NNLO and "Classic" Power Corrections

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Outline

- NP power corrections
- Universal low-scale $\alpha_{\rm S}$ hypothesis
- NLL resummation
- Including power correction
- NNLO+NLL+NP fit to thrust

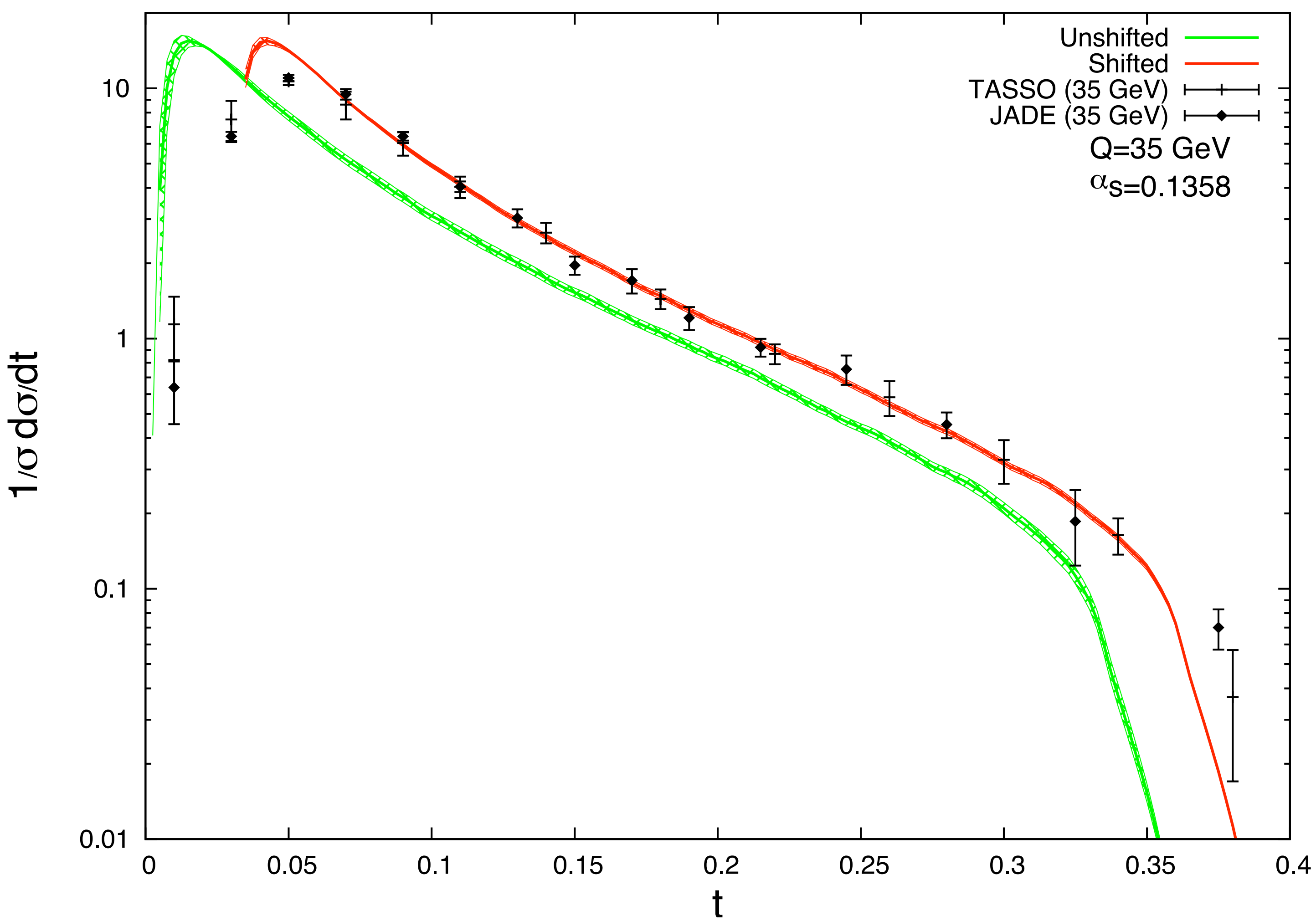
S Catani, L Trentadue, G Turnock & BW, NPB407(1993)3

Y Dokshitzer, G Marchesini & BW, NPB469(1996)93

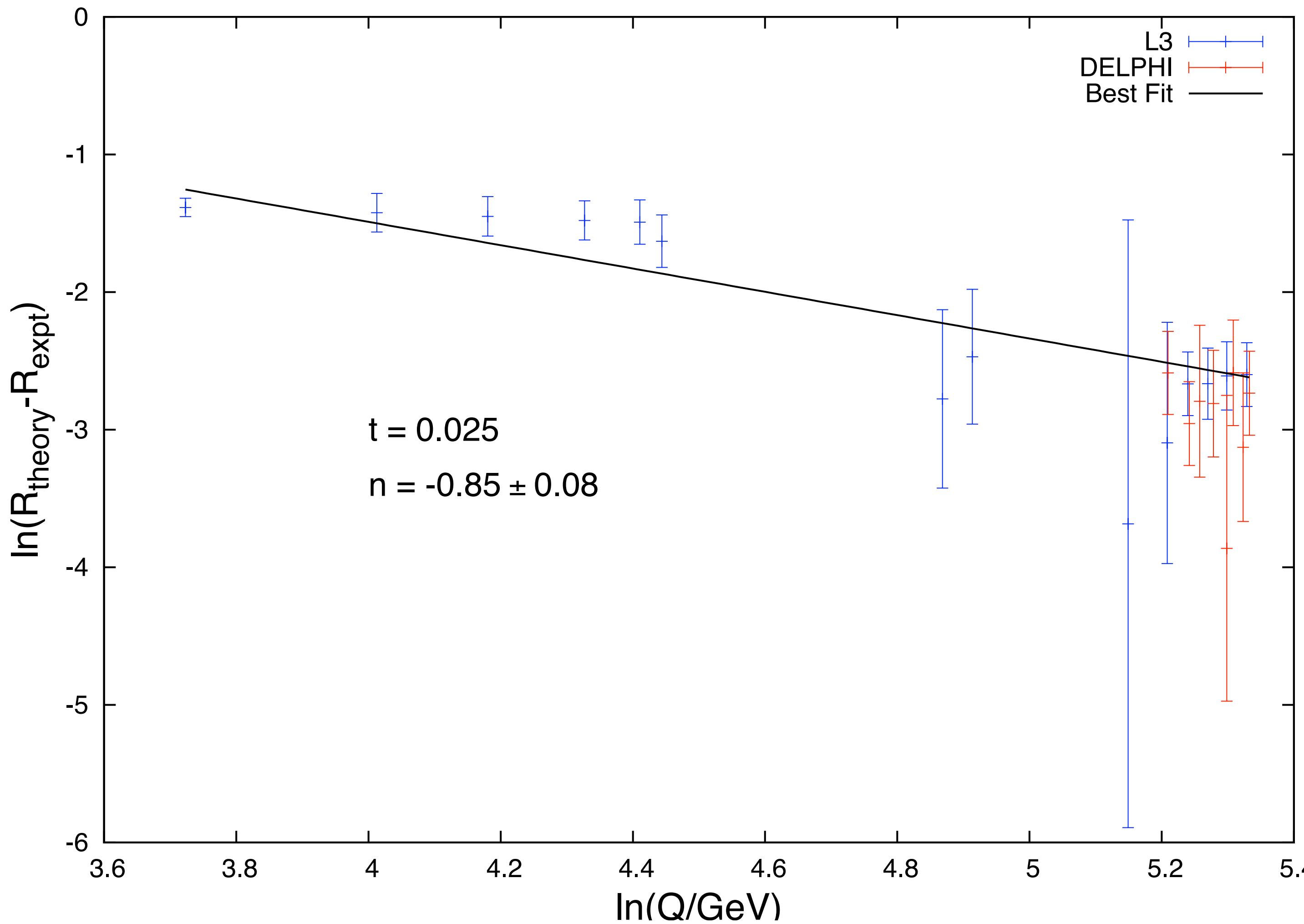
R Davison & BW, EPJC 59(2009)13; 69(2010)341

NP shift in Thrust

$$T = \max_{\vec{n}} \left(\frac{\sum_{i=1}^{N} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{N} |\vec{p_i}|} \right) \equiv 1 - t$$

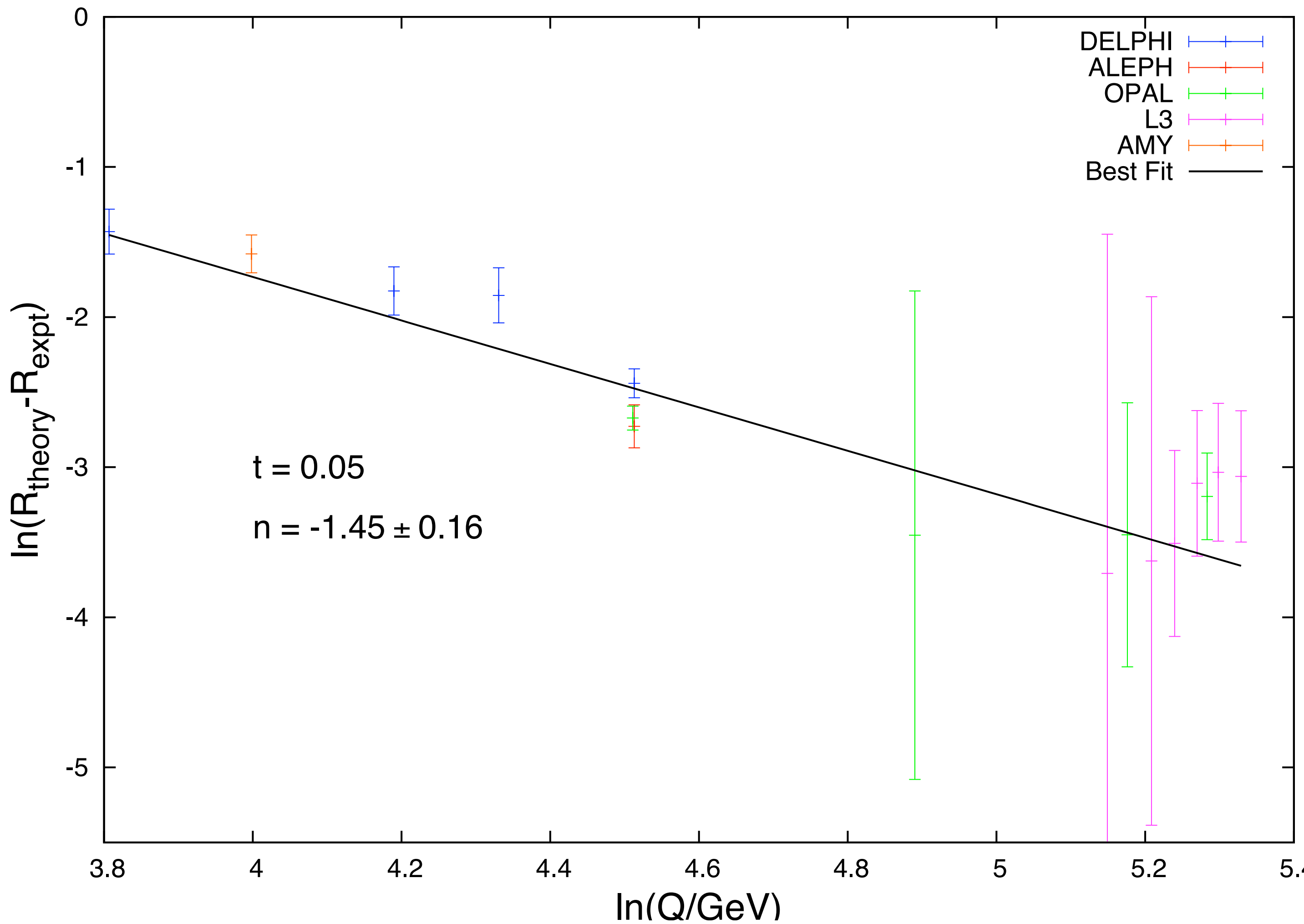


NP power dependence



- $R_{theory} - R_{expt} = AQ^n$

NP power dependence



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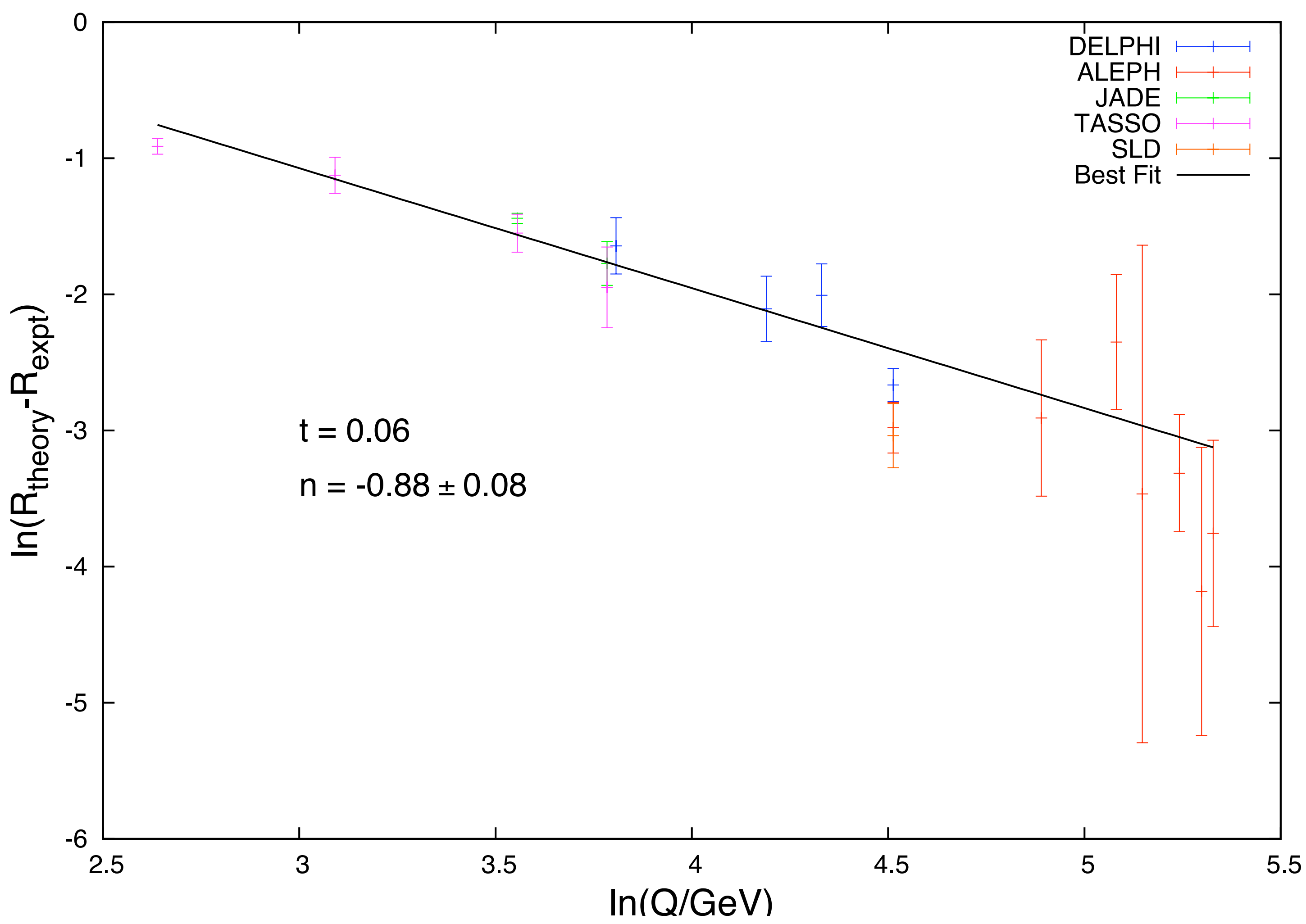

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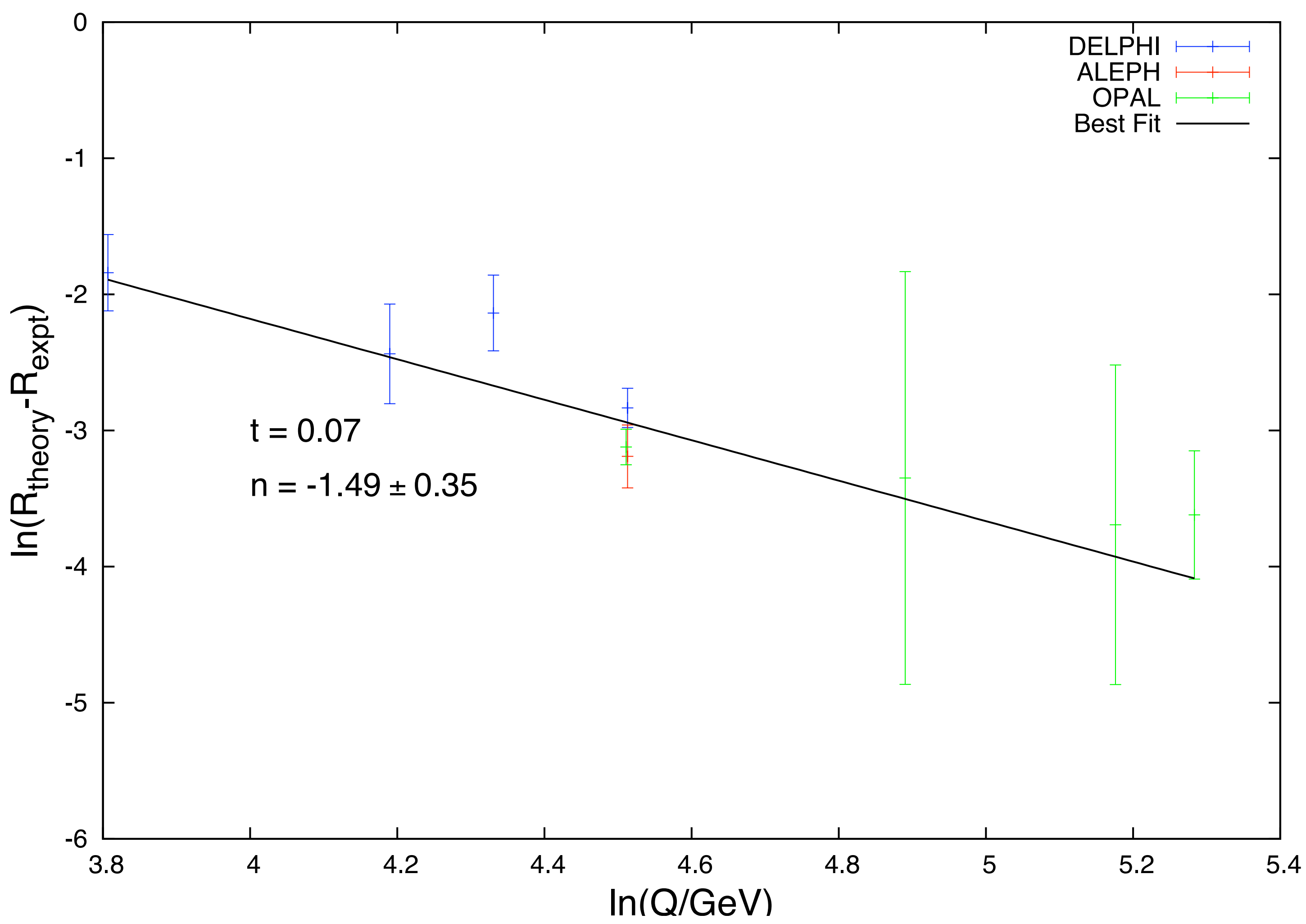

