Alpha_S from PACS-CS

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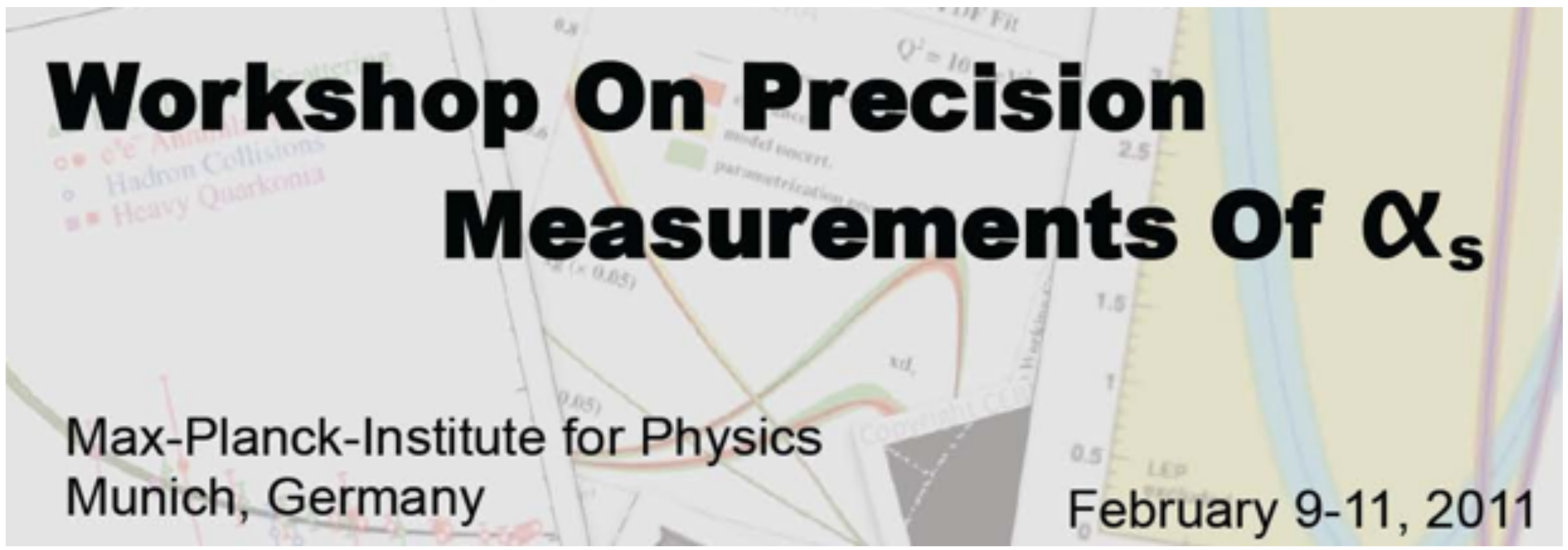


Reference: JHEP 0910 (2009) 053.

“Precise determination of the strong coupling constant in N_f=2+1 lattice QCD with the Schroedinger functional scheme”

S. Aoki, K.-I. Ishikawa, N. Ishizuka, T. Izubuchi, D. Kadoh, K. Kanaya, Y. Kuramashi, K. Murano, Y. Namekawa, M. Okawa, Y. Taniguchi, A. Ukawa, N. Ukita and T. Yoshie (PACS-CS Collaboration)

Our Goal

$$\Lambda_{\rm SF} = \frac{1}{L}\left(b_0\overline{g}(L)\right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\overline{g}(L)}\right)\exp\left(-\int_0^{\overline{g}(L)} dg\left(\frac{1}{\beta(g)}+\frac{1}{b_0 g^3}-\frac{b_1}{b_0^2 g}\right)\right)$$

$$\beta(g) = -g^3\left(b_0 + b_1 g^2 + b_2 g^4 + \cdots\right),$$
$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_f\right),$$
$$b_1 = \frac{1}{(4\pi)^4}\left(102 - \frac{38}{3}N_f\right),$$
$$b_2 = \frac{1}{(4\pi)^3}\left(0.483(7) - 0.275(5)N_f + 0.0361(5)N_f^2 - 0.00175(1)N_f^3\right)$$

$\bar{g}(L)$: Schrödinger Functional (SF) coupling

L: special box size

Advantages

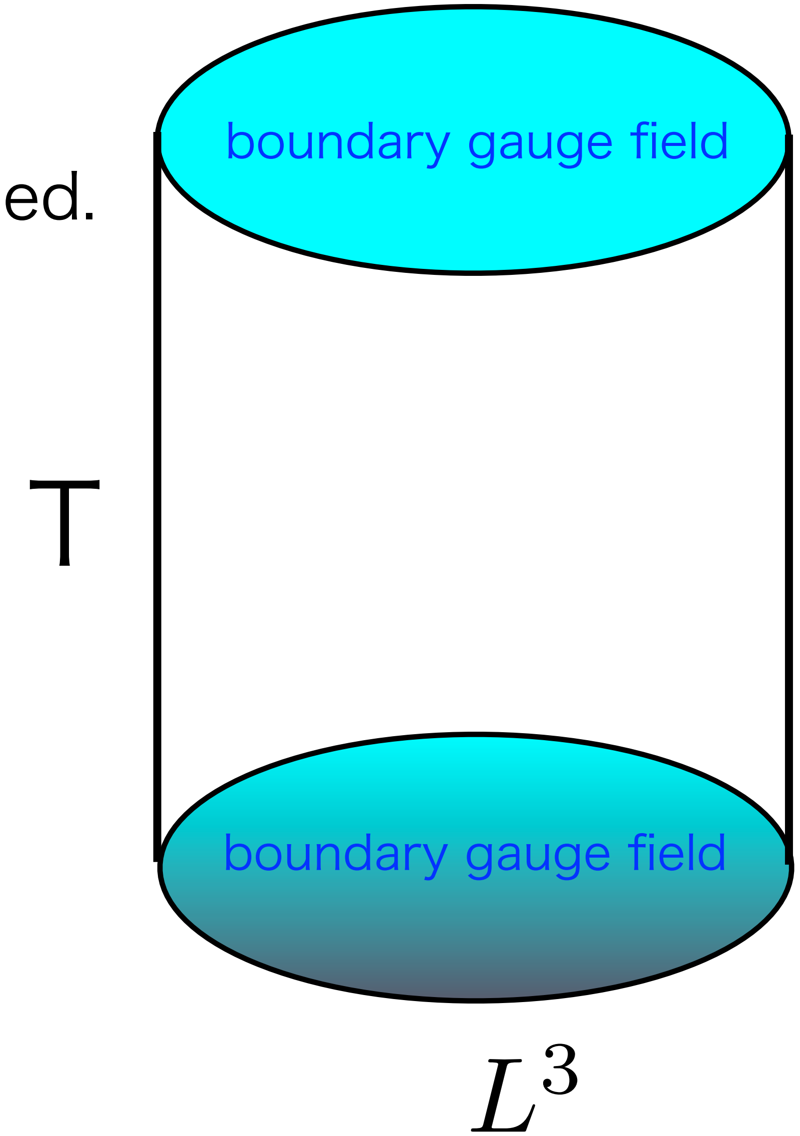
Non-perturbative.

