# NNLO and "Classic" Power Corrections

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## Outline

- NP power corrections
- Universal low-scale  $\alpha_{\rm S}$  hypothesis
- NLL resummation
- Including power correction
- NNLO+NLL+NP fit to thrust

S Catani, L Trentadue, G Turnock & BW, NPB407(1993)3
 Y Dokshitzer, G Marchesini & BW, NPB469(1996)93
 R Davison & BW, EPJC 59(2009)13; 69(2010)341

## NP shift in Thrust

$$T = \max_{\overrightarrow{n}} \left( \frac{\sum_{i=1}^{N} |\overrightarrow{p_i} \cdot \overrightarrow{n}|}{\sum_{i=1}^{N} |\overrightarrow{p_i}|} \right) \equiv 1 - t$$







• 
$$R_{theory}$$
 -  $R_{expt}$  =  $AQ^n$ 









• 
$$R_{theory} - R_{expt} = AQ^n$$







• 
$$R_{theory} - R_{expt} = AQ^n$$





• 
$$R_{\text{theory}} - R_{\text{expt}} = AQ^n$$





### Universal low-scale effective $\alpha_S$



### Power Corrections

- Renormalon is due to IR divergence of  $\alpha_s = \alpha_s(q)$
- Postulate universal IRregular  $\alpha_{eff}$
- Power corrections depend on



 $\alpha_0(\mu_{\mathrm{I}}) = \frac{1}{\mu_{\mathrm{I}}} \int_0^{\mu_{\mathrm{I}}} \alpha_{\mathrm{eff}}(p_t) \, dp_t$ 

• Match NP & PT at  $\ \mu_I \sim 2 \ {
m GeV}$ 

### NLL thrust resummation

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dT} = \frac{Q^2}{2\pi i} \int_C d\nu \, \mathrm{e}^{(1-T)\nu Q^2} \left[ \tilde{J}^q_\nu(Q^2) \right]^2 \,,$$
$$\ln \tilde{J}^q_\nu(Q^2) = \int_0^1 \frac{du}{u} \left( \mathrm{e}^{-u\nu Q^2} - 1 \right) \left[ \int_{u^2 Q^2}^{uQ^2} \frac{dq^2}{q^2} A(\alpha_{\mathrm{s}}(q)) + \frac{1}{2} B(\alpha_{\mathrm{s}}(\sqrt{u}Q)) \right]$$

- Leading PT contribution from  $q < \mu_{\rm I}$ 

## The 3-loop constant

- Coefficient of 3-loop cusp anomalous dimension
  - L = true value [used by Gehrmann et al., EPJC67(2010)57]
  - $K^2$  = effective scale change (used by RD&BW)
- Happen to agree at  $n_f = 5$  (but effect negligible anyway)



## Power correction to thrust

• Replace PT by NP for  $q < \mu_{\mathrm{I}}$ 

$$\delta \ln \tilde{J}^{q}_{\nu}(Q^{2})\Big|_{\text{n.p.}} = \frac{2C_{F}}{\pi} \int_{0}^{\mu_{I}} \frac{dq}{q} \alpha_{\text{eff}}(q) \int_{q^{2}/Q^{2}}^{q/Q} \frac{du}{u} \left(e^{-u\nu Q^{2}} - 1\right)$$

• For 
$$\mu_{\mathrm{I}}\nu Q \ll 1$$
, i.e.  $1 - T \gg \mu_{\mathrm{I}}/Q$   
 $\delta \ln \tilde{J}^{q}_{\nu}(Q^{2})\Big|_{\mathrm{n.p.}} \simeq -\frac{2C_{F}}{\pi} \int_{0}^{\mu_{\mathrm{I}}} dq \,\alpha_{\mathrm{eff}}(q) \,\nu Q \equiv -\frac{2C_{F}}{\pi} \frac{\mu_{\mathrm{I}}}{Q} \,\alpha_{0}(\mu_{\mathrm{I}}) \,\nu Q^{2}$ 

