UNBIASED α_s FROM GLOBAL FITS:
THE NNPDF APPROACH

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UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

WORKSHOP ON PRECISION MEASUREMENTS OF α_s

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α_s FROM GLOBAL PDF FITS

- THE GOOD: LOTS OF INFORMATION COMBINED $\Rightarrow$ SMALL STATISTICAL UNCERTAINTY
- THE BAD: DEPENDENCE ON PDFS $\Rightarrow$ POTENTIALLY LARGE THEORETICAL BIAS

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1ST EXAMPLE

THE GOTTFRIED SUM RULE

- $S_G(Q^2) = \int \frac{dx}{x} \left(F_2^P(x, Q^2) - F_2^n(x, Q^2)\right)$

- NMC DETERMINES $S_G(0.004 < x < 0.8, 4\ \mathrm{GeV}^2) = 0.228 \pm 0.020$
- A NEURAL NETWORK FIT TO NMC+BCDMS STRUCTURE FUNCTION DATA GETS $S_G(0.004 < x < 0.8, 4\ \mathrm{GeV}^2) = 0.228 \pm 0.044$ (Abbate, S.F., 2005)
- DIFFERENCE IN UNCERTAINTY COMES ENTIRELY FROM SMALLEST x BIN
  $\Rightarrow$ VERY NONLINEAR SHAPE OF STR FCTN AT SMALL x POSSIBLE

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2ND EXAMPLE

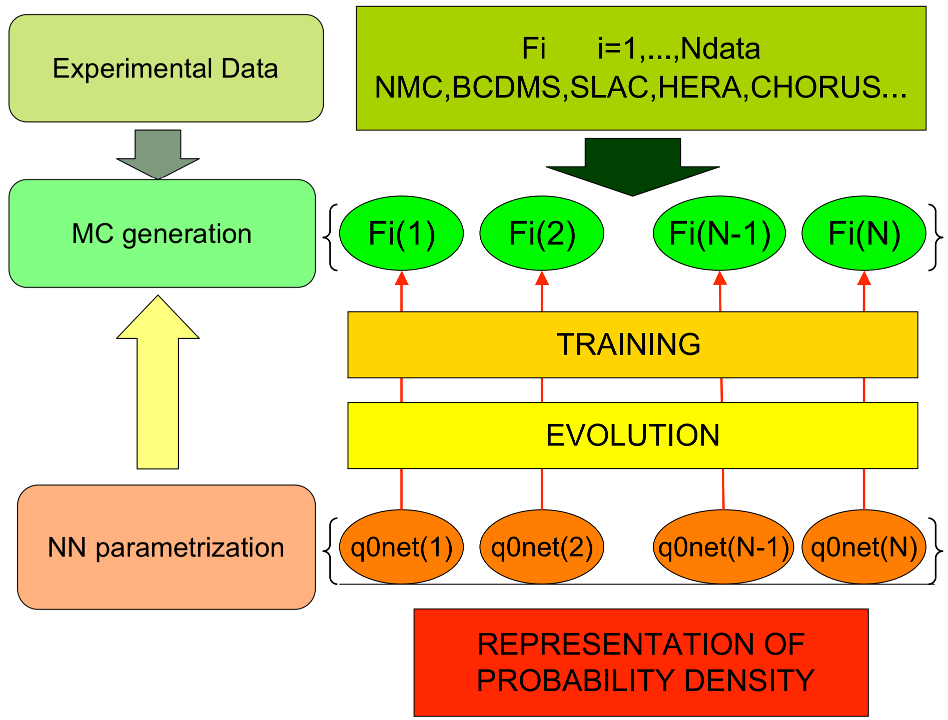
α_s FROM BCDMS & NMC DATA

- CAN DETERMINE NLO α_s FROM SCALING VIOLATIONS OF NONSINGLET TRUNCATED (MEASURED) MOMENTS
  $\Rightarrow$ NO DEPENDENCE ON PDFs OR EXTRAPOLATION
- $\alpha_s(M_Z) = 0.124\ ^{+0.005}_{-0.008}$ (s.f., Latorre, Magnea, Piccione, 2002)
- BCDMS FIND $\alpha_s(M_Z) = 0.113 \pm 0.005$ (Virchaux, Milsztajn, 1992)
  (NMC FIND $\alpha_s(M_Z) = 0.117\ ^{+0.011}_{-0.016}$ (Arneodo et al., 1993))
- BCDMS (& NMC) RESULTS BASED ON SIMULTANEOUS FIT OF α_s & PDFs
  $\Rightarrow$ DIFFERENCE LIKELY DUE TO PDF DEPENDENCE

UNBIASED PDF DETERMINATION: THE NNPDF APPROACH

BASIC IDEA: MONTE CARLO SAMPLING
OF THE PROBABILITY MEASURE IN THE (FUNCTION) SPACE OF PDFs

- START FROM MONTE CARLO SAMPLING OF DATA SPACE
- EACH PDF$\leftrightarrow$ NEURAL NETWORK PARAMETRIZED BY 37 PARAMETERS (NNPDF: $37 \otimes 7 = 259$ PARMS)
  "INFINITE" NUMBER OF PARAMETERS$\Rightarrow$ CAN REPRESENT ANY FUNCTION
- FIT STOPS WHEN QUALITY OF FIT TO RANDOMLY SELECTED "VALIDATION" DATA (NOT FITTED) STOPS IMPROVING



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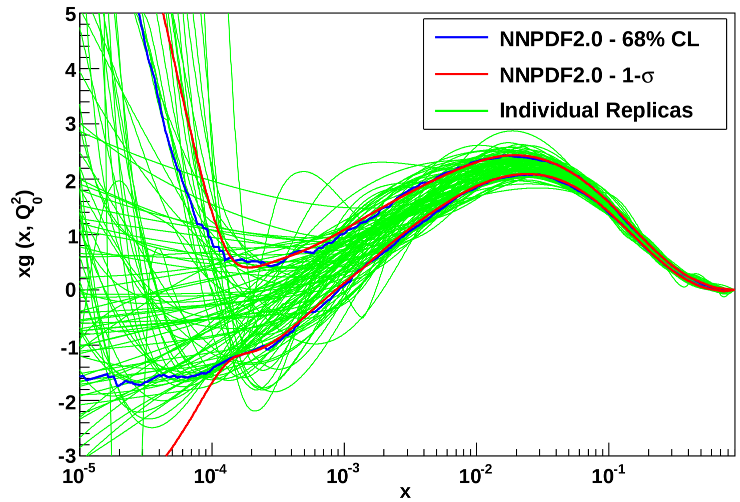
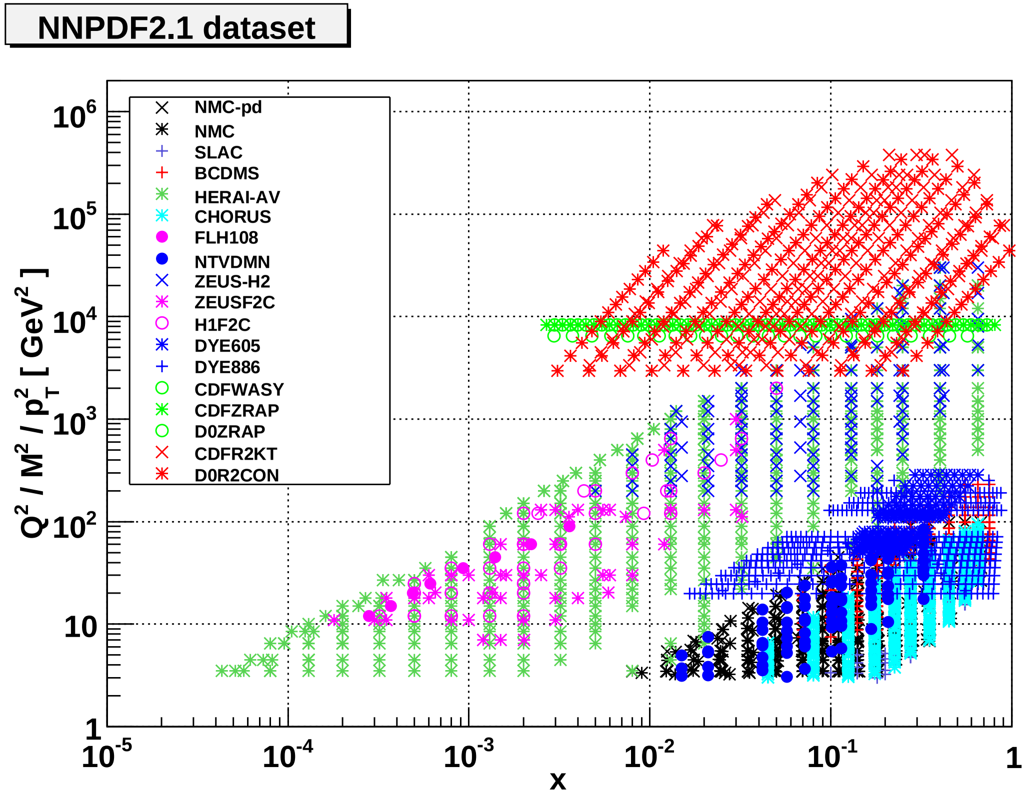
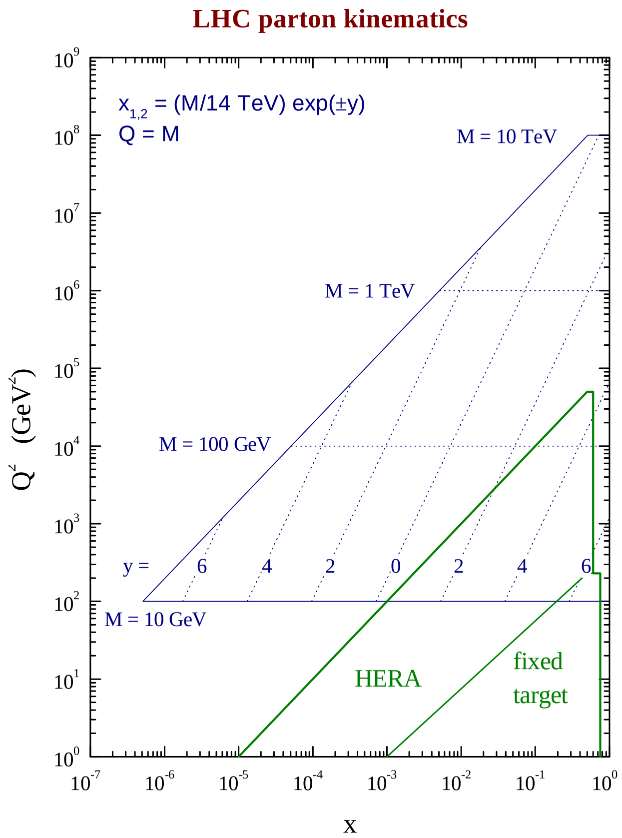
CAN DETERMINE BOTH 68C.L.& 1-σ