A box size L gives the scale. No other scale is needed.

The continuum limit (a->0) can be taken.

Disadvantages

Separate simulations have to be performed.



Strategy for Λ

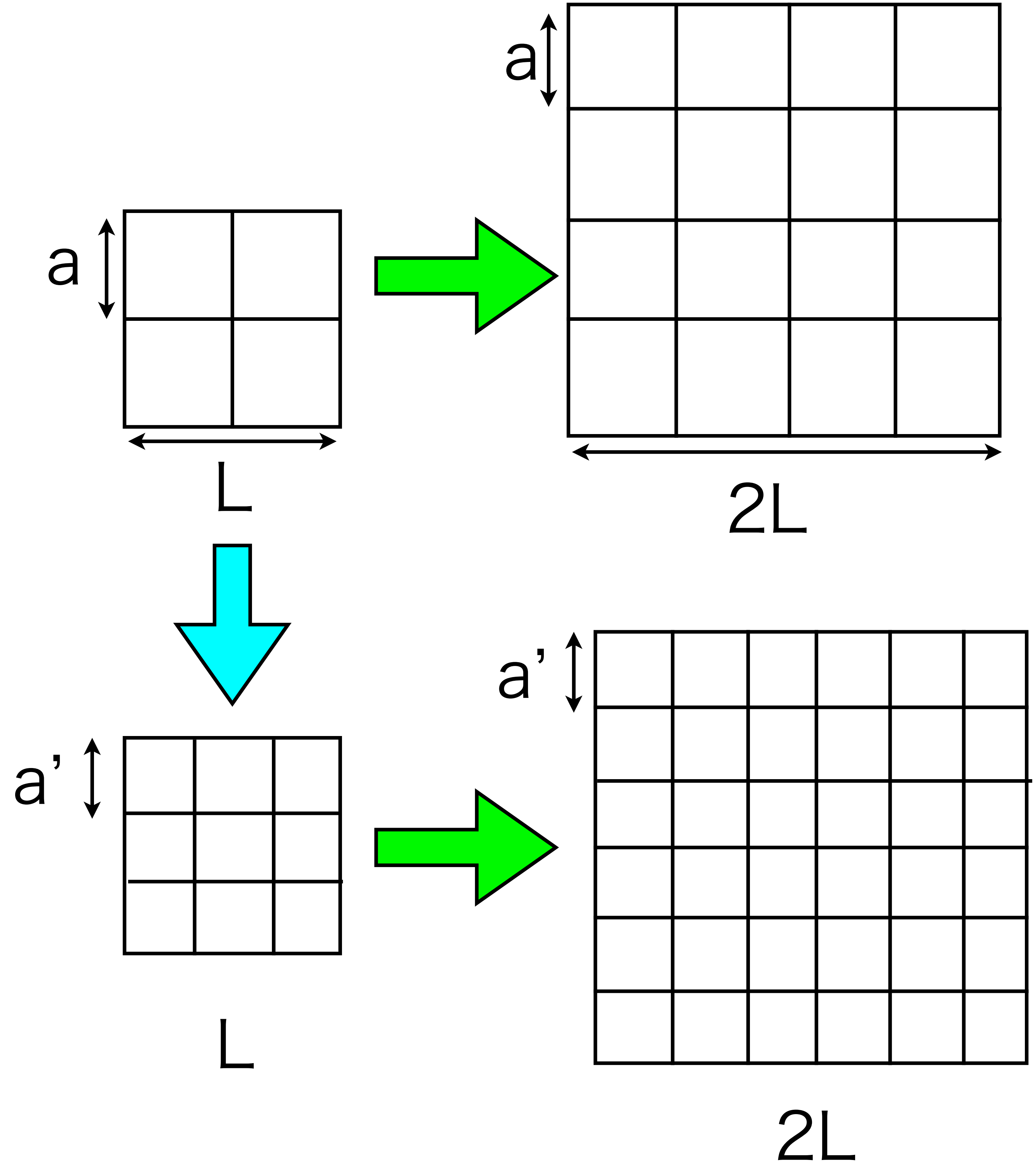
(1) Step Scaling Function (SSF)

$$\Sigma\left(u, \frac{a}{L}\right) = \bar{g}^2(2L)\Big|_{u=\bar{g}^2(L)}$$

continuum limit

$$\sigma(u) = \lim_{a/L \to 0} \Sigma\left(u, \frac{a}{L}\right)$$

polynomial fit in u for $\sigma(u)$



(2) define a reference scale $L_{\max}$ through a fixed value of $\bar{g}^2(L_{\max})$

non-perturbative $a/L_{\max} \ll 1$

non-perturbative

SSF(n times)

perturbative

$1/L_{\max} \sim 0.5$ GeV $\longrightarrow$ $1/L = 2^n/L_{\max} \sim 16$ GeV

3-loop

(3) $\Lambda_{\mathrm{SF}} L_{\max}$ from $\Lambda_{\mathrm{SF}} = \frac{1}{L}\left(b_0 \overline{g}(L)\right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\overline{g}(L)}\right) \exp\left(-\int_0^{\overline{g}(L)} dg \left(\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right)\right)$

$g^2(L)$ with $L = 2^{-n} L_{\max}$

(4) $L_{\max}$ in physical unit from hadron mass

an independent large scale simulation at some a is required.

$$\Lambda_{\overline{\mathrm{MS}}} = 2.61192\Lambda_{\mathrm{SF}}$$

for 3 flavors

Strategy for $\alpha^{(5)}_{\overline{\rm MS}}(M_Z)$

(1)$L_{\rm max}$ in unit of $\rm GeV^{-1}$

(2)$\alpha(q)$ at $q = 2^5/L{\rm max} \sim 16{\rm GeV}$ by $\sigma(u)$ continuum, non-perturbative

perturbative

(3)Change the scheme to $\overline{\rm MS}$ at 2-loop higher order correction is less than 0.1%

$$\alpha_{\overline{\rm MS}}(sq) = \alpha_{\rm SF}(q) + c_1(s)\alpha^2_{\rm SF}(q) + c_2(s)\alpha^3_{\rm SF}(q) + \cdots,$$

$$c_1(s) = -8\pi b_0 \ln(s) + 1.255621(2) + 0.0398629(2)N_f,$$ =0 at s=2.61192

$$c_2(s) = c_1(s)^2 - 32\pi^2 b_1 \ln(s) + 1.197(10) + 0.140(6)N_f - 0.0330(2){N_f}^2$$

(4)Run down to $\mu = m_c$ at 4-loop running with $N_f = 3$ in $\overline{\rm MS}$.