• `Milan Factor': 
$$\alpha_0 \rightarrow 2\mathcal{M} \, \alpha_0/\pi = 0.95 \, \alpha_0$$
  
Dokshitzer et al., JHEP05(1998)003

$$\bullet \delta \ln \tilde{J}^q_{\nu}(Q^2) = \delta \ln \tilde{J}^q_{\nu}(Q^2) \Big|_{\text{n.p.}} - \delta \ln \tilde{J}^q_{\nu}(Q^2) \Big|_{\text{pert}} = \delta T \,\nu Q^2$$

### Power corrections to event shapes

#### I/Q renormalon present in T & C, absent in y<sub>3</sub>



## NLO results from e<sup>+</sup>e<sup>-</sup>



Movilla Fernandez, Bethke, Biebel & Kluth, EPJ C22(2001)1

## NLO results from DIS



# NNLO+NLL matching

• At NNLO: 
$$[\bar{\alpha}_s \equiv \alpha_s(\mu_R)/2\pi]$$
 A Gehrmann-DeRidder et al.,  
 $JHEP II(2007)058$   
 $R(t) = \int_0^t dt \frac{1}{\sigma} \frac{d\sigma}{dt} = \int_{1-t}^1 dT \frac{1}{\sigma} \frac{d\sigma}{dT} = 1 + \bar{\alpha}_s R_1(t) + \bar{\alpha}_s^2 R_2(t) + \bar{\alpha}_s^3 R_3(t) + \dots$   
• At NLL:  $\ln R(t) = Lg_1(\bar{\alpha}_s L) + g_2(\bar{\alpha}_s L) = \sum_{n=1}^{\infty} \sum_{m=n}^{n+1} G_{nm} \bar{\alpha}_s^n L^m$   
• Log-R matching:  $\left[ L = \ln \left( \frac{1}{t} + 1 - \frac{1}{t_{max}} \right) \right]$   
 $\ln R(t) = Lg_1(\alpha_s L) + g_2(\alpha_s L) + \bar{\alpha}_s(R_1(t) - G_{11}L - G_{12}L^2) + \bar{\alpha}_s(R_1(t) - G_{11}L - G_{12}L^2) + \bar{\alpha}_s^2 \left( R_2(t) - \frac{1}{2} [R_1(t)]^2 - G_{22}L^2 - G_{23}L^3 \right) + \bar{\alpha}_s^3 \left( R_3(t) - R_1(t) R_2(t) + \frac{1}{3} [R_1(t)]^3 - G_{33}L^3 - G_{34}L^4 \right).$ 

