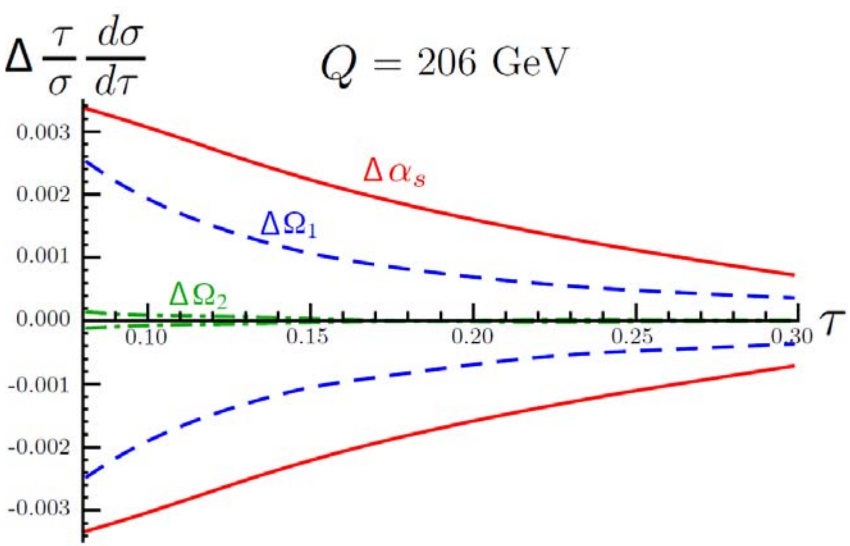
- $O(\alpha_s^3)$ fixed order (nonsingular). Event2 $O(\alpha_s^2)$ and EERAD3 $O(\alpha_s^3)$.
- $O(\alpha_s^3)$ matrix elements. Axial singlet anomaly. Full hard function at 3 loops.
- Resummation at N^{3}LL. Effective field theory (SCET).
- Correct theory in peak, tail and multijet (profile functions).
- Field theory matrix elements for power corrections.
- Removal of u=1/2 renormalon in leading power correction/soft function.
- QED effects in Sudadok & FSR @ NNLL $O(\alpha_s^2)$ with $\alpha\sim\alpha_s^2$.
- bottom mass corrections with factorization theorem.
- Computation of bin cumulants in a meaningful way.

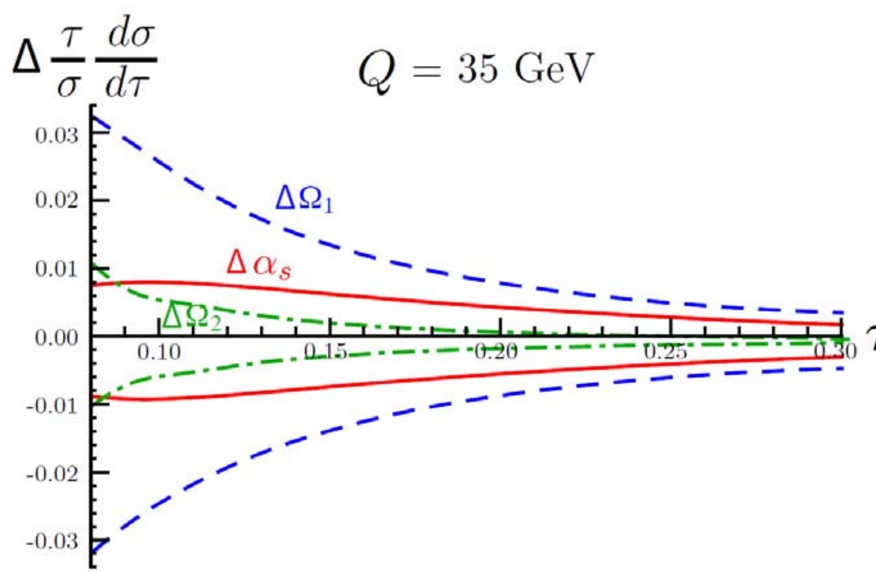
Why a global fit (many Q's)

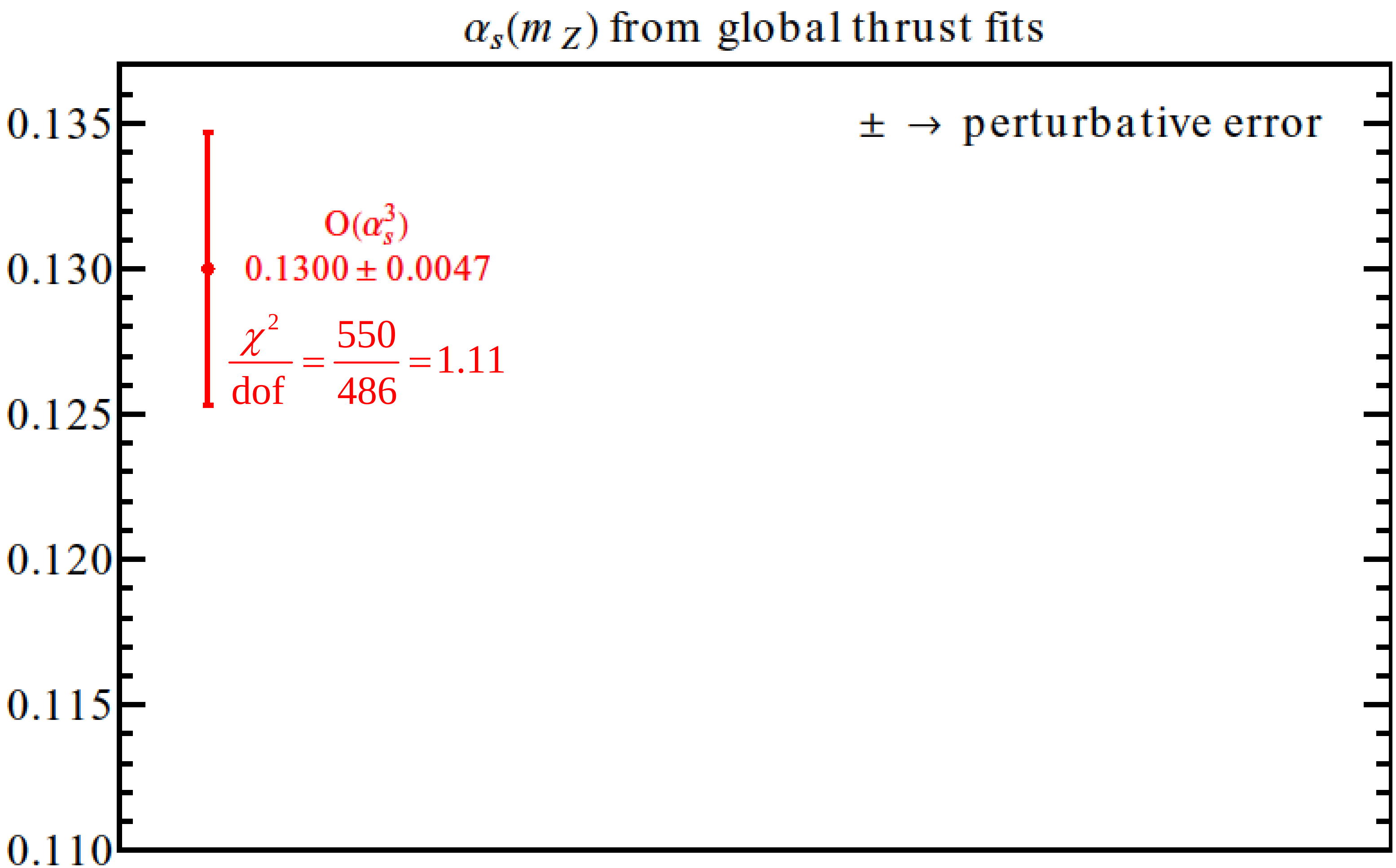
We fit for Ω_1 & $\alpha_s(m_Z)$ simultaneously. Strong degeneracy lifted by many Q's.