At $\mu = m_c$, $N_f = 3 \rightarrow N_f = 4$ at 3-loop

systematic error from 3-loop vs. 2-loop

$\mu = M(M) \rightarrow x = 0$

$$\frac{\alpha^{(N_f-1)}(\mu)}{\pi} = \frac{\alpha^{(N_f)}(\mu)}{\pi} F\left(\alpha^{(N_f)}(\mu), x\right), \quad x = \ln\frac{M(\mu)^2}{\mu^2},$$

$$F(\alpha, x) = 1 + \sum_{k=1}^{3} F_k(x)\left(\frac{\alpha}{\pi}\right)^k,$$

$$F_1(x) = \frac{1}{6}x,$$

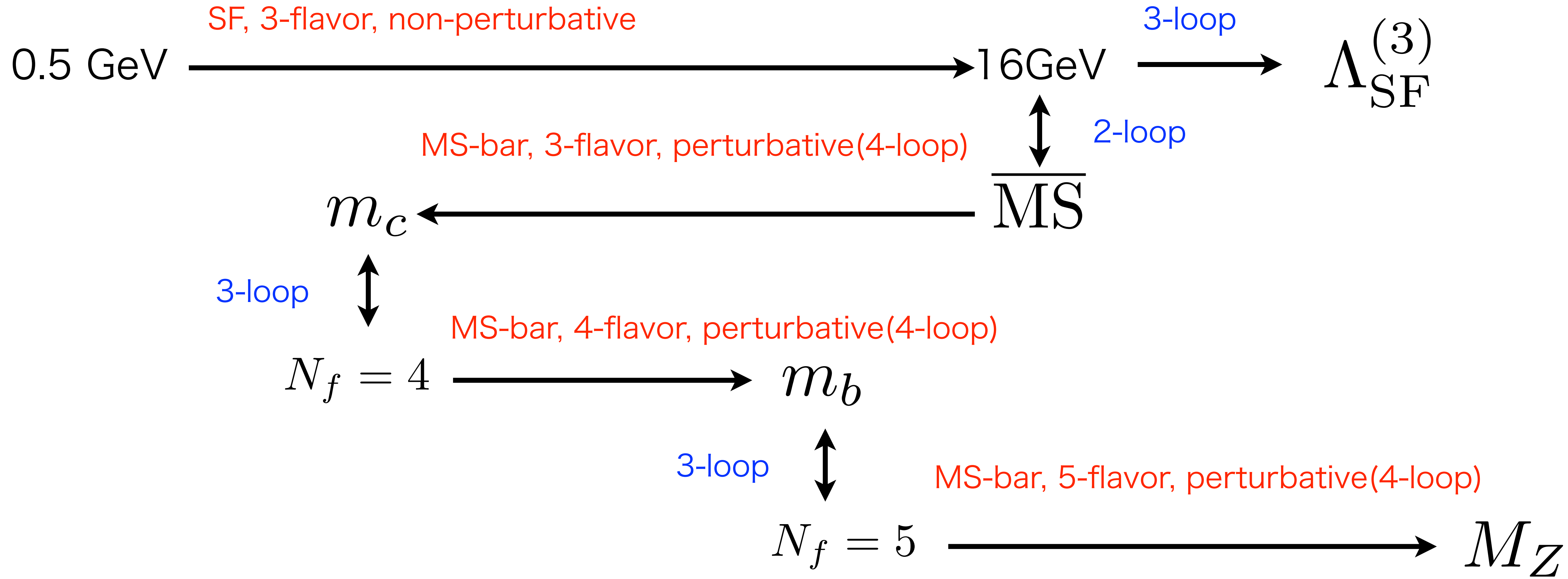
$$F_2(x) = F_1(x)^2 + \frac{11}{24}x + \frac{11}{72},$$

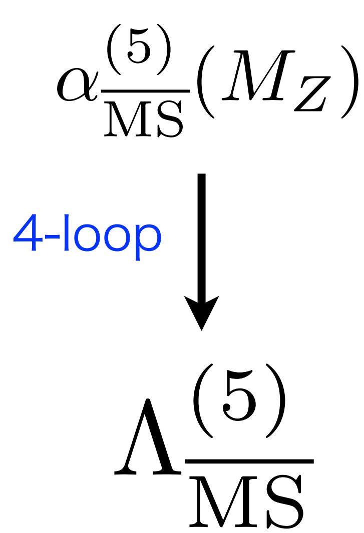
$$F_3(x) = \frac{564731}{124416} - \frac{82043}{27648}\zeta(3) + \frac{955}{576}x + \frac{53}{576}x^2 + \frac{1}{216}x^3 + (N_f - 1)\left(-\frac{2633}{31104} - \frac{67}{576}x - \frac{1}{36}x^2\right),$$

(5) Run up to $\mu = m_b(m_b)$, then $N_f = 4 \rightarrow N_f = 5$

(6) Run up to $\mu = M_z(M_Z)$, then $\alpha^{(5)}_{\overline{\rm MS}}(M_Z)$

(7) $\Lambda^{(5)}_{\overline{\rm MS}}$ from $\mu = M_z(M_z) = 1/L$, $\alpha_s(M_z)$ and 4-loop $\beta(g)$.