#### • Fit range $\max\{\mu_{I}/Q, 0.05\} \le t < 0.33$



























Results of NNLO+NLL+NP fit

$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Experiment	$Q/{ m GeV}$	Ref.	No. Pts.	$\chi^2$	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	TASSO	14.0	[14]	4	8.2	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	TASSO	22.0	[14]	6	2.8	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	TASSO	35.0	[14]	8	0.7	
L3       41.4 $[16]$ 8       3.4         JADE       44.0 $[15]$ 10       3.8         TASSO       44.0 $[14]$ 8       6.8         DELPHI       45.0 $[17]$ 11       11.6         AMY       54.5 $[18]$ 4       4.9         L3       55.3 $[16]$ 8       3.2         L3       65.4 $[16]$ 8       7.5         DELPHI       66.0 $[17]$ 11       14.5         L3       75.7 $[16]$ 8       1.9         DELPHI       76.0 $[17]$ 11       10.3         L3       85.1 $[16]$ 8       3.6         OPAL       91.0 $[19]$ 5       11.9         ALEPH       91.2 $[20]$ 27       16.1         DELPHI       91.2 $[21]$ 6       2.7         L3       130.1 $[16]$ 10       14.6         ALEPH       133.0 $[19]$ 5       6.5         L3       161.3 $[16]$ 10       2.1         OPAL	JADE	35.0	[15]	10	10.5	
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L3130.1 $[16]$ 1014.6ALEPH133.0 $[20]$ 67.2OPAL133.0 $[19]$ 56.5L3136.1 $[16]$ 1037.3ALEPH161.0 $[20]$ 65.5L3161.3 $[16]$ 104.0ALEPH172.0 $[20]$ 614.0L3172.3 $[16]$ 102.1OPAL177.0 $[19]$ 51.1L3182.8 $[16]$ 102.7ALEPH183.0 $[20]$ 64.0DELPHI183.0 $[17]$ 1333.1L3188.6 $[16]$ 103.4ALEPH189.0 $[20]$ 66.7DELPHI192.0 $[17]$ 1339.7OPAL197.0 $[19]$ 510.0ALEPH200.0 $[20]$ 621.0DELPHI196.0 $[17]$ 1339.7OPAL197.0 $[19]$ 510.0ALEPH200.0 $[20]$ 621.0DELPHI200.0 $[17]$ 137.1L3200.0 $[16]$ 96.5DELPHI205.0 $[17]$ 1314.9DELPHI205.0 $[17]$ 1312.6ALEPH206.0 $[20]$ 67.0L3206.2 $[16]$ 1010.0DELPHI207.0 $[17]$ 1311.7Total430 <t< td=""><td>SLD</td><td>91.2</td><td>[21]</td><td>6</td><td>2.7</td><td></td></t<>	SLD	91.2	[21]	6	2.7	
ALEPH $133.0$ $[20]$ $6$ $7.2$ OPAL $133.0$ $[19]$ $5$ $6.5$ L3 $136.1$ $[16]$ $10$ $37.3$ ALEPH $161.0$ $[20]$ $6$ $5.5$ L3 $161.3$ $[16]$ $10$ $4.0$ ALEPH $172.0$ $[20]$ $6$ $14.0$ L3 $172.3$ $[16]$ $10$ $2.1$ OPAL $177.0$ $[19]$ $5$ $1.1$ L3 $182.8$ $[16]$ $10$ $2.7$ ALEPH $183.0$ $[20]$ $6$ $4.0$ DELPHI $183.0$ $[17]$ $13$ $33.1$ L3 $188.6$ $[16]$ $10$ $3.4$ ALEPH $189.0$ $[20]$ $6$ $6.7$ DELPHI $192.0$ $[17]$ $13$ $22.7$ DELPHI $192.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $7.1$ L3 $200.0$ $[17]$ $13$ $14.9$ DELPHI $200.0$ $[17]$ $13$ $14.9$ DELPHI $206.0$ $[20]$ $6$ $7.0$ L3 $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$	L3	130.1	[16]	10	14.6	
OPAL $133.0$ $[19]$ $5$ $6.5$ L3 $136.1$ $[16]$ $10$ $37.3$ ALEPH $161.0$ $[20]$ $6$ $5.5$ L3 $161.3$ $[16]$ $10$ $4.0$ ALEPH $172.0$ $[20]$ $6$ $14.0$ L3 $172.3$ $[16]$ $10$ $2.1$ OPAL $177.0$ $[19]$ $5$ $1.1$ L3 $182.8$ $[16]$ $10$ $2.7$ ALEPH $183.0$ $[20]$ $6$ $4.0$ DELPHI $183.0$ $[17]$ $13$ $33.1$ L3 $188.6$ $[16]$ $10$ $3.4$ ALEPH $189.0$ $[20]$ $6$ $6.7$ DELPHI $199.0$ $[17]$ $13$ $22.7$ DELPHI $199.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $196.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $14.9$ DELPHI $206.0$ $[20]$ $6$ $7.0$ L3 $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$	ALEPH	133.0	[20]	6	7.2	