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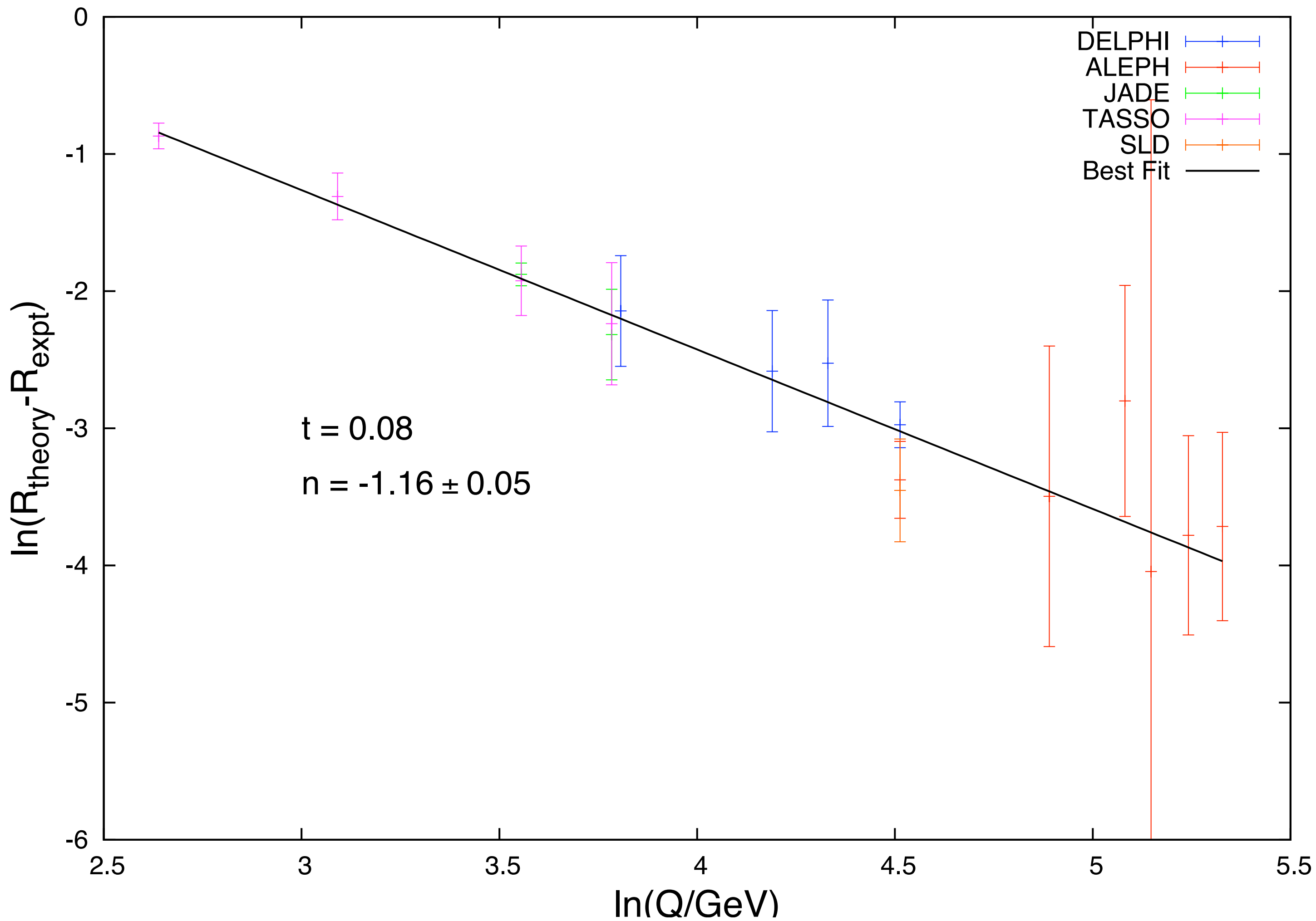

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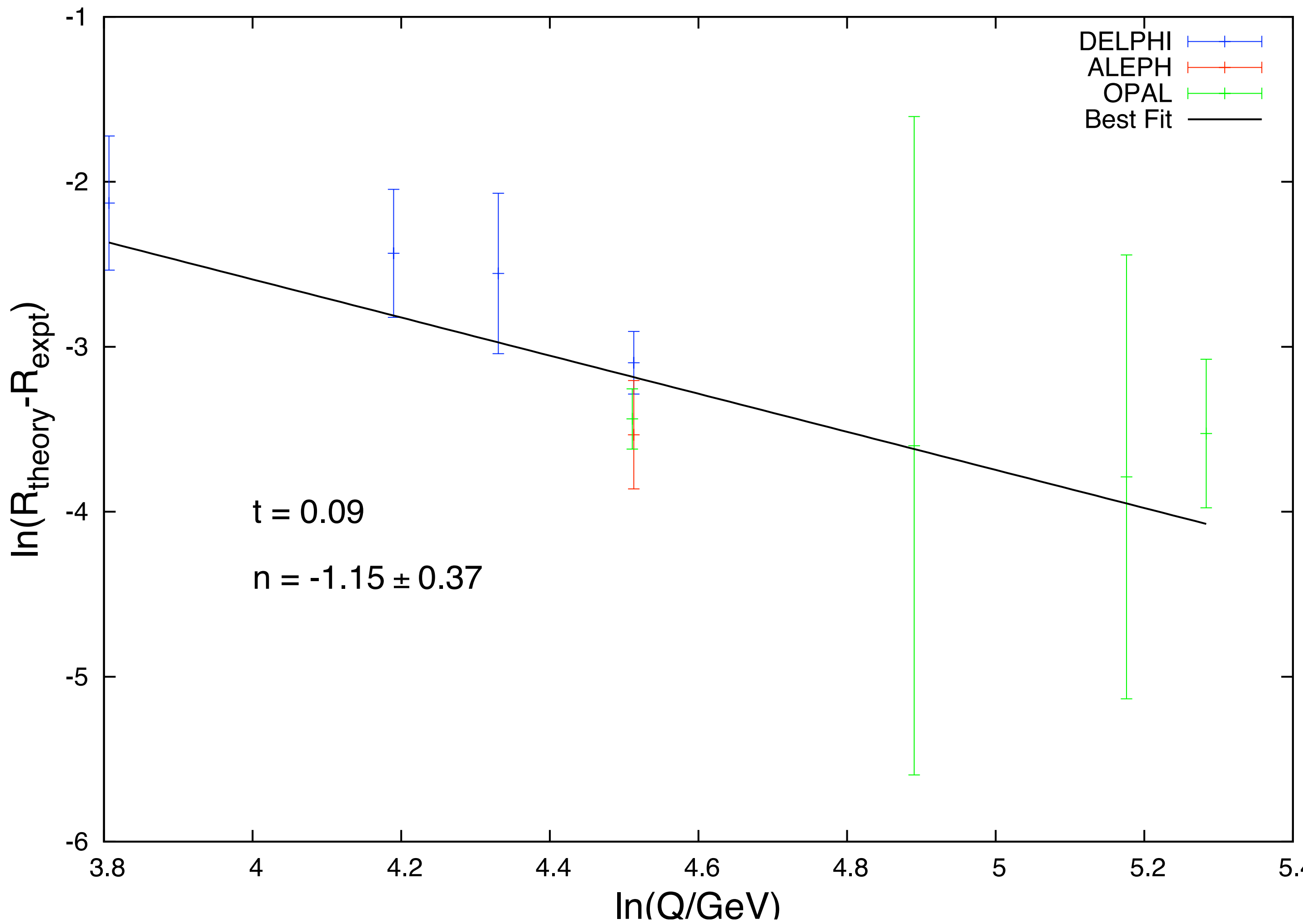

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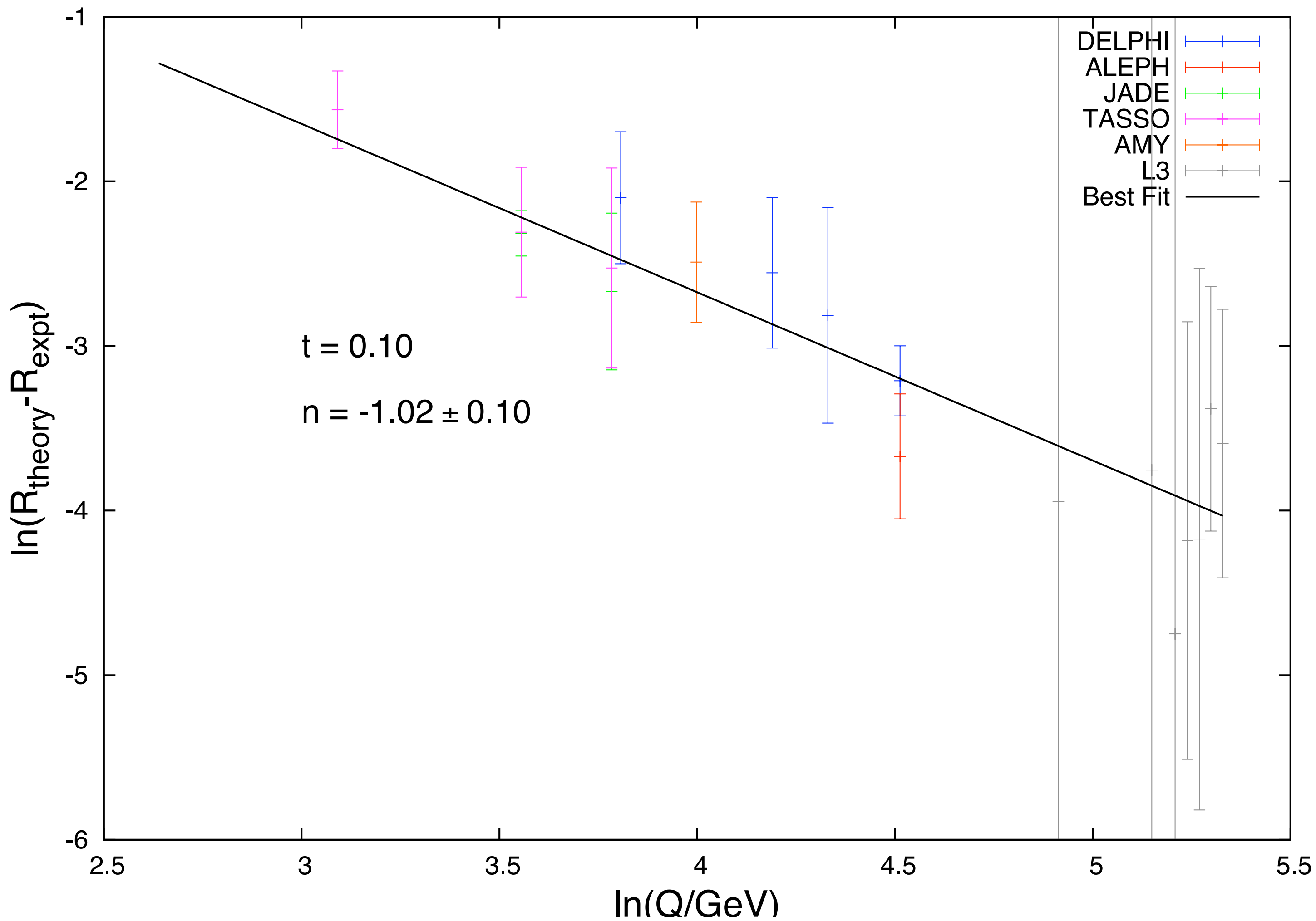

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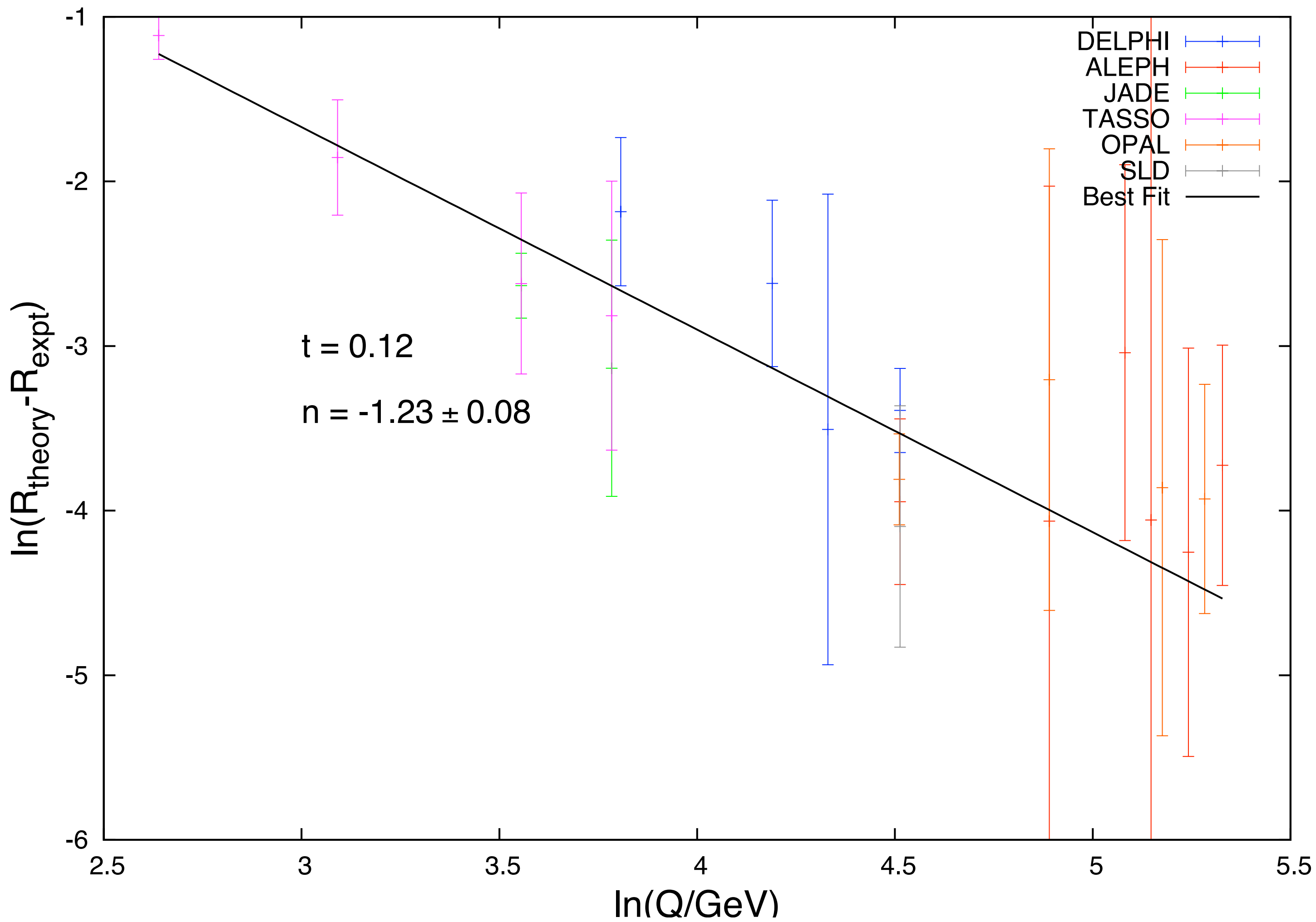