Power correction needed with 20% accuracy to get α_s at the 1% level

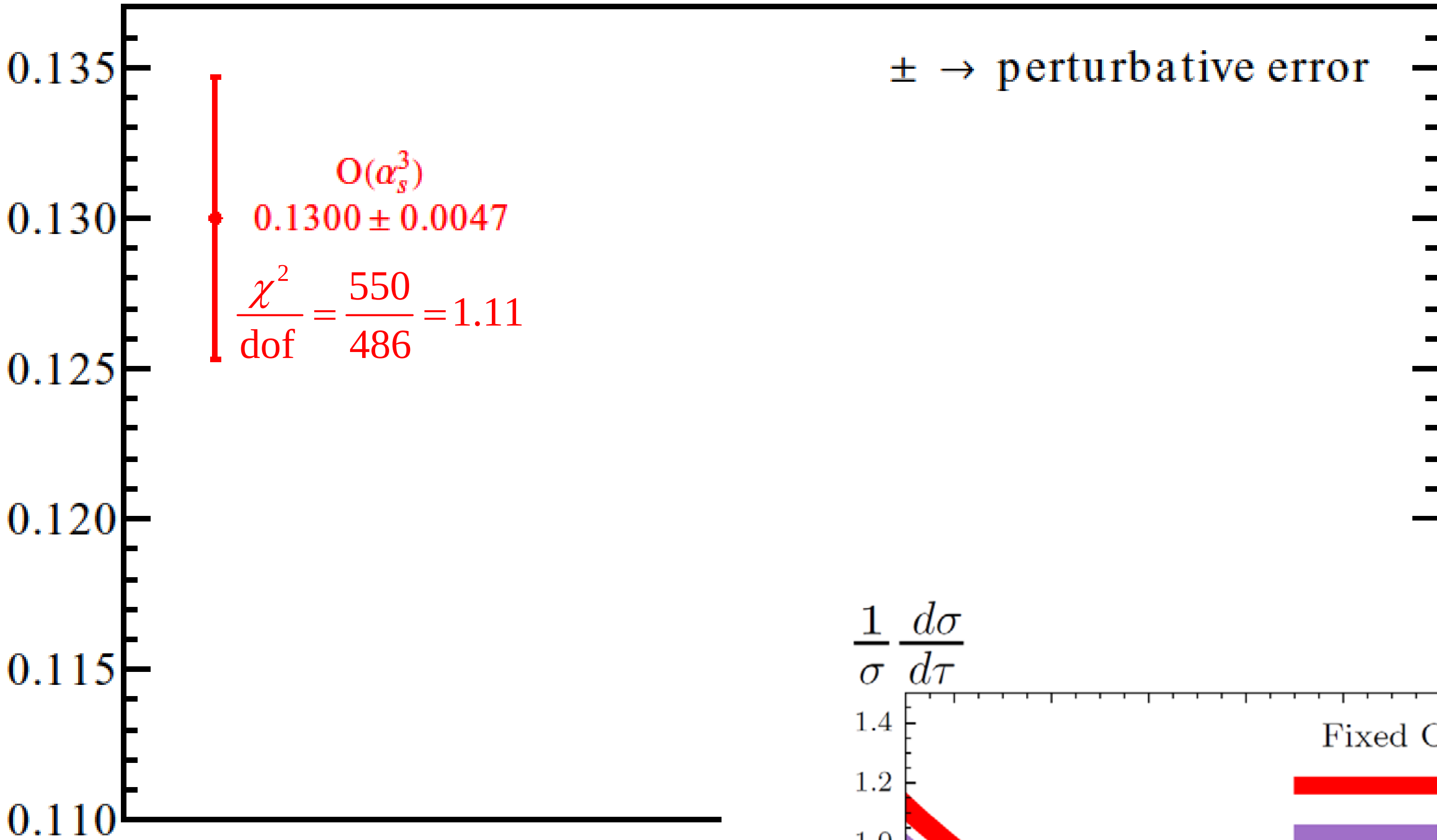






• Pure Fixed order

$\alpha_s(m_Z)$ from global thrust fits

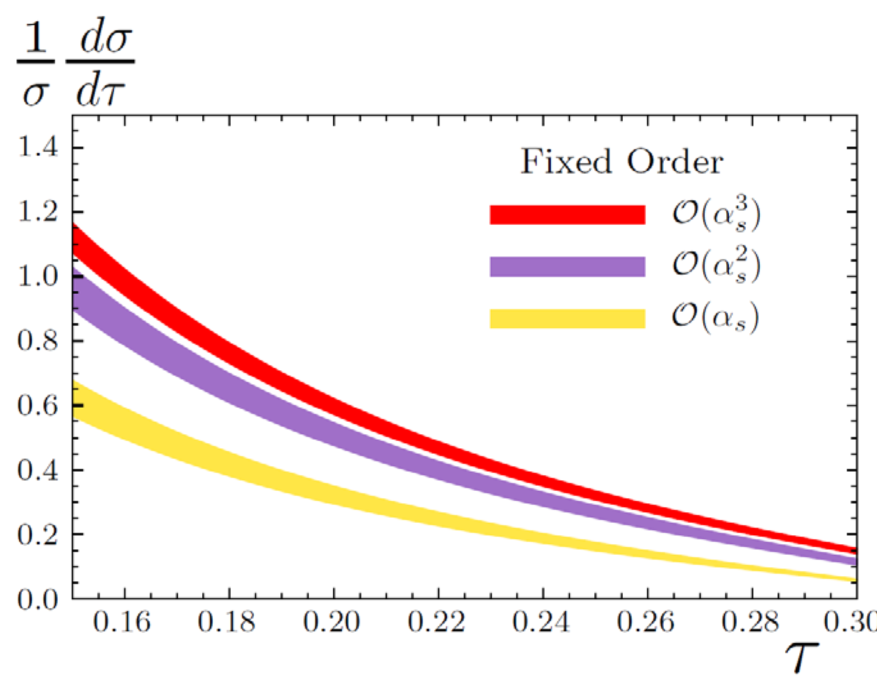


- Pure Fixed order

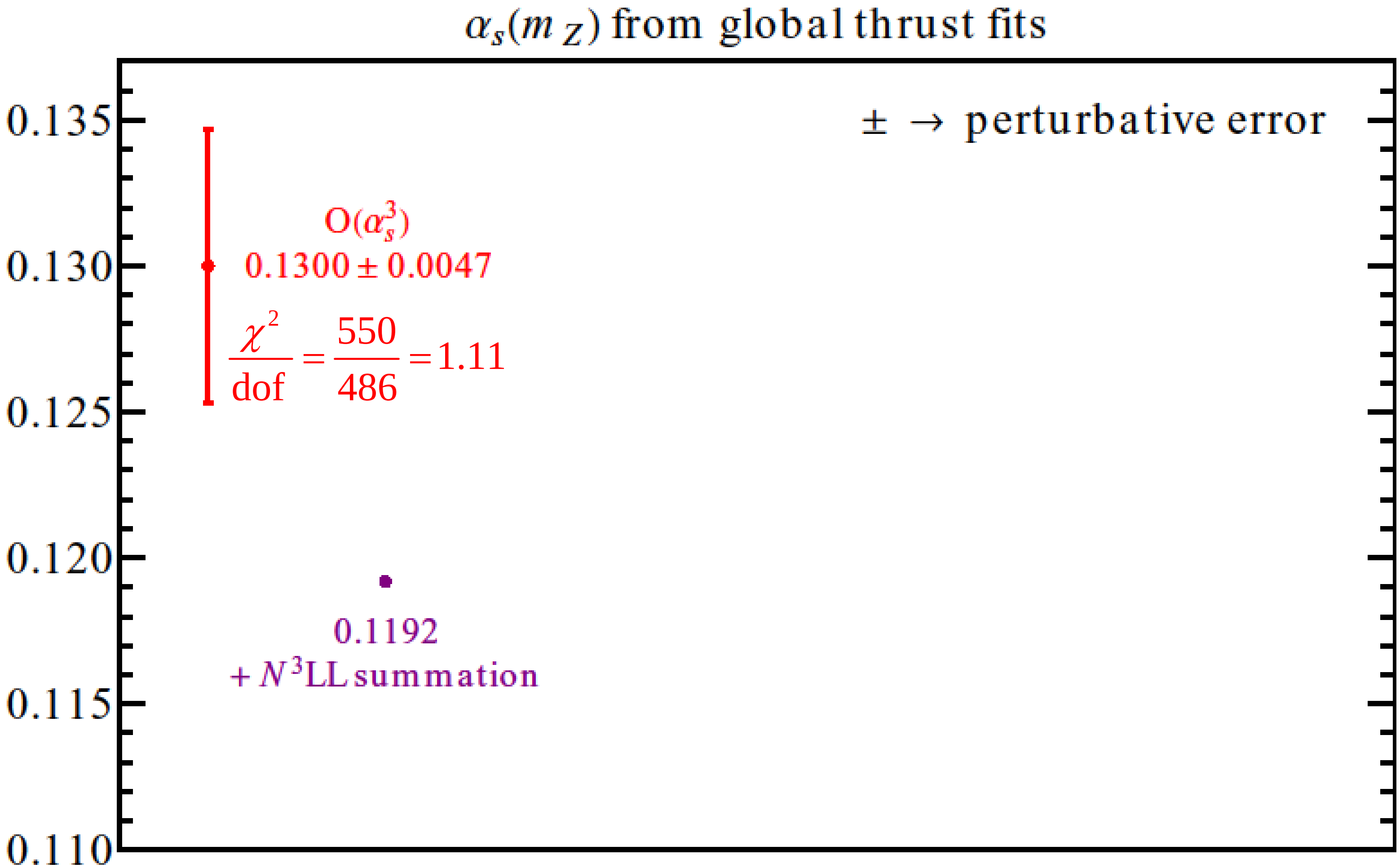
ALEPH Q=m_Z thrust

$0.1274 \pm (0.0042)_{\text{pert}}$

Dissertory et al '07

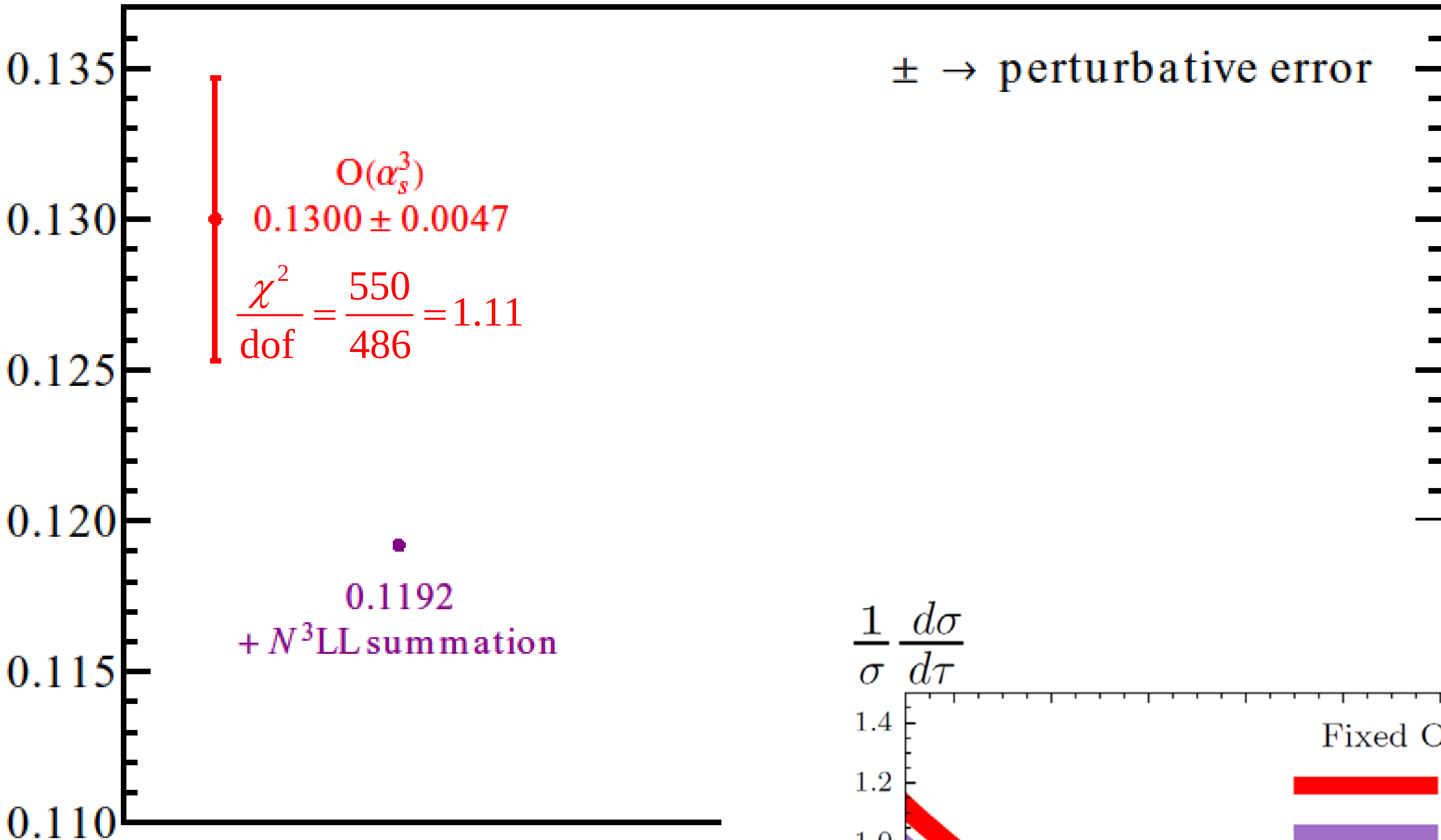


ALEPH Q's all event shapes $0.1240 \pm (0.0029)_{\text{pert}}$



- Resummation at N^3LL

$\alpha_s(m_Z)$ from global thrust fits



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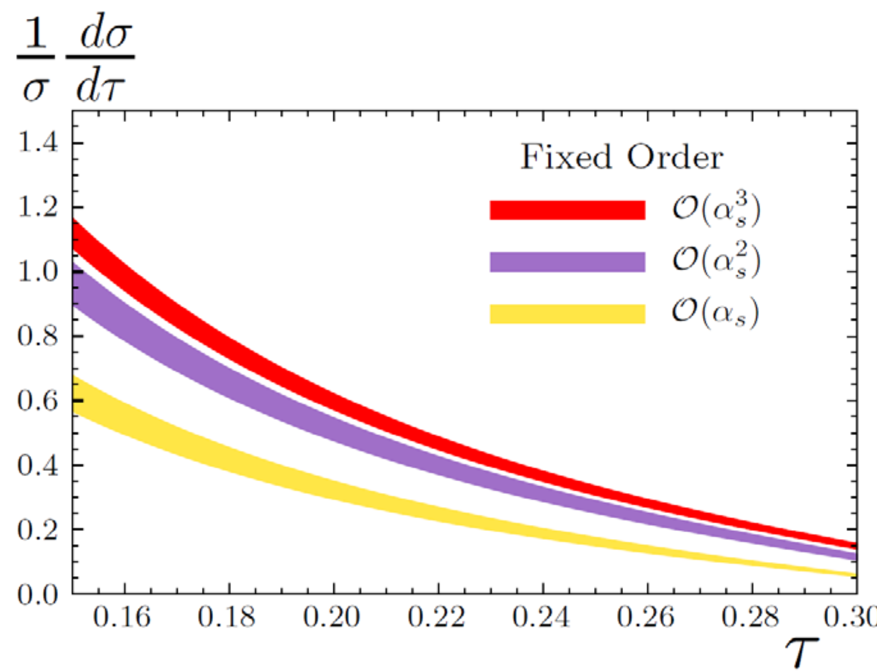
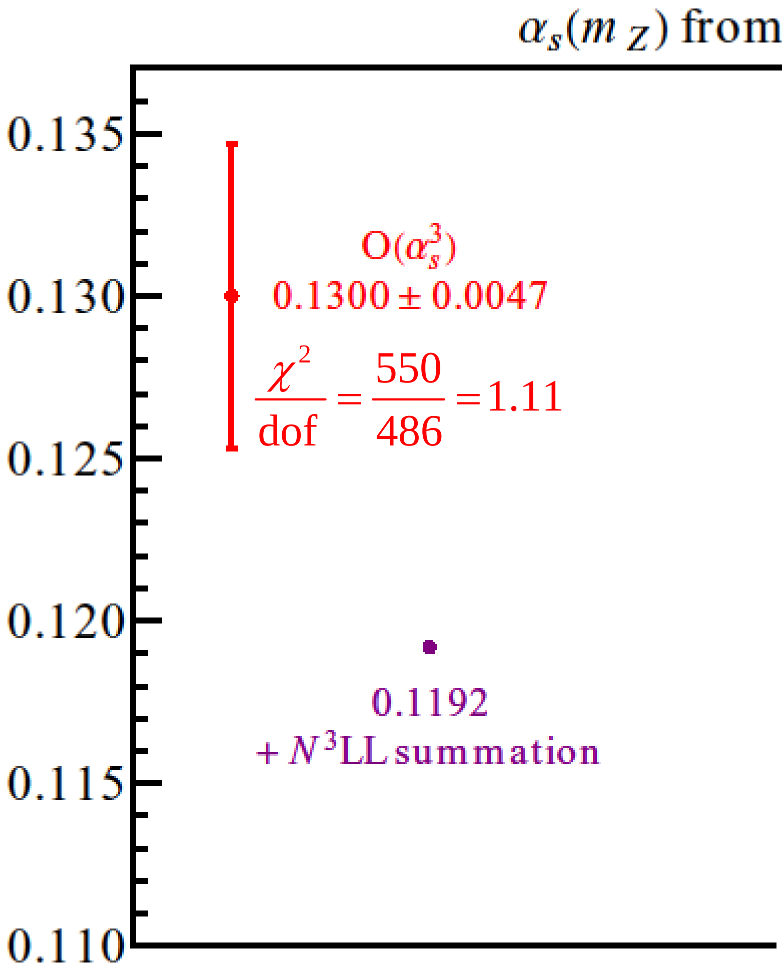


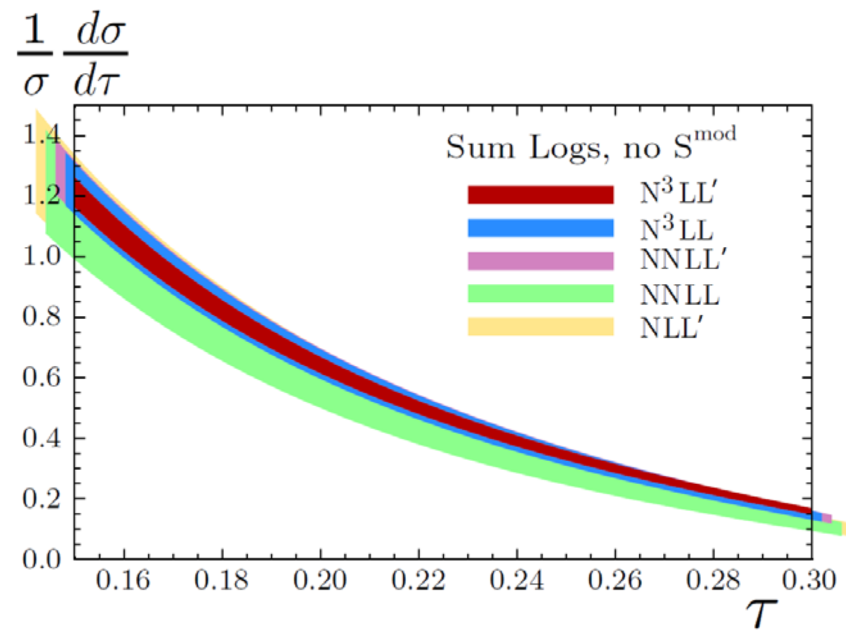


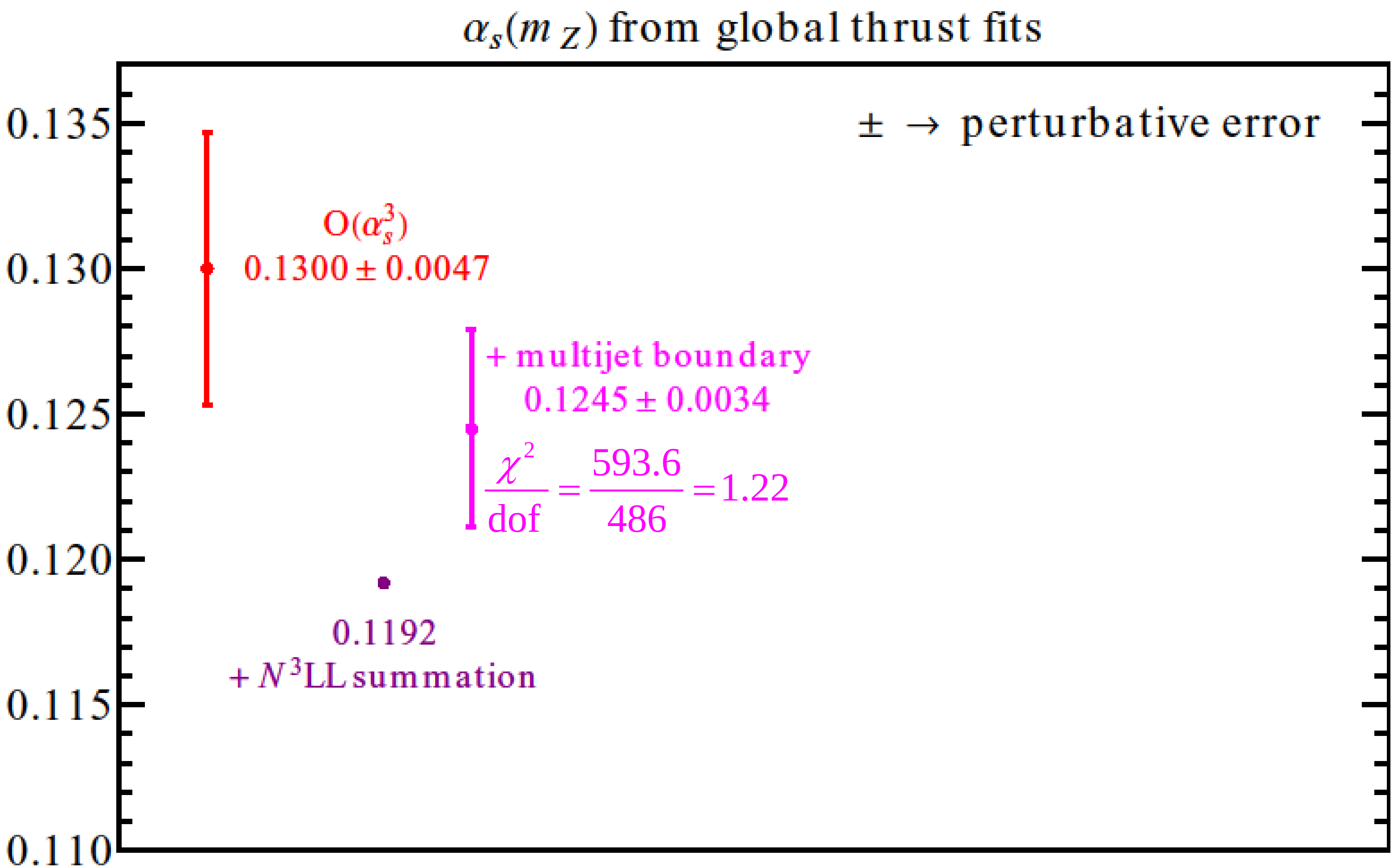
• Resummation at N³LL

Fit to ALEPH and OPAL

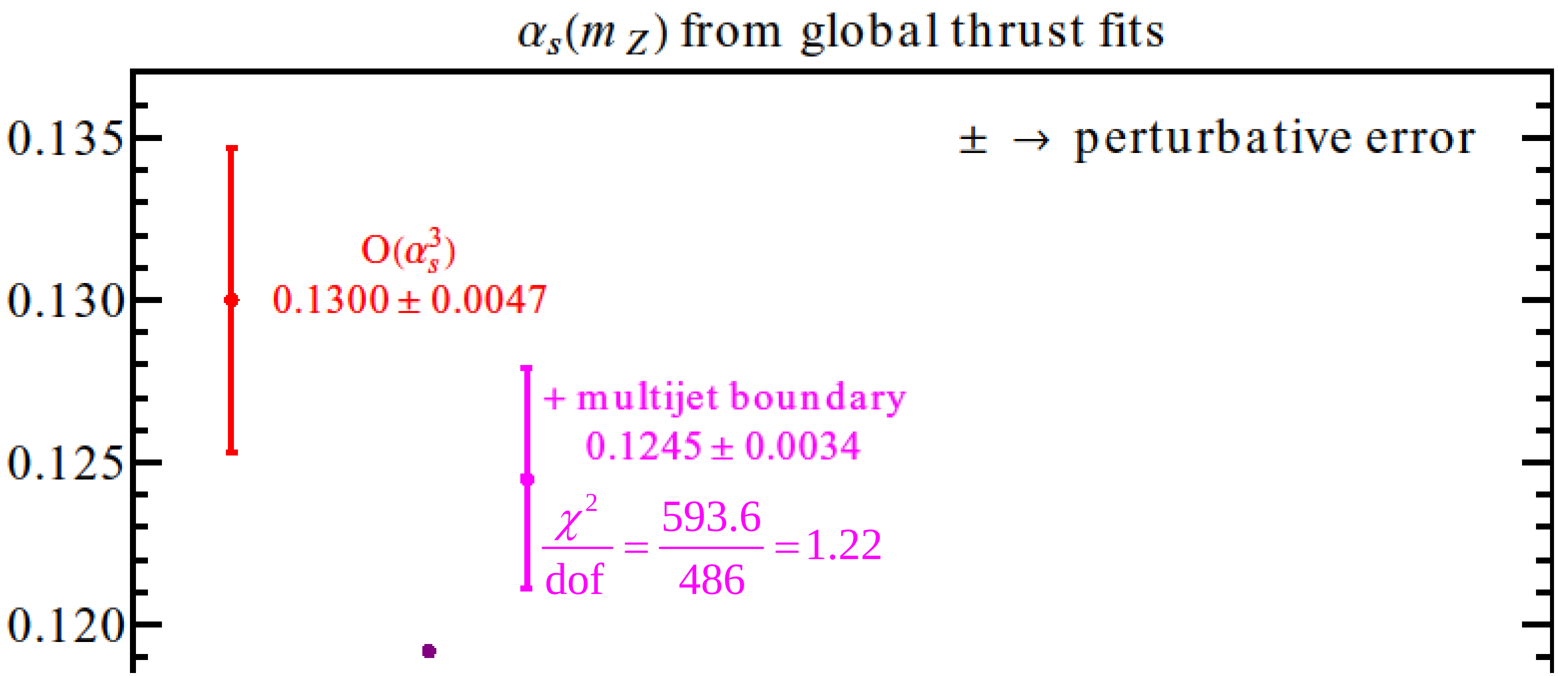
$0.1172 \pm (0.0012)_{\text{pert}}$

Becher & Schwartz '08

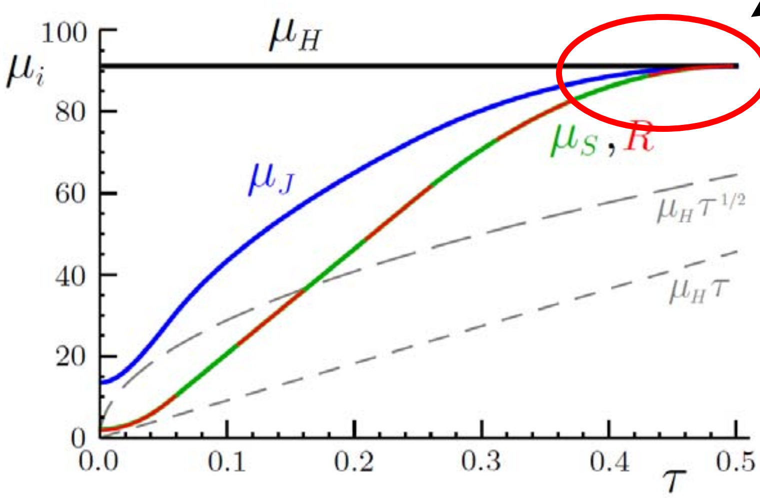


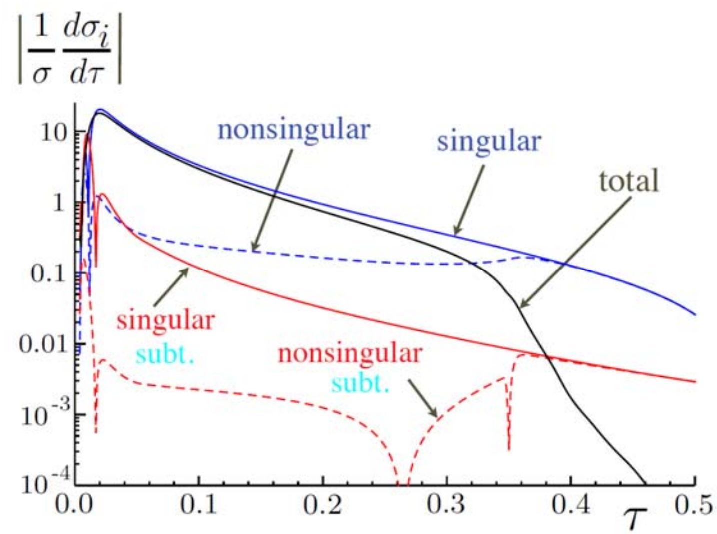


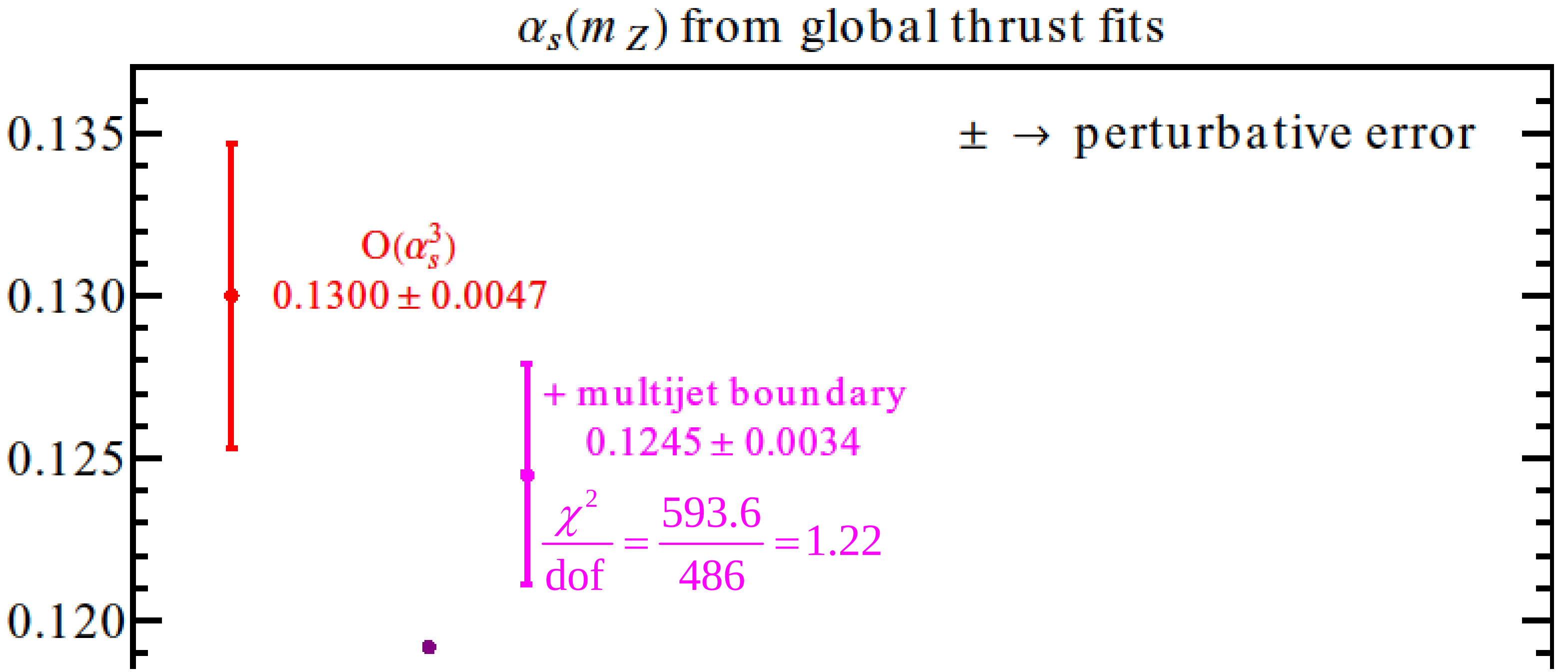
- Resummation at N^3LL
- Multijet boundary condition
- No power corrections
- No renormalon subtraction