THE NNPDF2.1 PDF DETERMINATION





- GLOBAL PDF FIT, INCLUDES
  - DIS: NEUTRAL AND CHARGED CURRENT, CHARGED LEPTON AND NEUTRINO BEAMS, INCLUSIVE AND CHARM-TAGGED
  - DRELL-YAN: FIXED TARGET AND COLLIDER, NEUTRAL γ^* AND Z AND CHARGED CURRENT W PRODUCTION
  - INCLUSIVE JETS
- NLO QCD, HQ MASSES INCLUDED TO $O(\alpha_s)$
- 7 PDFS PARAMETRIZED INDEPENDENTLY, HQ GENERATED DYNAMICALLY
- SETS WITH DIFFERENT VALUES OF α_s, m_c, m_b AVAILABLE

DETERMINING α_s FROM NNPDF2.1

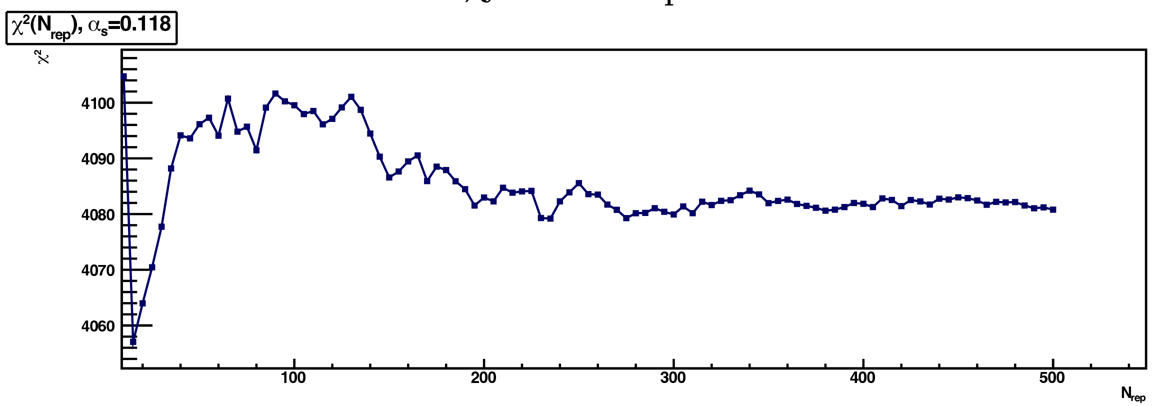
ADVANTAGES

- CAN CHECK STATISTICAL FEATURES OF RESULTS:
  - INDEP. OF PARAMETRIZATION $\Rightarrow$ ABSENCE OF BIAS
  - DEPENDENCE OF UNCERTAINTIES ON SIZE OF MC SAMPLE $\Rightarrow$ ERROR AND ERROR ON THE ERROR
  - BAYESIAN IMPACT OF NEW DATA ON FIT $\Rightarrow$ CONSISTENCY WITH STAT INFERENCE
- CAN STUDY IMPACT OF INDIVIDUAL DATA ON RESULTS

CAVEATS

- χ^2 IS A RANDOM VARIABLE $\Rightarrow$ FLUCTUATES FOR FINITE SAMPLE SIZE $\Delta\chi^2 \sim \frac{\sqrt{N_{\rm dat}}}{N_{\rm rep}}$ $\Rightarrow$ ADDITIONAL UNCERTAINTY DUE TO FINITE-SIZE FLUCTUATIONS
- χ^2 OF AVERAGE ALWAYS LOWER THAN AVERAGE OF χ^2 (FIT LEARNS UNDERLYING LAW) $\Rightarrow$ FLUCTUATIONS OF χ^2 SUPERPOSED TO DECREASE OF AVERAGE