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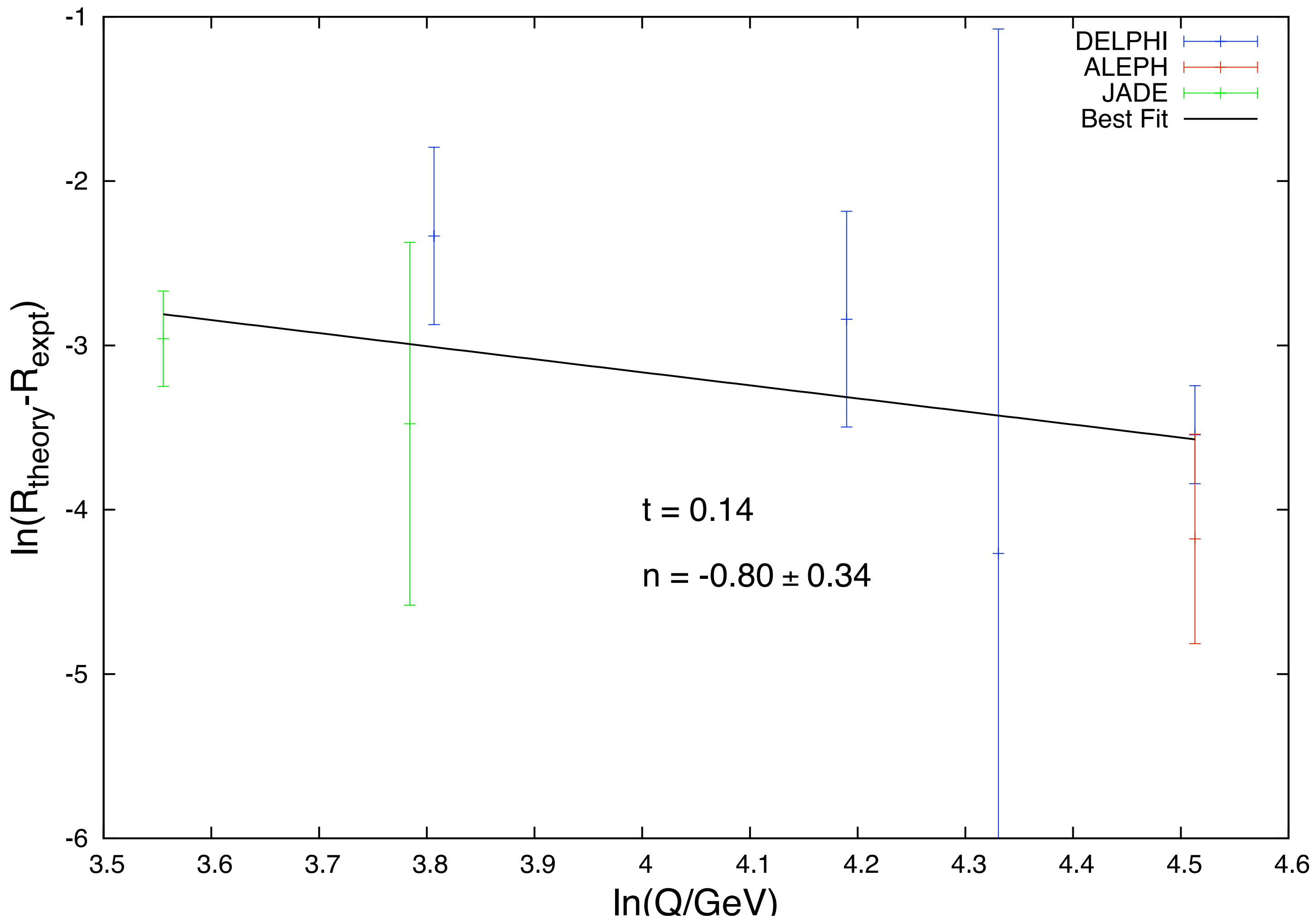

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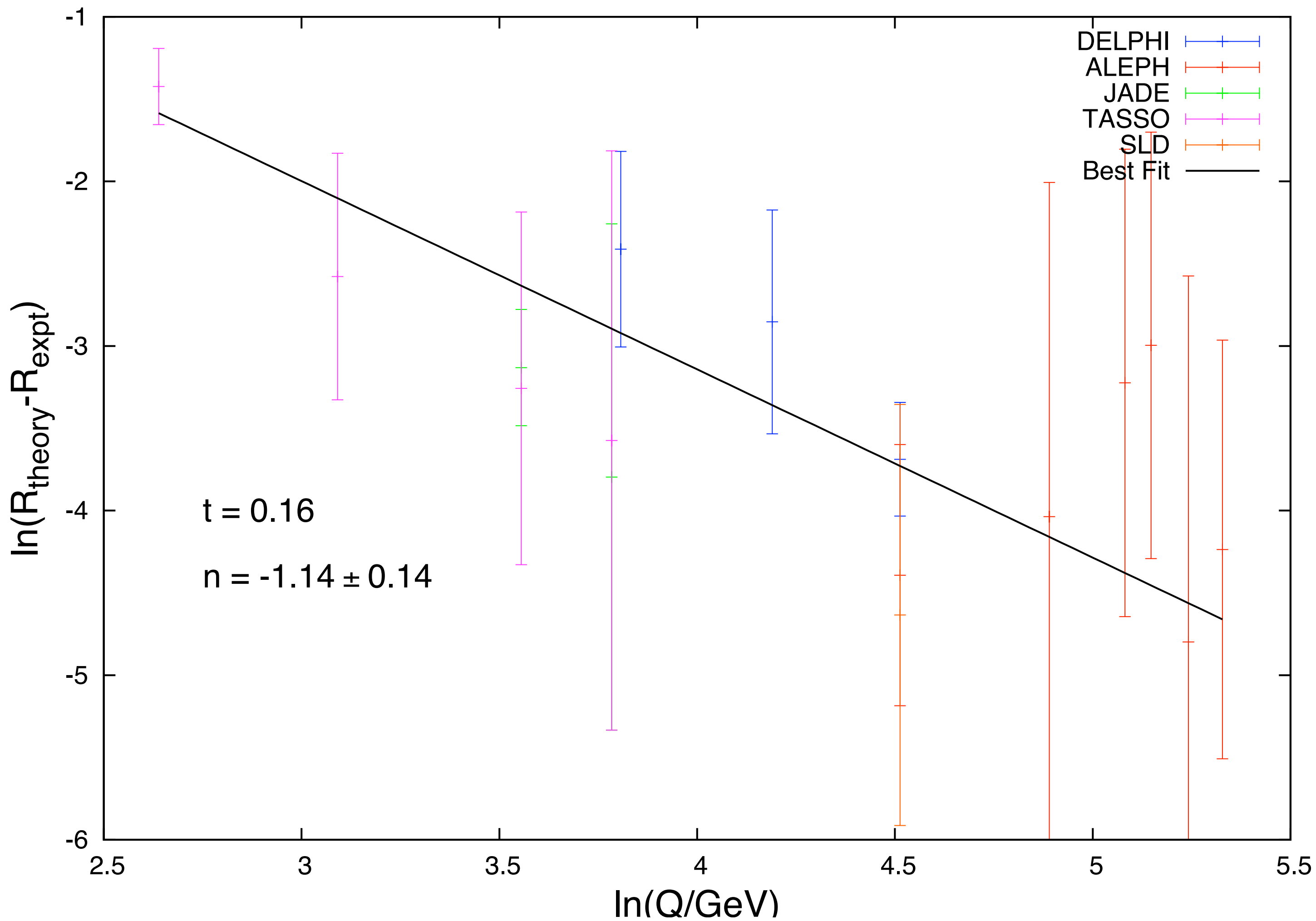

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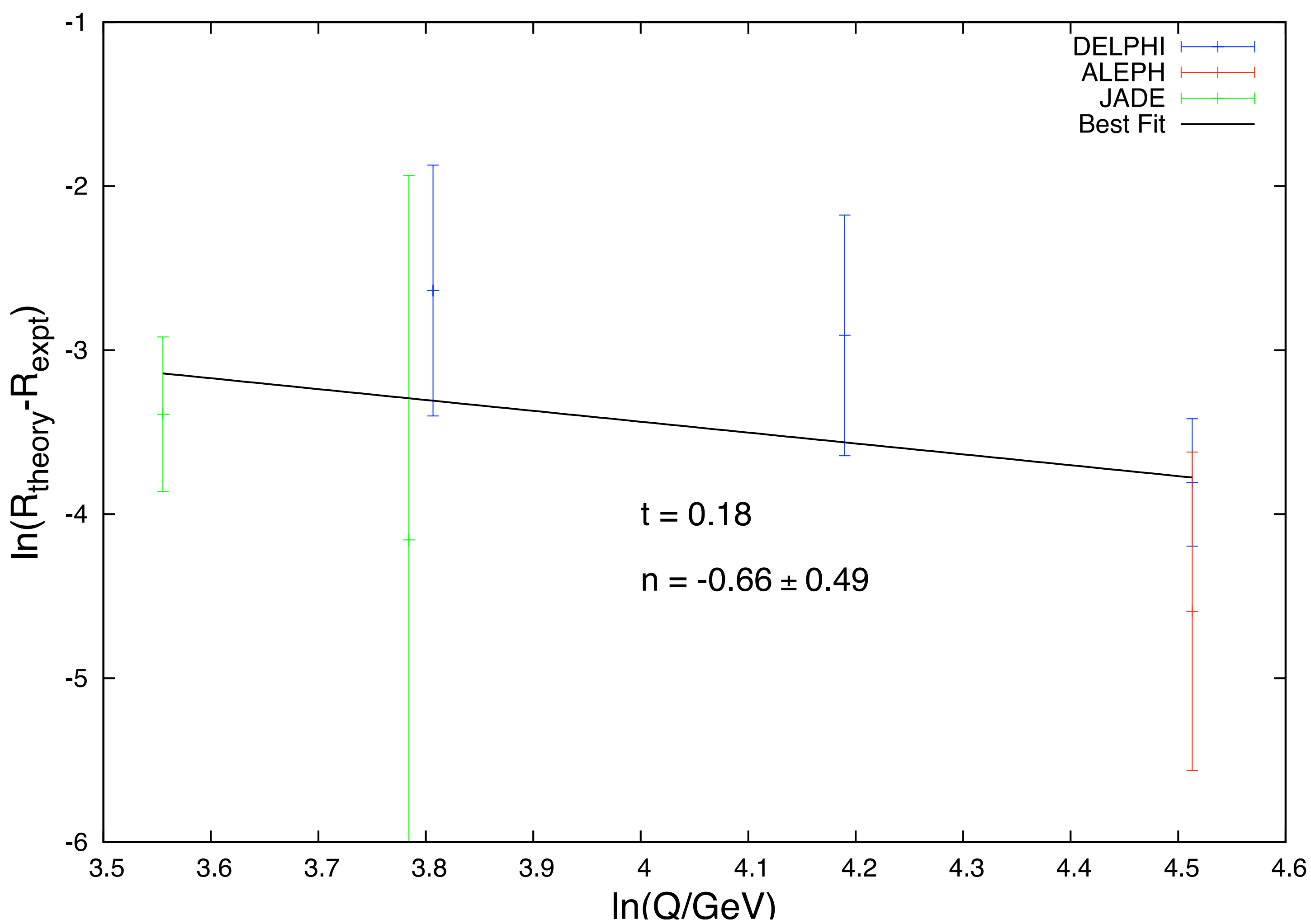

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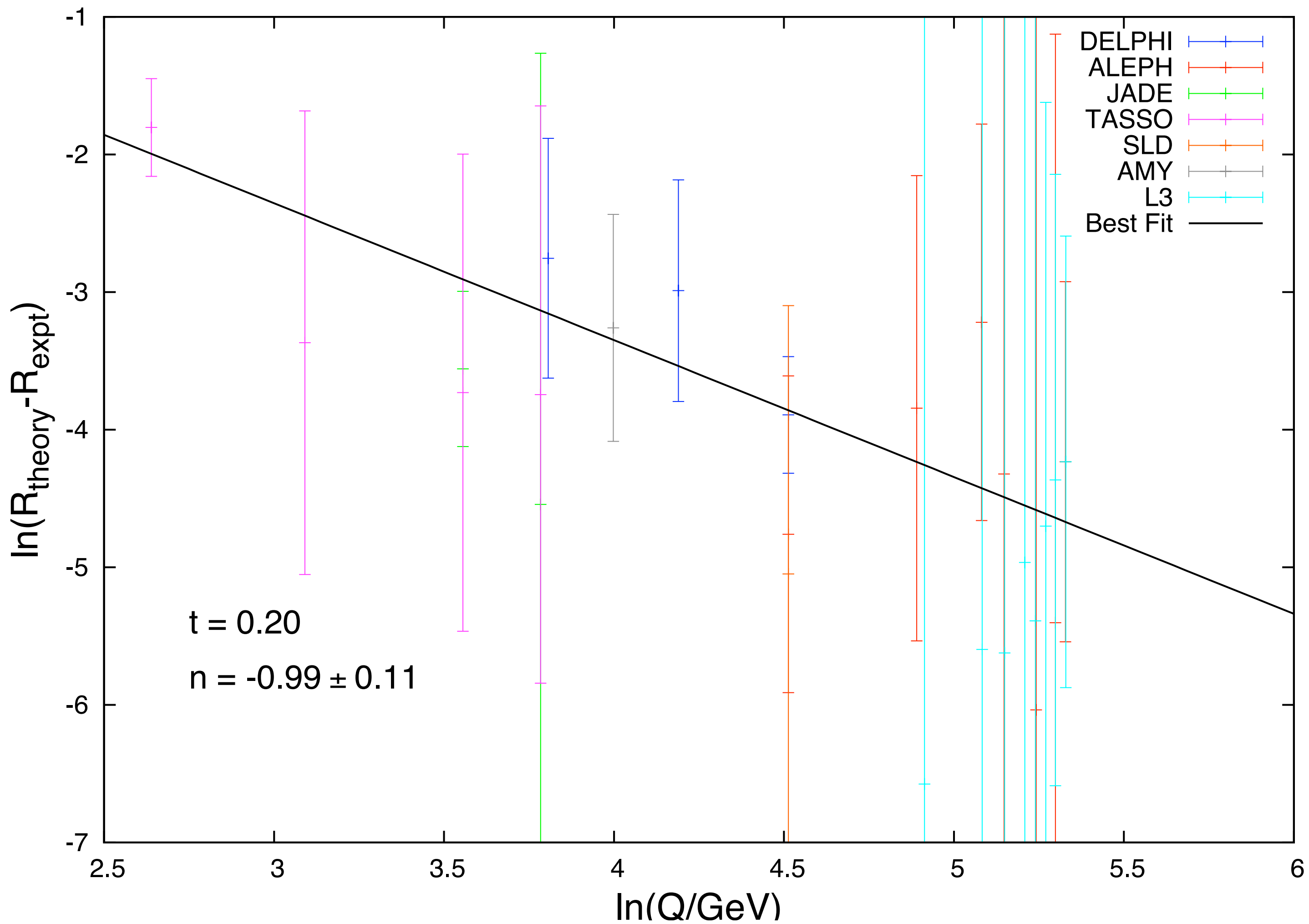

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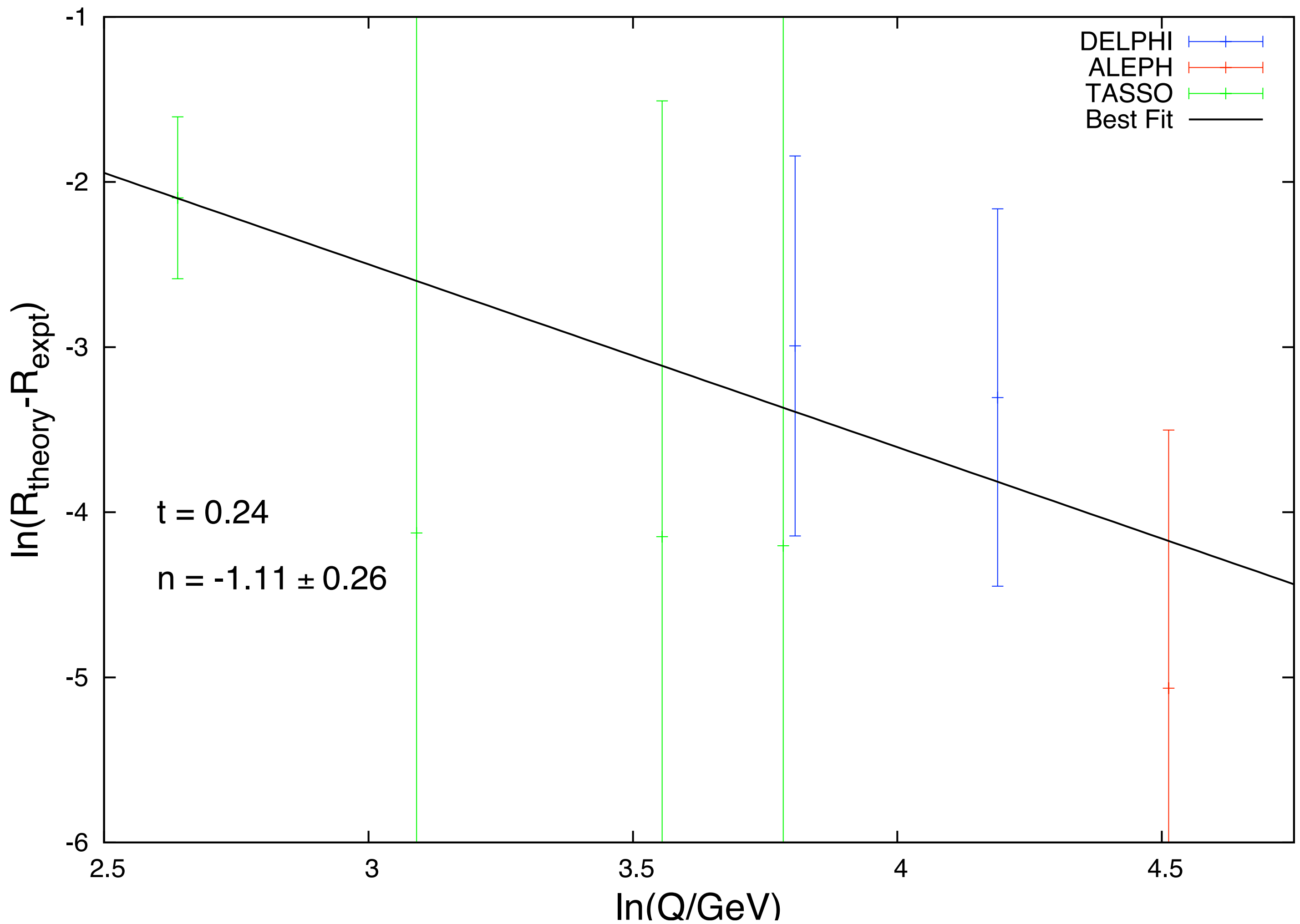