L3136.1 $[16]$ 1037.3ALEPH161.0 $[20]$ 65.5L3161.3 $[16]$ 104.0ALEPH172.0 $[20]$ 614.0L3172.3 $[16]$ 102.1OPAL177.0 $[19]$ 51.1L3182.8 $[16]$ 102.7ALEPH183.0 $[20]$ 64.0DELPHI183.0 $[17]$ 1333.1L3188.6 $[16]$ 103.4ALEPH189.0 $[20]$ 66.7DELPHI192.0 $[17]$ 1322.7DELPHI192.0 $[17]$ 1339.7OPAL197.0 $[19]$ 510.0ALEPH200.0 $[20]$ 621.0DELPHI196.0 $[17]$ 1339.7OPAL197.0 $[19]$ 510.0ALEPH200.0 $[20]$ 621.0DELPHI200.0 $[17]$ 1314.9DELPHI200.0 $[17]$ 1314.9DELPHI205.0 $[17]$ 1312.6ALEPH206.0 $[20]$ 67.0L3206.2 $[16]$ 1010.0DELPHI207.0 $[17]$ 1311.7Total430466.010	OPAL	133.0	[19]	5	6.5	
ALEPH $161.0$ $[20]$ $6$ $5.5$ L3 $161.3$ $[16]$ $10$ $4.0$ ALEPH $172.0$ $[20]$ $6$ $14.0$ L3 $172.3$ $[16]$ $10$ $2.1$ OPAL $177.0$ $[19]$ $5$ $1.1$ L3 $182.8$ $[16]$ $10$ $2.7$ ALEPH $183.0$ $[20]$ $6$ $4.0$ DELPHI $183.0$ $[17]$ $13$ $33.1$ L3 $188.6$ $[16]$ $10$ $3.4$ ALEPH $189.0$ $[20]$ $6$ $6.7$ DELPHI $189.0$ $[17]$ $13$ $22.7$ DELPHI $199.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $196.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $14.9$ DELPHI $200.0$ $[17]$ $13$ $14.9$ DELPHI $206.0$ $[20]$ $6$ $7.0$ L3 $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$	L3	136.1	[16]	10	37.3	
L3161.3 $\begin{bmatrix} 16 \\ 20 \end{bmatrix}$ 104.0ALEPH172.0 $\begin{bmatrix} 20 \end{bmatrix}$ 614.0L3172.3 $\begin{bmatrix} 16 \end{bmatrix}$ 102.1OPAL177.0 $\begin{bmatrix} 19 \end{bmatrix}$ 51.1L3182.8 $\begin{bmatrix} 16 \end{bmatrix}$ 102.7ALEPH183.0 $\begin{bmatrix} 20 \end{bmatrix}$ 64.0DELPHI183.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1333.1L3188.6 $\begin{bmatrix} 16 \end{bmatrix}$ 103.4ALEPH189.0 $\begin{bmatrix} 20 \end{bmatrix}$ 66.7DELPHI189.0 $\begin{bmatrix} 20 \end{bmatrix}$ 66.7DELPHI192.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1322.7DELPHI196.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1339.7OPAL194.4 $\begin{bmatrix} 16 \end{bmatrix}$ 101.2DELPHI196.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1339.7OPAL197.0 $\begin{bmatrix} 19 \end{bmatrix}$ 510.0ALEPH200.0 $\begin{bmatrix} 20 \end{bmatrix}$ 621.0DELPHI200.0 $\begin{bmatrix} 17 \end{bmatrix}$ 137.1L3200.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1314.9DELPHI205.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1312.6ALEPH206.0 $\begin{bmatrix} 20 \end{bmatrix}$ 67.0L3206.2 $\begin{bmatrix} 16 \end{bmatrix}$ 1010.0DELPHI207.0 $\begin{bmatrix} 17 \end{bmatrix}$ 1311.7Total430466.0	ALEPH	161.0	[20]	6	5.5	
ALEPH $172.0$ $\begin{bmatrix} 20 \\ 20 \end{bmatrix}$ 6 $14.0$ L3 $172.3$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $2.1$ OPAL $177.0$ $\begin{bmatrix} 19 \\ 19 \end{bmatrix}$ $5$ $1.1$ L3 $182.8$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $2.7$ ALEPH $183.0$ $\begin{bmatrix} 20 \\ 20 \end{bmatrix}$ $6$ $4.0$ DELPHI $183.0$ $\begin{bmatrix} 17 \\ 1 \end{bmatrix}$ $13$ $33.1$ L3 $188.6$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $3.4$ ALEPH $189.0$ $\begin{bmatrix} 20 \\ 20 \end{bmatrix}$ $6$ $6.7$ DELPHI $189.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $22.7$ DELPHI $192.0$ $\begin{bmatrix} 17 \\ 17 \end{bmatrix}$ $13$ $22.7$ DELPHI $199.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $39.7$ OPAL $194.4$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $1.2$ DELPHI $196.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $39.7$ OPAL $197.0$ $\begin{bmatrix} 19 \\ 9 \end{bmatrix}$ $5$ DELPHI $200.0$ $\begin{bmatrix} 20 \\ 16 \end{bmatrix}$ $6$ DELPHI $200.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $14.9$ DELPHI $200.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $14.9$ DELPHI $200.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $14.9$ DELPHI $206.0$ $\begin{bmatrix} 20 \\ 6 \end{bmatrix}$ $6$ ALEPH $206.0$ $\begin{bmatrix} 20 \\ 6 \end{bmatrix}$ $6$ DELPHI $207.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $11.7$ Total $430$ $466.0$	L3	161.3	[16]	10	4.0	