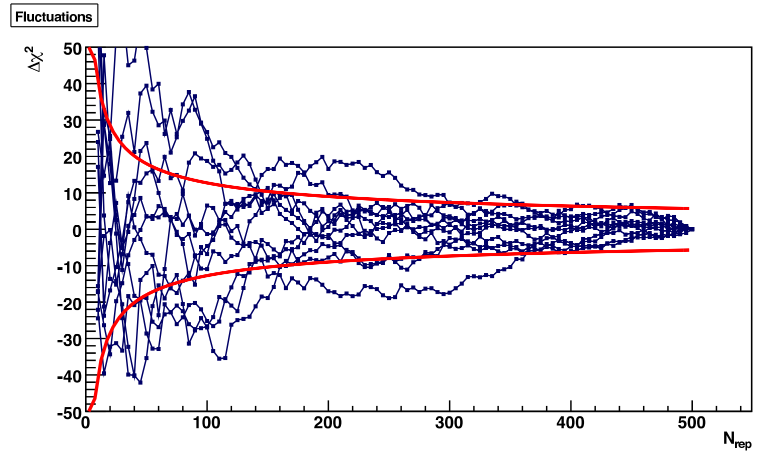
χ^2 VS $N_{\rm rep}$



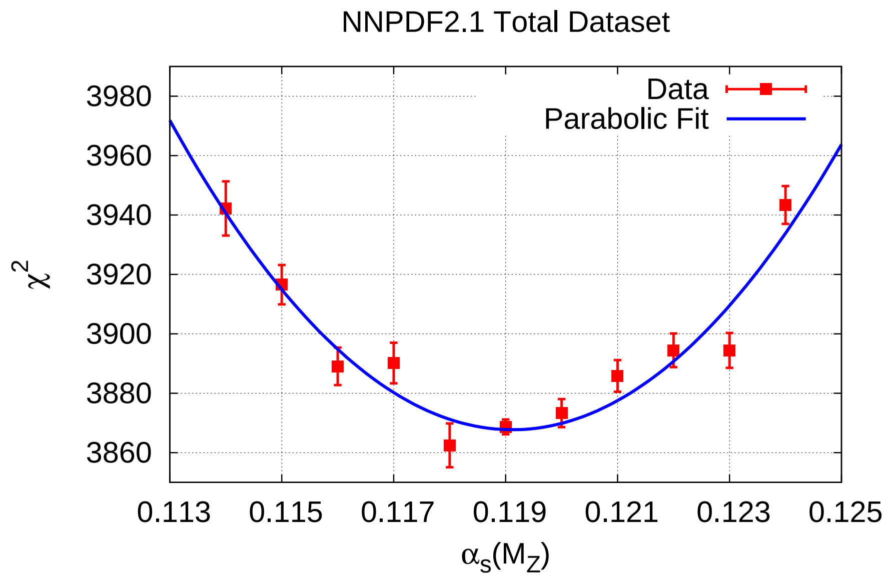
THE PROCEDURE

- PRODUCE $N_{\rm rep}$ REPLICAS FOR A RANGE OF VALUES OF α_s & DETERMINED THE χ^2
- DETERMINE THE STATISTICAL FINITE-SIZE UNCERTAINTY ON EACH χ^2 VALUE (E.G. BY BOOTSTRAP)
- PERFORM A PARABOLIC FIT TO χ^2 PROFILE, POSSIBLY DISCARD OUTER NON-PARABOLIC POINTS BASED ON FIT QUALITY
- DETERMINE α_s AND STATISTICAL UNCERTAINTY BY STANDARD $\Delta\chi^2 = 1$ ABOUT THE MINIMUM
- DETERMINE FURTHER PROCEDURAL UNCERTAINTY DUE TO UNCERTAINTY ON BEST-FIT PARMS $\Rightarrow$ REPEAT FOR LARGER $N_{\rm rep}$ UNTIL PROCEDURAL UNCERTAINTY NEGLIGIBLE COMPARED TO STATISTICAL

χ^2 FLUCTUATIONS VS. $N_{\rm rep}$



χ^2 PROFILE VS. α_s

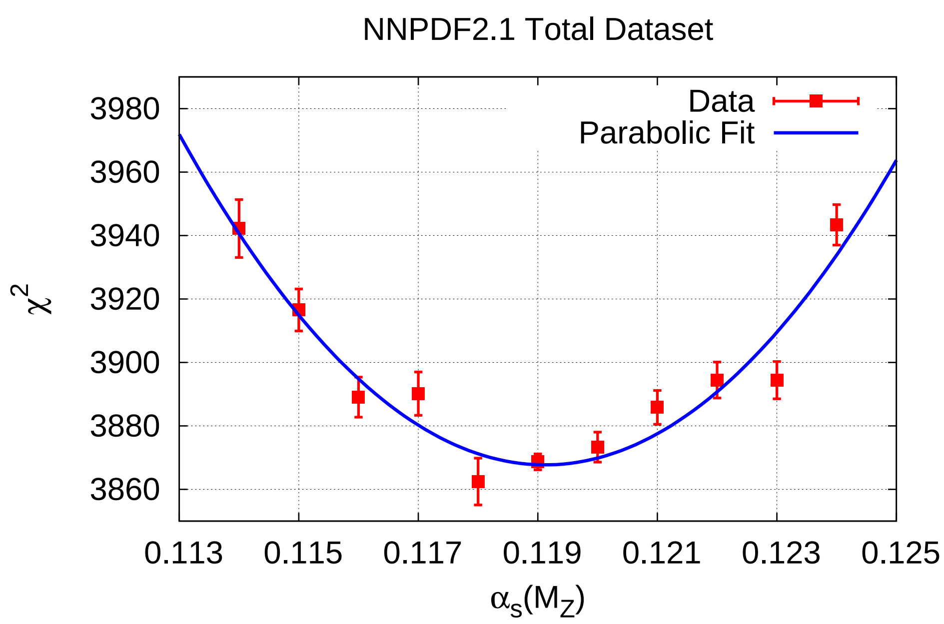


THE RESULT: GLOBAL FIT

NLO NNPDF2.1 GLOBAL DETERMINATION (ONLY STAT. ERROR KNOWN)

$N_{\mathrm{rep}} = 300$ PER VALUE OF α_s

$$\alpha_s(M_z) = 0.1191 \pm 0.0006(\mathrm{stat.}) \pm 0.0001(\mathrm{proc.}) = 0.1191 \pm 0.0006(\mathrm{stat.})$$



$\chi^2/\mathrm{d.o.f.} = 1.4$ for the parabolic fit

THE RESULT: DIS ONLY

DO DIS DATA PREFER A SMALLER VALUE?

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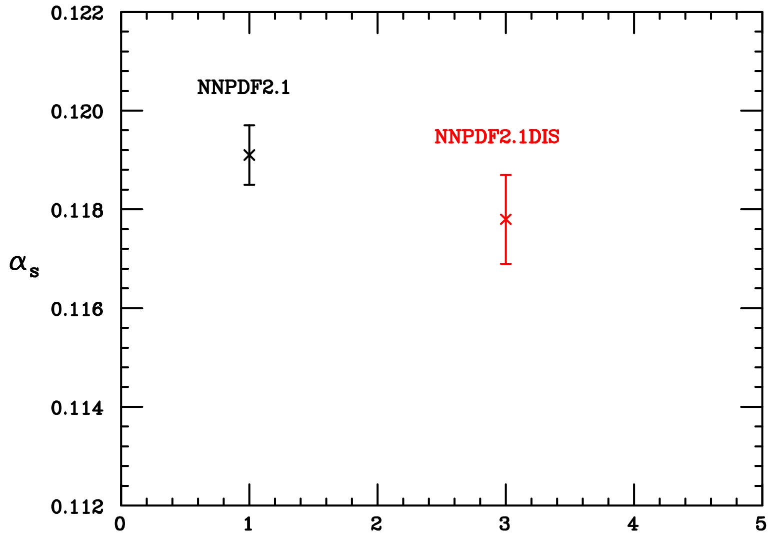
DO DIS DATA PREFER A SMALLER VALUE?

NLO NNPDF2.1 DEEP-INELASTIC DATA (ONLY STAT. ERROR KNOWN)

$N_{\mathrm{rep}} = 500$ PER VALUE OF α_s

$$\alpha_s(M_z) = 0.1178 \pm 0.0009(\mathrm{stat.})$$



- YES
- BUT NOT MUCH SMALLER & WITH LARGER UNCERTAINTY (COMPATIBLE WITHIN UNCERTAINTIES, AS IT OUGHT TO)

THE RESULTS: WHAT ABOUT THEORETICAL UNCERTAINTIES?

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HEAVY QUARKS

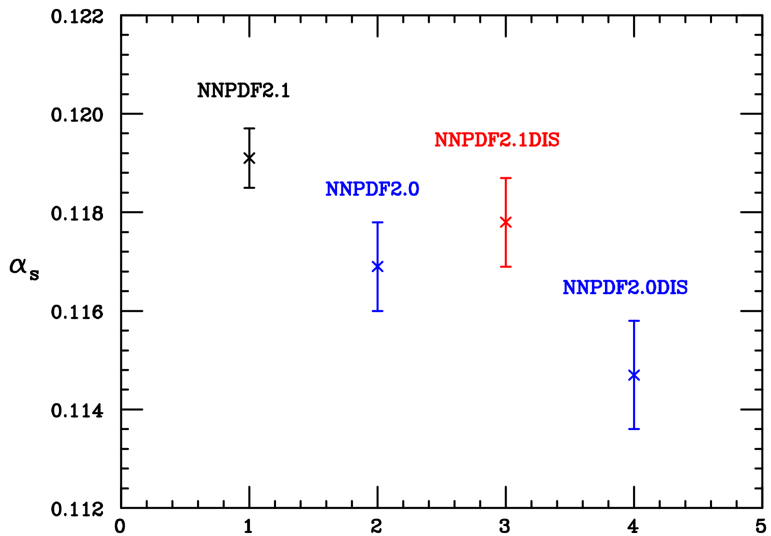
NEGLECT ALL $O(m_c)$ & $O(m_b)$ CORRECTIONS ⇒ ZERO-MASS SCHEME (NNPDF2.0)