Simulations

3-flavor massless QCD

non-perturbatively O(a) improved Wilson quark action

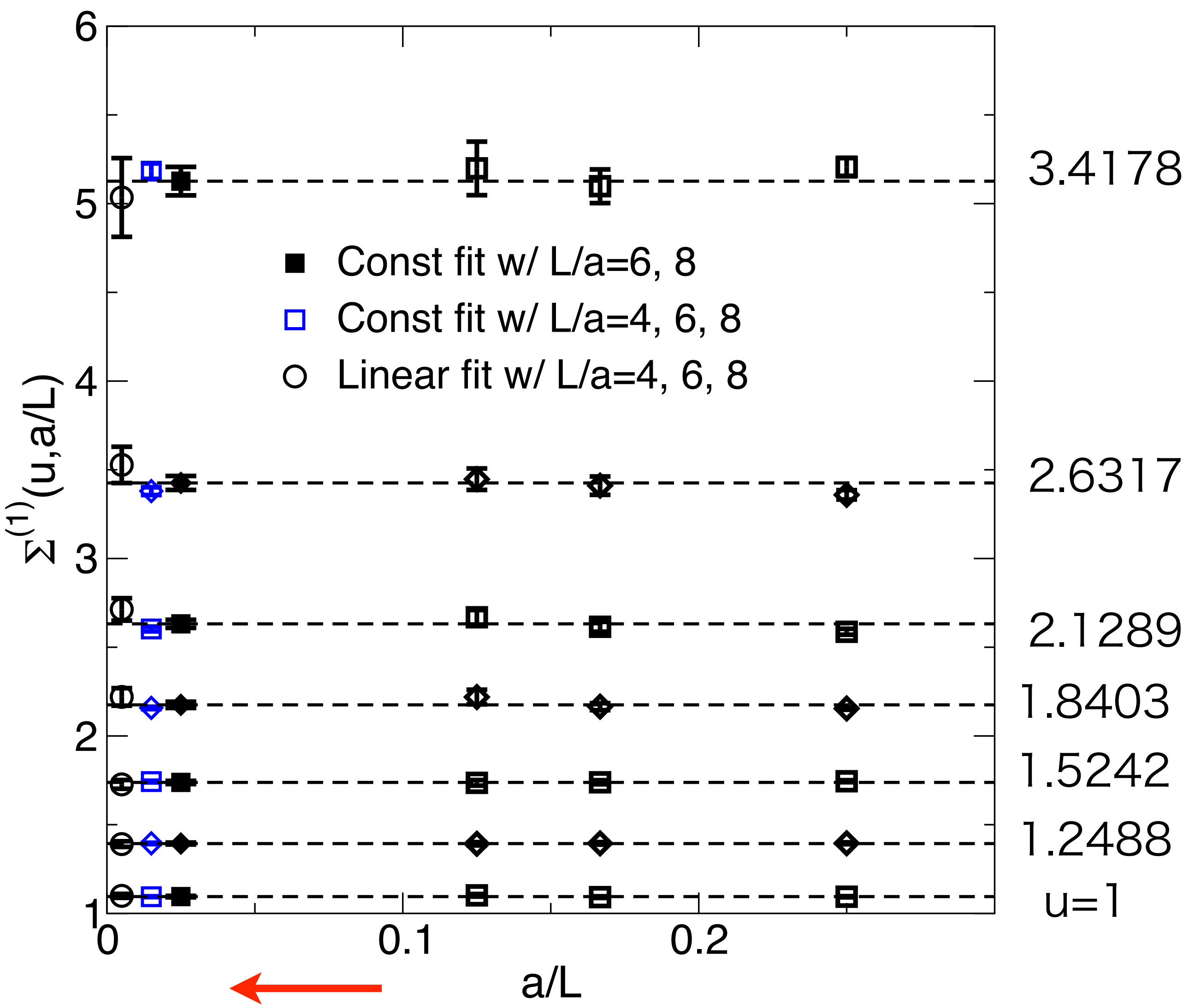
RG improved gauge action

non-perturbative c_A

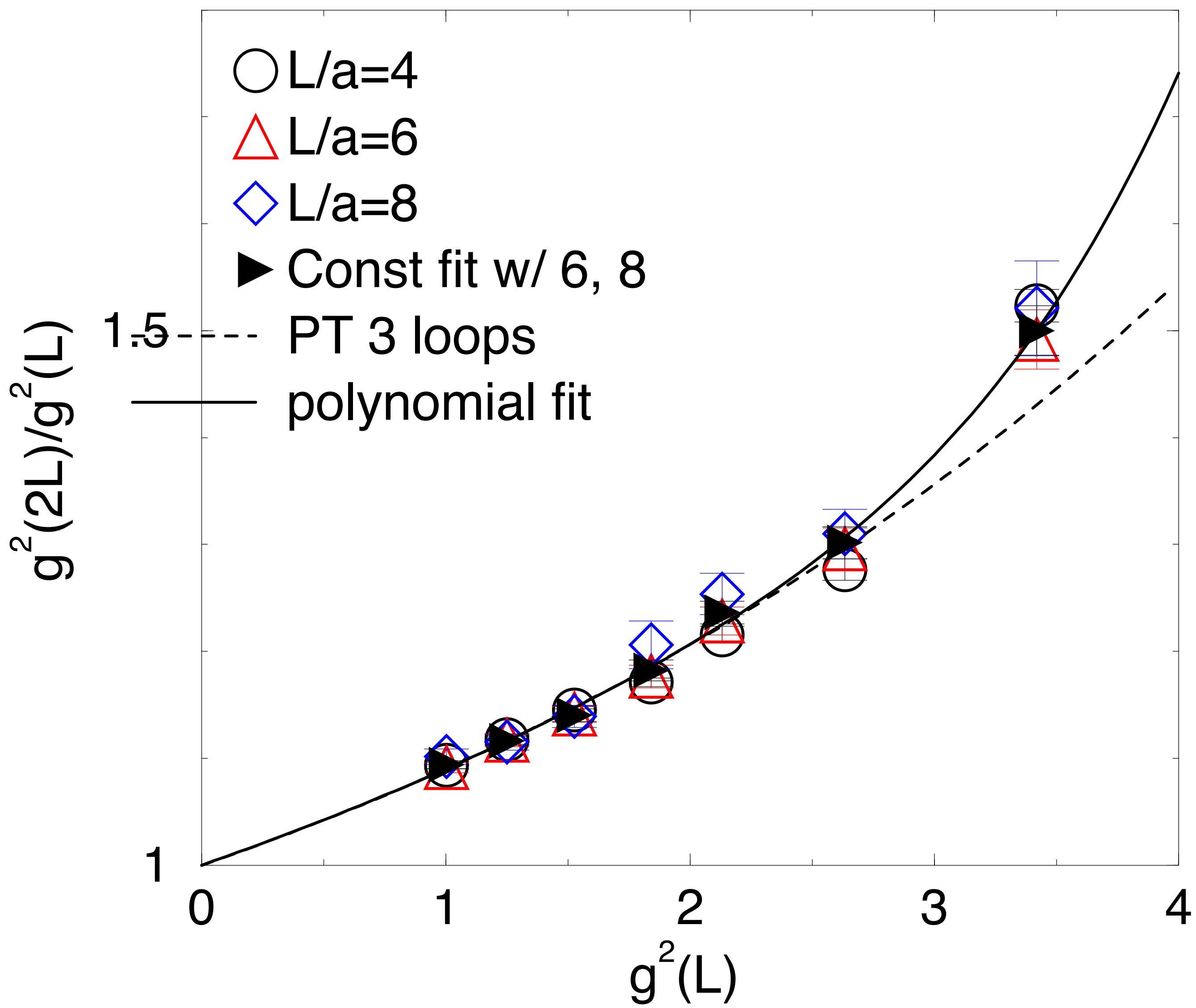
tree-level boundary terms

Results

SSF for coupling



continuum limit of SSF