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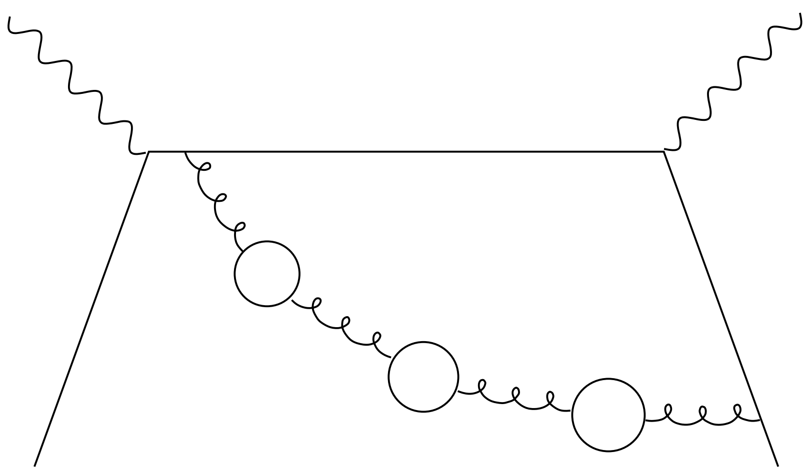
Universal low-scale effective α_S

- Infrared renormalon



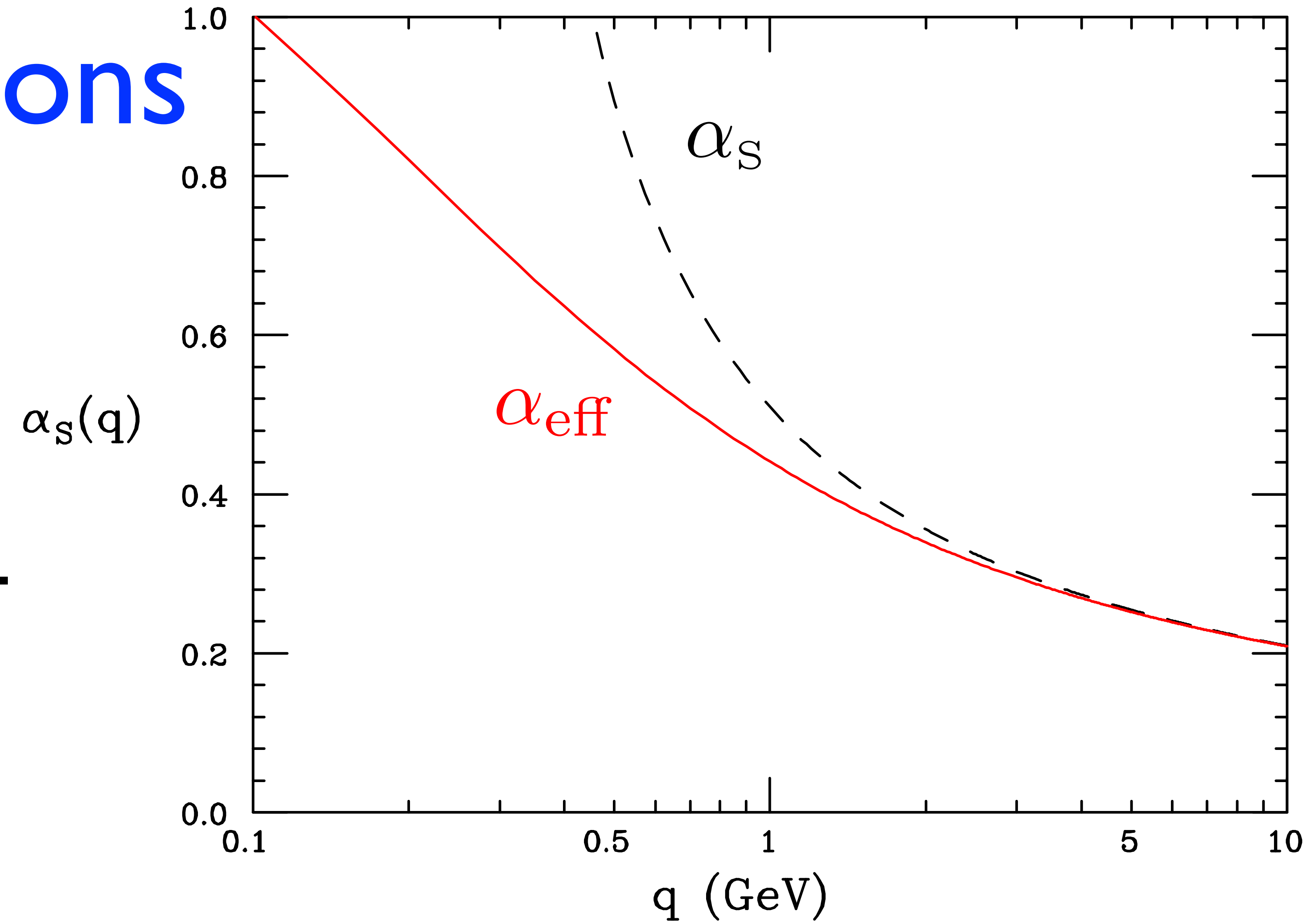
$$F \sim \int_0^Q \frac{dp_t}{Q} \alpha_S(p_t)$$
$$= \alpha_S(Q) \sum_n \int_0^Q \frac{dp_t}{Q} \left[b\alpha_S(Q) \ln \frac{Q^2}{p_t^2} \right]^n$$
$$= \alpha_S(Q) \sum_n n! [2b\alpha_S(Q)]^n$$

- Divergent series: truncate at smallest term

($n_m = [2b\alpha_S(Q)]^{-1}$) $\Rightarrow$ uncertainty

$$\delta F \sim n_m! [2b\alpha_S(Q)]^{n_m} \sim e^{-n_m} = \frac{\Lambda}{Q}$$

Power Corrections



- Renormalon is due to IR divergence of α_S
- Postulate universal IR-regular $\alpha_{\rm eff}$
- Power corrections depend on

$$\alpha_0(\mu_{\rm I}) = \frac{1}{\mu_{\rm I}} \int_0^{\mu_{\rm I}} \alpha_{\rm eff}(p_t)\, dp_t$$

- Match NP & PT at $\mu_I \sim 2$ GeV

NLL thrust resummation

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dT} = \frac{Q^2}{2\pi i}\int_C d\nu\, {\rm e}^{(1-T)\nu Q^2}\left[\tilde{J}^q_\nu(Q^2)\right]^2 ,$$

$$\ln \tilde{J}^q_\nu(Q^2) = \int_0^1 \frac{du}{u}\left({\rm e}^{-u\nu Q^2}-1\right)\left[\int_{u^2Q^2}^{uQ^2}\frac{dq^2}{q^2}A(\alpha_{\rm S}(q)) + \frac{1}{2}B(\alpha_{\rm S}(\sqrt{u}Q))\right]$$

- Leading PT contribution from $q < \mu_{\rm I}$

0.116 + 0.087 + 0.071 = 0.274

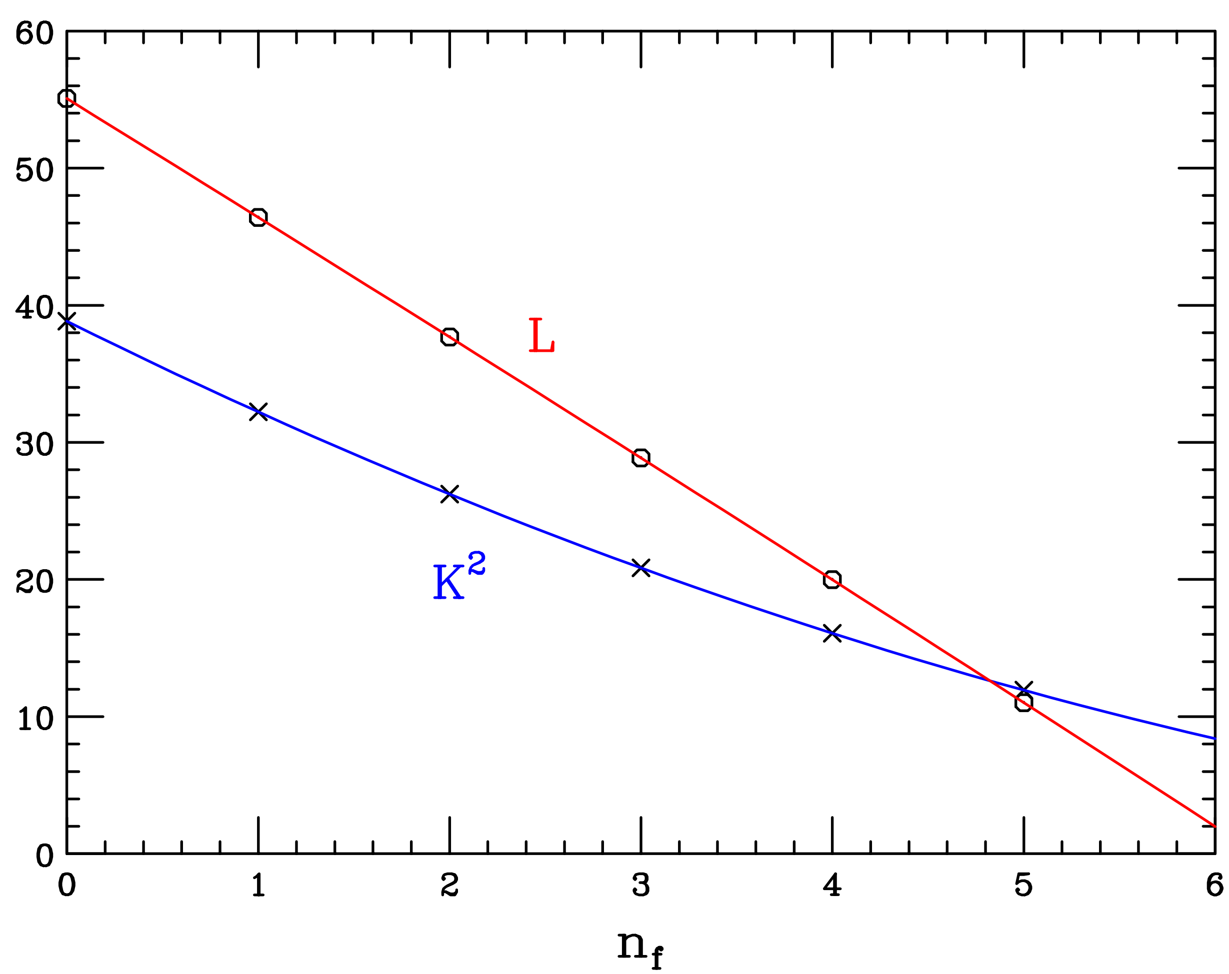
$$\delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\rm pert} = -\frac{2C_F}{\pi}\frac{\mu_{\rm I}}{Q}\left\{\alpha_{\rm S}(\mu_R) + \alpha_s^2(\mu_R)\frac{\beta_0}{\pi}\left(\ln\frac{\mu_R}{\mu_I} + \frac{K}{2\beta_0} + 1\right)\right.$$
$$\left. + \alpha_s^3(\mu_R)\left(\frac{\beta_0}{\pi}\right)^2\left[\ln^2\frac{\mu_R}{\mu_I} + \left(\ln\frac{\mu_R}{\mu_I}+1\right)\left(2+\frac{\beta_1}{2\beta_0^2}+\frac{K}{\beta_0}\right) + \frac{K^2}{4\beta_0^2}\right]\right\}\nu Q^2.$$

NNLO

$$K = C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right) - \frac{5}{9}n_f$$ ➡ K^2=11.9 vs L=11.0

The 3-loop constant

- Coefficient of 3-loop cusp anomalous dimension
  - ➡ L = true value [used by Gehrmann et al., EPJC67(2010)57]
  - ➡ K^2 = effective scale change (used by RD&BW)
- Happen to agree at $n_f = 5$ (but effect negligible anyway)



Power correction to thrust

- Replace PT by NP for $q < \mu_\mathrm{I}$

$$\delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\mathrm{n.p.}} = \frac{2C_F}{\pi}\int_0^{\mu_\mathrm{I}} \frac{dq}{q}\alpha_{\mathrm{eff}}(q)\int_{q^2/Q^2}^{q/Q}\frac{du}{u}\left(\mathrm{e}^{-u\nu Q^2}-1\right)$$

- For $\mu_\mathrm{I}\nu Q \ll 1$, i.e. $1 - T \gg \mu_\mathrm{I}/Q$

$$\delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\mathrm{n.p.}} \simeq -\frac{2C_F}{\pi}\int_0^{\mu_\mathrm{I}} dq\,\alpha_{\mathrm{eff}}(q)\,\nu Q \equiv -\frac{2C_F}{\pi}\frac{\mu_\mathrm{I}}{Q}\,\alpha_0(\mu_\mathrm{I})\,\nu Q^2$$

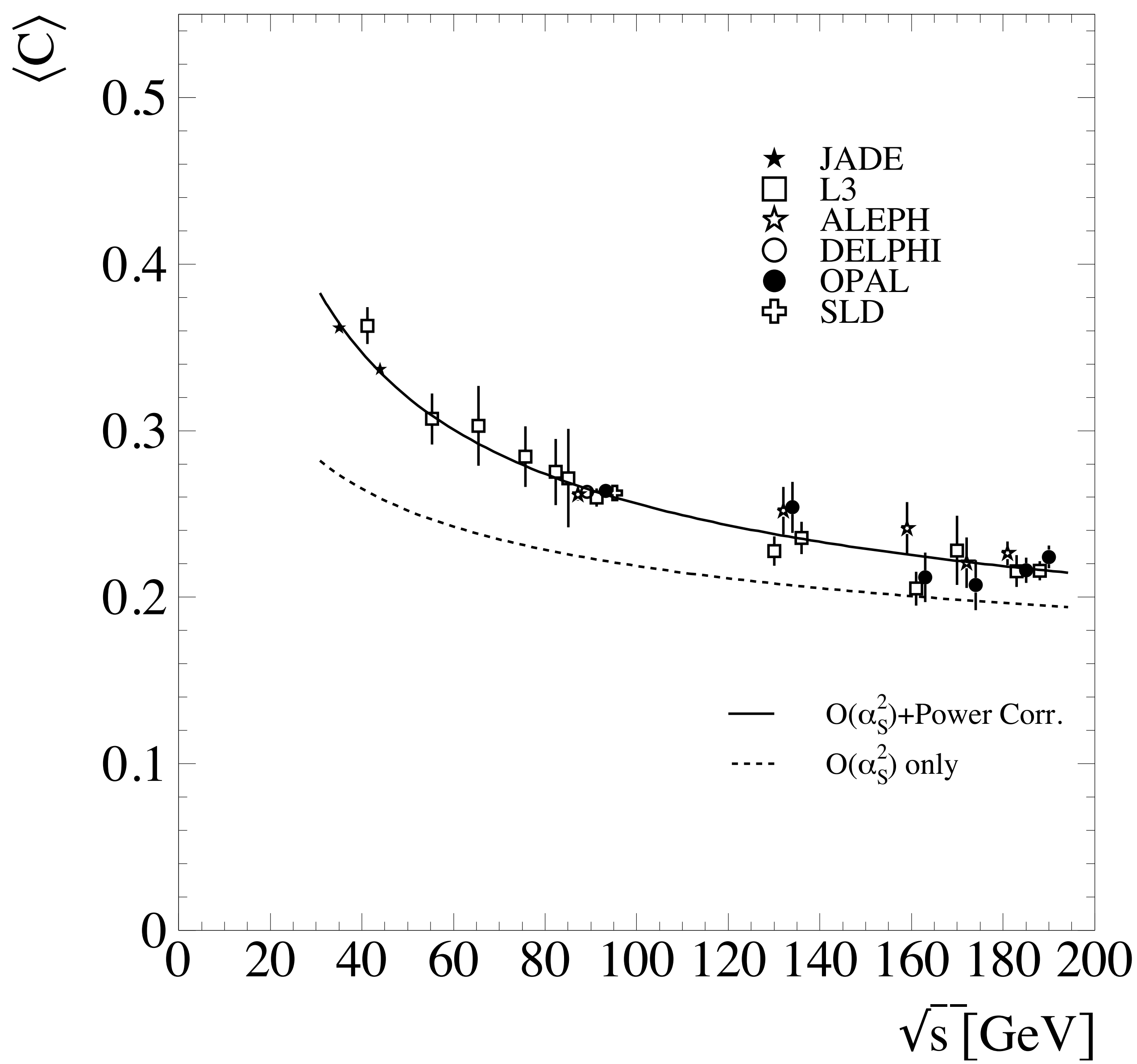
- \`Milan Factor': $\alpha_0 \to 2\mathcal{M}\,\alpha_0/\pi = 0.95\,\alpha_0$