NLO NNPDF2.0 GLOBAL

$$\alpha_s(M_z) = 0.1169 \pm 0.0009(\text{stat.})$$

NLO NNPDF2.0 DEEP-INELASTIC

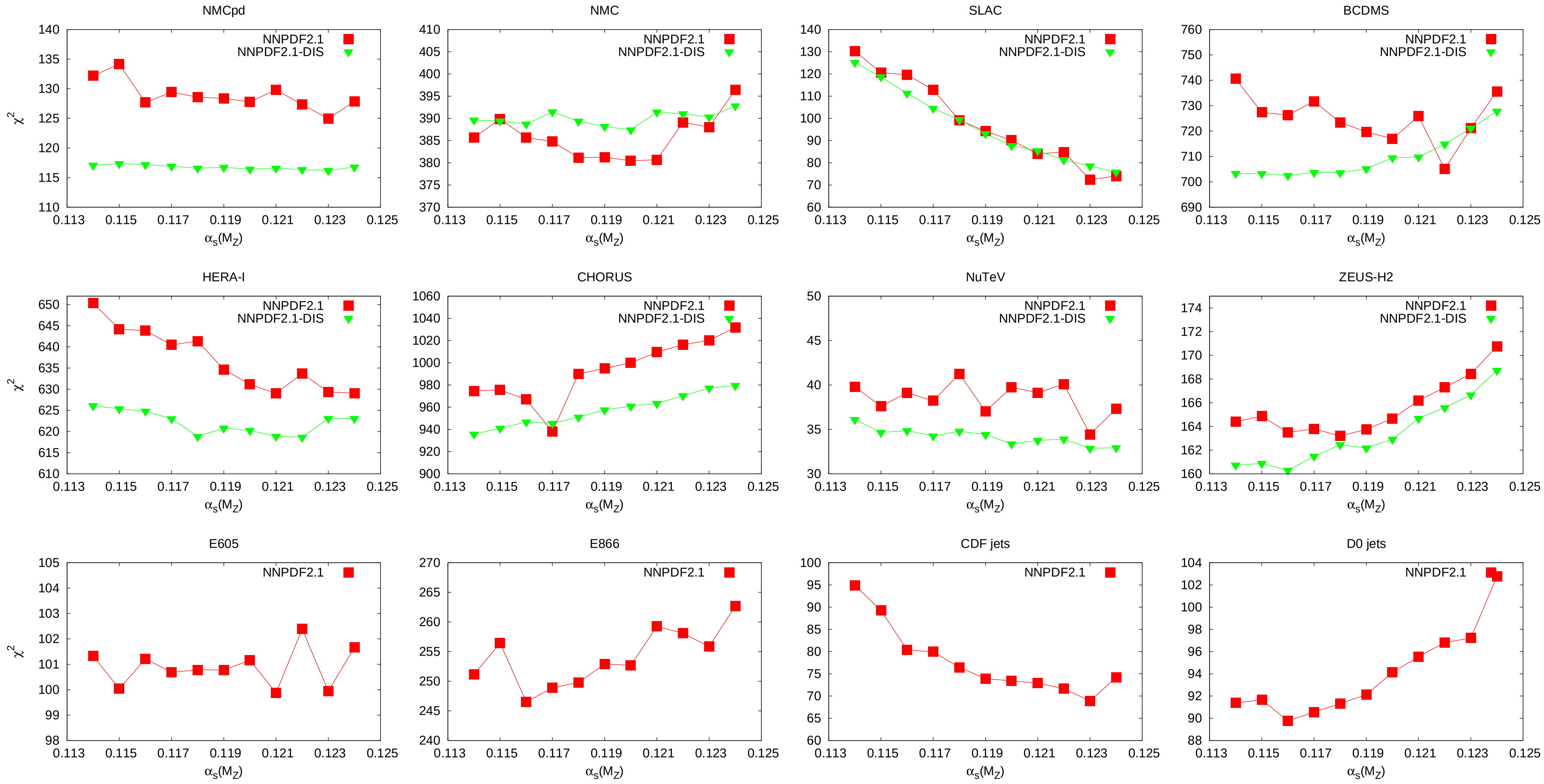
$$\alpha_s(M_z) = 0.1147 \pm 0.0011(\text{stat.})$$



- EFFECT RATHER LARGER THAN STATISTICAL UNCERTAINTY
  note old BCDMS & NMC determinations performed in this scheme
- OTHER TH. UNCERTAINTIES:
  - VALUES OF HQ MASSES
  - HIGHER ORDERS (SCALE VARIATION)
- TH. UNCERTAINTY LIKELY DOMINANT

INDIVIDUAL EXPERIMENTS: WHAT'S GOING ON?

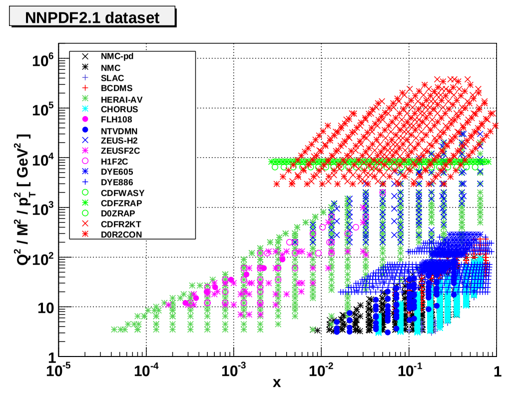
χ^2 PROFILE FOR INDIVIDUAL EXPERIMENTS IN GLOBAL & DIS FITS



- BCDMS SIMILAR TO HERA, NMC PRETTY FLAT
- DIS EXPERIMENTS (BCDMS+HERA) IN DIS FIT HAVE A "RUNAWAY DIRECTION" AT SMALL α_s,
  ABSENT IN GLOBAL FIT
- JET EXPERIMENTS EXCLUDE SMALL α_s

IS THERE A PROBLEM WITH DIS?

CONJECTURE



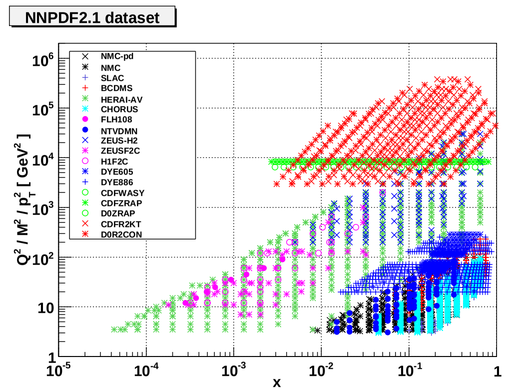
- IN DIS, GLUON DETERMINED BY SCALING VIOLATIONS $\Rightarrow$ CAN COMPENSATE SMALLER α_s WITH LARGER GLUON (OR CONVERSELY)
  $\Rightarrow$ RUNAWAY DIRECTIONS POSSIBLE IN DIS FIT
- JET DATA ARE AT MUCH LARGER SCALE, IF α_s TOO SMALL OR LARGE HIGH-SCALE GLUON WILL COME OUT WRONG
  $\Rightarrow$ RUNAWAY DIRECTION QUENCHED IN GLOBAL FIT
  χ^2-PDF CORRELN.

THE EVIDENCE

- COMPUTE THE CORRELATION BETWEEN χ^2 & PDFs AT THE GLOBAL BEST FIT
  LARGE (POSITIVE OR NEGATIVE) CORRN. $\Leftrightarrow$ RUNAWAY DIRECTION
- OPPOSITE SIGN CORRELATIONS $\Leftrightarrow$ DATA PULLING IN OPPOSITE DIRECTIONS

IS THERE A PROBLEM WITH DIS?