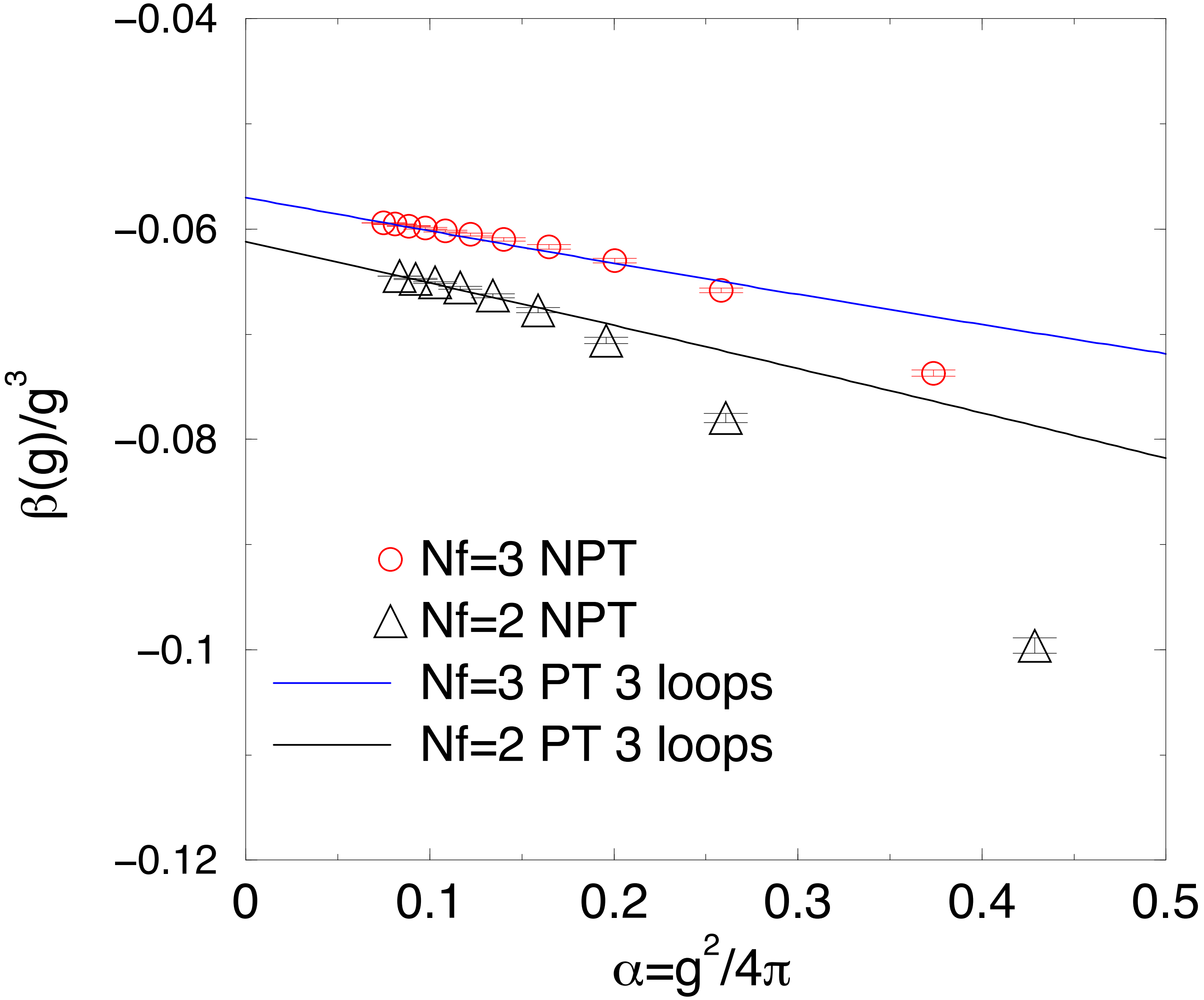
polynomial fit

$$\Sigma^{(2)}\left(u, \frac{a}{L}\right) = \frac{\Sigma\left(u, a/L\right)}{1 + \delta_1(a/L)u + d_2(a/L)u^2}.$$

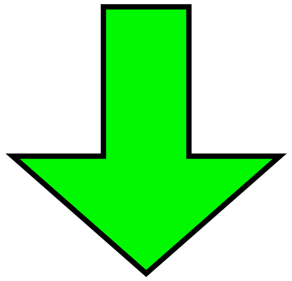
$$\sigma(u) = u + s_0u^2 + s_1u^3 + s_2u^4 + s_3u^5 + s_4u^6$$
$$s_3 = -0.000673, \quad s_4 = 0.0003434.$$