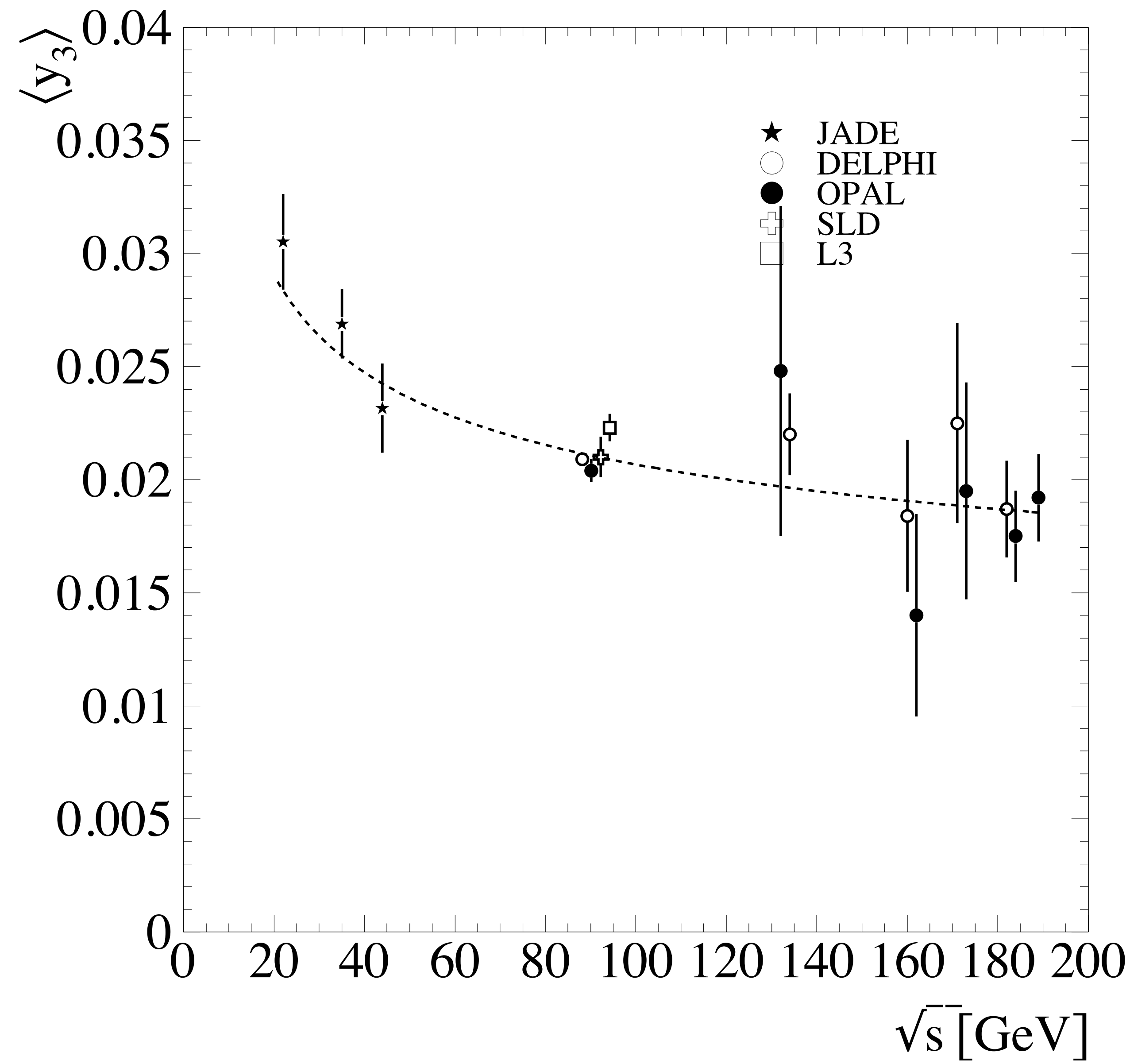
Dokshitzer et al., JHEP05(1998)003

$$\Rightarrow \delta \ln \tilde{J}^q_\nu(Q^2) = \delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\mathrm{n.p.}} - \delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\mathrm{pert}} = \delta T\,\nu Q^2$$

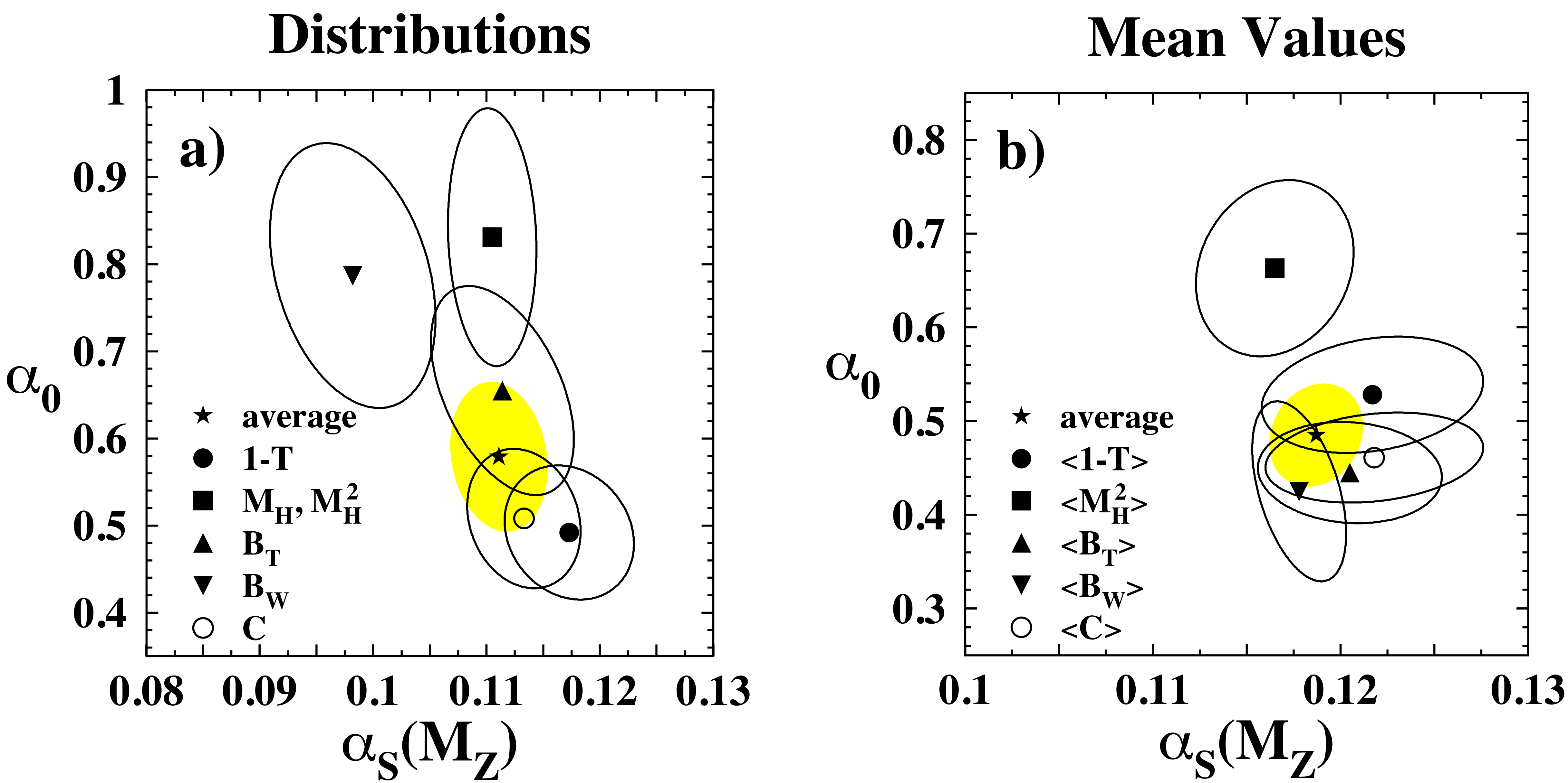
Power corrections to event shapes

- 1/Q renormalon present in T & C, absent in y_3



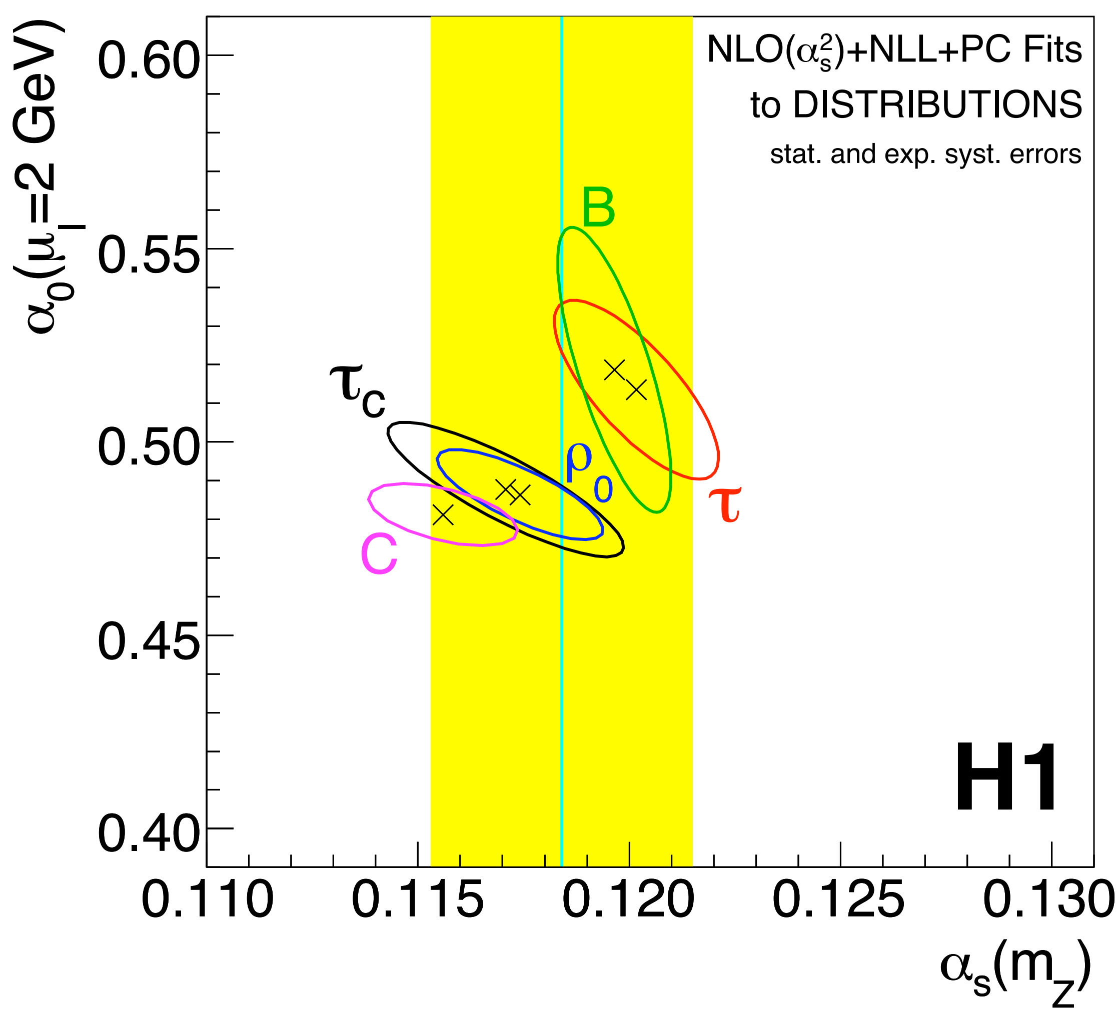


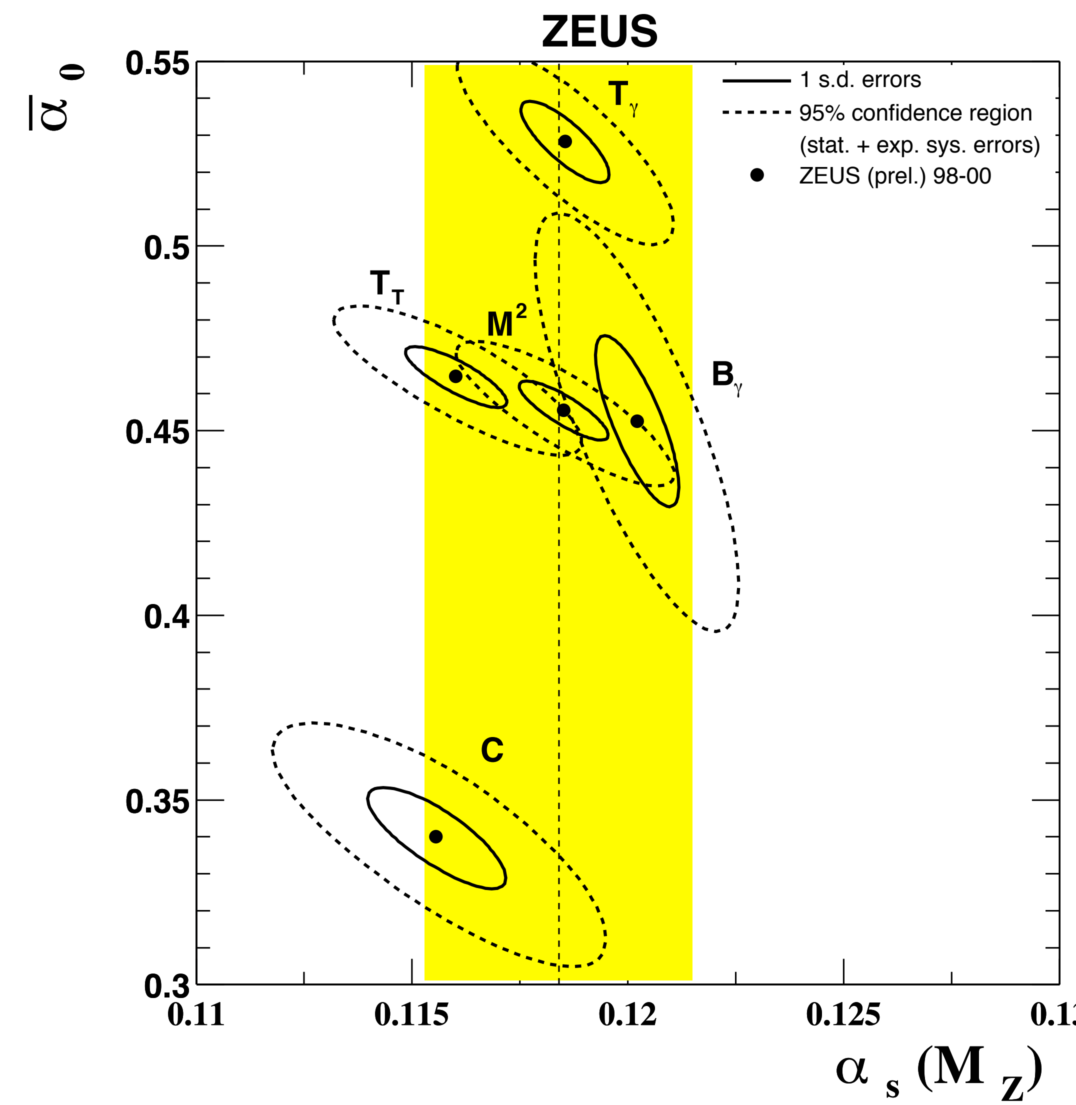
NLO results from e^+e^-



Movilla Fernandez, Bethke, Biebel & Kluth, EPJ C22(2001)1

NLO results from DIS





NNLO+NLL matching

- **At NNLO:** $[\bar{\alpha}_s \equiv \alpha_s(\mu_R)/2\pi]$

A Gehrmann-DeRidder et al., JHEP 11(2007)058

$$R\left(t\right) = \int_0^t dt \frac{1}{\sigma}\frac{d\sigma}{dt} = \int_{1-t}^1 dT \frac{1}{\sigma}\frac{d\sigma}{dT} = 1 + \bar{\alpha}_s R_1(t) + \bar{\alpha}_s^2 R_2(t) + \bar{\alpha}_s^3 R_3(t) + \ldots$$

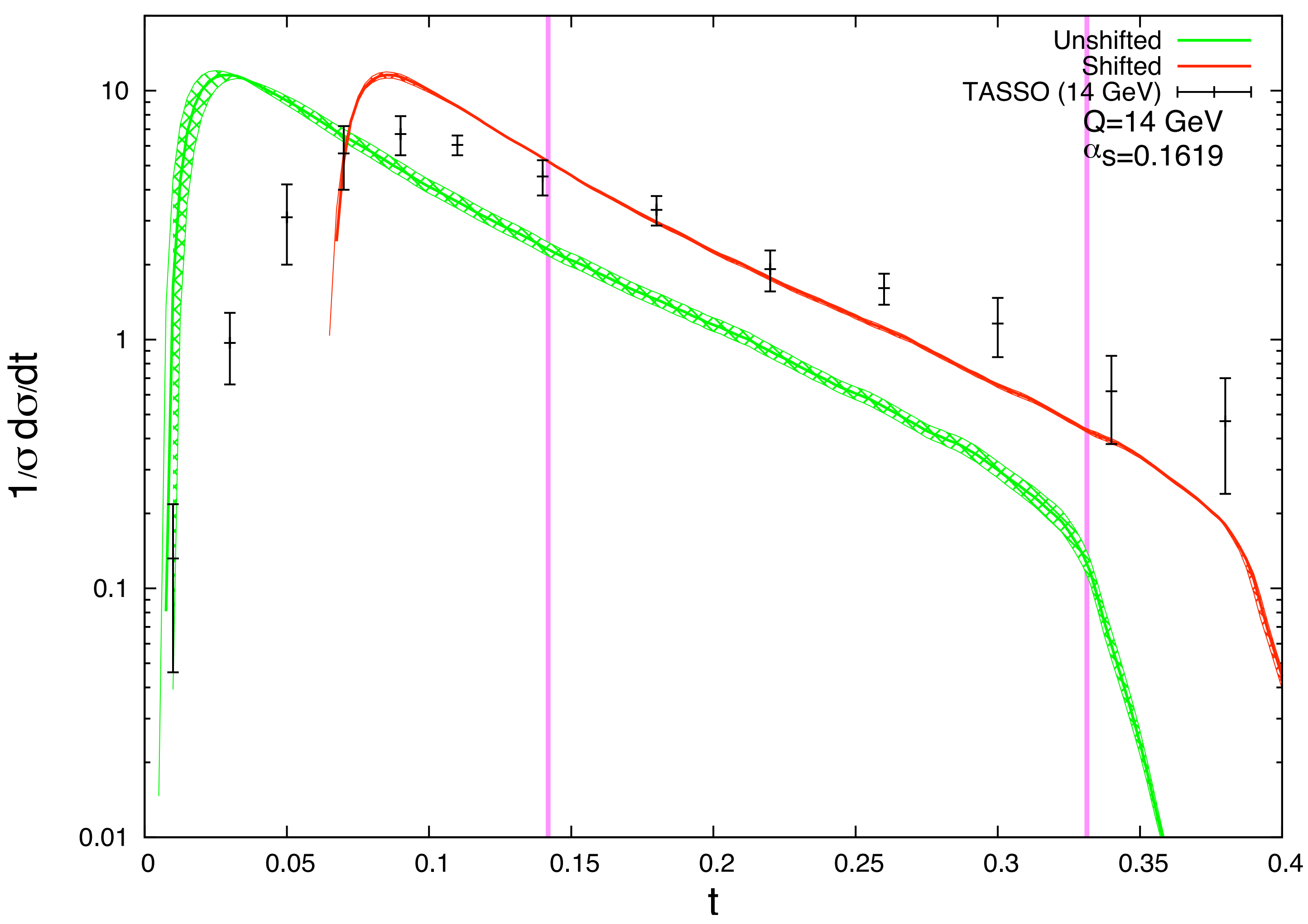
- **At NLL:** $\ln R(t) = L g_1(\bar{\alpha}_s L) + g_2(\bar{\alpha}_s L) = \sum_{n=1}^{\infty}\sum_{m=n}^{n+1} G_{nm}\,\bar{\alpha}_s^n L^m$