CONJECTURE



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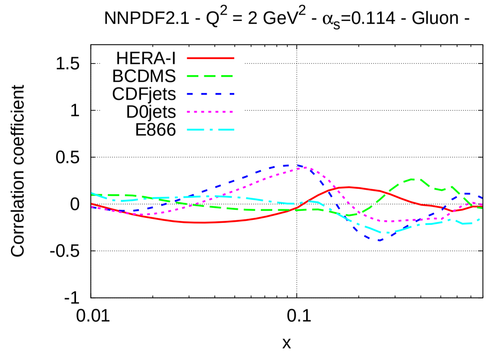
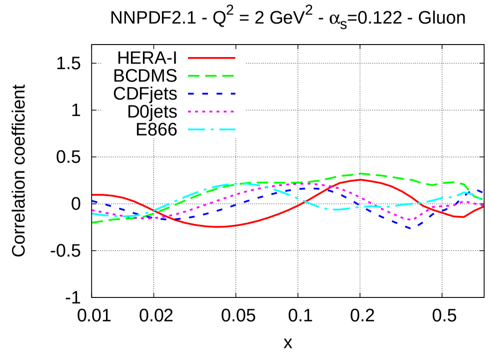
SMALL α_s



LARGE α_s



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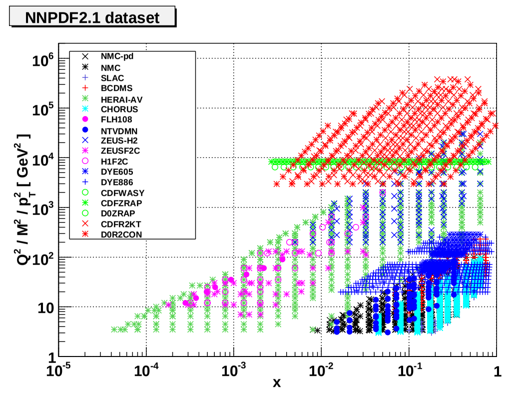
- OPPOSITE SIGN CORRELATIONS $\Leftrightarrow$ DATA PULLING IN OPPOSITE DIRECTIONS
- AT LOW $\alpha_s = 0.114$ DIS FIT HAS A RUNAWAY DIRECTION STABILIZED BY GLUON
- AT HIGH $\alpha_s = 0.122$ FIT IS GENERALLY STABLE, THOUGH STILL SOME PULL IN HERA LOW x

IS THERE A PROBLEM WITH DIS?

CONJECTURE



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χ^2-PDF CORRELN.

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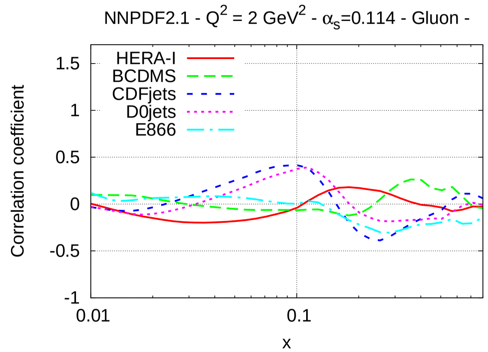
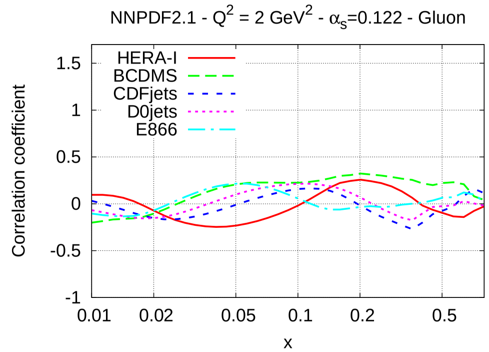


LARGE α_s



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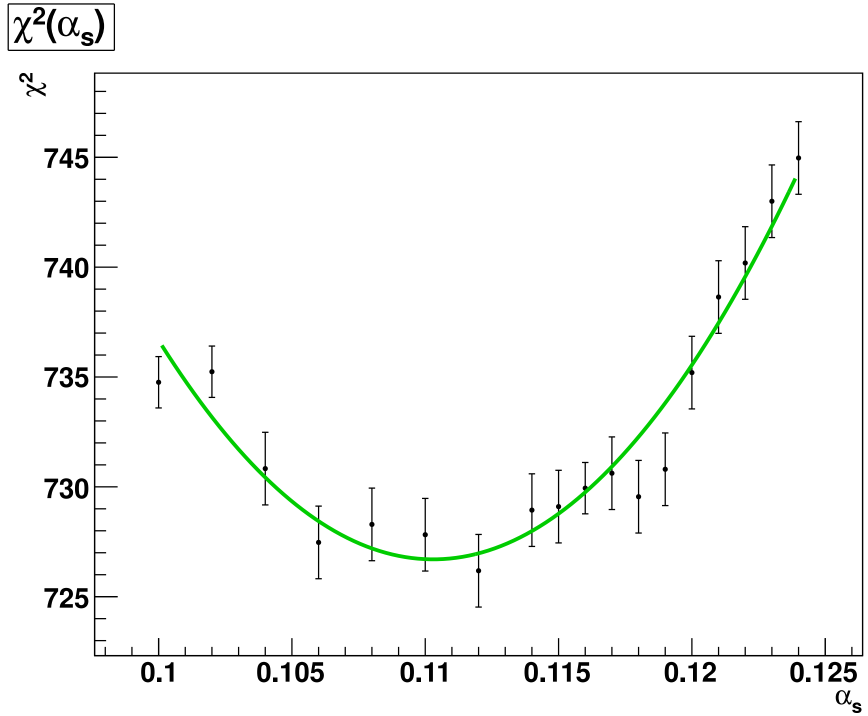
- NO PROBLEM WITH DIS DATA, PERHAPS SOME PROBLEM WITH HERA LOW x

ONE MORE RESULT: HERA DATA ONLY

NLO NNPDF2.1 HERA DATA ONLY

$N_{\rm rep} = 500/1000$ PER VALUE OF α_s

$$\alpha_s(M_z) = 0.1103 \pm 0.0033(\text{stat.}) \pm 0.0003(\text{proc.}) = 0.110 \pm 0.003(\text{stat.})$$

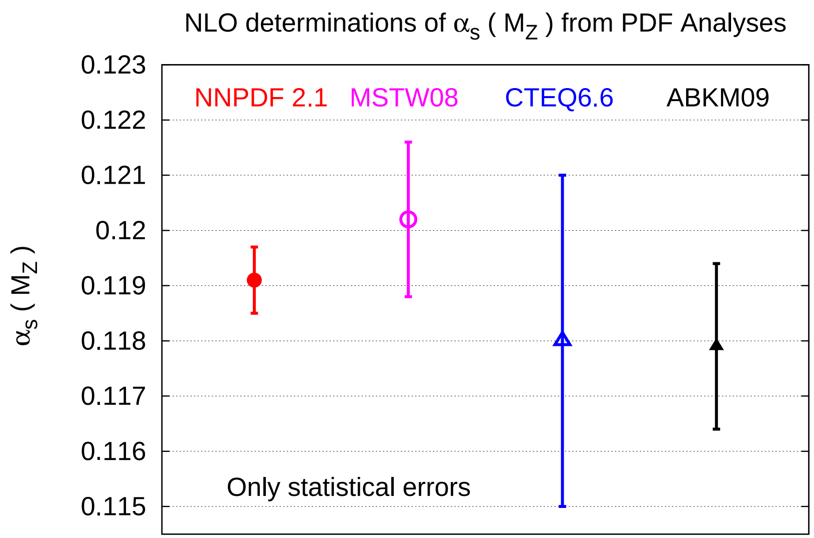


χ^2/d.o.f/ = 1.1 for the parabolic fit

- HERA DATA FAVOR LOWER α_s WITH LARGER UNCERTAINTY
- LIKELY CONSISTENT WITH GLOBAL WITHIN TOTAL UNCERTAINTY
- POSSIBLE ISSUES WITH INADEQUATE NLO THEORY FOR SMALL x HERA DATA? (Caola, s.f., Rojo, 2010, HERAPDF)
  WOULD BE WORSE AT NNLO

CONCLUSIONS

α_s FROM NLO PDF FITS



NNPDF METHODOLOGY LEADS TO DETERMINATION OF α_s WITH

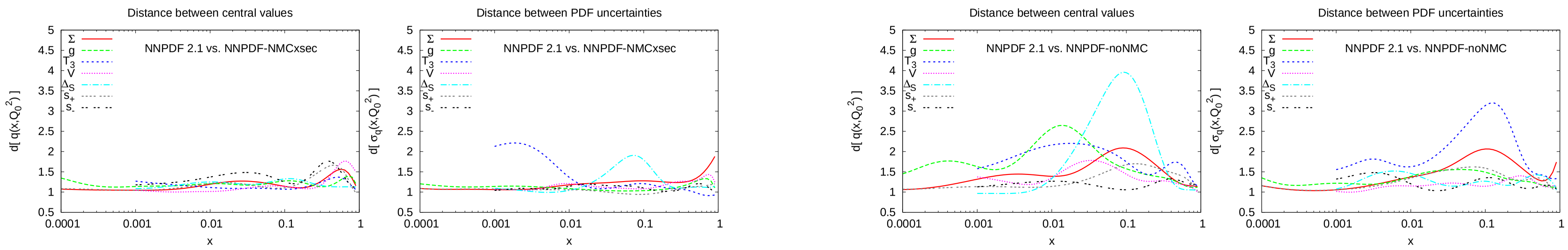
- UNUSUALLY SMALL STATISTICAL UNCERTAINTY
- SMALL ("NO") PARAMETRIZATION BIAS
- POSSIBILITY TO STUDY IMPACT OF INDIVIDUAL DATA:
  - DIS AND HADRON COLLIDER VALUES CONSISTENT
  - NO PROBLEM WITH BCDMS & NMC
  - HERA DATA (LOW x?) PREFER LOWER CENTRAL VALUE