L3 $172.3$ $\begin{bmatrix} 16 \\ 19 \end{bmatrix}$ $5$ $1.1$ L3 $182.8$ $\begin{bmatrix} 16 \\ 19 \end{bmatrix}$ $5$ $1.1$ L3 $182.8$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $2.7$ ALEPH $183.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $4.0$ DELPHI $183.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $33.1$ L3 $188.6$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $3.4$ ALEPH $189.0$ $\begin{bmatrix} 20 \\ 20 \end{bmatrix}$ $6$ $6.7$ DELPHI $189.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $22.7$ DELPHI $192.0$ $\begin{bmatrix} 17 \\ 17 \end{bmatrix}$ $13$ $22.7$ DELPHI $192.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $12.1$ L3 $194.4$ $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$ $1.2$ DELPHI $196.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $39.7$ OPAL $197.0$ $\begin{bmatrix} 19 \\ 9 \end{bmatrix}$ $5$ $10.0$ ALEPH $200.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $7.1$ $13$ L3 $200.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $14.9$ $9$ DELPHI $205.0$ $\begin{bmatrix} 17 \\ 13 \end{bmatrix}$ $14.9$ $14.0$ DELPHI $206.0$	ALEPH	172.0	[20]	6	14.0	
OPAL $177.0$ $[19]$ $5$ $1.1$ L3 $182.8$ $[16]$ $10$ $2.7$ ALEPH $183.0$ $[20]$ $6$ $4.0$ DELPHI $183.0$ $[17]$ $13$ $33.1$ L3 $188.6$ $[16]$ $10$ $3.4$ ALEPH $189.0$ $[20]$ $6$ $6.7$ DELPHI $189.0$ $[17]$ $13$ $22.7$ DELPHI $192.0$ $[17]$ $13$ $12.1$ L3 $194.4$ $[16]$ $10$ $1.2$ DELPHI $196.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $7.1$ L3 $200.0$ $[16]$ $9$ $6.5$ DELPHI $200.0$ $[17]$ $13$ $14.9$ DELPHI $200.0$ $[17]$ $13$ $14.9$ DELPHI $205.0$ $[17]$ $13$ $14.9$ DELPHI $206.2$ $[16]$ $10$ $10.0$ DELPHI $206.0$ $[20]$ $6$ $7.0$ L3 $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$	L3	172.3	[16]	10	2.1	
L3 $182.8$ $[16]$ $10$ $2.7$ ALEPH $183.0$ $[20]$ $6$ $4.0$ DELPHI $183.0$ $[17]$ $13$ $33.1$ L3 $188.6$ $[16]$ $10$ $3.4$ ALEPH $189.0$ $[20]$ $6$ $6.7$ DELPHI $189.0$ $[17]$ $13$ $22.7$ DELPHI $192.0$ $[17]$ $13$ $12.1$ L3 $194.4$ $[16]$ $10$ $1.2$ DELPHI $196.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $7.1$ L3 $200.0$ $[16]$ $9$ $6.5$ DELPHI $202.0$ $[17]$ $13$ $14.9$ DELPHI $205.0$ $[17]$ $13$ $12.6$ ALEPH $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$ $466.0$	OPAL	177.0	[19]	5	1.1	
ALEPH $183.0$ $[20]$ $6$ $4.0$ DELPHI $183.0$ $[17]$ $13$ $33.1$ L3 $188.6$ $[16]$ $10$ $3.4$ ALEPH $189.0$ $[20]$ $6$ $6.7$ DELPHI $189.0$ $[17]$ $13$ $22.7$ DELPHI $192.0$ $[17]$ $13$ $12.1$ L3 $194.4$ $[16]$ $10$ $1.2$ DELPHI $196.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $7.1$ L3 $200.0$ $[16]$ $9$ $6.5$ DELPHI $202.0$ $[17]$ $13$ $14.9$ DELPHI $205.0$ $[17]$ $13$ $12.6$ ALEPH $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$ $466.0$	L3	182.8	[16]	10	2.7	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	ALEPH	183.0	[20]	6	4.0	
L3 $188.6$ $\begin{bmatrix} 16 \\ 20 \end{bmatrix}$ $10$ $3.4$ ALEPH $189.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $6.7$ DELPHI $189.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $22.7$ DELPHI $192.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $12.1$ L3 $194.4$ $\begin{bmatrix} 16 \end{bmatrix}$ $10$ $1.2$ DELPHI $196.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $39.7$ OPAL $197.0$ $\begin{bmatrix} 19 \end{bmatrix}$ $5$ $10.0$ ALEPH $200.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $21.0$ DELPHI $200.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $7.1$ L3 $200.0$ $\begin{bmatrix} 16 \end{bmatrix}$ $9$ $6.5$ DELPHI $202.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $14.9$ DELPHI $205.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $12.6$ ALEPH $206.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $7.0$ L3 $206.2$ $\begin{bmatrix} 16 \end{bmatrix}$ $10$ $10.0$ DELPHI $207.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $11.7$ Total $430$ $466.0$ $466.0$	DELPHI	183.0	[17]	13	33.1	