- **Log-R matching:** $\left[L = \ln\left(\frac{1}{t} + 1 - \frac{1}{t_{\max}}\right)\right]$

$$\boxed{\begin{aligned}\ln R\left(t\right) = {} & L g_1\left(\alpha_s L\right) + g_2\left(\alpha_s L\right) \\ & + \bar{\alpha}_s\left(R_1\left(t\right) - G_{11}L - G_{12}L^2\right) \\ & + \bar{\alpha}_s^2\left(R_2\left(t\right) - \frac{1}{2}\left[R_1\left(t\right)\right]^2 - G_{22}L^2 - G_{23}L^3\right) \\ & + \bar{\alpha}_s^3\left(R_3\left(t\right) - R_1\left(t\right)R_2\left(t\right) + \frac{1}{3}\left[R_1\left(t\right)\right]^3 - G_{33}L^3 - G_{34}L^4\right).\end{aligned}}$$

NNLO+NLL+NP fit to Thrust

- Fit range $\max\{\mu_{\mathrm{I}}/Q, 0.05\} \leq t < 0.33$



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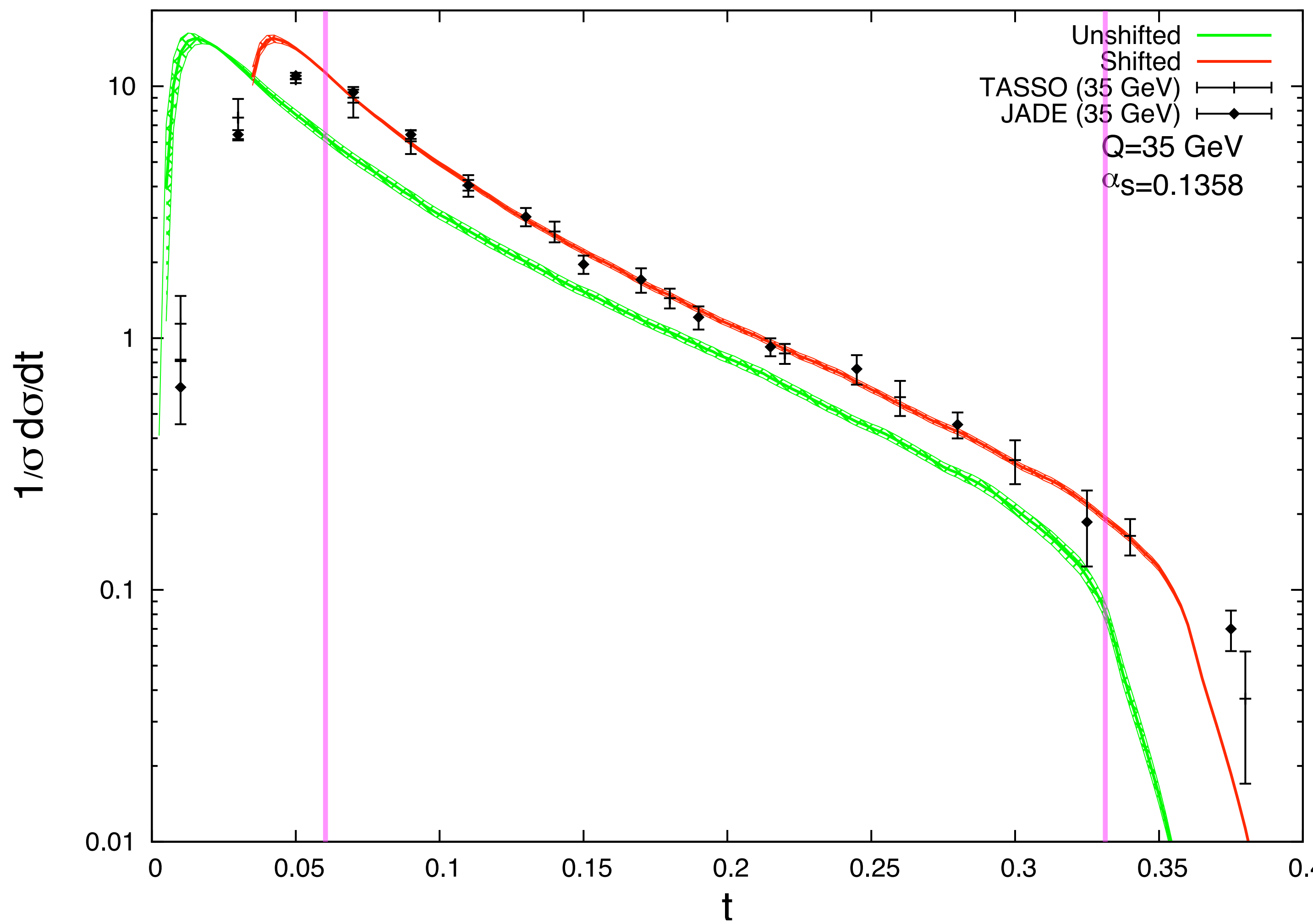

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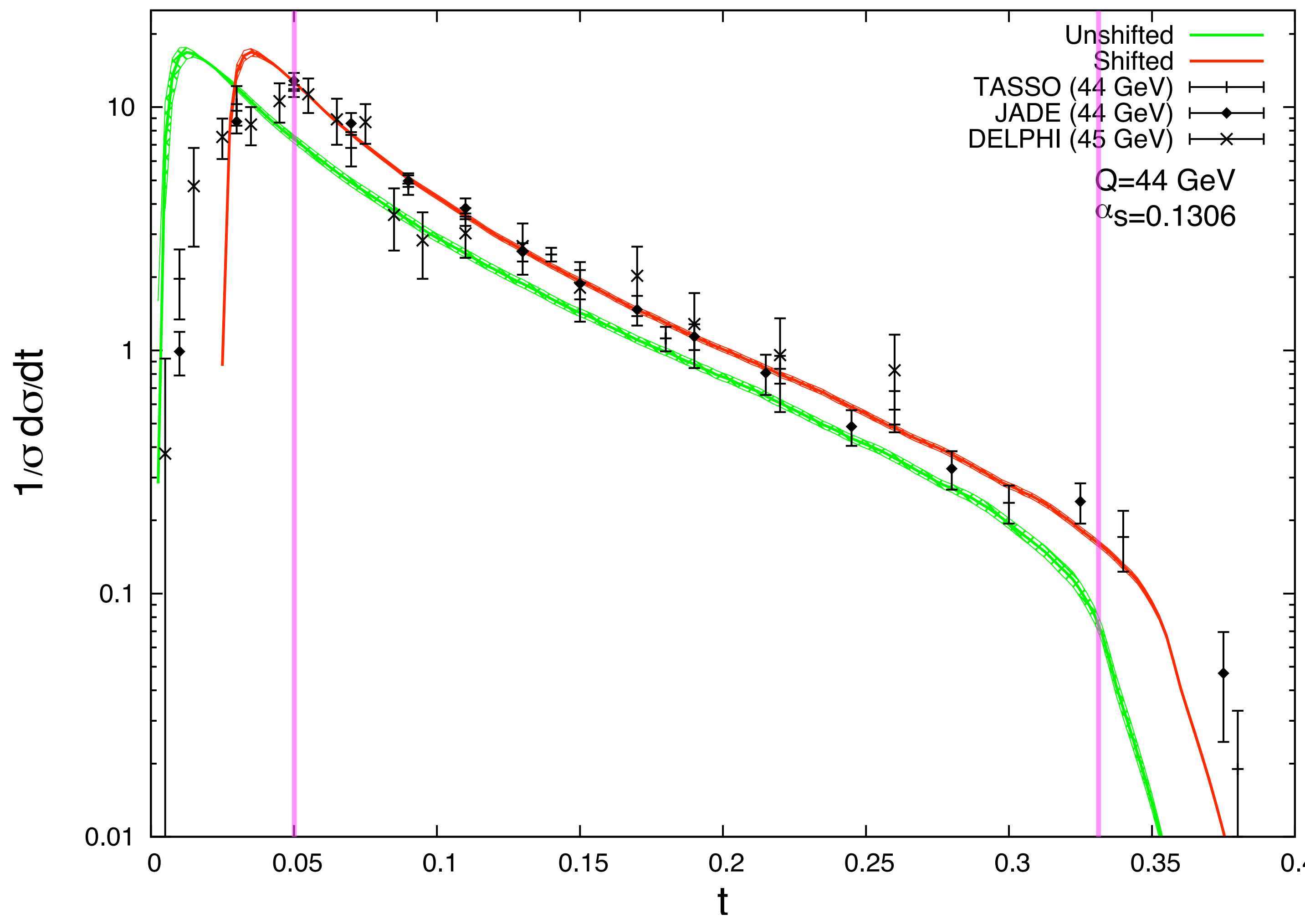

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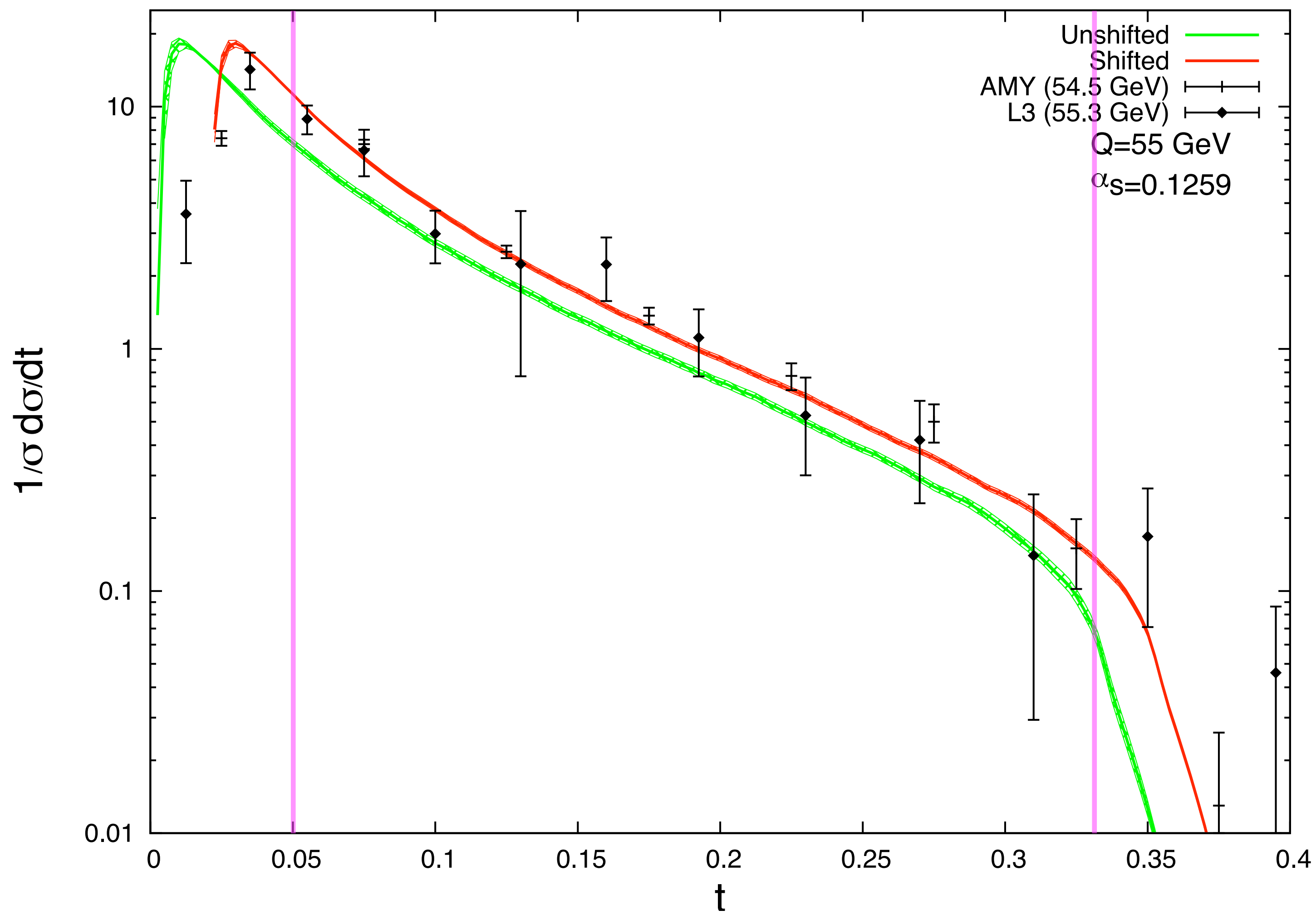

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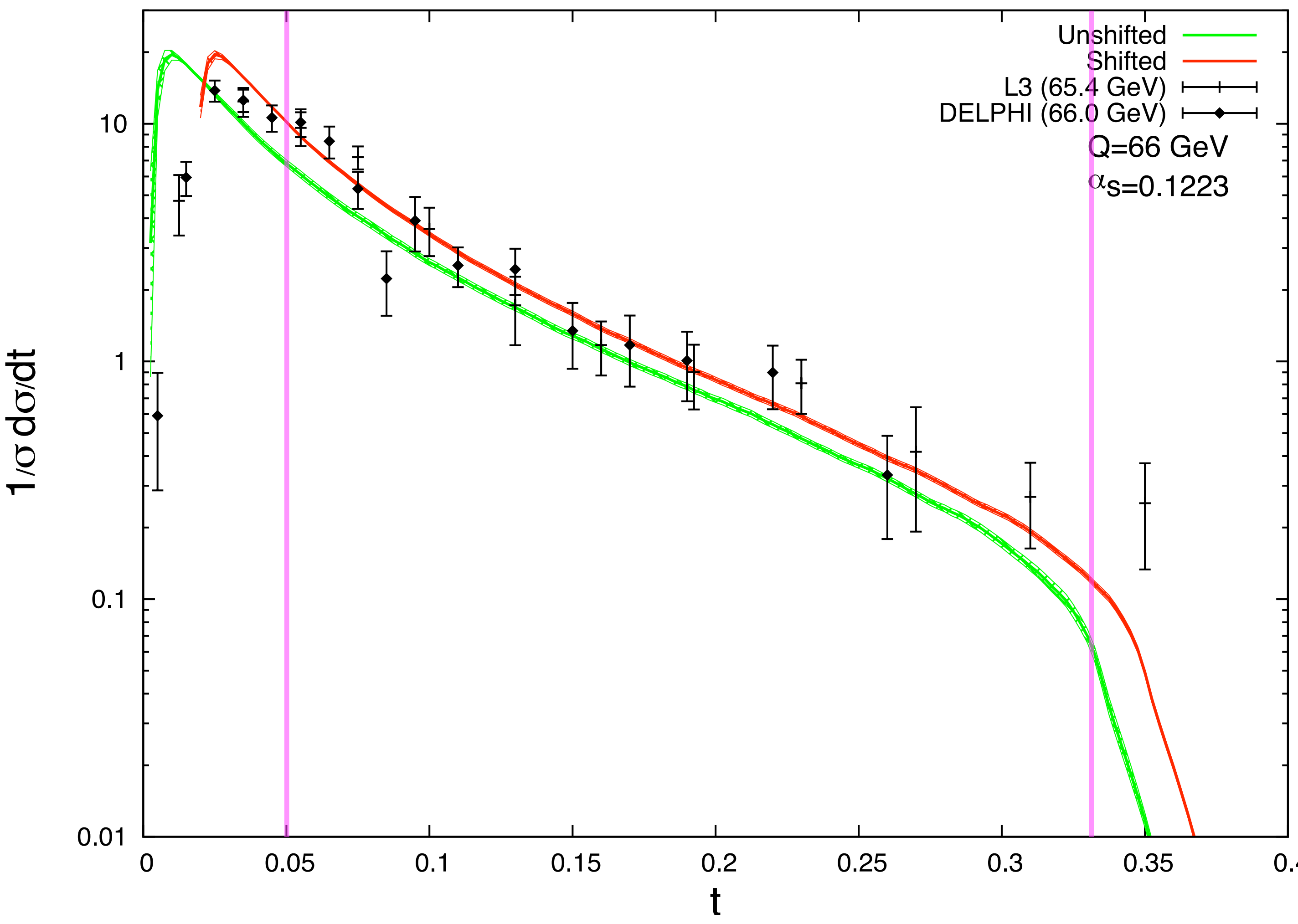