We must turn off the resummation in the multijet region







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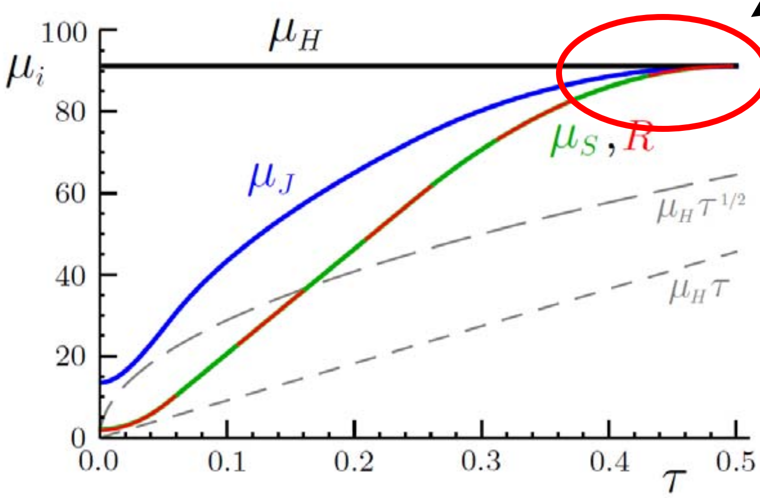
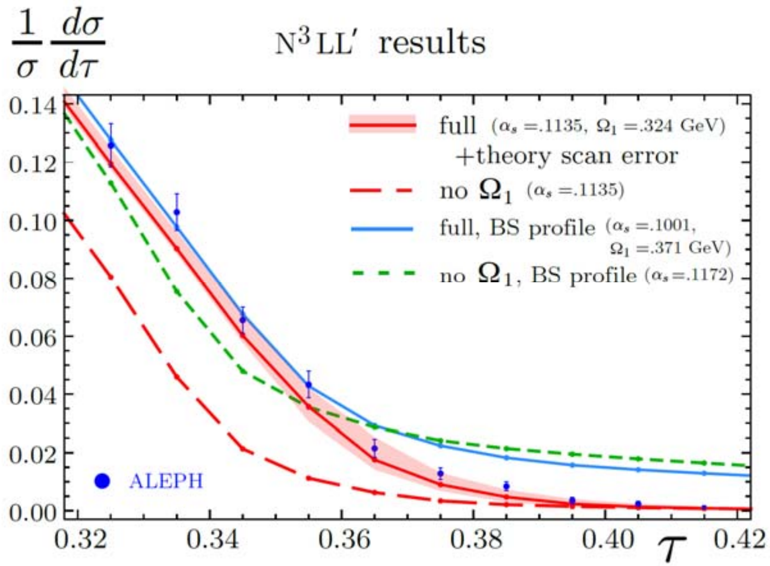
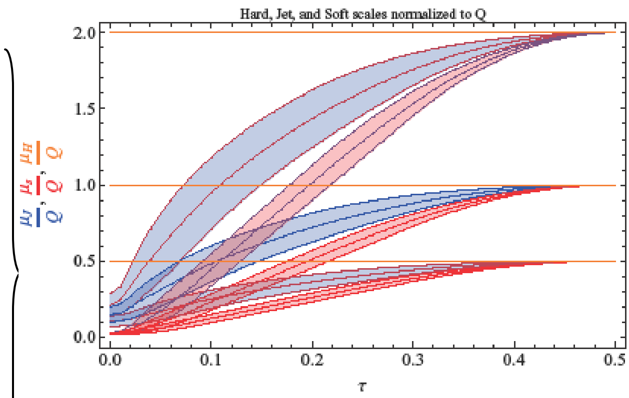




Estimate of perturbative uncertainties

parameter	default value	range of values
μ_0	2 GeV	1.5 to 2.5 GeV
n_1	5	2 to 8
t_2	0.25	0.20 to 0.30
e_J	0	-1,0,1
e_H	1	0.5 to 2.0
n_s	0	-1,0,1
s_2	-39.1	-36.6 to -41.6
$\Gamma_3^{\rm cusp}$	1553.06	-1553.06 to $+4569.18$
j_3	0	-3000 to $+3000$
s_3	0	-500 to $+500$
ϵ_2	0	-1,0,1
ϵ_3	0	-1,0,1



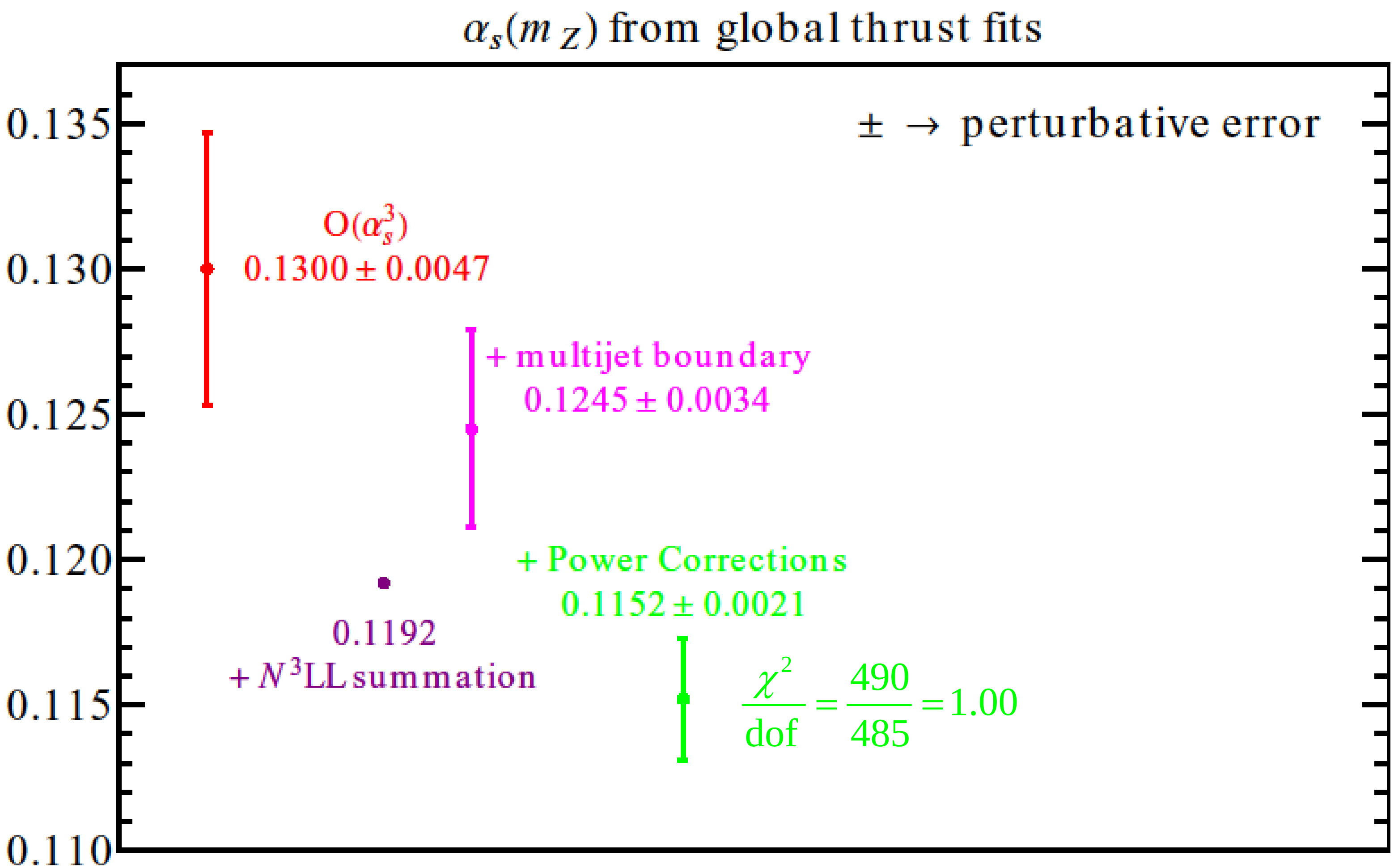
Profile functions

$h_3 = 8998.05$

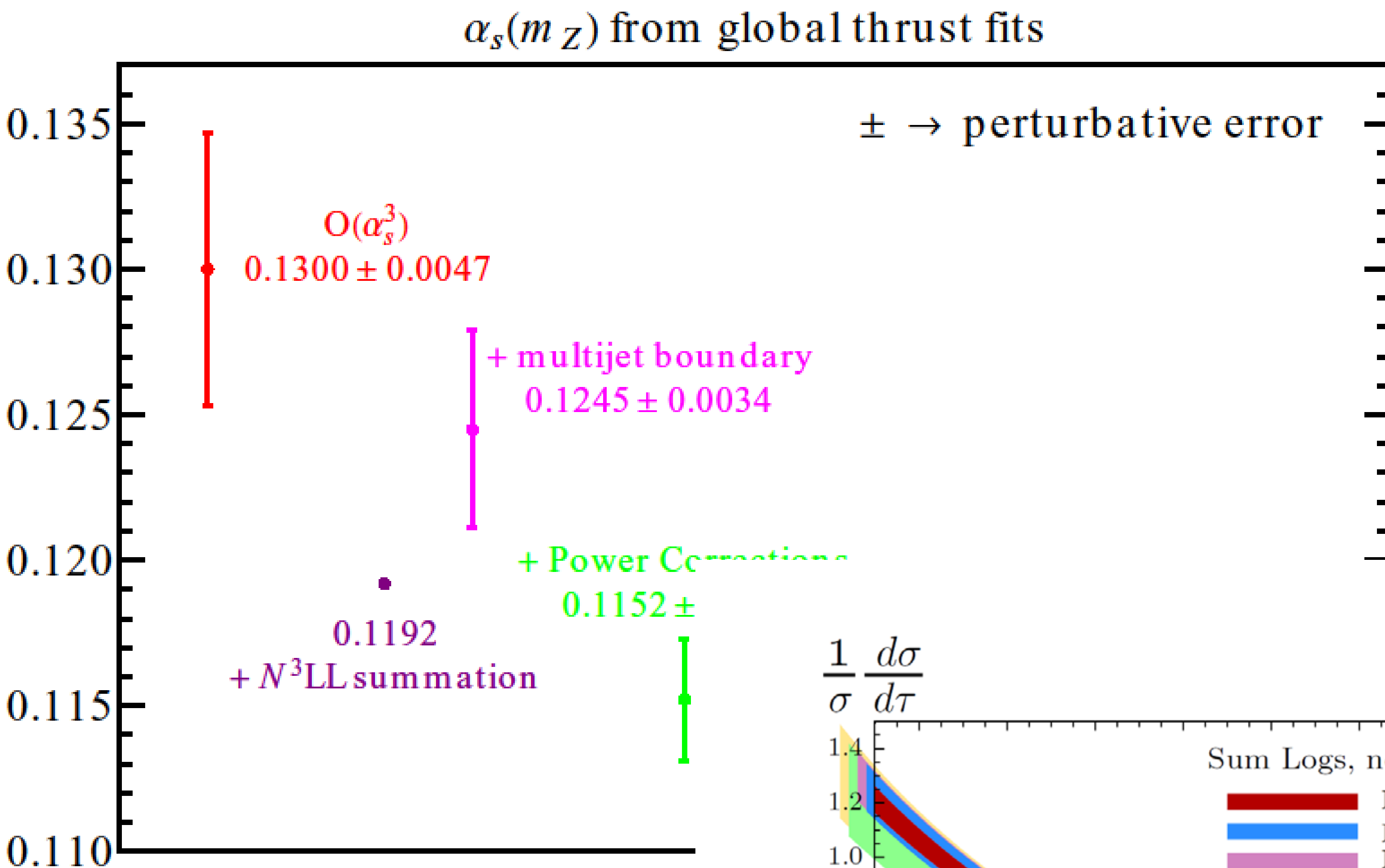
Baikov et al

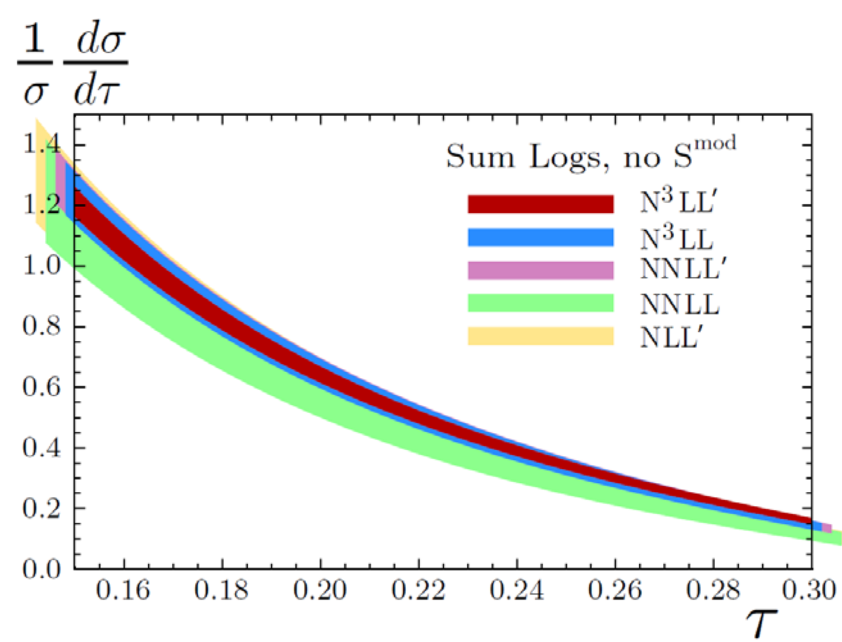
Padè approximants for range

Nonsingular statistical error



- Resummation at N^3LL
- Multijet boundary condition
- Power corrections give -7.5% shift





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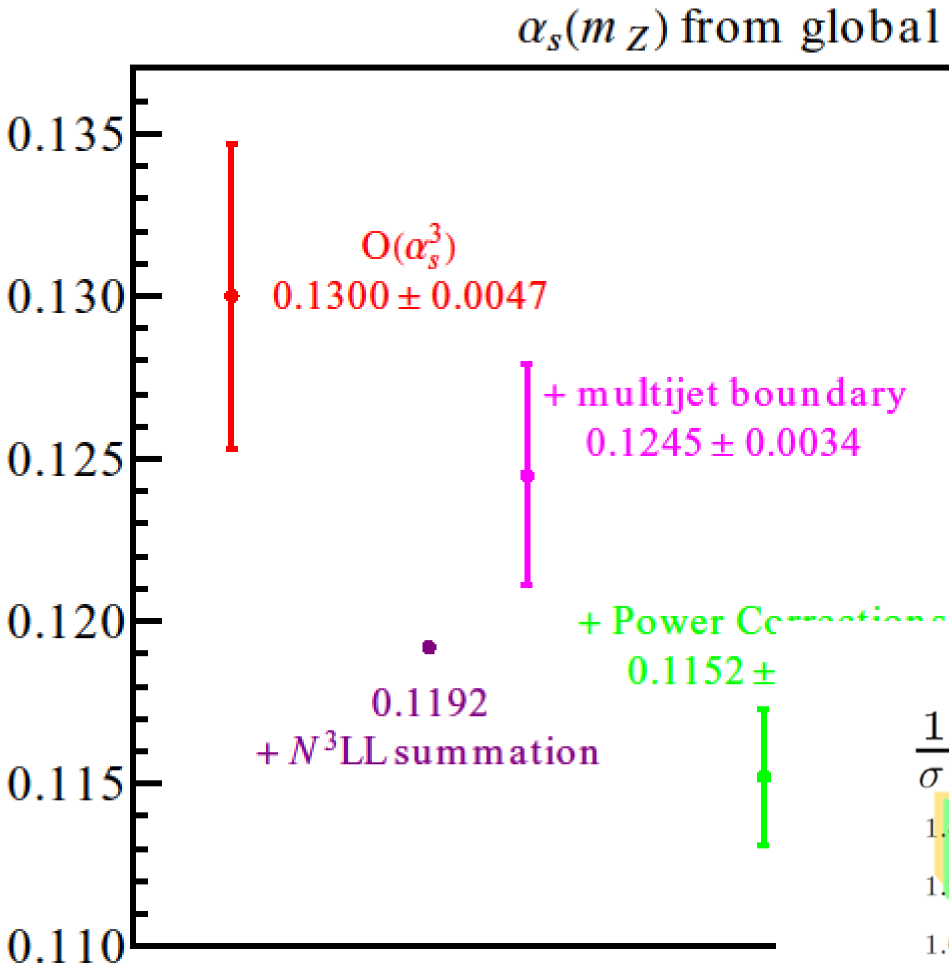