perturbative scaling violation

Non-perturbative beta-function



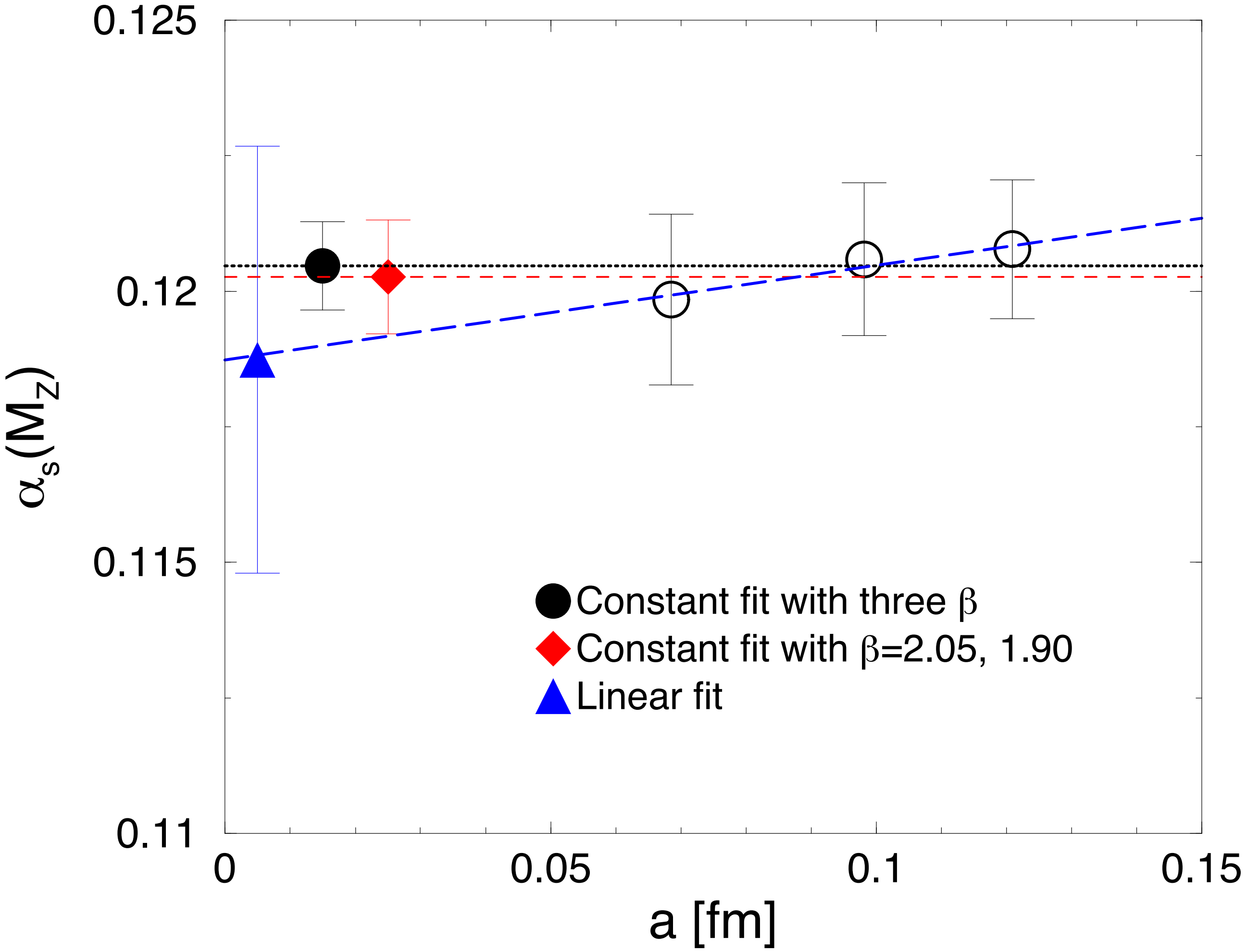
$$-L\frac{\partial u(L)}{\partial L} = 2\sqrt{u}\beta(\sqrt{u}), \quad u = \overline{g}^2(L)$$

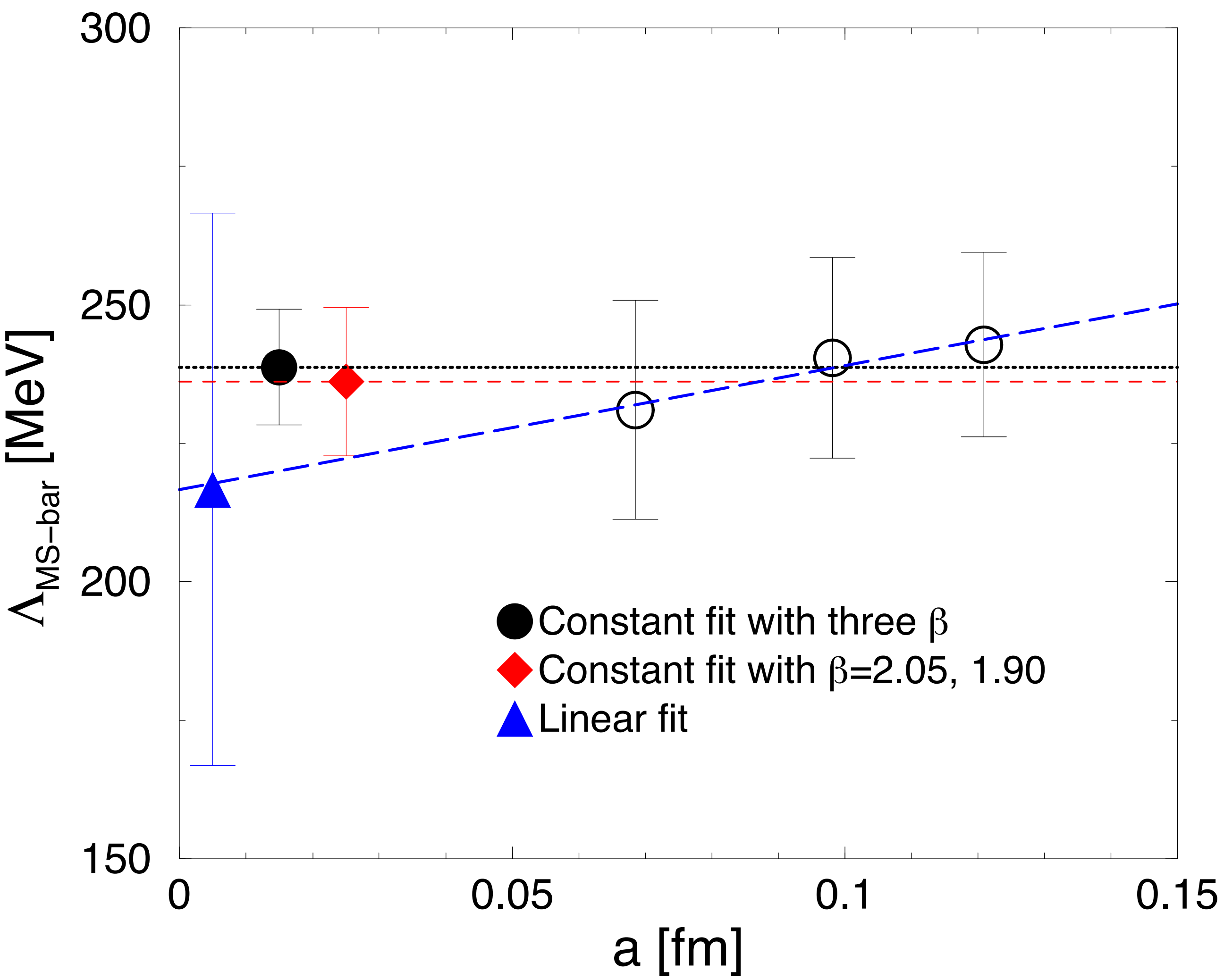


$$\beta\left(\sqrt{\sigma(u)}\right) = \beta(\sqrt{u})\sqrt{\frac{u}{\sigma(u)}}\frac{\partial\sigma(u)}{\partial u}.$$