ALEPH $189.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $6.7$ DELPHI $189.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $22.7$ DELPHI $192.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $12.1$ L3 $194.4$ $\begin{bmatrix} 16 \end{bmatrix}$ $10$ $1.2$ DELPHI $196.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $39.7$ OPAL $197.0$ $\begin{bmatrix} 19 \end{bmatrix}$ $5$ $10.0$ ALEPH $200.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $21.0$ DELPHI $200.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $7.1$ L3 $200.0$ $\begin{bmatrix} 16 \end{bmatrix}$ $9$ $6.5$ DELPHI $202.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $14.9$ DELPHI $205.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $12.6$ ALEPH $206.0$ $\begin{bmatrix} 20 \end{bmatrix}$ $6$ $7.0$ L3 $206.2$ $\begin{bmatrix} 16 \end{bmatrix}$ $10$ $10.0$ DELPHI $207.0$ $\begin{bmatrix} 17 \end{bmatrix}$ $13$ $11.7$ Total $430$ $466.0$	L3	188.6	[16]	10	3.4	
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	ALEPH	189.0	[20]	6	6.7	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	DELPHI	189.0	[17]	13	22.7	
L3 $194.4$ $[16]$ $10$ $1.2$ DELPHI $196.0$ $[17]$ $13$ $39.7$ OPAL $197.0$ $[19]$ $5$ $10.0$ ALEPH $200.0$ $[20]$ $6$ $21.0$ DELPHI $200.0$ $[17]$ $13$ $7.1$ L3 $200.0$ $[16]$ $9$ $6.5$ DELPHI $202.0$ $[17]$ $13$ $14.9$ DELPHI $205.0$ $[17]$ $13$ $12.6$ ALEPH $206.2$ $[16]$ $10$ $10.0$ DELPHI $207.0$ $[17]$ $13$ $11.7$ Total $430$ $466.0$	DELPHI	192.0	[17]	13	12.1	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	L3	194.4	[16]	10	1.2	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	DELPHI	196.0	[17]	13	39.7	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	OPAL	197.0	[19]	5	10.0	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	ALEPH	200.0	[20]	6	21.0	
L3     200.0     [16]     9     6.5       DELPHI     202.0     [17]     13     14.9       DELPHI     205.0     [17]     13     12.6       ALEPH     206.0     [20]     6     7.0       L3     206.2     [16]     10     10.0       DELPHI     207.0     [17]     13     11.7       Total     430     466.0	DELPHI	200.0	[17]	13	7.1	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	L3	200.0	[16]	9	6.5	
DELPHI         205.0         [17]         13         12.6           ALEPH         206.0         [20]         6         7.0           L3         206.2         [16]         10         10.0           DELPHI         207.0         [17]         13         11.7           Total         430         466.0         466.0	DELPHI	202.0	[17]	13	14.9	
ALEPH     206.0     [20]     6     7.0       L3     206.2     [16]     10     10.0       DELPHI     207.0     [17]     13     11.7       Total     430     466.0	DELPHI	205.0	[17]	13	12.6	
L3         206.2         [16]         10         10.0           DELPHI         207.0         [17]         13         11.7           Total         430         466.0	ALEPH	206.0	[20]	6	7.0	
DELPHI         207.0         [17]         13         11.7           Total         430         466.0         466.0	L3	206.2	[16]	10	10.0	
Total 430 466.0	DELPHI	207.0	[17]	13	11.7	
	Total			430	466.0	