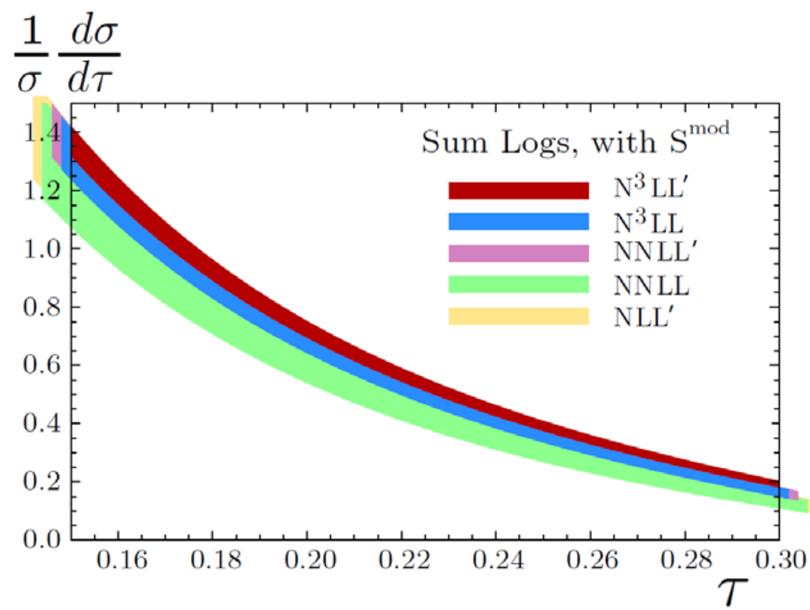

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In the tail region $\ell_{\rm soft} \sim Q\,\tau \gg \Lambda_{QCD}$

and we can expand the soft function

Shifts distributions to the right !

$$S(\tau) = S_{\rm pert}(\tau) - S'_{\rm pert}(\tau)\frac{2\Omega_1}{Q} \approx S_{\rm pert}\left(\tau - \frac{2\Omega_1}{Q} \right) \qquad \Omega_1 \sim \Lambda_{QCD}$$

Is a nonperturbative parameter

Ω_1 is defined in field theory

$$\bar{\Omega}_1 \equiv \frac{1}{2N_C} \langle 0 | \mathrm{tr} \bar{Y}_{\bar{n}}(0) Y_n(0)\, i\partial_\tau Y_n^\dagger(0) \bar{Y}_{\bar{n}}^\dagger(0) | 0 \rangle \qquad \overline{\rm MS}$$

$$i\partial_\tau \equiv \theta(i\bar{n}\cdot\partial - i n\cdot\partial)\, i n\cdot\partial + \theta(i\bar{n}\cdot\partial - i n\cdot\partial)\, i\bar{n}\cdot\partial$$

Consistency check

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - \frac{2\Lambda}{Q}\right)$$

Perturbative expression — Power correction

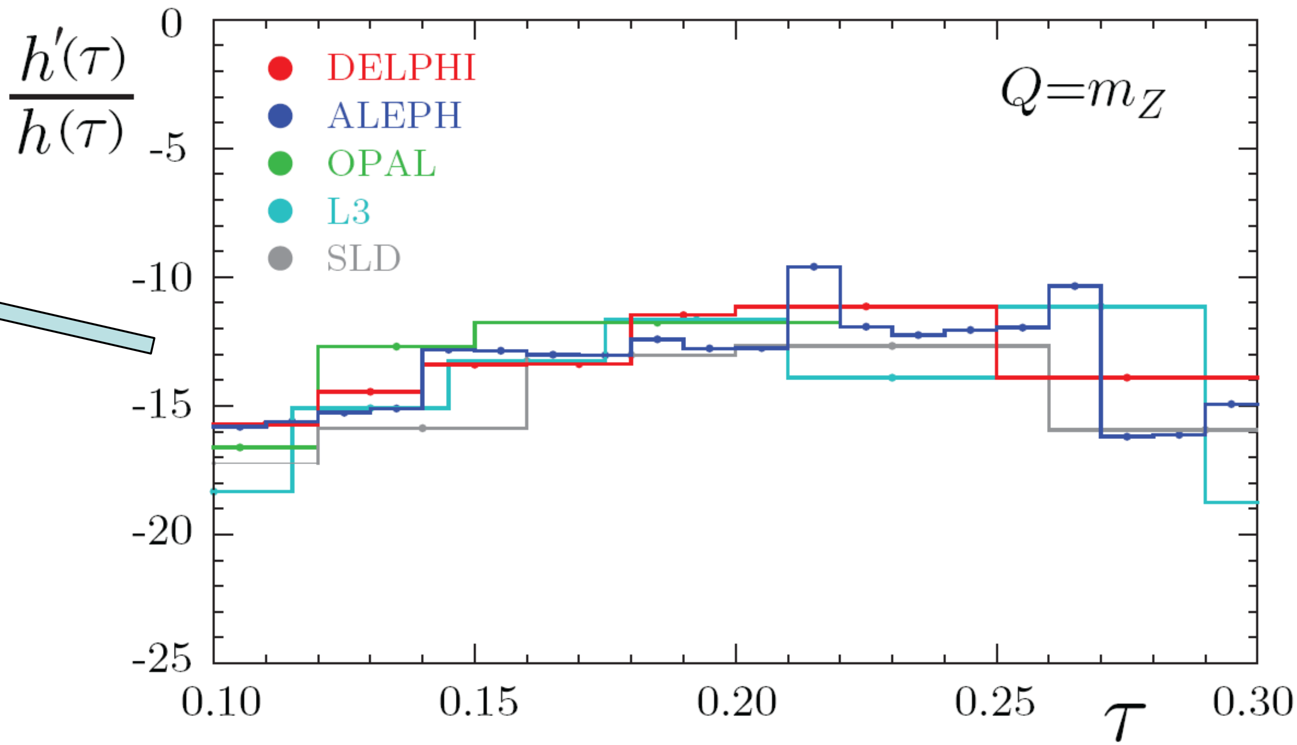
assuming that $h \sim \alpha_s$

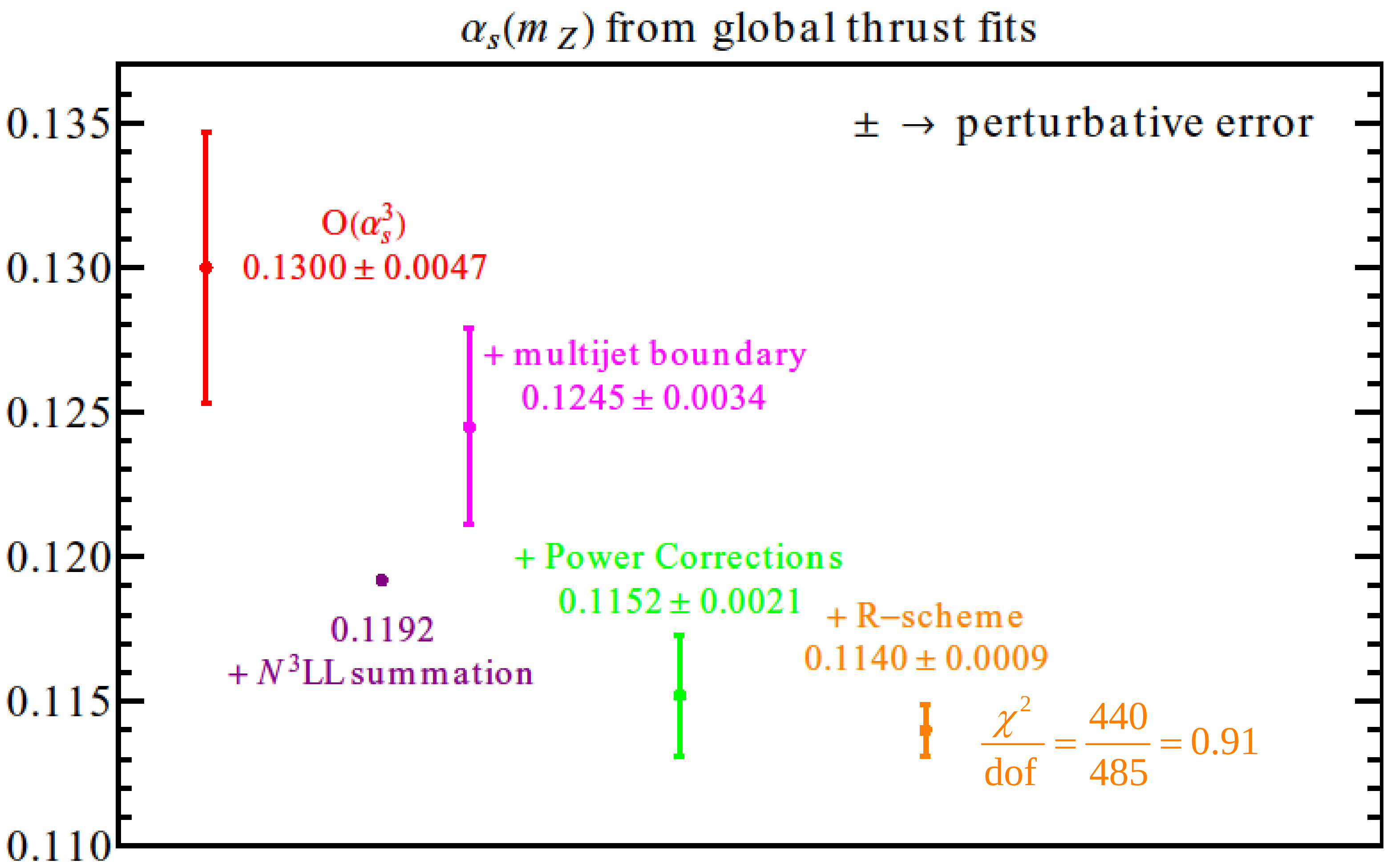
$$\frac{\delta\alpha_s}{\alpha_s} \approx \frac{2\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$

$$\frac{h'(\tau)}{h(\tau)} \approx -14 \pm 4$$

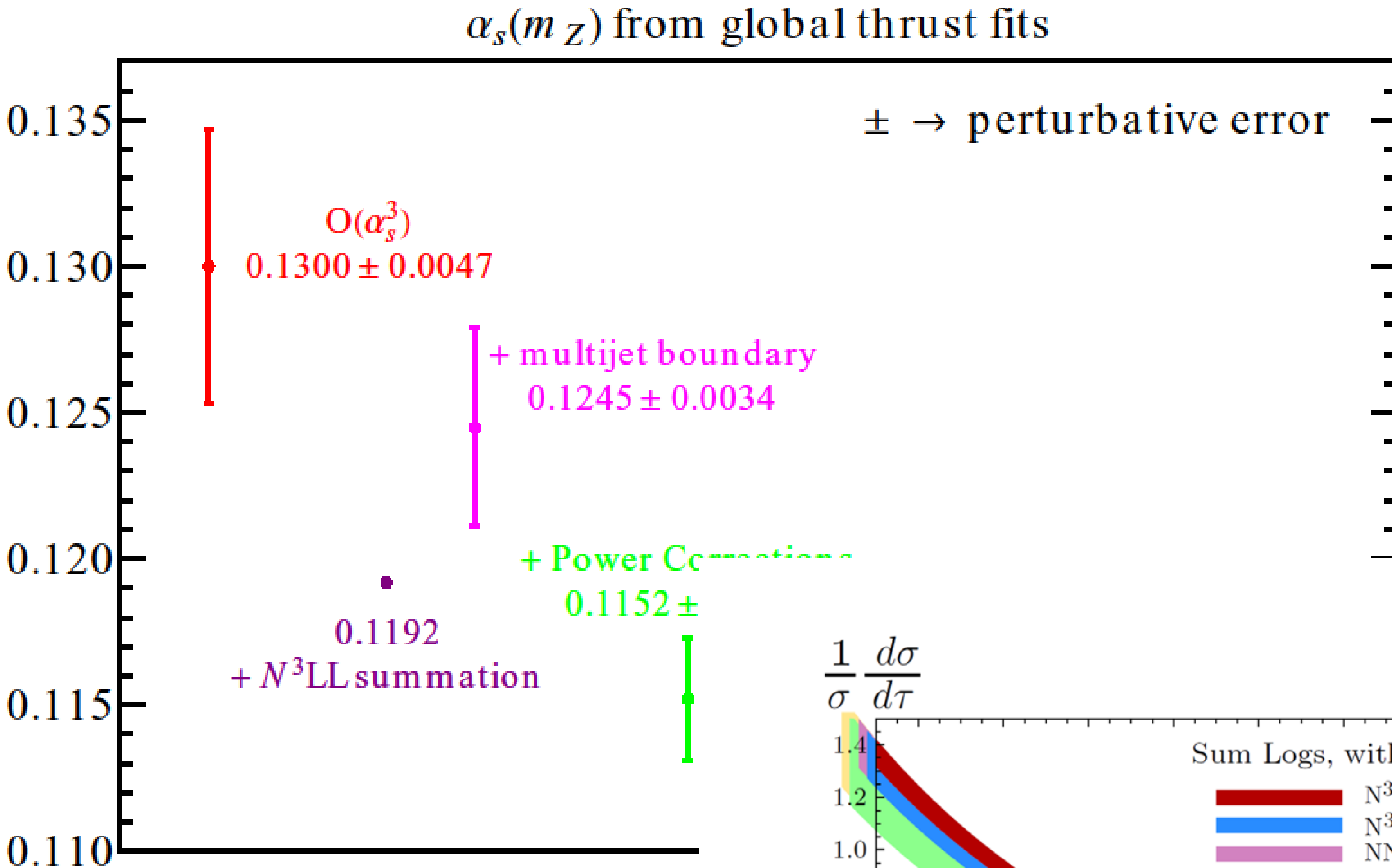
assuming $\Lambda \sim 0.3\,\mathrm{GeV}$

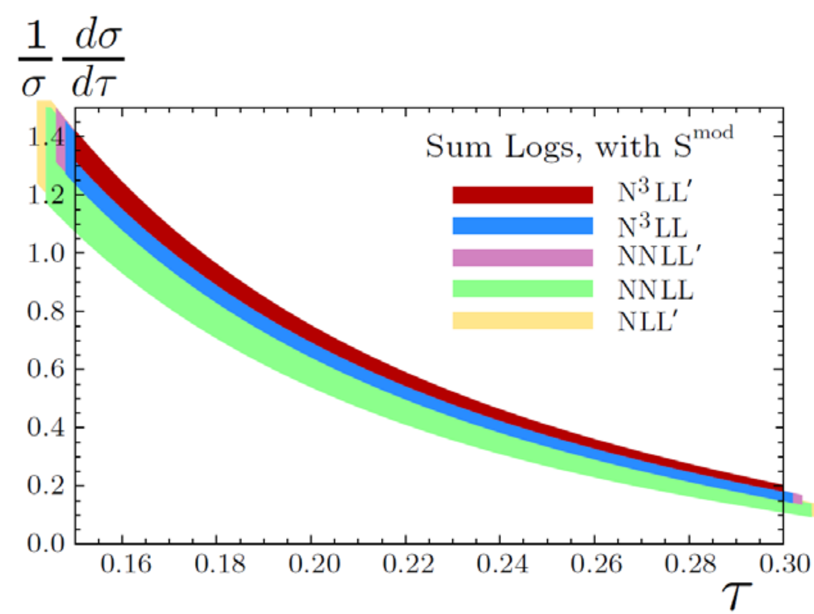
$$\frac{\delta\alpha_s}{\alpha_s} \approx -(9 \pm 3)\,\%$$





- Resummation at N^3LL
- Multijet boundary condition
- Power correction, in a scheme free of the $O(\Lambda_{QCD})$ renormalon





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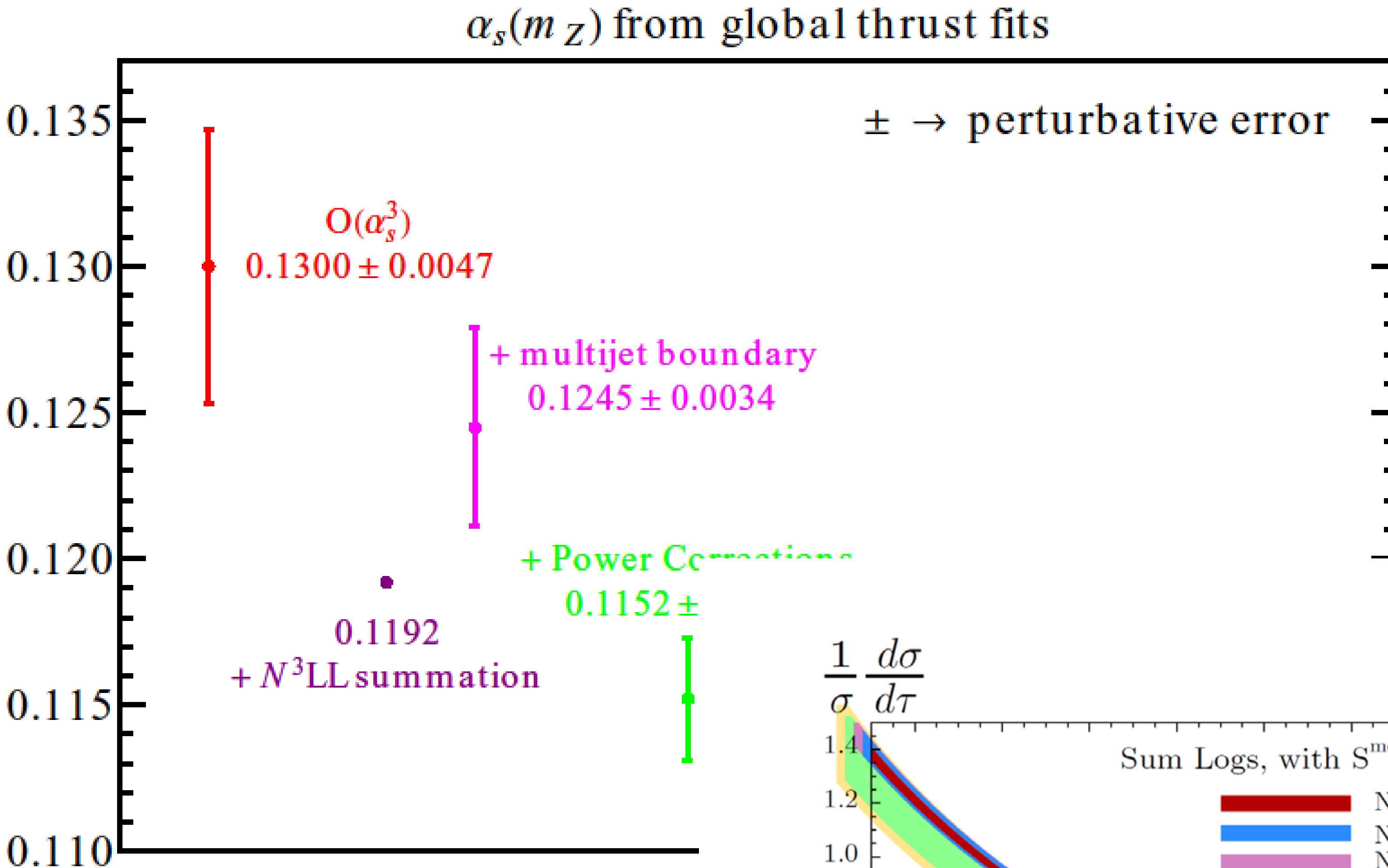