EXTRAS

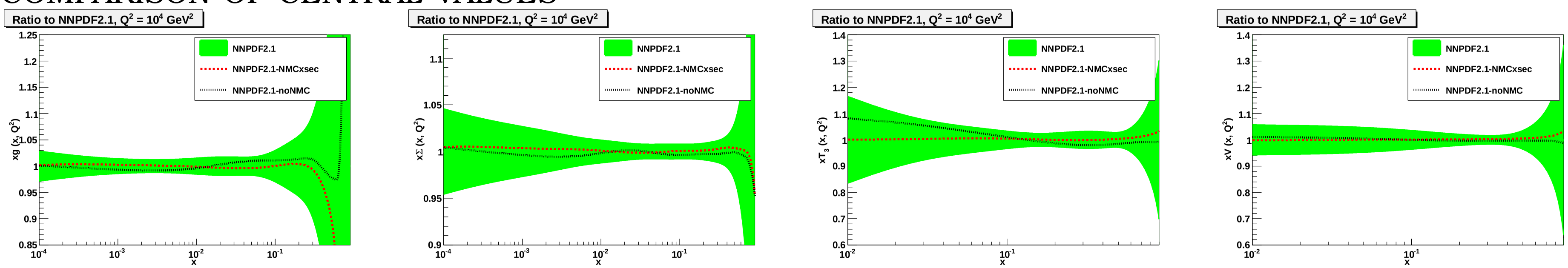
IMPACT OF NMC DATA ON NNPDF2.1

- REPEAT NNPDF2.1 WITH DIFFERENT TREATMENT OF NMC DATA (XSECT INSTEAD OF STRUCTURE FUNCTION)
- REPEAT NNPDF2.1 WITHOUT NMC DATA

STATISTICAL DISTANCES



COMPARISON OF CENTRAL VALUES



- XSECT VS STR FCTN $\Rightarrow$ STATISTICALLY ALMOST UNDISTINGUISHABLE
- NO NMC $\Rightarrow$ ALMOST NO EFFECT ON CENTRAL VALUE, SLIGHT INCREASE IN UNCERTAINTIES

Monte Carlo data generation

- BCDMS+ NMC proton & deuteron F_2 data (full correlated systematics available), taken at 4 beam energies

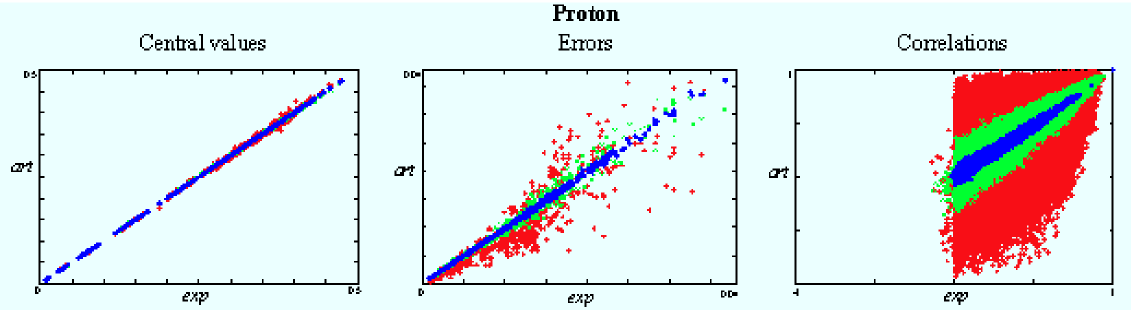
- on top of stat. errors, 4 systematics + 1 normalization (NMC) or 6 systematics + 1 absolute & 2 relative normalizations (BCDMS), with various forms of correlation (full, or for each target, or for each beam energy)

Generate data according to a multigaussian distribution

$$F_i^{(art)\,(k)} =$$

$$(1 + r_5^{(k)}\,\sigma_N)\sqrt{1 + r_{i,6}^{(k)}\,\sigma_{N_t}}\sqrt{1 + r_{i,7}^{(k)}\,\sigma_{N_b}}\left[F_i^{(exp)} + \frac{r_{i,1}^{(k)}\,f_b + r_{i,2}^{(k)}\,f_{i,s} + r_{i,3}^{(k)}\,f_{i,r}}{100}F_i^{(exp)} + r_{i,s}^{(k)}\,\sigma_s^i\right]$$

r univariate gaussian random nos., one $r_{i,s}$ for each data, but single $r_{i,j}$ for all correlated data

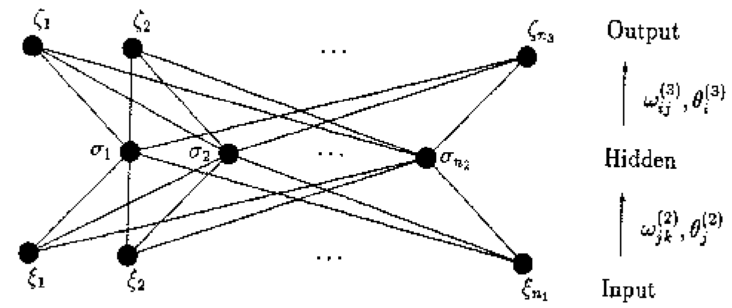


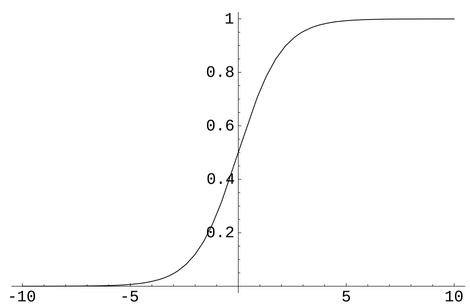
SCATTER PLOT ART. VS. EXP. FOR 10 (RED) 100 (GREEN) AND 1000 (BLUE) REPLICAS

NEED 1000 REPLICAS TO REPRODUCE CORRELATIONS TO PERCENT ACCURACY

NEURAL NETWORKS

A NONLINEAR FUNCTIONAL FORM





MULTILAYER FEED-FORWARD NETWORKS

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by weights and thresholds
$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right)$$
- Sigmoid activation function
$g(x) = \frac{1}{1+e^{-\beta x}}$

EXAMPLE: A 1-2-1 NN

$$f(x) = \cfrac{1}{1+e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}}}}$$

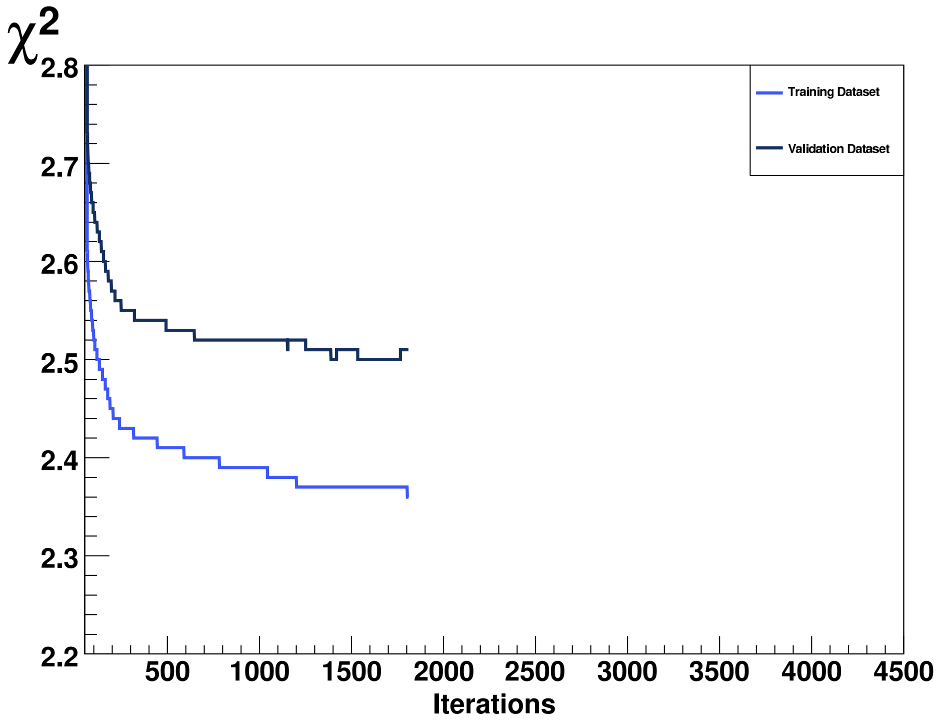
- ANY FUNCTION CAN BE REPRESENTED BY A SUFFICIENTLY BIG NEURAL NETWORK $\Rightarrow$ CAN CHECK INDEPENDECE OF SIZE
- MINIMZATION PERFORMED BY GENETIC ALGORITHM
- INFORMATION CONTAINED ("COARSENESS") IN NN INCREASES DURING MINIMIZATION