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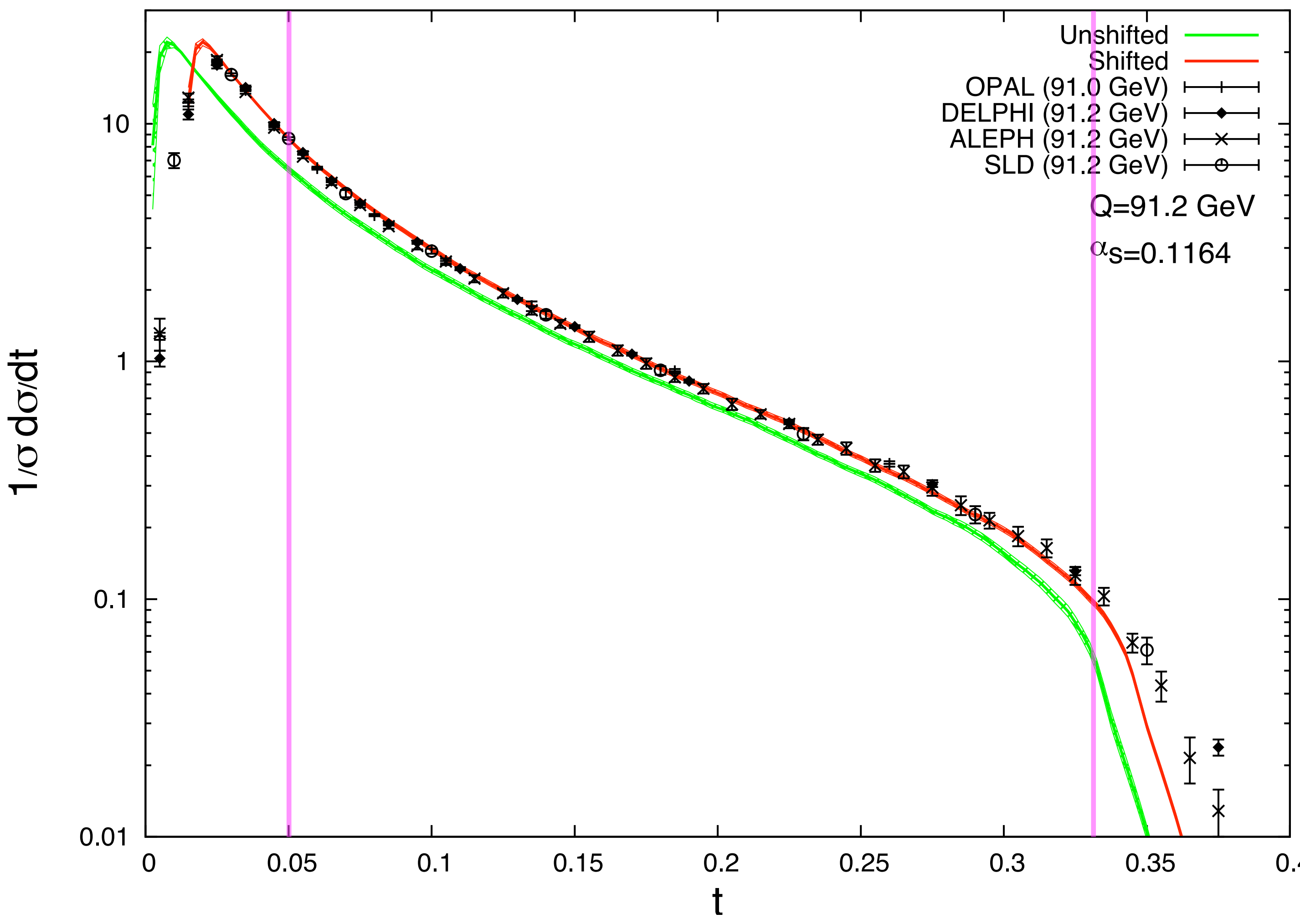

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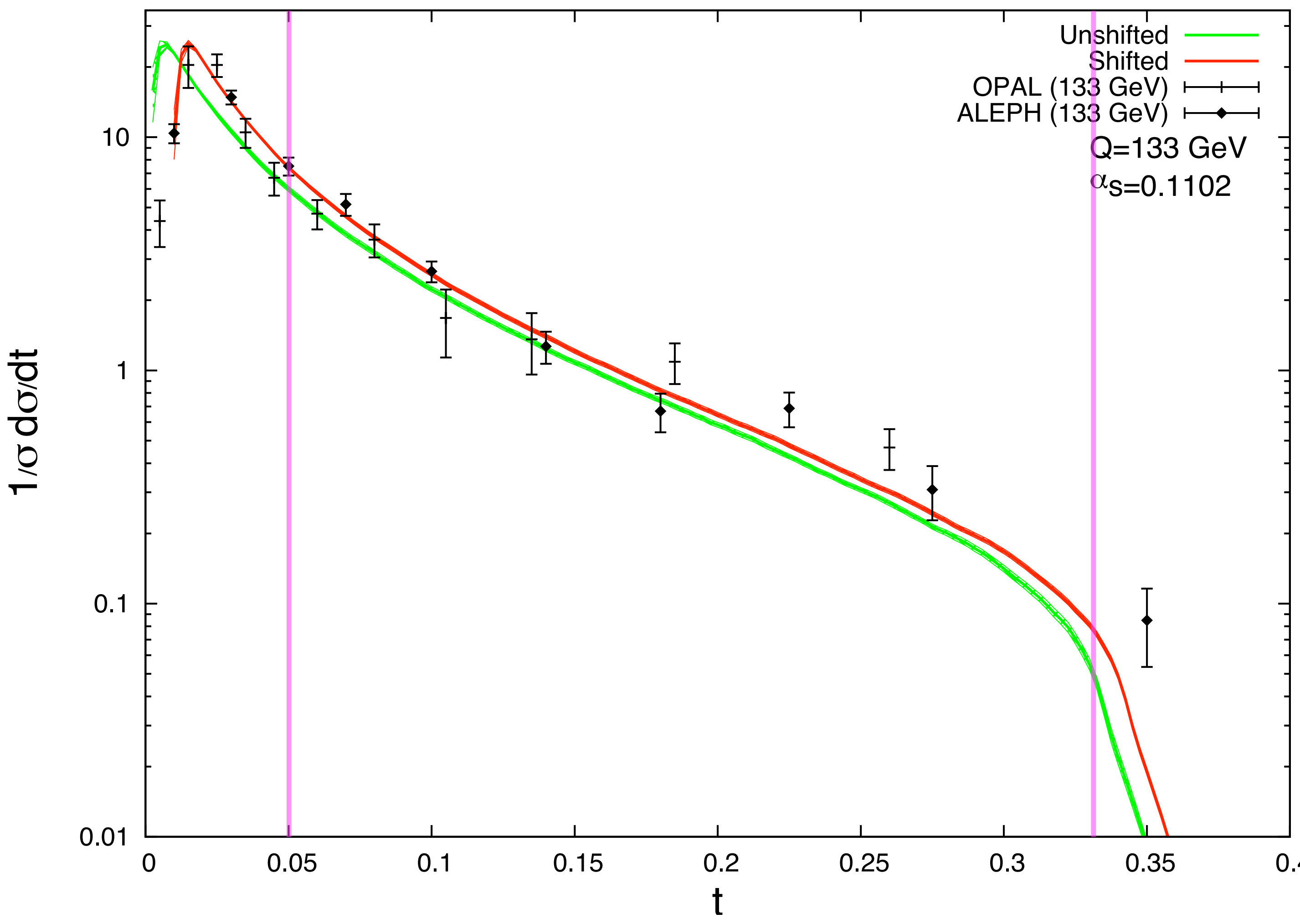

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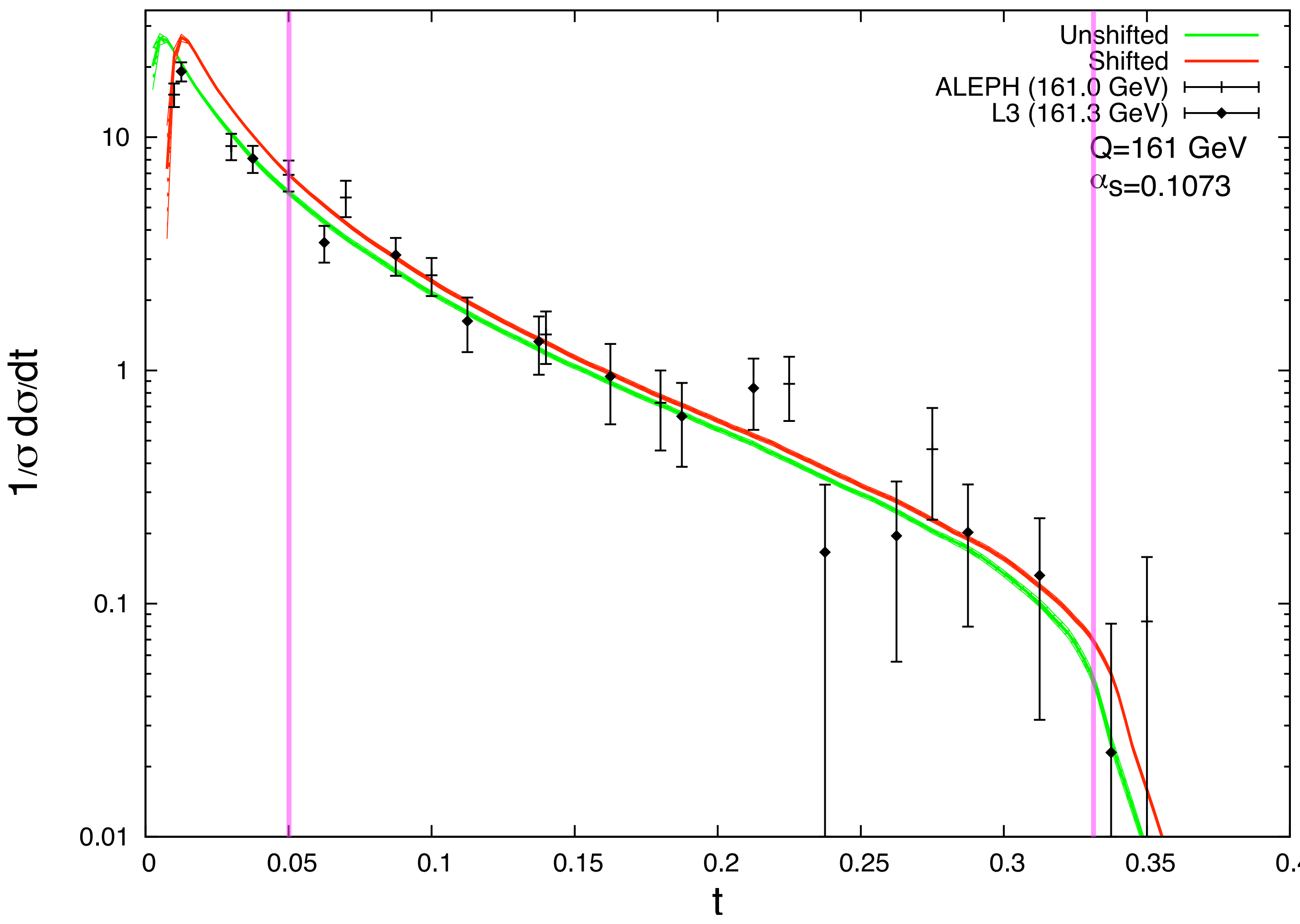

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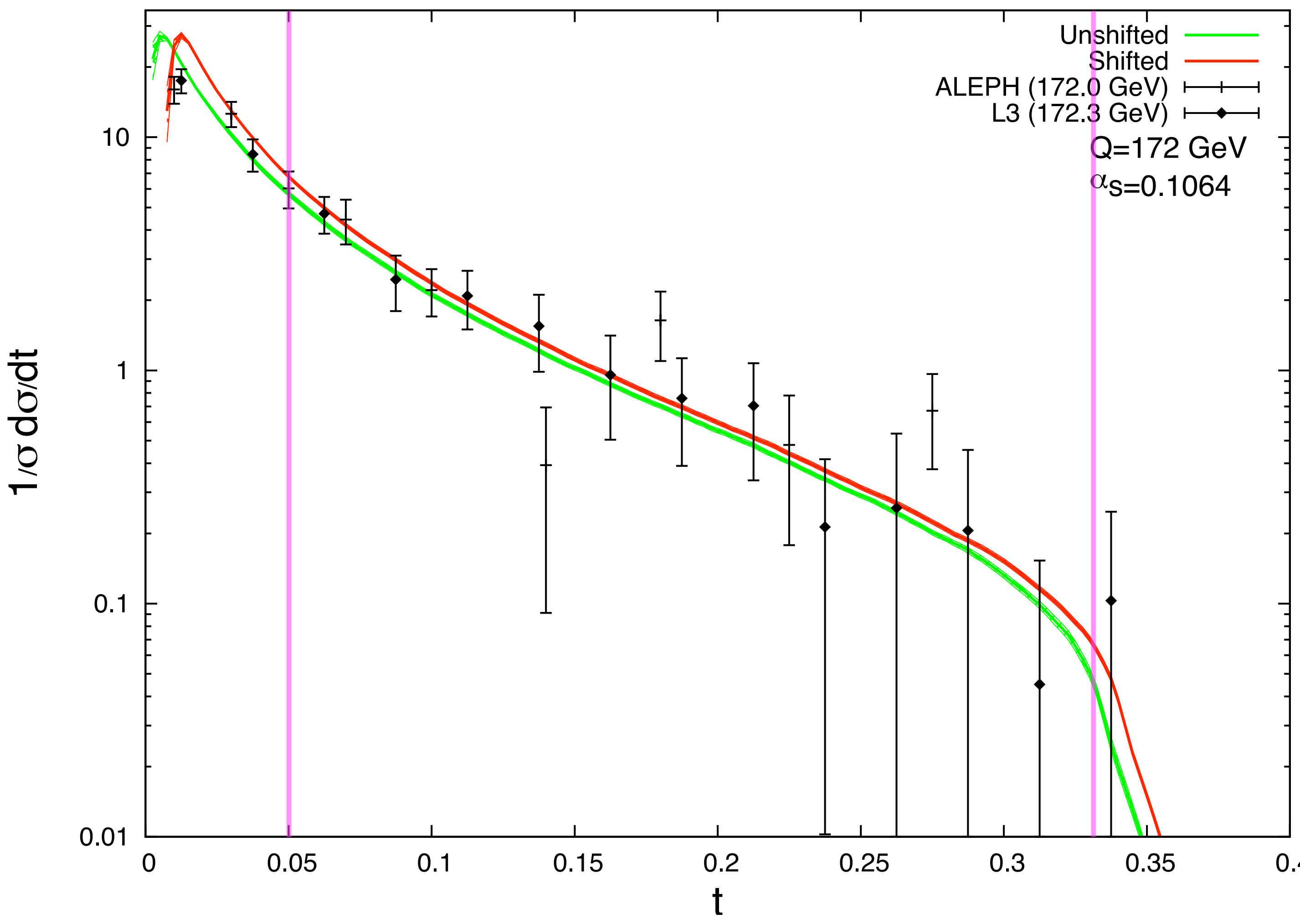

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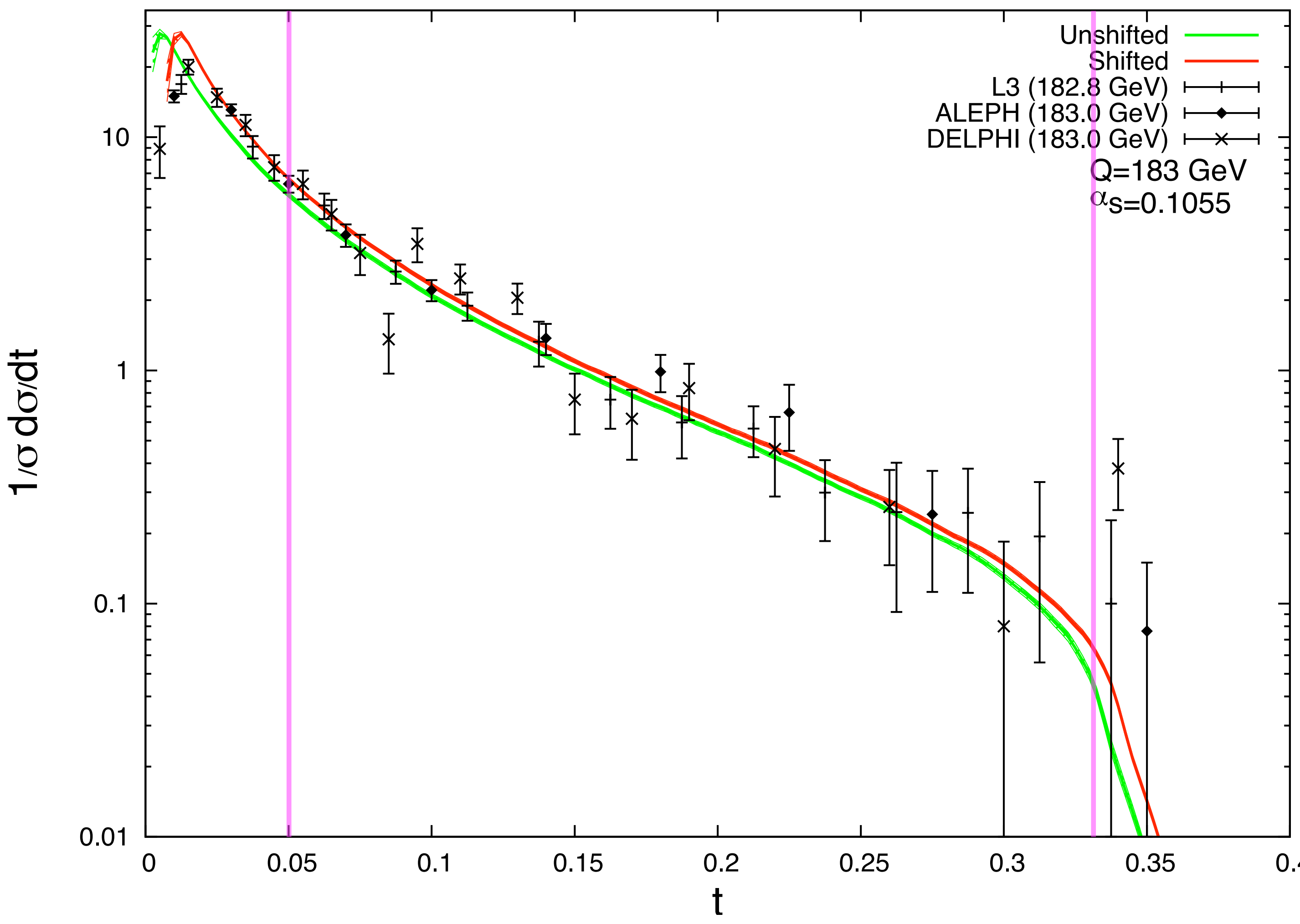