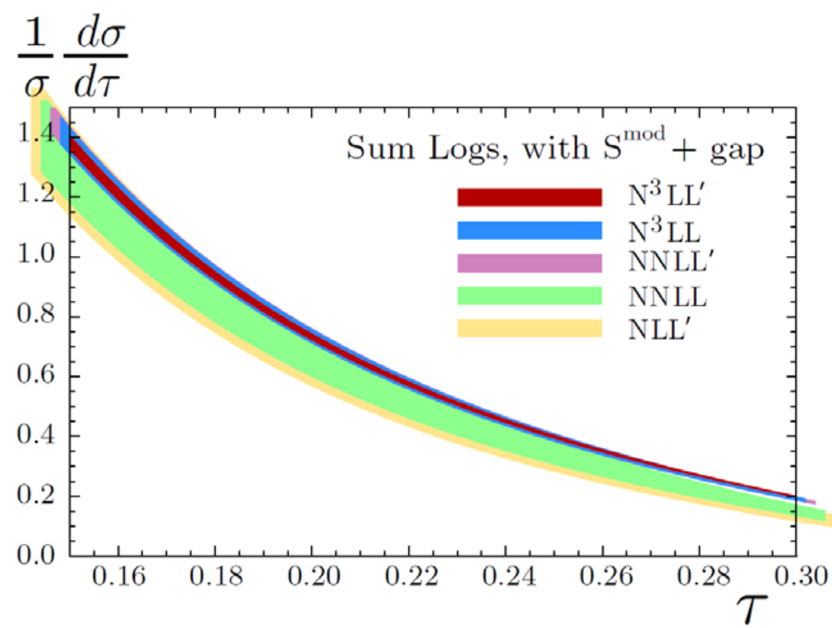

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R-scheme

$$\Omega_1(R,\mu_s) = \bar{\Omega}_1 - \delta(R,\mu_s)$$

$$\delta(R,\mu) = \sum_n \left(\frac{\alpha}{4\pi}\right)^n \delta_n(R,\mu)$$

Renormalon free

perturbative non perturbative

$$S(l,\mu) = \int \mathrm{d}\,l' \left[e^{-2\delta(R,\mu)\partial/\partial l} S_{\text{part}}(l,\mu) \right] S_\tau^{\text{mod}}(l' - 2\,\bar{\Delta}(R,\mu), R)$$

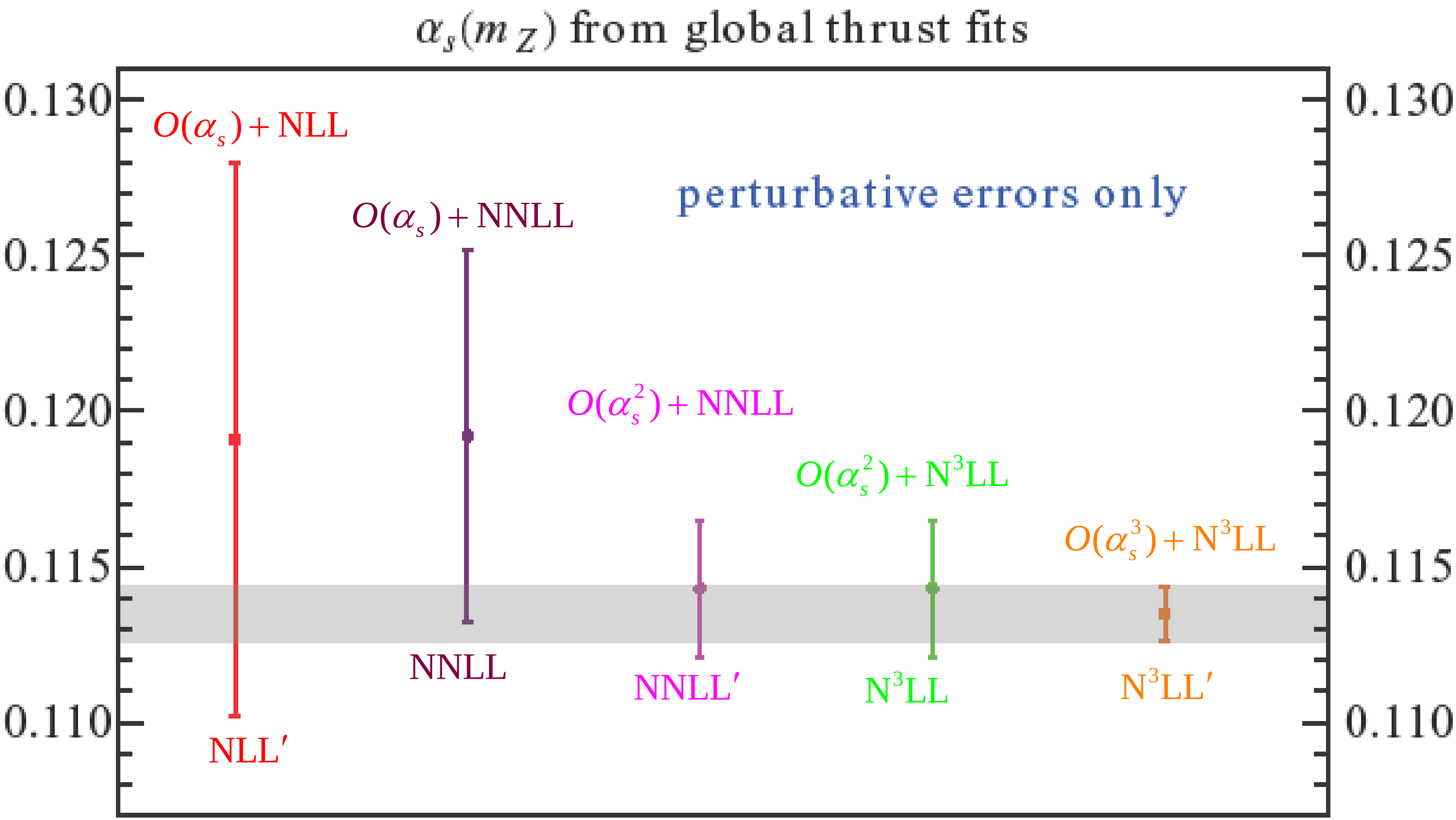
$$\delta_n(R,\mu) = \mathrm{R}\, e^{\gamma_R} \sum_m \delta_n^m \log^m\left(\frac{R}{\mu}\right) \longrightarrow$$

Can become large

Keep them O(1) $\longrightarrow$ $R \sim \mu$

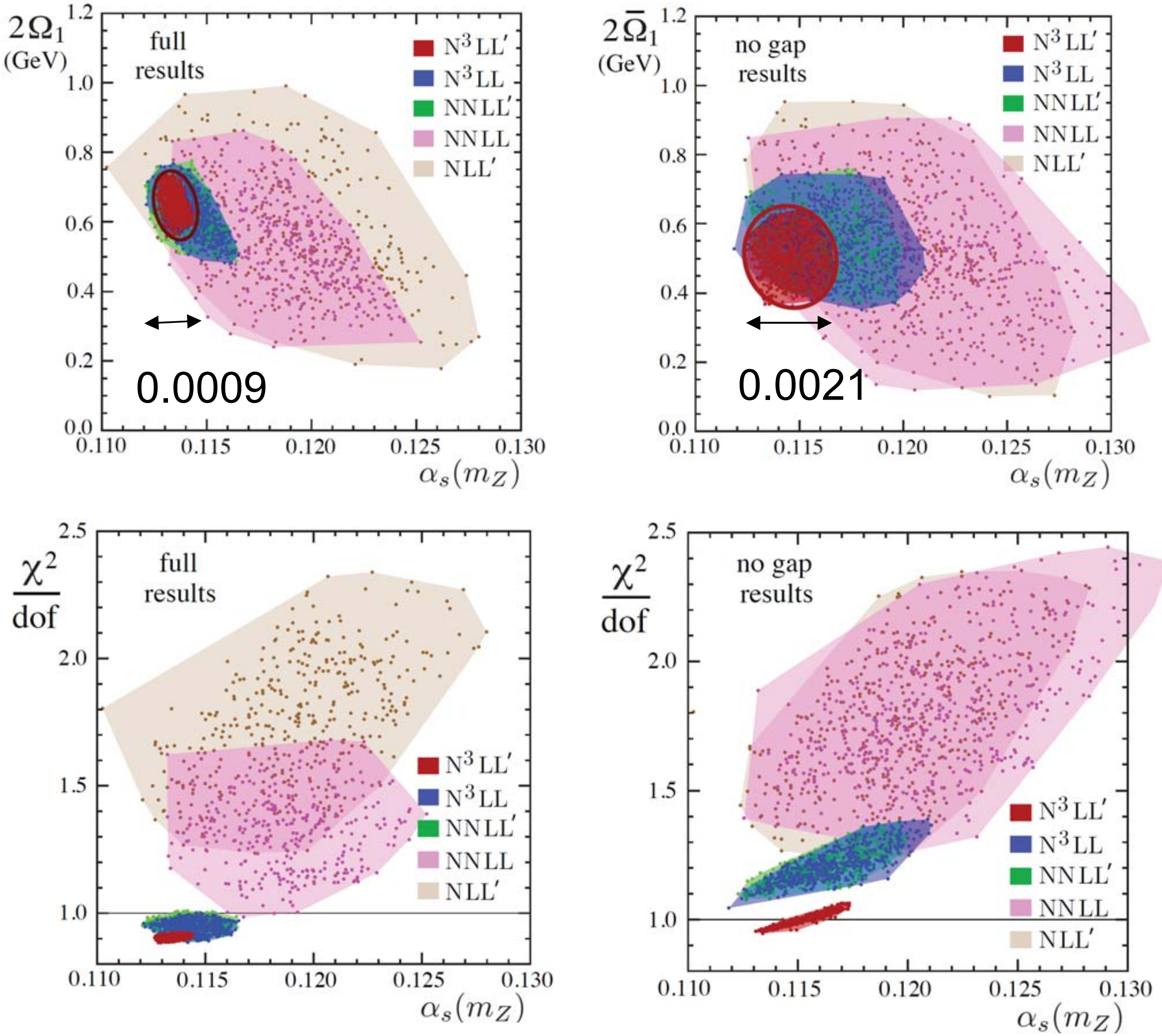
Running equations for $\bar{\Delta}(R,\mu)$ $\begin{cases} \text{R-running} \\ \mu\text{-running} \end{cases}$

Convergence of results



Theory uncertainty is from a flat scan

Renormalon free | **With renormalon**



Renormalon-free results have smaller theory errors and better fits

Ω_1 determined to 16% accuracy

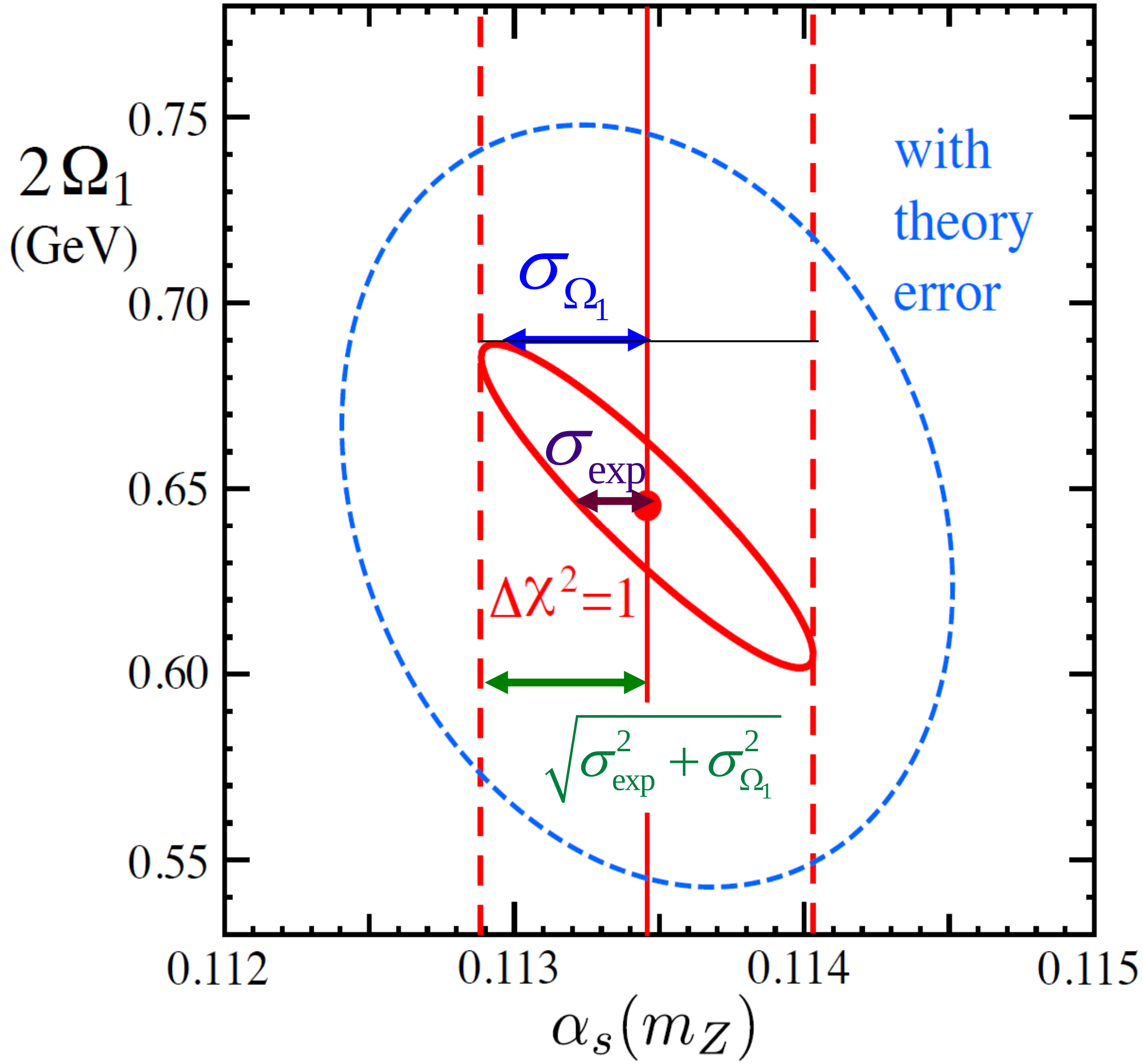
500 points random scan per order

Adding individual errors in quadrature gives similar (but smaller) error

Experimental error

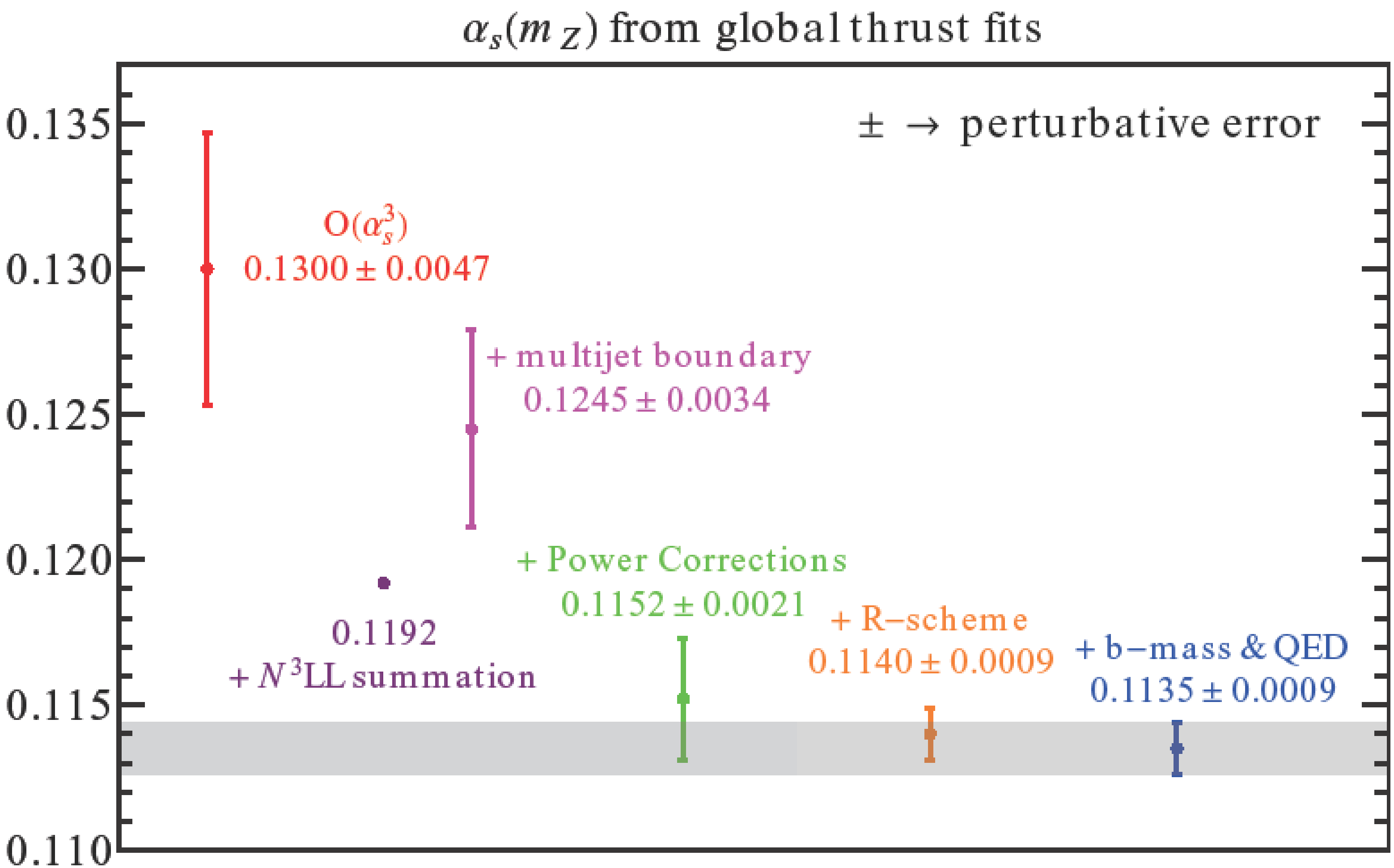
$$\frac{\chi^2}{\text{dof}} = \frac{440}{485} = 0.91$$

1σ (39% confidence level)