DATA MONTE CARLO $\Rightarrow$ PDF MONTE CARLO
CROSS-VALIDATION

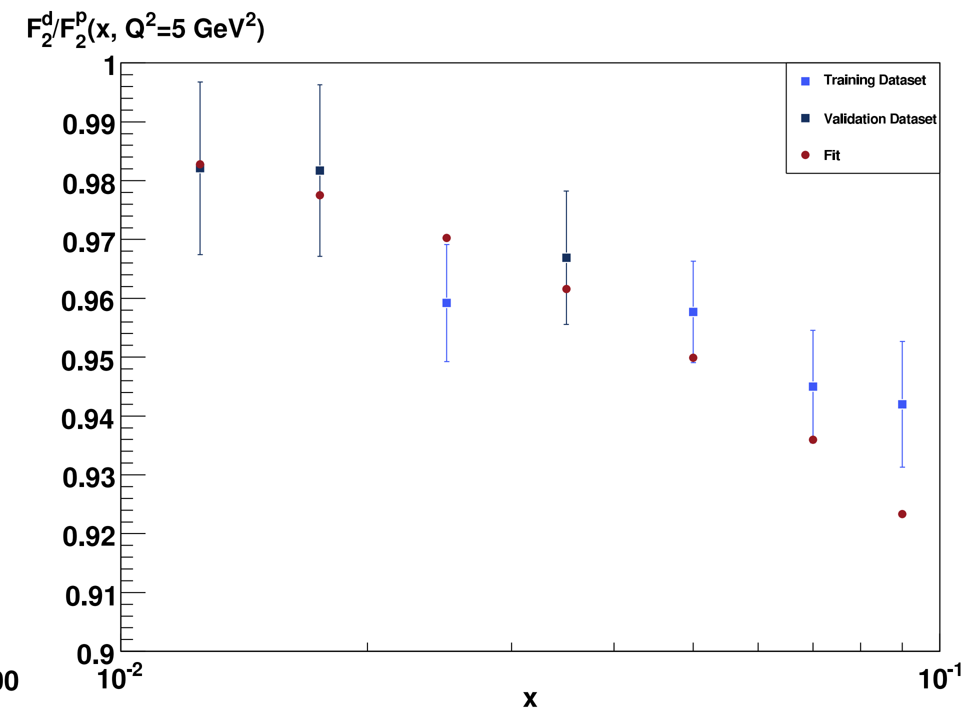
- REPLICAS ARE FITTED TO A DATA SUBSET
- A DIFFERENT SUBSET OF DATA USE FOR EACH REPLICA
- OPTIMAL FIT WHEN FIT TO VALIDATION (CONTROL) DATA STOPS IMPROVING
-

OPTIMAL FITTING

χ^2



FIT TO DATA

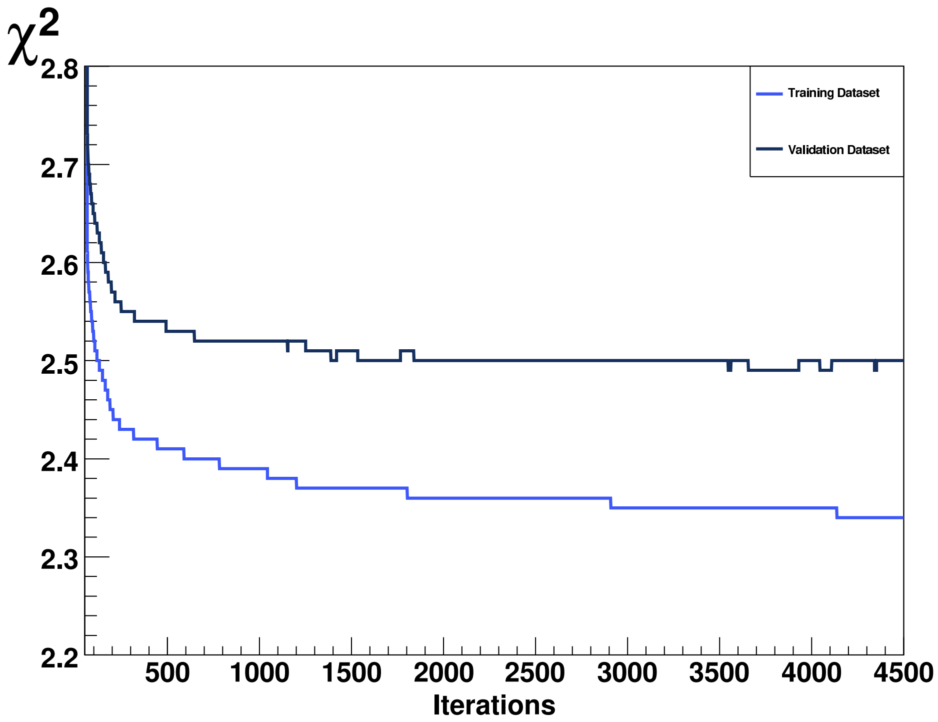


DATA MONTE CARLO ⇒ PDF MONTE CARLO
CROSS-VALIDATION

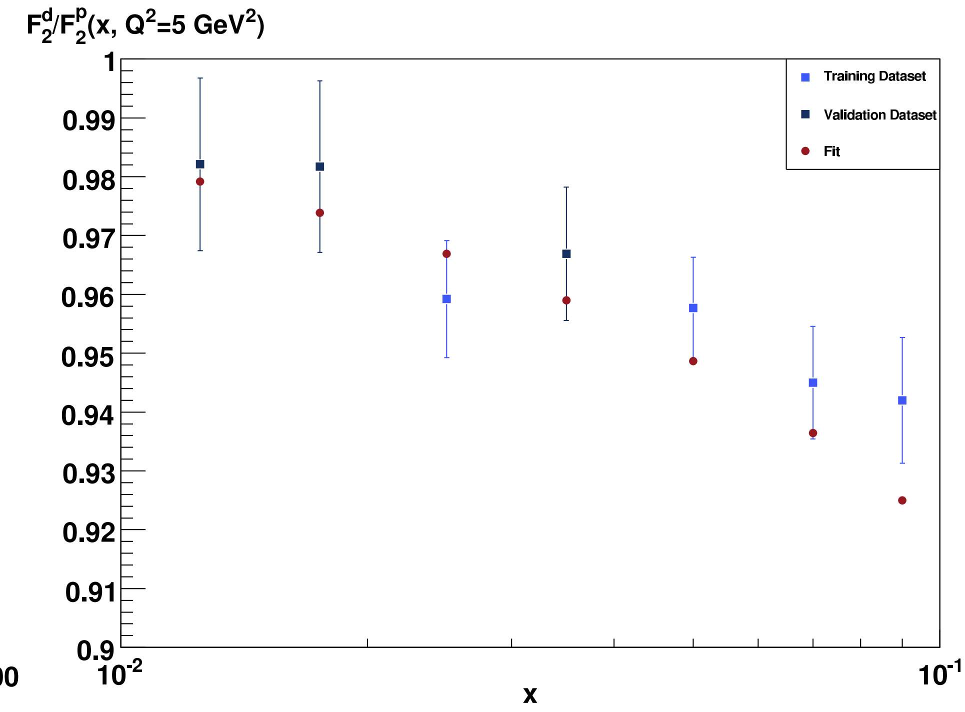
- REPLICAS ARE FITTED TO A DATA SUBSET
- A DIFFERENT SUBSET OF DATA USE FOR EACH REPLICA
- OPTIMAL FIT WHEN FIT TO VALIDATION (CONTROL) DATA STOPS IMPROVING
- THE BEST FIT IS NOT AT THE MINIMUM OF THE χ^2