start from u=0.9381

continuum limit





scale from m_π, m_K and m_Ω (2+1 full QCD, CP-PACS/JLQCD)

extrapolation from heavier pion mass

Final Result

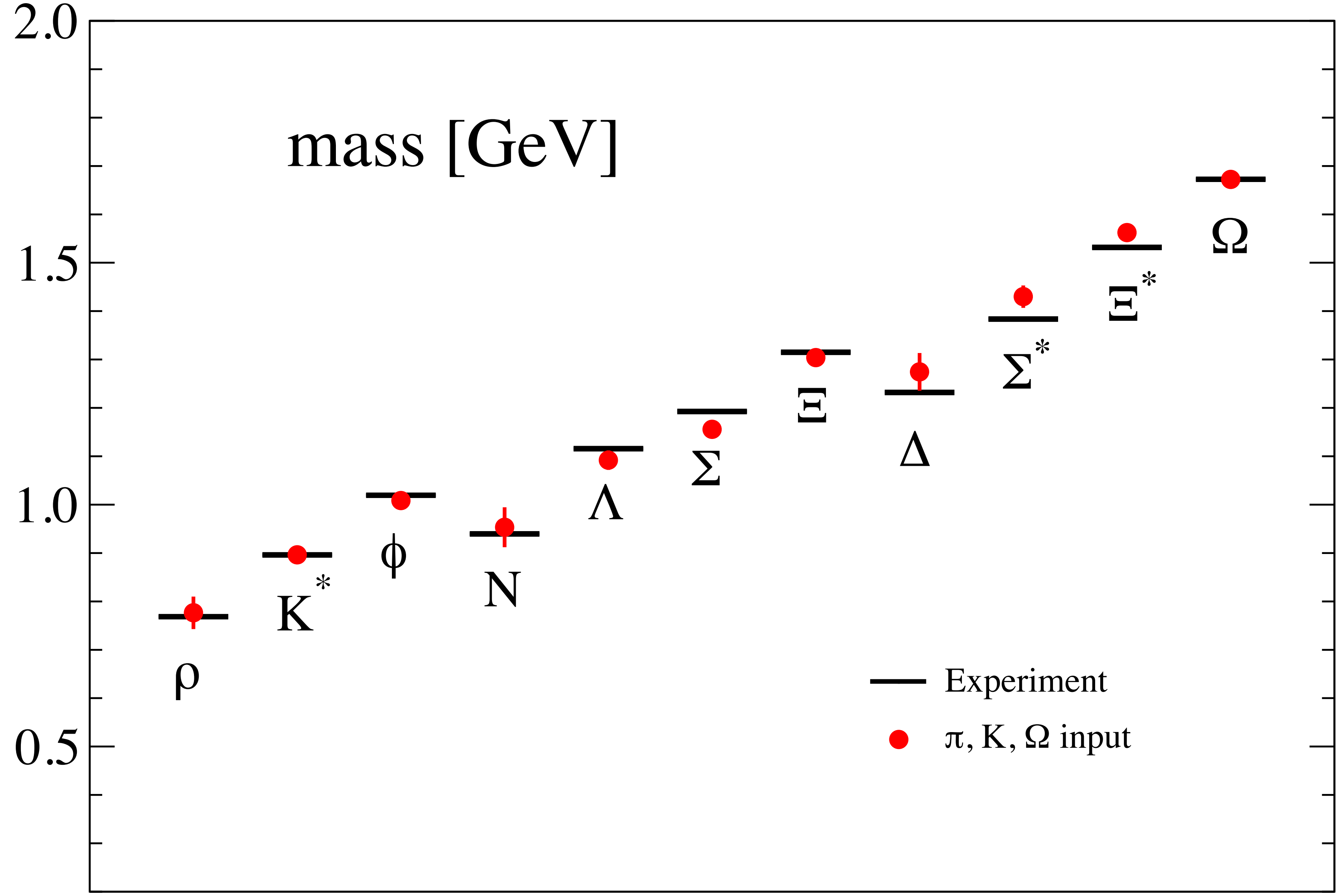
matching

$$\alpha_s(M_Z) = 0.12047(81)(48)(^{+0}_{-173}),$$

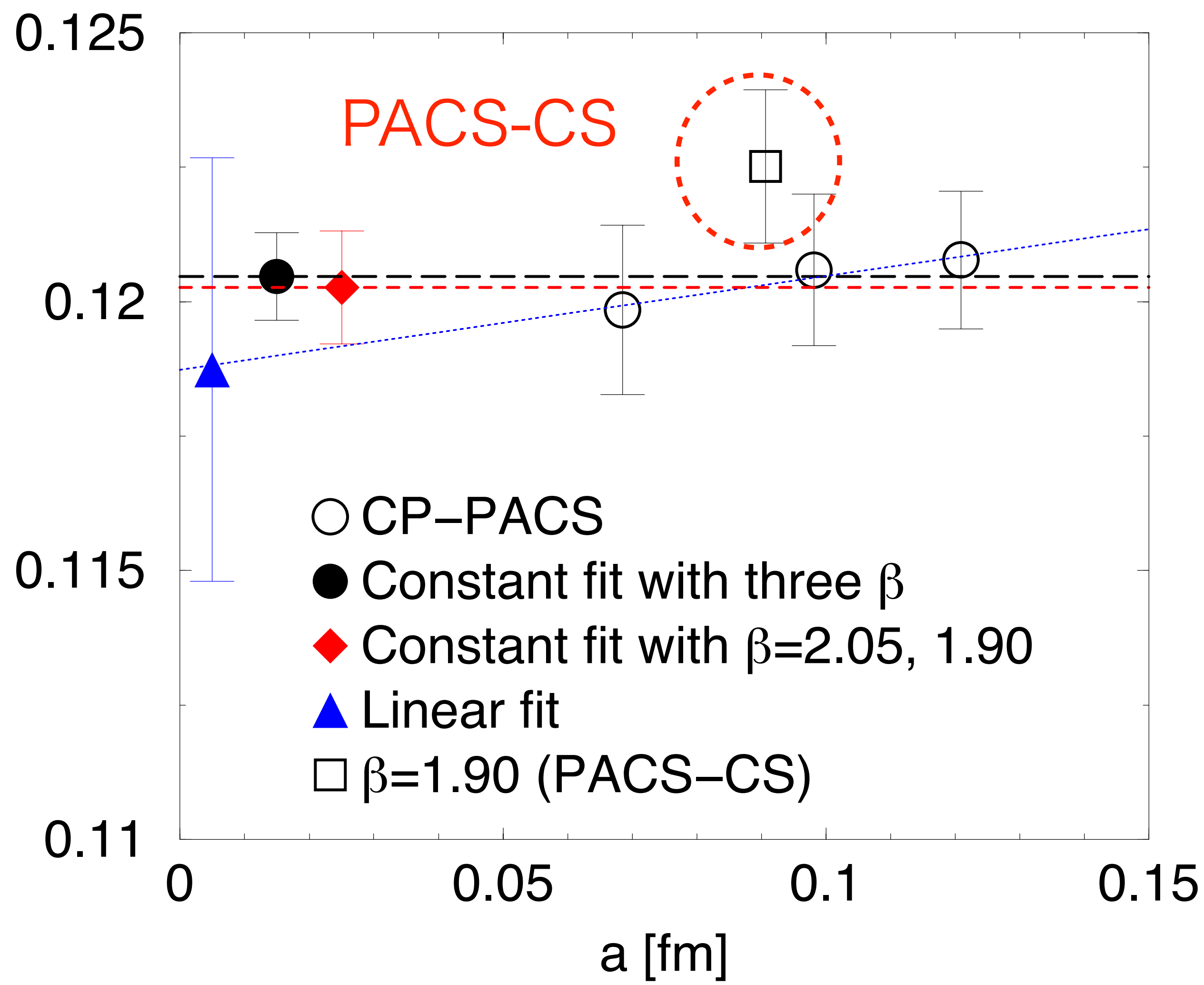
a->0

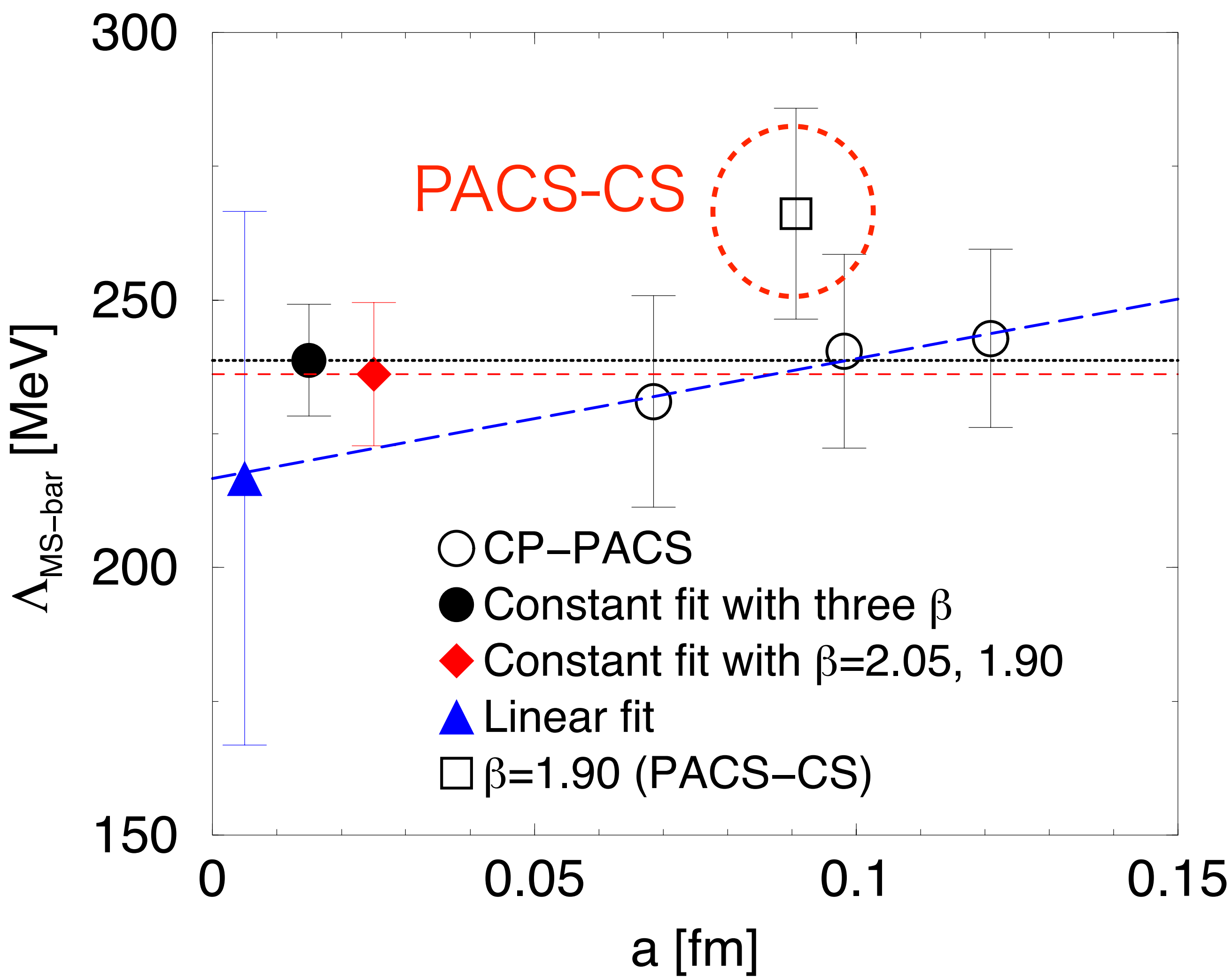
$$\Lambda^{(5)}_{\overline{\rm MS}} = 239(10)(6)(^{+0}_{-22})\ \mathrm{MeV},$$

The latest results in 2+1 full QCD from PACS-CS Collaboration



$a \simeq 0.09\,\mathrm{fm}$ $m_\pi^{\mathrm{min.}} \simeq 156\,\mathrm{MeV}$





$$\alpha_s(M_Z) = 0.1225(14)(5), \quad \Lambda_{\overline{\mathrm{MS}}}^{(5)} = 266(20)(7)\ \mathrm{MeV}$$

No continuum limit (a=0.09 fm)