Varying the renormalisation scale  $\mu_R^2 \in [Q^2/2, 2Q^2]$ gave best fit values in the range  $\alpha_0 (2 \text{ GeV}) = 0.585$ ,  $\Lambda_{\overline{MS}}^{(5)} = 0.173 \text{ GeV}$  to  $\alpha_0 (2 \text{ GeV}) = 0.598$ ,  $\Lambda_{\overline{MS}}^{(5)} = 0.210$ GeV with no significant change in the quality of fit. Thus we find  $\Lambda_{\overline{MS}}^{(5)} = 0.100 \pm 0.025 \pm 0.020$  G M (20)

$$\Lambda_{\overline{MS}}^{(5)} = 0.190_{-0.022-0.017}^{+0.025+0.020} \text{ GeV}$$
(39)

where the first error is the combined experimental statistical and systematic error and the second is due to the theoretical renormalisation scale uncertainty. The corresponding strong coupling constant is

$$\alpha_s (91.2 \text{ GeV}) = 0.1164^{+0.0022+0.0017}_{-0.0021-0.0016} , \qquad (40)$$

or, combining all the errors in quadrature,

$$\alpha_s (91.2 \text{ GeV}) = 0.1164^{+0.0028}_{-0.0026} ,$$
 (41)

Alpha\_s Workshop

+0.0040 - 0.0038

 $2 \rightarrow 4$