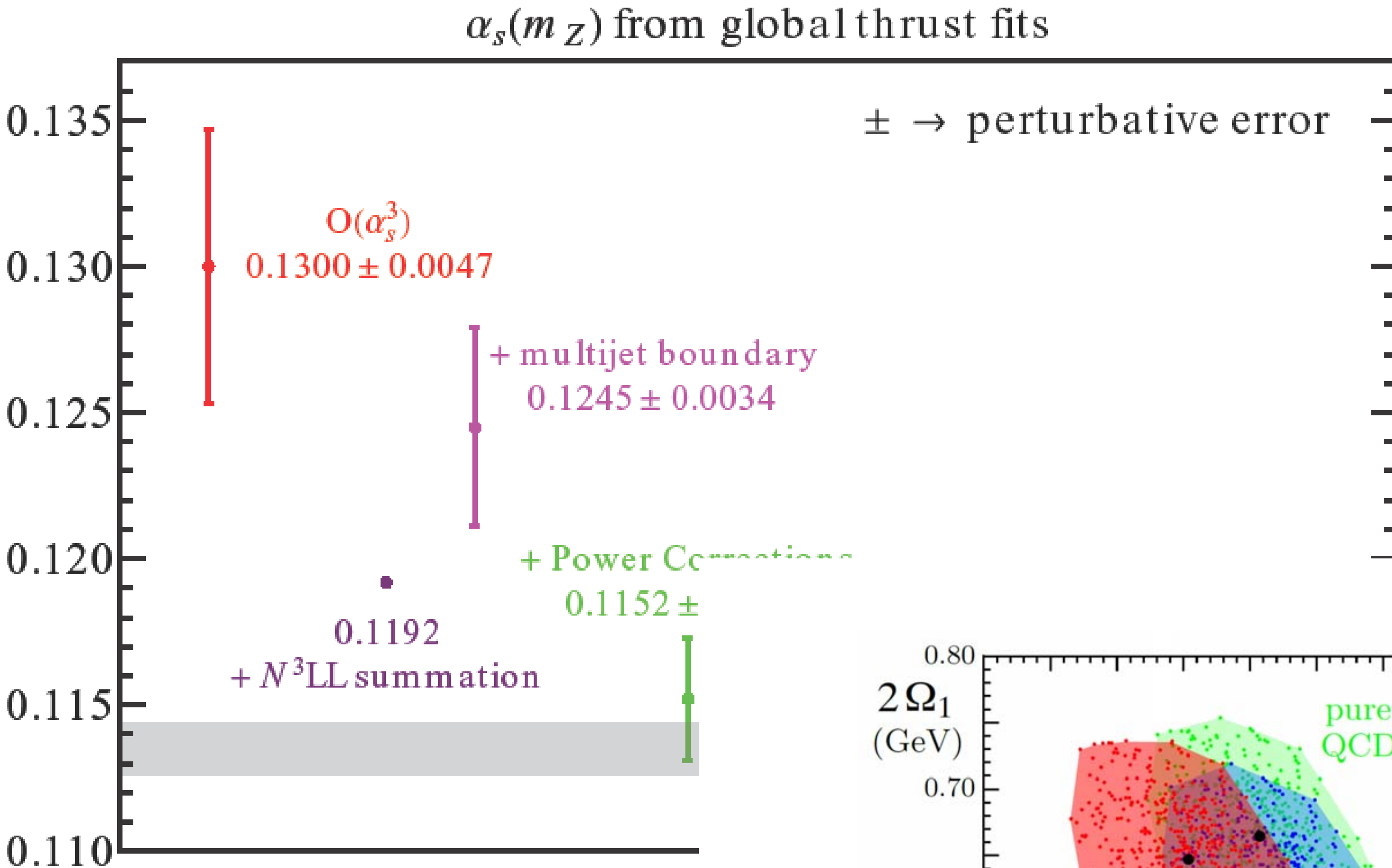


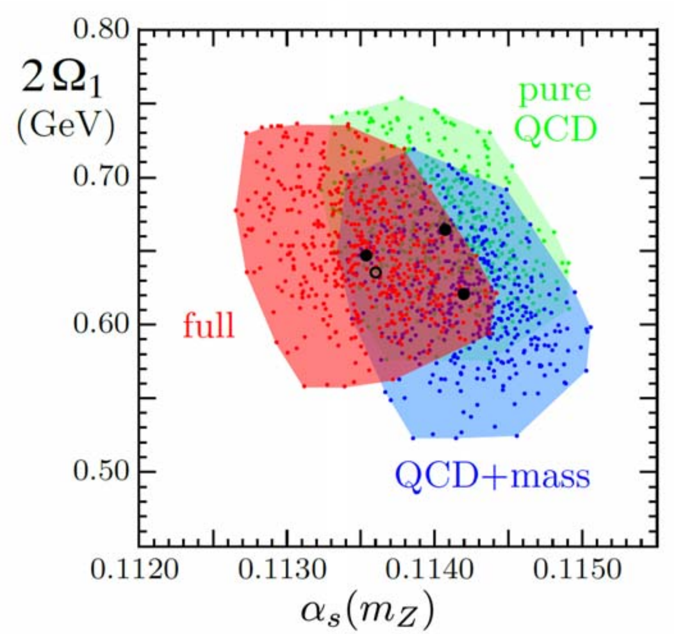
$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

mostly Ω_1, and includes Ω_2 — scan



- Resummation at N^3LL
- Multijet boundary condition
- Power correction, in a scheme free of the $O(\Lambda_{QCD})$ renormalon
- QED & bottom mass corrections





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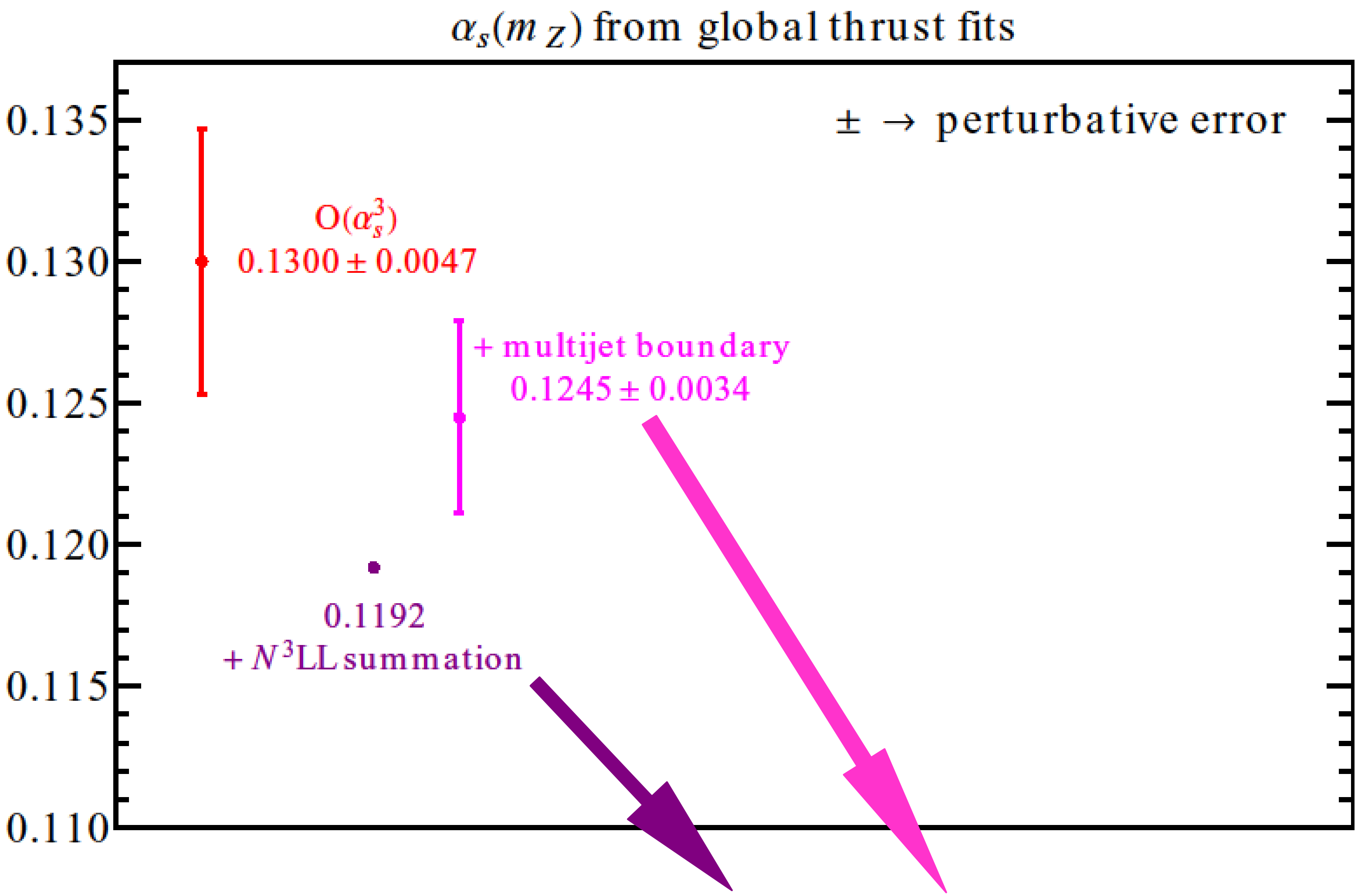


Preliminary Fits to ALEPH data:			
	thrust	0.1169	0.1223
	heavy jet mass	0.1175	0.1220

Perturbative N^3LL agrees*
with Chien & Schwartz '10

Final thrust result

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

- Use of the latest $O(\alpha_s^3)$ results for fixed order and matrix elements.
- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets:
    - Resummation of logs at N^3LL.
    - Field theoretical treatment of nonperturbative effects.
- Theory valid in peak, tail and multijet.
- Renormalon free Ω_1 provides very stable results → improves perturbative errors.
- Inclusion of QED, b-mass and axial singlet corrections.
- Global fit of all data with all Q's. Simultaneous fits to α_s & Ω_1 .

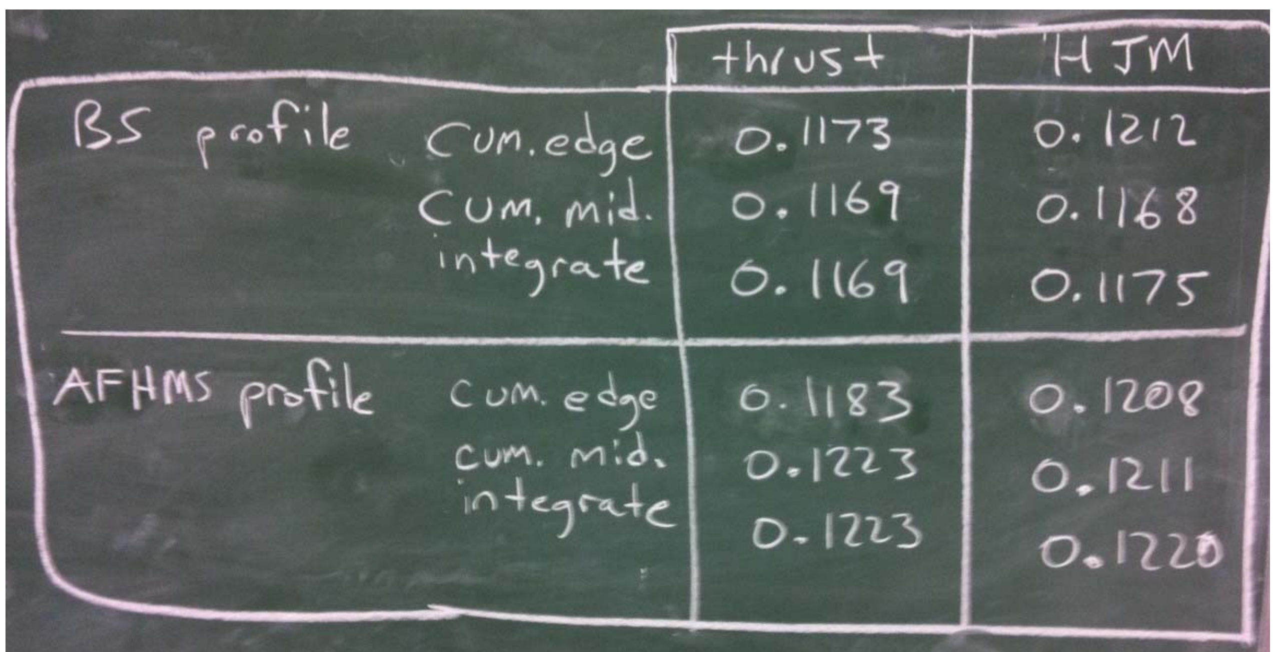
Back up slides

Scale setting in cumulants

$$\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))$$

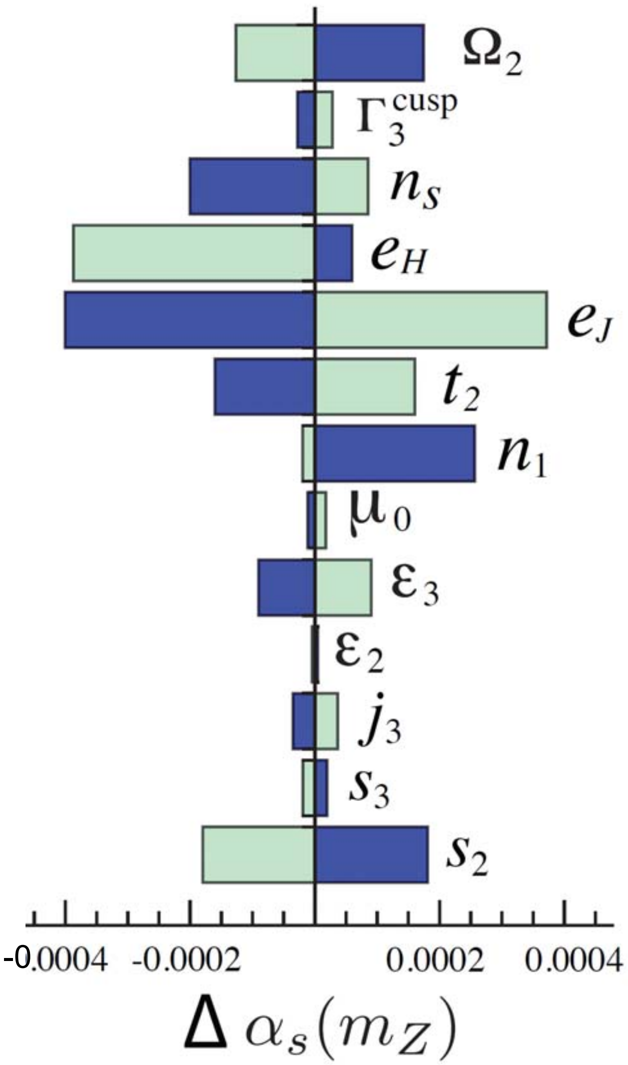
$$= \underbrace{\left[\int_{\tau_1}^{\tau_2} d\tau' \, \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau', \mu_i(\tau_2)) \right]}_{\simeq \left[\int_{\tau_1}^{\tau_2} d\tau' \, \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau', \mu_i(\tau')) \right]} + \underbrace{\Sigma(\tau_1, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))}_{\simeq (\tau_2 - \tau_1) \frac{d\mu_i(\tau_1)}{d\tau} \frac{\partial}{\partial \mu_i} \int_0^{\tau_1} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau_1))}$$

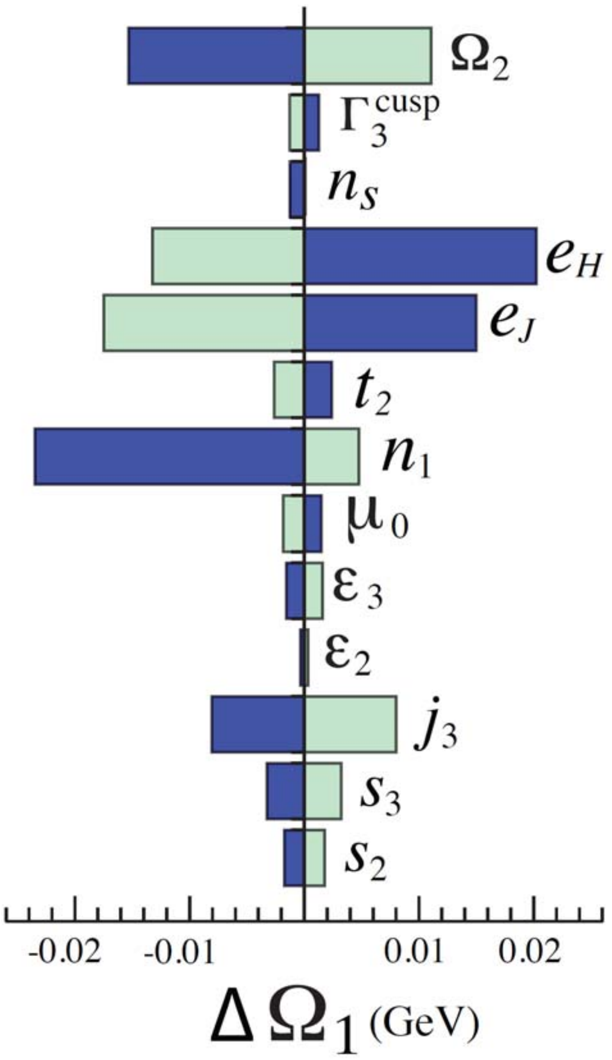
Preliminary numbers



		thrust	HJM
BS profile	cum. edge	0.1173	0.1212
	cum. mid.	0.1169	0.1168
	integrate	0.1169	0.1175
AFHMS profile	cum. edge	0.1183	0.1208
	cum. mid.	0.1223	0.1211
	integrate	0.1223	0.1220

Effect of the various scan parameters

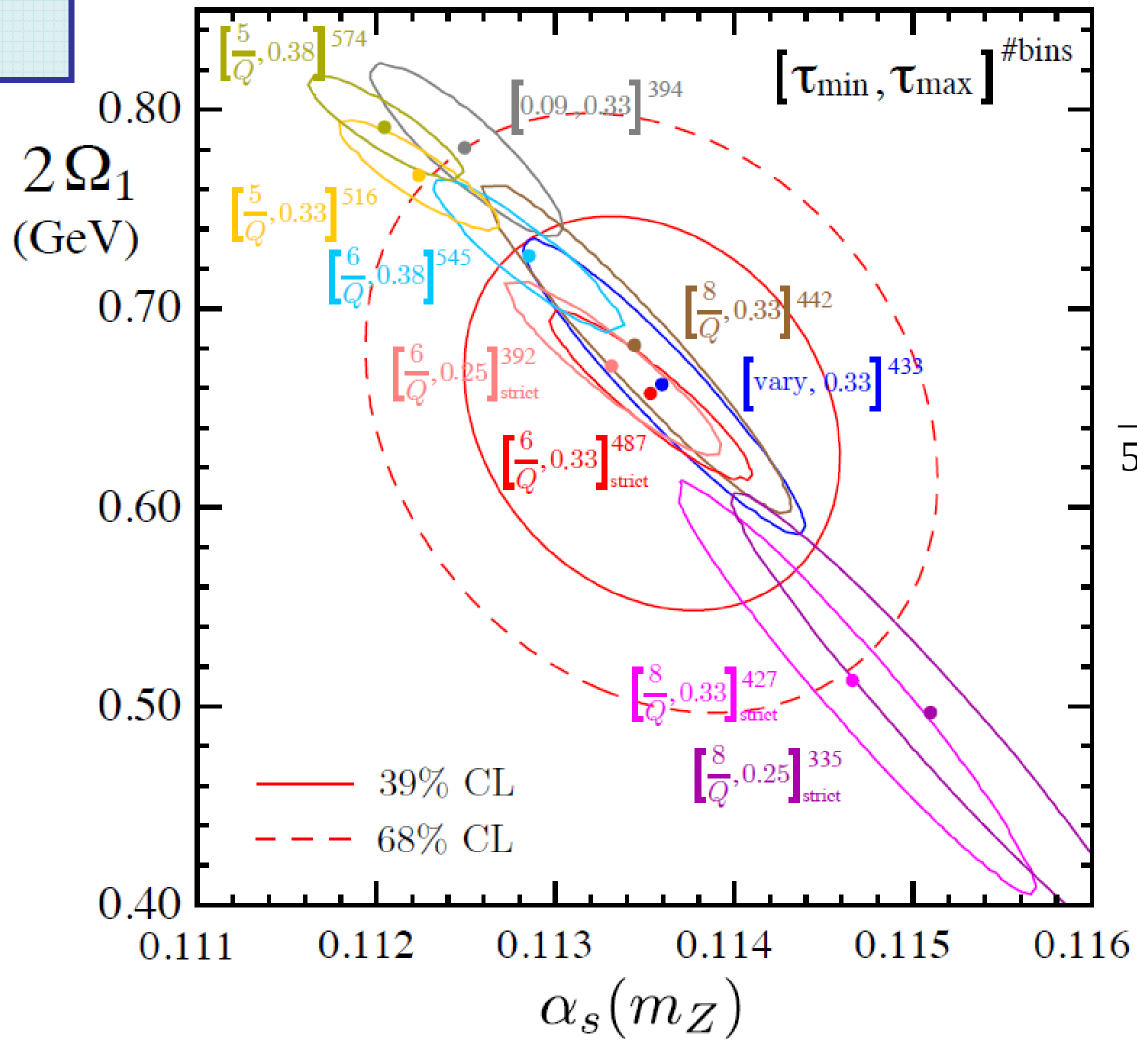




Fit for bins: different data sets

Ω_2 effects increase

Renormalon free



$$\frac{\Omega_1}{50.2\mathrm{GeV}} = 0.1200 - \alpha_s(m_Z)$$

statistical errors decrease

Theoretical motivation

$$\ln\left(\frac{1}{\sigma}\frac{d\sigma}{d\tau}\right) \sim (\ln\tau)\sum_{k=0}(\alpha_s \ln\tau)^{k+1} + \sum_{k=0}(\alpha_s \ln\tau)^{k+1} + \alpha_s\sum_{k=0}(\alpha_s \ln\tau)^{k} + \alpha_s^2\sum_{k=0}(\alpha_s \ln\tau)^{k} + \ldots$$