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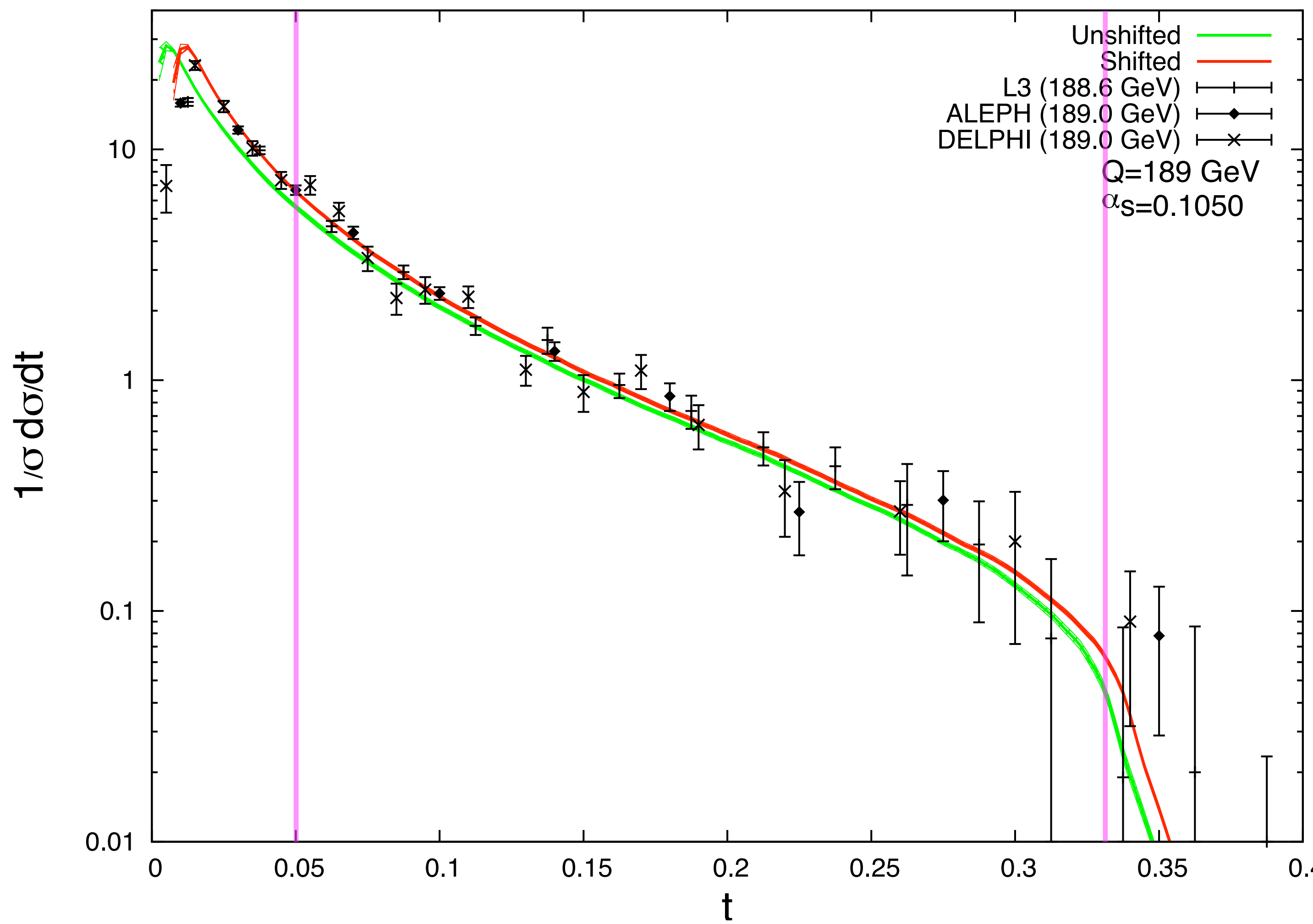

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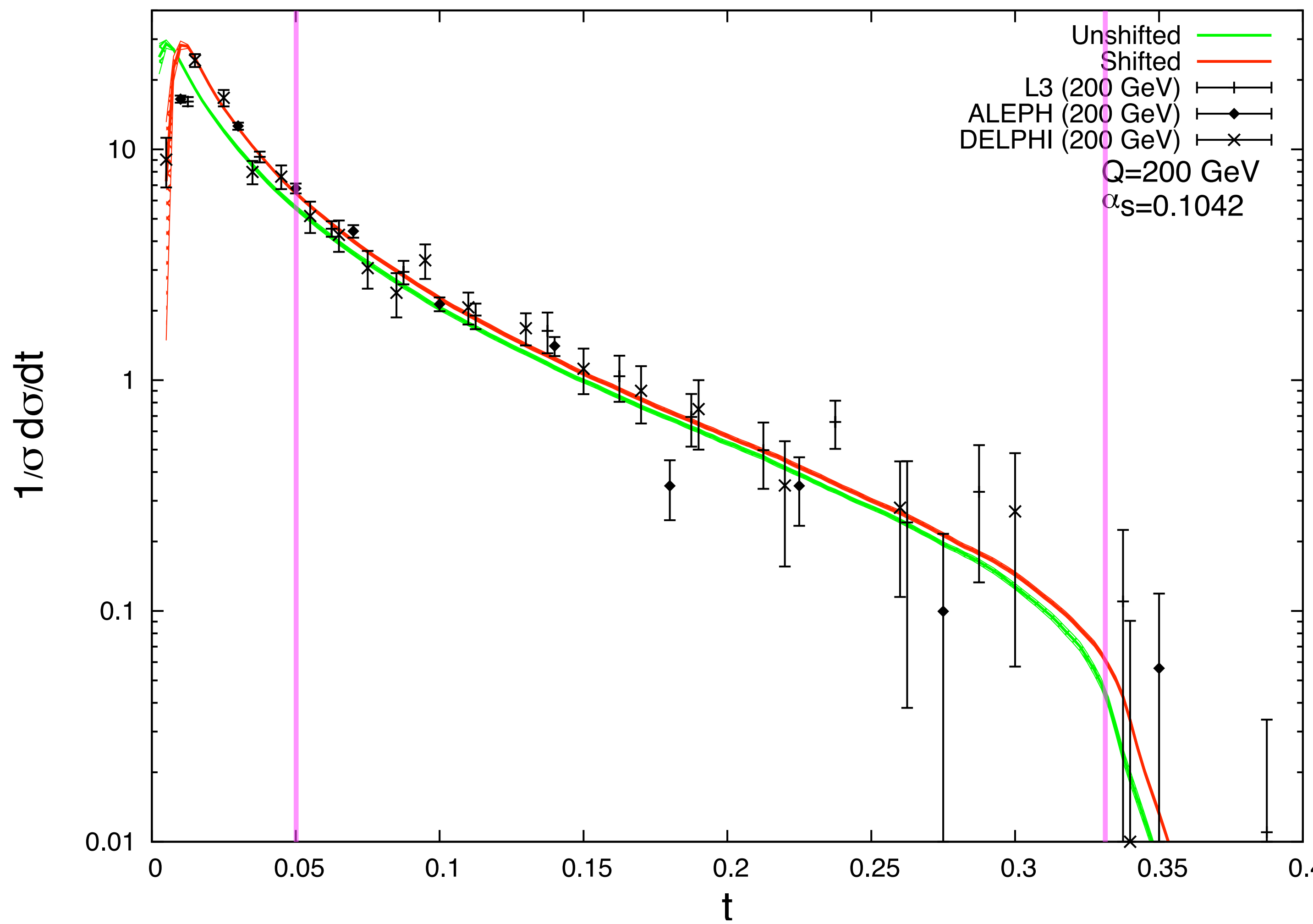

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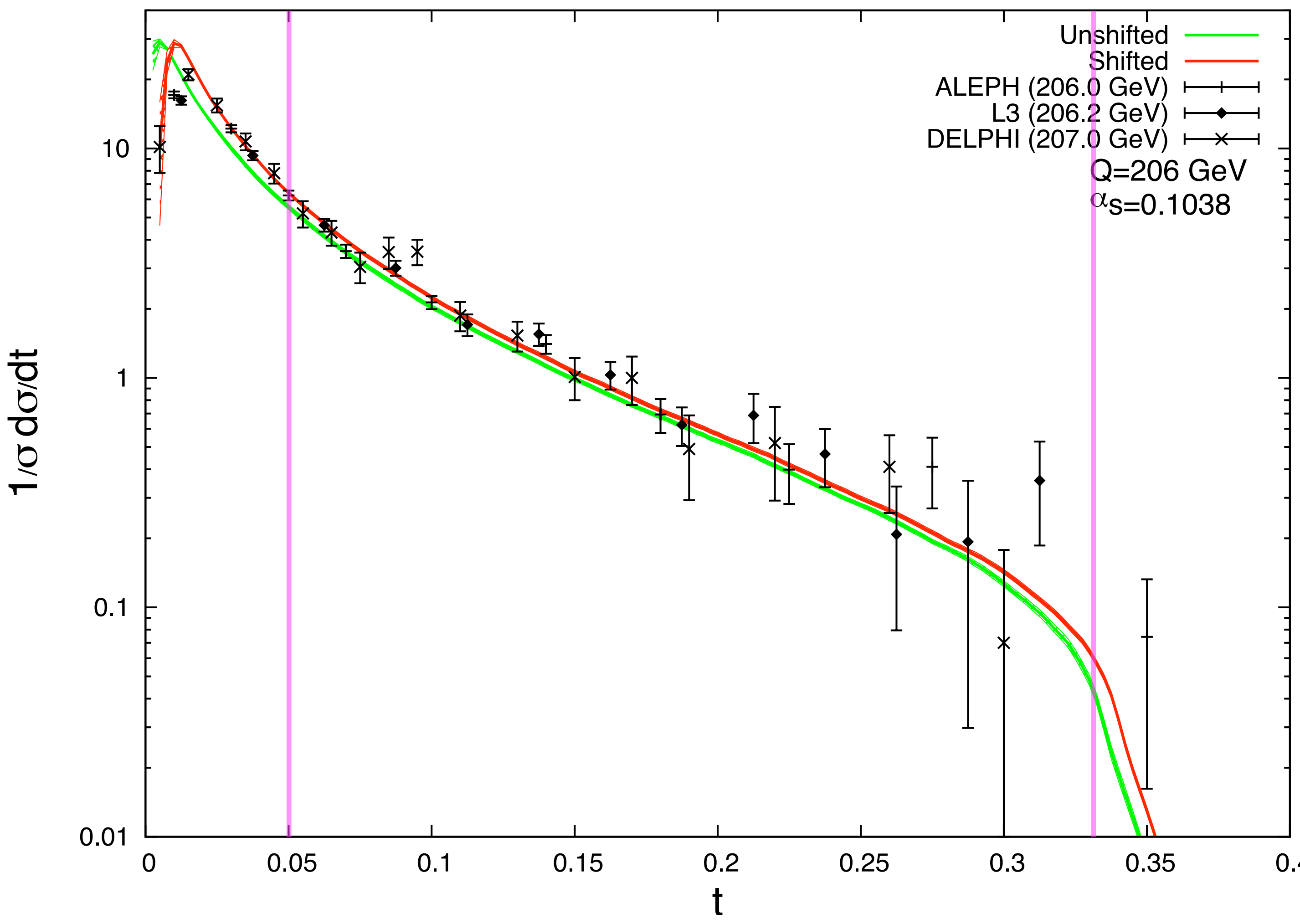
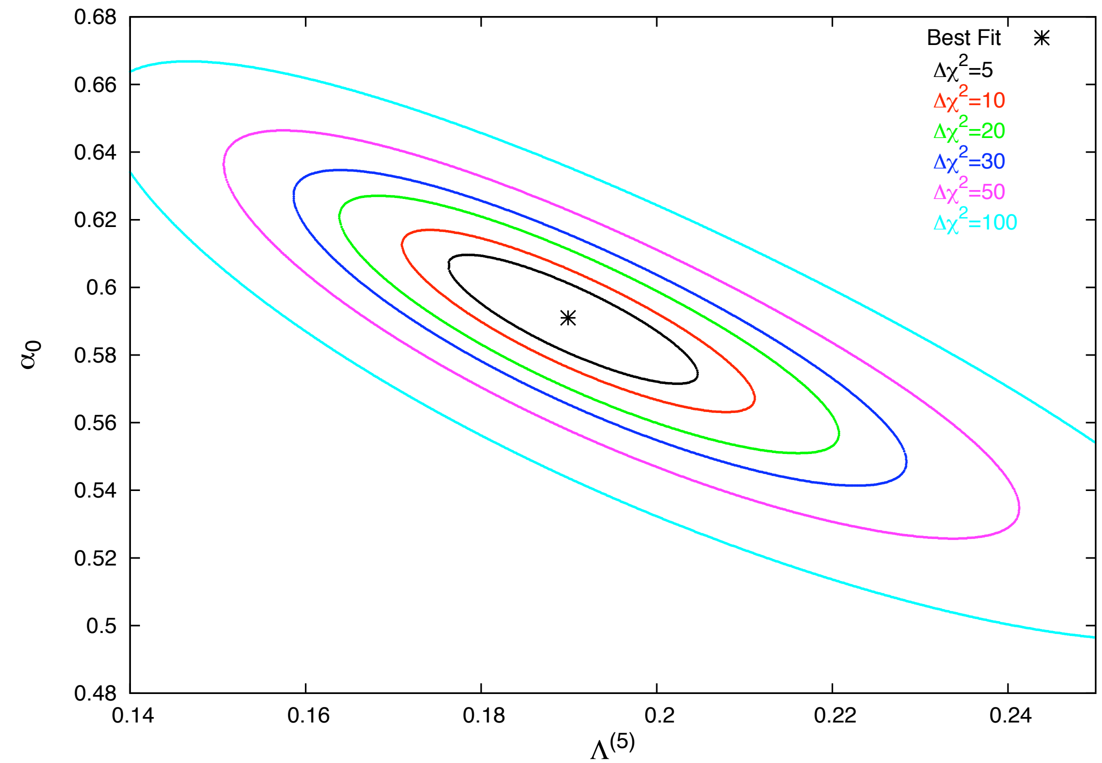


Results of NNLO+NLL+NP fit

Experiment	Q/GeV	Ref.	No. Pts.	χ^2
TASSO	14.0	[14]	4	8.2
TASSO	22.0	[14]	6	2.8
TASSO	35.0	[14]	8	0.7
JADE	35.0	[15]	10	10.5
L3	41.4	[16]	8	3.4
JADE	44.0	[15]	10	3.8
TASSO	44.0	[14]	8	6.8
DELPHI	45.0	[17]	11	11.6
AMY	54.5	[18]	4	4.9
L3	55.3	[16]	8	3.2
L3	65.4	[16]	8	7.5
DELPHI	66.0	[17]	11	14.5
L3	75.7	[16]	8	1.9
DELPHI	76.0	[17]	11	10.3
L3	82.3	[16]	8	4.0
L3	85.1	[16]	8	3.6
OPAL	91.0	[19]	5	11.9
ALEPH	91.2	[20]	27	16.1
DELPHI	91.2	[17]	11	18.8
SLD	91.2	[21]	6	2.7
L3	130.1	[16]	10	14.6
ALEPH	133.0	[20]	6	7.2
OPAL	133.0	[19]	5	6.5
L3	136.1	[16]	10	37.3
ALEPH	161.0	[20]	6	5.5
L3	161.3	[16]	10	4.0
ALEPH	172.0	[20]	6	14.0
L3	172.3	[16]	10	2.1
OPAL	177.0	[19]	5	1.1
L3	182.8	[16]	10	2.7
ALEPH	183.0	[20]	6	4.0
DELPHI	183.0	[17]	13	33.1
L3	188.6	[16]	10	3.4
ALEPH	189.0	[20]	6	6.7
DELPHI	189.0	[17]	13	22.7
DELPHI	192.0	[17]	13	12.1
L3	194.4	[16]	10	1.2
DELPHI	196.0	[17]	13	39.7
OPAL	197.0	[19]	5	10.0
ALEPH	200.0	[20]	6	21.0
DELPHI	200.0	[17]	13	7.1
L3	200.0	[16]	9	6.5
DELPHI	202.0	[17]	13	14.9
DELPHI	205.0	[17]	13	12.6
ALEPH	206.0	[20]	6	7.0
L3	206.2	[16]	10	10.0
DELPHI	207.0	[17]	13	11.7
Total			430	466.0



Varying the renormalisation scale $\mu_R^2 \in [Q^2/2, 2Q^2]$ gave best fit values in the range $\alpha_0\,(2\text{ GeV}) = 0.585$, $\Lambda^{(5)}_{\overline{MS}} = 0.173$ GeV to $\alpha_0\,(2\text{ GeV}) = 0.598$, $\Lambda^{(5)}_{\overline{MS}} = 0.210$ GeV with no significant change in the quality of fit. Thus we find

$$\Lambda^{(5)}_{\overline{MS}} = 0.190^{+0.025+0.020}_{-0.022-0.017}\ \text{GeV} \qquad (39)$$

where the first error is the combined experimental statistical and systematic error and the second is due to the theoretical renormalisation scale uncertainty. The corresponding strong coupling constant is

$$\alpha_s\,(91.2\text{ GeV}) = 0.1164^{+0.0022+0.0017}_{-0.0021-0.0016}\,, \qquad (40)$$

or, combining all the errors in quadrature,

$$\boxed{\alpha_s\,(91.2\text{ GeV}) = 0.1164^{+0.0028}_{-0.0026}\,,} \qquad (41)$$

$2 \longrightarrow 4$

+0.0040
- 0.0038