LL NLL NNLL N^3LL

For low τ, $\ln(\tau) \sim \frac{1}{\alpha_s}$ ⇒ Rearrange the perturbative series and resum logs

In fact logs dominate even for moderate τ

Theoretical motivation

$$\ln\left(\frac{1}{\sigma}\frac{d\sigma}{d\tau}\right) \sim (\ln\tau)\sum_{k=0}(\alpha_s \ln\tau)^{k+1} + \sum_{k=0}(\alpha_s \ln\tau)^{k+1} + \alpha_s\sum_{k=0}(\alpha_s \ln\tau)^{k} + \alpha_s^2\sum_{k=0}(\alpha_s \ln\tau)^{k} + \ldots$$

LL NLL NNLL N^3LL

For low τ, $\ln(\tau) \sim \frac{1}{\alpha_s}$ ⟹ Rearrange the perturbative series and resum logs

In fact logs dominate even for moderate τ

singular — nonsingular

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} \sim \underbrace{\sum_n \alpha_s^n \delta(\tau) + \sum_{n,l}\alpha_s^n \frac{\ln^l \tau}{\tau}}_{\text{singular}} + \underbrace{\sum_{n,l}\alpha_s^n \ln^l \tau + \sum_n \alpha_s^n f_n(\tau)}_{\text{nonsingular}}$$

perturbative part

$$+ f\left(\tau, \frac{\Lambda_{QCD}}{Q}\right)$$

Nonperturbative power corrections

Classical resummation

[CTTW] Catani et al

$$\int_0^{\tau} \mathrm{d}\tau \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = C(\alpha_s)\Sigma(\alpha_s,\tau) + D(\alpha_s,\tau)$$

matching condition

resummation of singular terms (exponenciates)

finite terms as $\tau \to 0$

$$C(\alpha_s) = 1 + \sum_n C_n \alpha_s^n$$

$$\log\Sigma(\alpha_s,\tau) = \log(\tau)\sum_n [\alpha_s \log(\tau)]^n + \sum_n [\alpha_s \log(\tau)]^n + \alpha_s \sum_n [\alpha_s \log(\tau)]^n + \cdots$$

LL NLL NNLL

- Resumation is performed at the level of the cross section.
- It is produced by multiple collinear and soft gluon emisions.

$$\left.\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_{\text{resumation singular}} = \frac{Q^2}{2\pi i}\int_C \mathrm{d}s\, e^{\tau s Q^2}\left[\bar{J}_C(Q^2,s)\right]^2$$

Classical resummation

[CTTW] Catani et al

$$\int_0^\tau d\tau \frac{1}{\sigma}\frac{d\sigma}{d\tau} = C(\alpha_s)\Sigma(\alpha_s,\tau) + D(\alpha_s,\tau)$$

matching condition → $C(\alpha_s)$

resummation of singular terms (exponenciates) → $\Sigma(\alpha_s,\tau)$

finite terms as $\tau \to 0$ → $D(\alpha_s,\tau)$

$$C(\alpha_s) = 1 + \sum_n C_n \alpha_s^n$$

$$\log\Sigma(\alpha_s,\tau) = \log(\tau)\sum_n[\alpha_s \log(\tau)]^n + \sum_n[\alpha_s \log(\tau)]^n + \alpha_s\sum_n[\alpha_s \log(\tau)]^n + \cdots$$

LL NLL NNLL

• Resumation is performed at the level of the cross section.

• It is produced by multiple collinear and soft gluon emisions.

$$\left.\frac{1}{\sigma}\frac{d\sigma}{d\tau}\right|_{\text{resumation singular}} = \frac{Q^2}{2\pi i}\int_C ds\, e^{\tau s Q^2}\left[\bar{J}_C(Q^2,s)\right]^2$$

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{\rm ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int ds \; J_\tau(s,\mu) \, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\rm part}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

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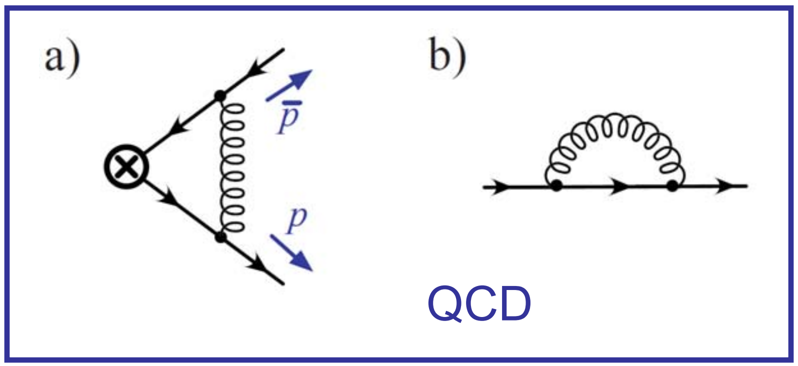
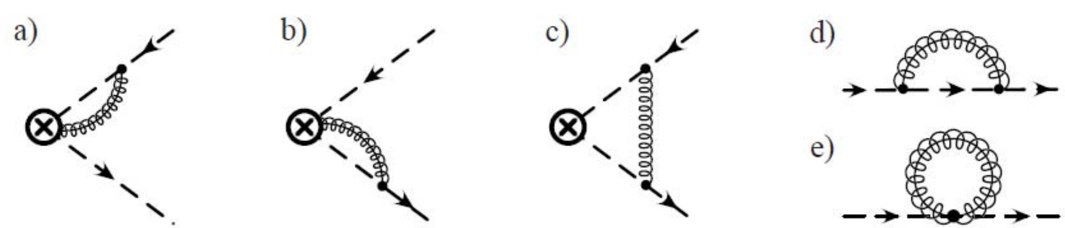
- Electroweak factor: family (flavor) and current (vector/axial-vector) dependent
- Lowest order cross section

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I H_Q^I(Q,\mu) \int ds\ J_\tau(s,\mu)\, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{part}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

Hard Wilson coefficient (function) $H(Q,\mu_h) = |C(Q,\mu_h)|^2$





- Known up to $O(\alpha_s^3)$ N³LO
- Anomalous dimension known up to $O(\alpha_s^3)$
- Cusp anomalous dimension known up to $O(\alpha_s^3)$

Baikov et al

Lee et al

Moch et al

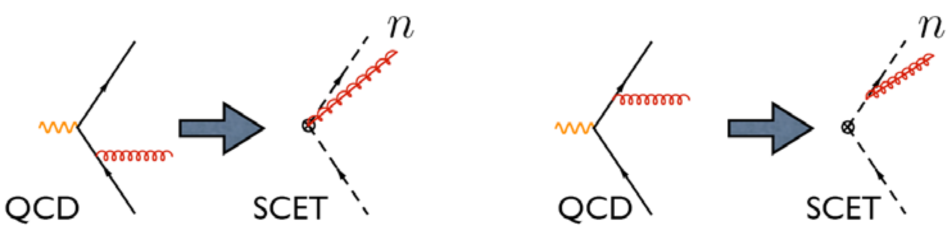
Adding the $O(\alpha_s^4)$ cusp with a Padè approximation $\longrightarrow$ N³LL analysis

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{b}}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_{\tau}^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

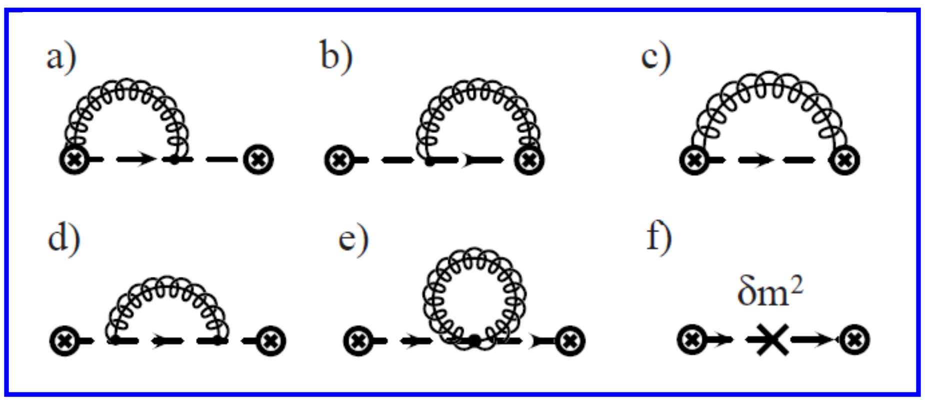
$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int \mathrm{d}s \, J_\tau(s,\mu) \, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial \tau}} S_\tau^{\mathrm{part}}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

Jet function

$$J_n(Q r_n^+, \mu) = \frac{-1}{8\pi N_c Q} \, \mathrm{Disc} \int d^4x \, e^{i r_n \cdot x} \, \langle 0 | \mathrm{T} \, \bar{\chi}_{n,Q}(0) \bar{\not{n}} \chi_n(x) | 0 \rangle$$



Discontinuity of quark self-energy in the light-cone gauge



- Known up to $O(\alpha_s^2)$ NNLO — Becher & Neubert
- Anomalous dimension known up to $O(\alpha_s^3)$ — Moch, Vermaresen & Vogt

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I H_Q^I(Q,\mu) \int ds \; J_\tau(s,\mu) e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{part}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

Renormalon subtraction

- Reduces sensitivity to low momenta in the soft function
- Removes an $O(\Lambda_{QCD})$ renormalon from the first moment of the soft function

$$\Delta = \delta(R,\mu) + \bar{\Delta}(R,\mu)$$ ← Renormalon-free gap shift

$$\delta(R,\mu) = \sum_n \left(\frac{\alpha}{4\pi} \right)^n \delta_n(R,\mu)$$ ← Infrared scheme parameter

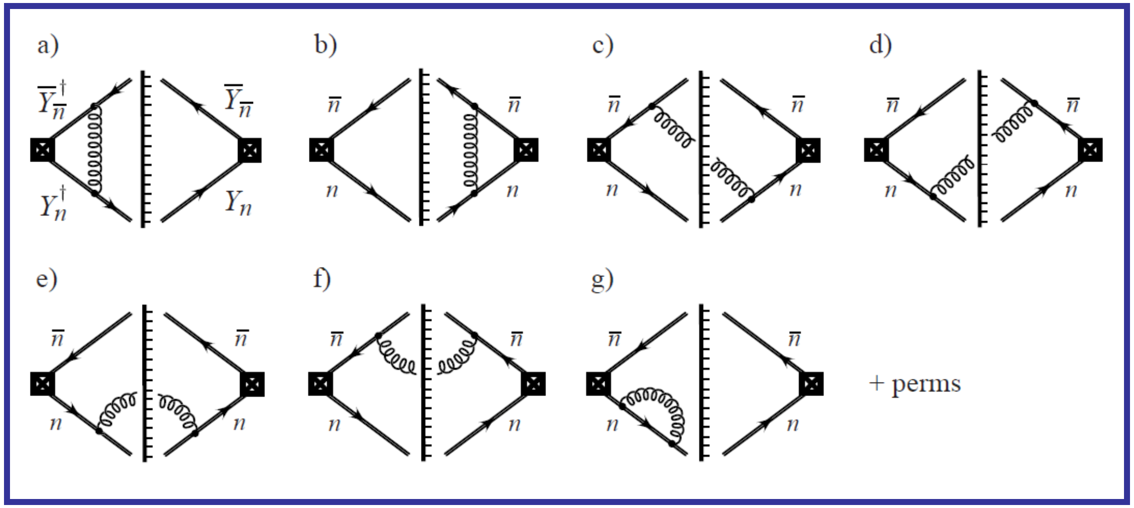
$$\delta_n(R,\mu) = R\, e^{\gamma_R} \sum_m \delta_n^m \log^m\left(\frac{R}{\mu} \right)$$ → Can become large, Keep them O(1) → $R \sim \mu$

Running equations for $\bar{\Delta}(R,\mu)$:
- R-running (infrared evolution)
- μ-running (ultraviolet evolution)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{b}}{\mathrm{d}\tau}\right)\left(\tau - \frac{k}{Q}\right) S_{\tau}^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q}\right)$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int \mathrm{d}s \; J_\tau(s,\mu)\, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial \tau}} S_{\tau}^{\mathrm{part}}\left(Q\tau - \frac{s}{Q}, \mu\right)$$

Soft Function



$$S(\ell^+,\ell^-,\mu) \equiv \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\,\delta(\ell^- - k_s^{-b})\,\langle 0|(\overline{Y}_{\bar{n}})^{cd}\,(Y_n)^{ce}(0)|X_s\rangle\langle X_s|(Y_n^\dagger)^{ef}\,(\overline{Y}_{\bar{n}}^\dagger)^{df}(0)|0\rangle$$

- Analytically known up to $O(\alpha_s)$ — Schwartz; Fleming et al
- Numerically known up to $O(\alpha_s^2)$ NNLO — Becher & Schwartz; Hoang & Kluth

$$\frac{d\sigma}{d\tau} = \int dk \left(\boxed{\frac{d\hat{\sigma}}{d\tau}} + \frac{d\hat{\sigma}_{\mathrm{ns}}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I H_Q^I(Q,\mu) \int ds \; J_\tau(s,\mu)\, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

Still has large logs

Resummation of large logs!

No large logs any more

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I H_Q^I(Q,\mu_H) U_Q(Q,\mu_H,\mu_S) \int ds\,ds' \, U_J(s-s',\mu_S,\mu_J) J_\tau(s',\mu_J)\, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}}\left(Q\tau - \frac{s}{Q}, \mu_S \right)$$

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

$$\frac{d\hat{\sigma}_{ns}}{d\tau} \equiv \left. \frac{d\sigma}{d\tau} \right|_{\text{fixed order}} - \left. \frac{d\sigma}{d\tau} \right|_{\text{SCET(no resummation)}}$$

Known analytically to three-loop accuracy*

Known analitically at 1 loop — Ellis et al

Known numerically at 2 loops — EVENT2

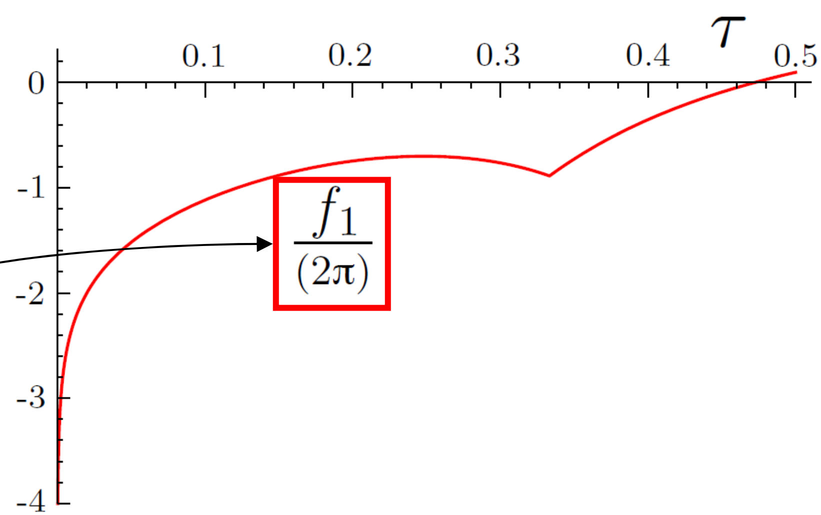
3 loops — EERAD3

Gets very noisy at small !

We use a fit function at small τ and an interpolating function for the rest

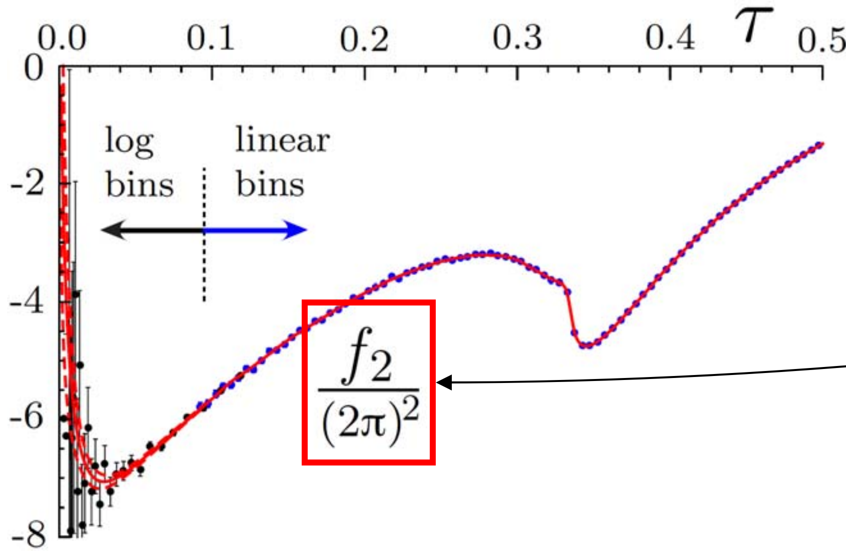
$$\frac{d\hat{\sigma}_{ns}}{d\tau} = \sum_I \sigma_0^I e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} f^I\left(\tau, \frac{\mu_{ns}}{Q} \right)$$

$$f^I(\tau,1) = \frac{\alpha_s(Q)}{2\pi} f_1(\tau) + \left[\frac{\alpha_s(Q)}{2\pi} \right]^2 f_2(\tau) + \left[\frac{\alpha_s(Q)}{2\pi} \right]^3 f_3(\tau) + \cdots$$



$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \boxed{\frac{d\hat{\sigma}_{ns}}{d\tau}} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

$$f^I(\tau,1) = \frac{\alpha_s(Q)}{2\pi} f_1(\tau) + \left[\frac{\alpha_s(Q)}{2\pi}\right]^2 \boxed{f_2(\tau)} + \left[\frac{\alpha_s(Q)}{2\pi}\right]^3 \boxed{f_3(\tau)} + \cdots$$

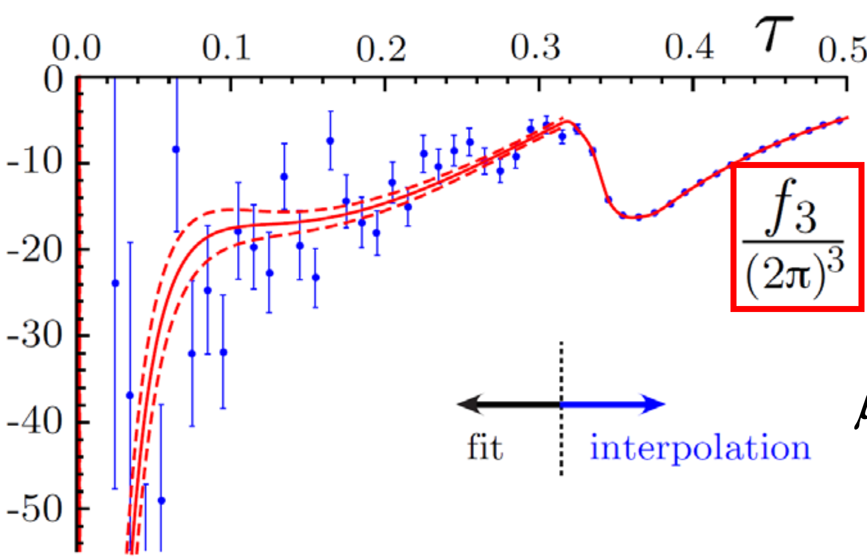


2 loops EVENT2

$$f_2(\tau) + \varepsilon_2\, \delta_2(\tau)$$

3 loops EERAD3

$$f_3(\tau) + \varepsilon_3\, \delta_3(\tau)$$



$$\mu_{ns} = \begin{cases} \mu_H & (n_s = 1) \\ \mu_J & (n_s = 0) \\ \frac{1}{2}(\mu_J + \mu_S) & (n_s = -1) \end{cases}$$

Varied in our a scan through parameter n_s

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{\rm ns}}{d\tau} + \boxed{\frac{d\hat{\sigma}_b}{d\tau}} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

Include b mass effects in factorization theorem (2% effect)

$$\frac{d\hat{\sigma}_b}{d\tau} = \frac{d\hat{\sigma}^{\rm NNLL}_{\rm massive}}{d\tau} - \frac{d\hat{\sigma}^{\rm NNLL}_{\rm massless}}{d\tau}$$

- At this order affects only jet function and τ limits.
- Use massive SCET factorization theorem. Flemming, Hoang, Matry, Stewart

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\rm ns}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right)\left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

Include QED effects in factorization theorem

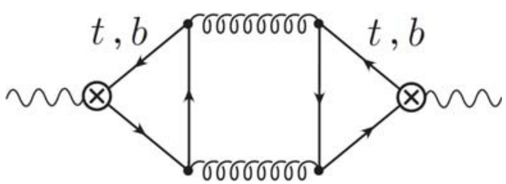
(2% effect)

- Count $\alpha_{\rm qed} \sim \alpha_s^2$, include only final state radiation.
- Include O($\alpha_s^2 \alpha_{\rm qed}$) corrections to QCD β function.
- Include one loop QED corrections to matrix elements and nonsingular.

Include axial anomaly contribution

(1% effect)

- Affects the bottom axial hard coefficient at O(α_s^2).
- Affects the nonsingular at O(α_s^2).
- Due to large top-bottom mass splitting.



Kniehl, Kuhn
Hagiwara, Kuruma, Yamada

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{\rm ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

$$\frac{\Lambda_{QCD}}{\mu_s} \approx \frac{\Lambda_{QCD}}{Q\tau}$$

$$\frac{\mu_s^2}{\mu_J^2} \approx \tau$$

$$\frac{\Lambda_{QCD}}{\mu_h} \approx \frac{\Lambda_{QCD}}{Q}$$

$$H\ J_T \otimes S_T^{pert} \otimes S_\tau^{\rm mod}$$

$$\frac{d\hat{\sigma}_{\rm ns}}{d\tau} \otimes S_\tau^{\rm mod}$$

It is the same model function!

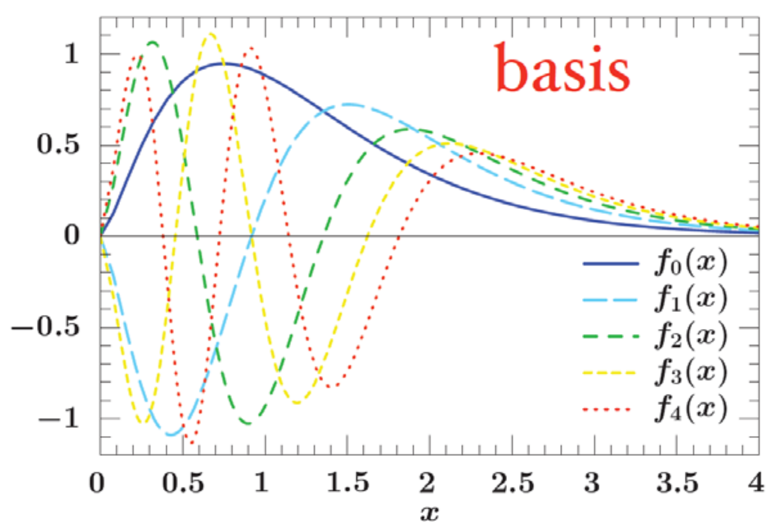
- In the peak region one cannot expand
- Tail and far tail: expansion causes shift

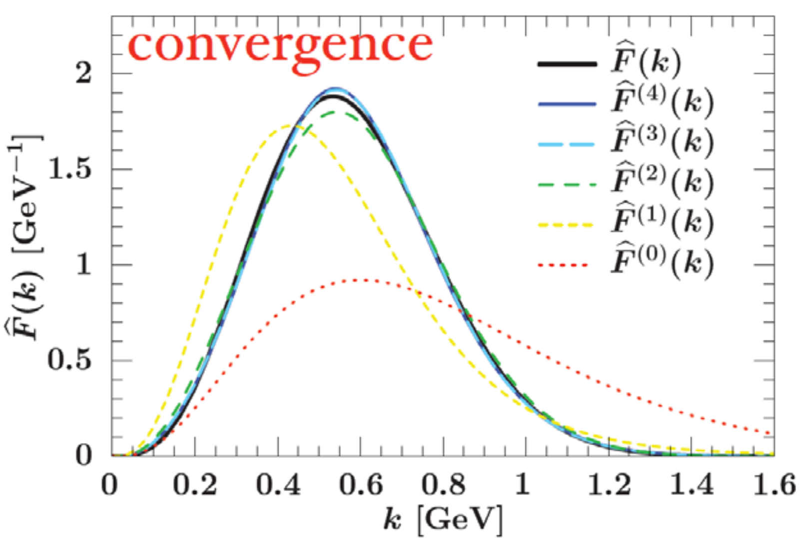
- Requires SCET sub-leading calculation
- Numerically irrelevant

Same effect for all tau

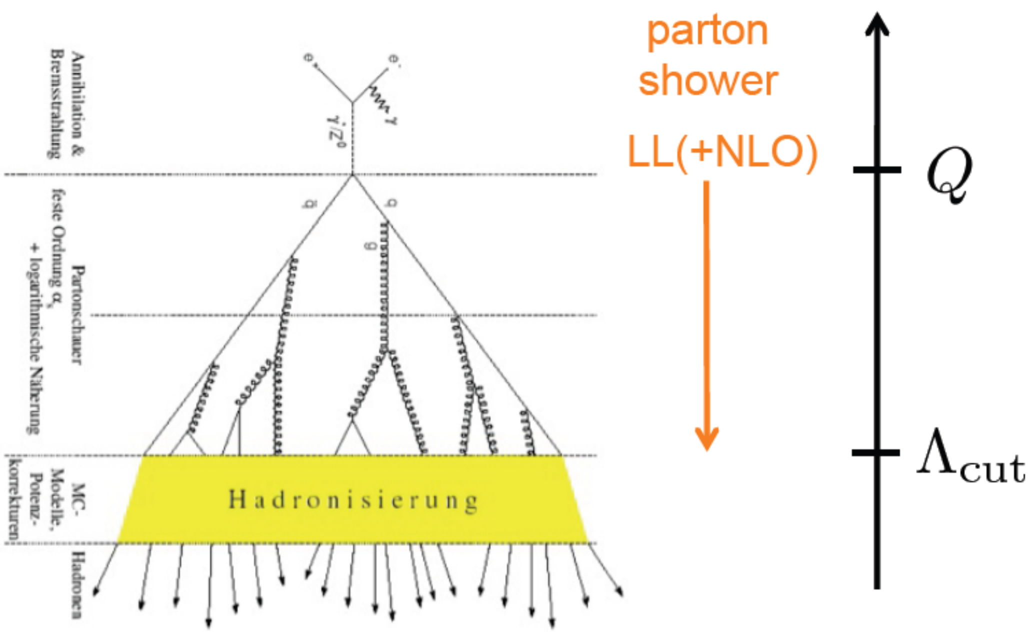
$$S_\tau^{\rm mod}(\ell) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n\, f_n\!\left(\frac{\ell}{\lambda} \right) \right]^2$$

Ligeti , Stewart & Tackmann





Hadronization Corrections from QCD Monte Carlos



Monte Carlo QCD:

Partonic MC results are in some (yet unspecified) scheme with an IR cutoff $\Lambda_{\rm cut} = 1\ \mathrm{GeV}$

→ free of IR renormalons

Analytic (multiloop) QCD:

Dim. reg. used to regularize IR momentum contributions

→ IR renormalons

→ Hadronization corrections in MC's cannot be used to estimate nonperturbative corrections for multiloop results based on dim. reg.

→ All analyses using hadronization corrections from MC's essentially fit the perturbative multiloop results to the LL(+NLO) partonic MC predictions.

Ingredients for the calculation

Order or the analysis

Anomalous dimension

Matrix elements

α_s running

Fixed order

gap subtractions

	cusp	non-cusp	matching	$\beta[\alpha_s]$	nonsingular	$\gamma_\Delta^{\mu,R}$	δ
LL	1	-	tree	1	-	-	-
NLL	2	1	tree	2	-	-	-
NNLL	3	2	1	3	1	1	1
N^3LL	4^{pade}	3	2	4	2	2	2
NLL′	2	1	1	2	1	1	1
NNLL′	3	2	2	3	2	2	2
N^3LL'	4^{pade}	3	3	4	3	3	3

The LL, NLL, NNLL and N^3LL rows are labelled "standard counting"; the NLL′, NNLL′ and N^3LL' rows are labelled "primed counting".

s_2

j_3

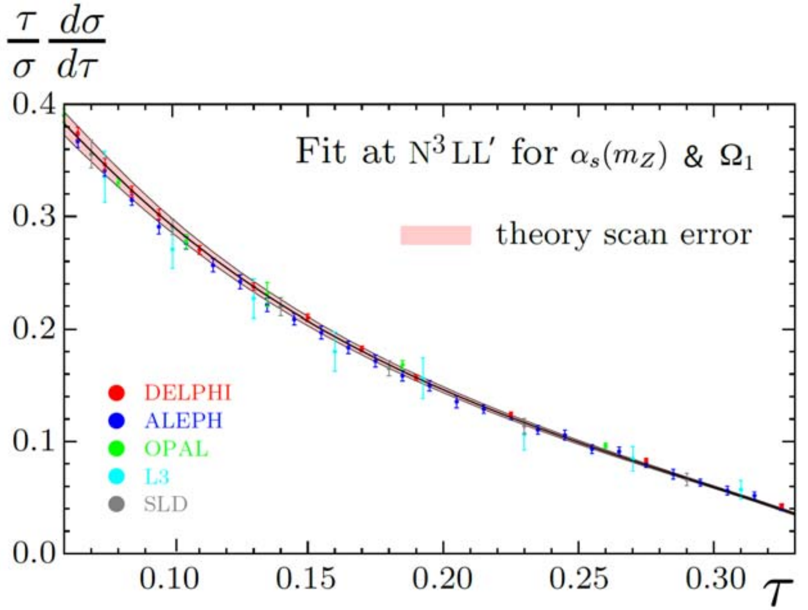
s_3

From a Padè approximant $\Gamma_{\text{cusp}}^{(3)}$

For jet and soft: log information known, and sum of non-log terms known.

When fixed order resuls are important primed counting is better

Peak and multijet regions

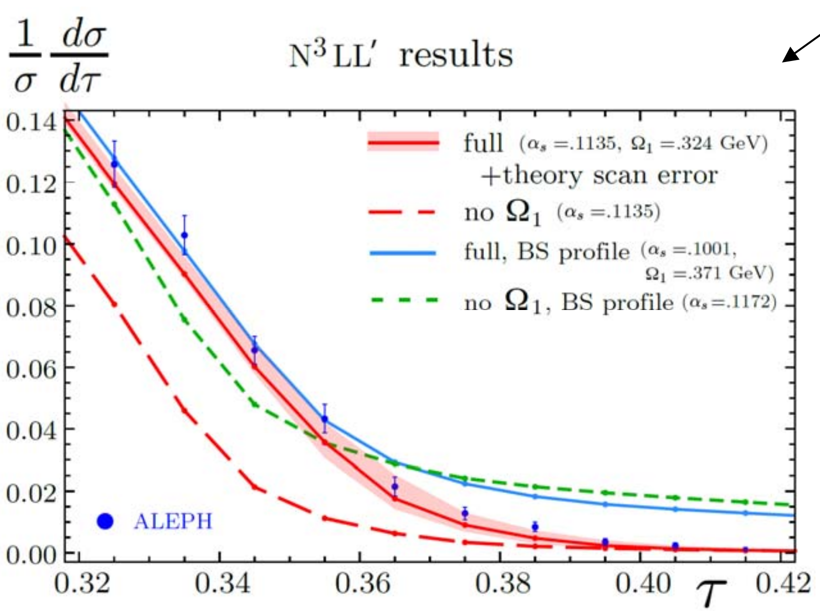


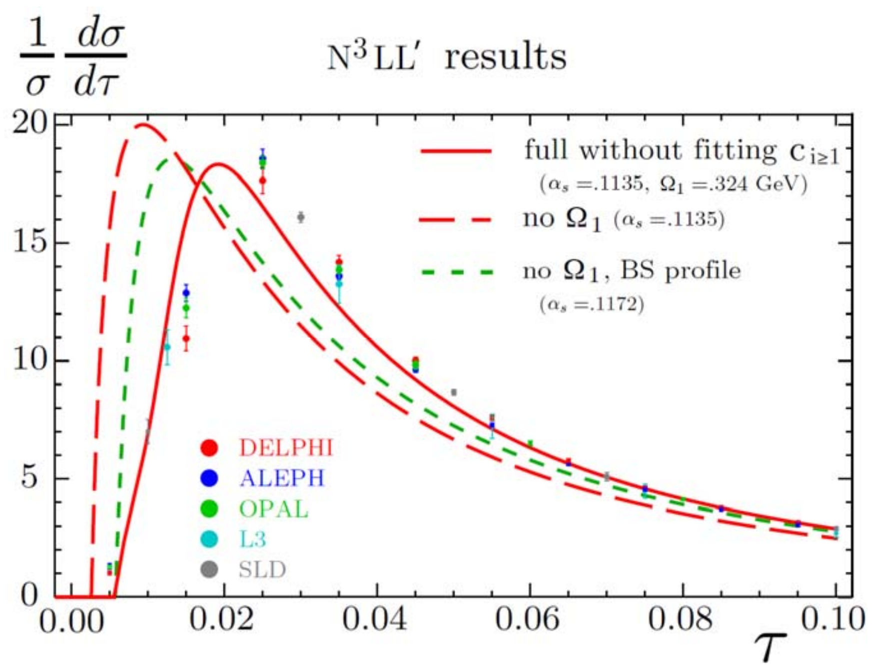
$Q = m_Z$

Fit tail

Predict multijet

Predict peak





Comparison with recent analyses

Becher & Schwartz 0803.0342

Our code removing:

- Model function (***)
- Renormalon subtraction (***)
- QED and mass corrections (**)
- Axial singlet (*)
- Full hard coefficient (*)

Our number full analysis:

$$\alpha_s(m_Z) = 0.1135 \pm 0.0006 \pm 0.0009$$

Their number:

$$\alpha(m_Z) = 0.1172 \pm 0.0013 \pm 0.0012 \pm 0.0012$$

Our reproduction using our pruned code:

$$\alpha(m_Z) = 0.1184 \pm 0.0013 \pm 0.0012 \pm 0.0012$$

They also use different profile functions (***)

Understimates the size of hadronization errors

- Data: Aleph & Opal
- Resummation: In the amplitude (EFT RG Equations)
- No nonperturbative effects in central value (Monte Carlo) (***)

(*) irrelevant (**) important (***) essential

Comparison with recent analyses

Davison & Webber 0809.3326

Our code removing:

- Log resumation in renormalon subtraction (**)
- Resummation beyond NLL (**)
- QED and mass corrections (**)
- Axial singlet (*)

Our number full analysis:

$$\alpha_s(m_Z) = 0.1135 \pm 0.0006 \pm 0.0009$$

Their number:

$$\alpha(m_Z) = 0.1164 \pm 0.0022 \pm 0.0017$$

Compatible !

- Data: Many Q's
- Resummation: In the distribution (classical approach)
- Nonperturbative effects in a model

Comparison with recent analyses

Dissertori et al 0712.0327

Our number full analysis:

$$\alpha_s(m_Z) = 0.1135 \pm 0.0006 \pm 0.0009$$

Their number:

$$\alpha(m_Z) = 0.1224 \pm 0.0013 \pm 0.0011 \pm 0.0028$$

Our code removing:

- Logs resummation (***)
- Model function (***)
- Renormalon subtraction (***)
- QED and mass corrections (**)
- Axial singlet (*)

Experiment	Energy	Dissertori et al. results [22]	Our fixed order code
ALEPH	91.2 GeV	0.1274(3)	0.1281
ALEPH	133 GeV	0.1197(35)	0.1289
ALEPH	161 GeV	0.1239(54)	0.1391
ALEPH	172 GeV	0.1101(72)	0.1117
ALEPH	183 GeV	0.1132(32)	0.1247
ALEPH	189 GeV	0.1140(20)	0.1295
ALEPH	200 GeV	0.1094(22)	0.1260
ALEPH	206 GeV	0.1075(21)	0.1214

- Data: Aleph
- No resummation (***)
- Nonperturbative effects treatment: Monte Carlo generator (***)

Heavy Jet Mass

The thrust axis defines two hemispheres $\quad s_i = \left(\sum_{\text{hem i}} p_j\right)^2 \quad \rightarrow \quad \rho = \mathrm{Max}(s_1, s_2)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = 2Q^2 \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int_0^{Q^2\rho} \mathrm{ds}_1 \int_0^{s_1/Q} \mathrm{dl}_1 \int_0^{Q\rho} \mathrm{dl}_2 \, J(s_1 - Ql_1, \mu) \, J(Q^2\rho - Ql_2, \mu) \, S(l_1, l_2, \mu)$$

- The heavy jet mass involves two nontrivial convolutions
- To hemispheres get entangled by non-perturbative effects
- For the nonsingular one would need the entire double differential hemiphere mass distribution $\longrightarrow$ highly nontrivial

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\rho}\left(\rho - \frac{\Omega_1}{Q}\right)$$

However in the far tail the OPE expansion works similarly