OVERFITTING

χ^2



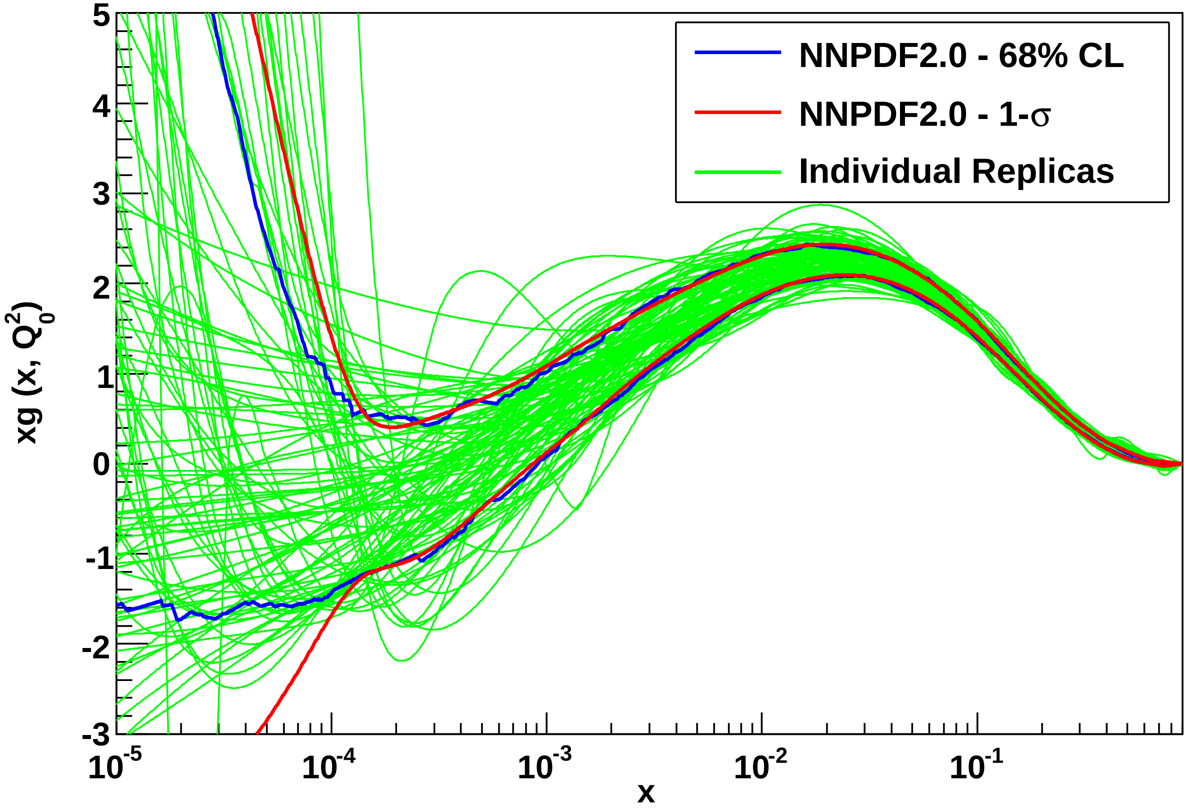
FIT TO DATA



LIKELIHOOD CONTOURS AND UNCERTAINTIES

NNPDF: ONE σ VS. CENTRAL 68% FOR THE MC DISTRIBUTION OF PDFS

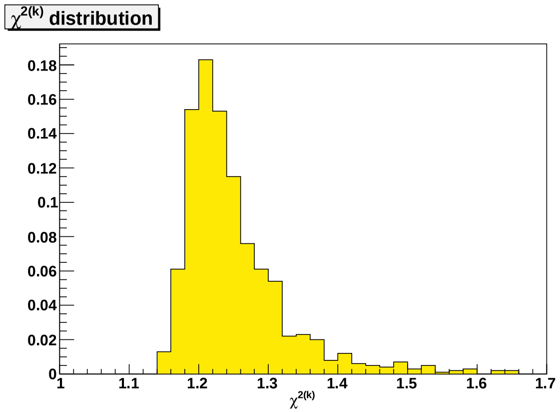
Example: the gluon distribution in the NNPDF2.0 set



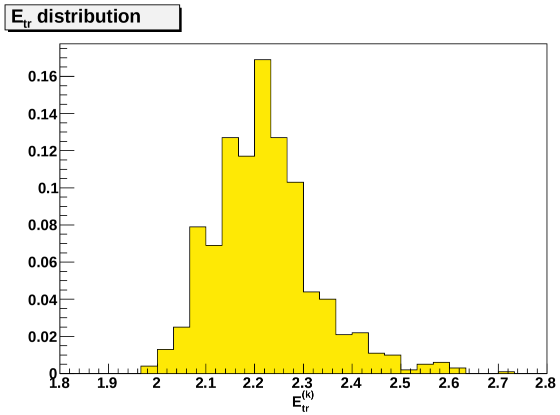
- ENSEMBLE OF REPLICAS $\leftrightarrow$ PROBABILITY DISTRIBUTION OF PDFS
- EXPECTED CENTRAL VALUE $\leftrightarrow$ MEAN; UNCERTAINTY $\leftrightarrow$ STANDARD DEVIATION
- ANY FEATURES OF DISTRIBUTION CAN BE DETERMINED (C.L., CORRELATIONS...)
- DISTRIBUTION NEED NOT BE GAUSSIAN $\rightarrow$ STANDARD DEVIATION $\neq$ 68% C.L. (GLUON $\Leftrightarrow$ STRUCTURE FUNCTION POSITIVITY CONSTRAINTS)

NNPDF2.1 STATISTICAL FEATURES

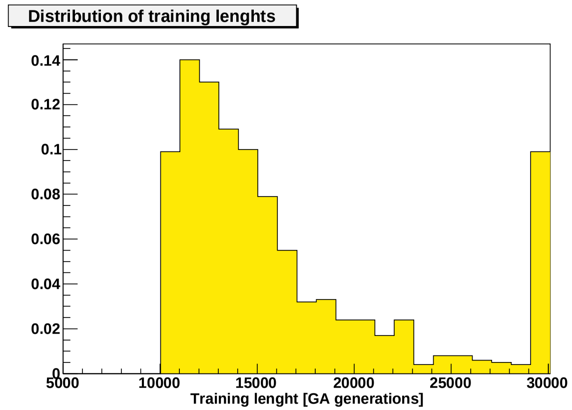
χ^2 TO DATA



χ^2 TO REPLICA



TRAINING LENGTHS



- χ^2 TO REPLICA PEAKED AROUND 2,
  χ^2 TO DATA PEAKED AROUND 1
- χ^2 OF AVERAGE SMALLER THAN AVERAGE OF χ^2
- AVERAGE UNCERTAINTY OF PREDICTION
  SMALLER THAN AVERAGE UNCERTAINTY ON DATA

$\Rightarrow$ FIT "LEARNS" UNDERLYING LAW

$\chi^2_{\rm tot}$	1.16
$\langle E\rangle \pm \sigma_E$	2.24 ± 0.09
$\langle E_{\rm tr}\rangle \pm \sigma_{E_{\rm tr}}$	2.22 ± 0.11
$\left\langle E_{\rm val}\right\rangle \pm \sigma_{E_{\rm val}}$	2.28 ± 0.12
$\langle {\rm TL}\rangle \pm \sigma_{\rm TL}$	$(1.6 \pm 0.6)\ 10^4$
$\left\langle \chi^{2(k)}\right\rangle \pm \sigma_{\chi^2}$	1.25 ± 0.09
$\left\langle \sigma^{(\rm exp)}\right\rangle_{\rm dat}$ (%)	11.3%
$\left\langle \sigma^{(\rm net)}\right\rangle_{\rm dat}$ (%)	4.4%
$\left\langle \rho^{(\rm exp)}\right\rangle_{\rm dat}$	0.18
$\left\langle \rho^{(\rm net)}\right\rangle_{\rm dat}$	0.56

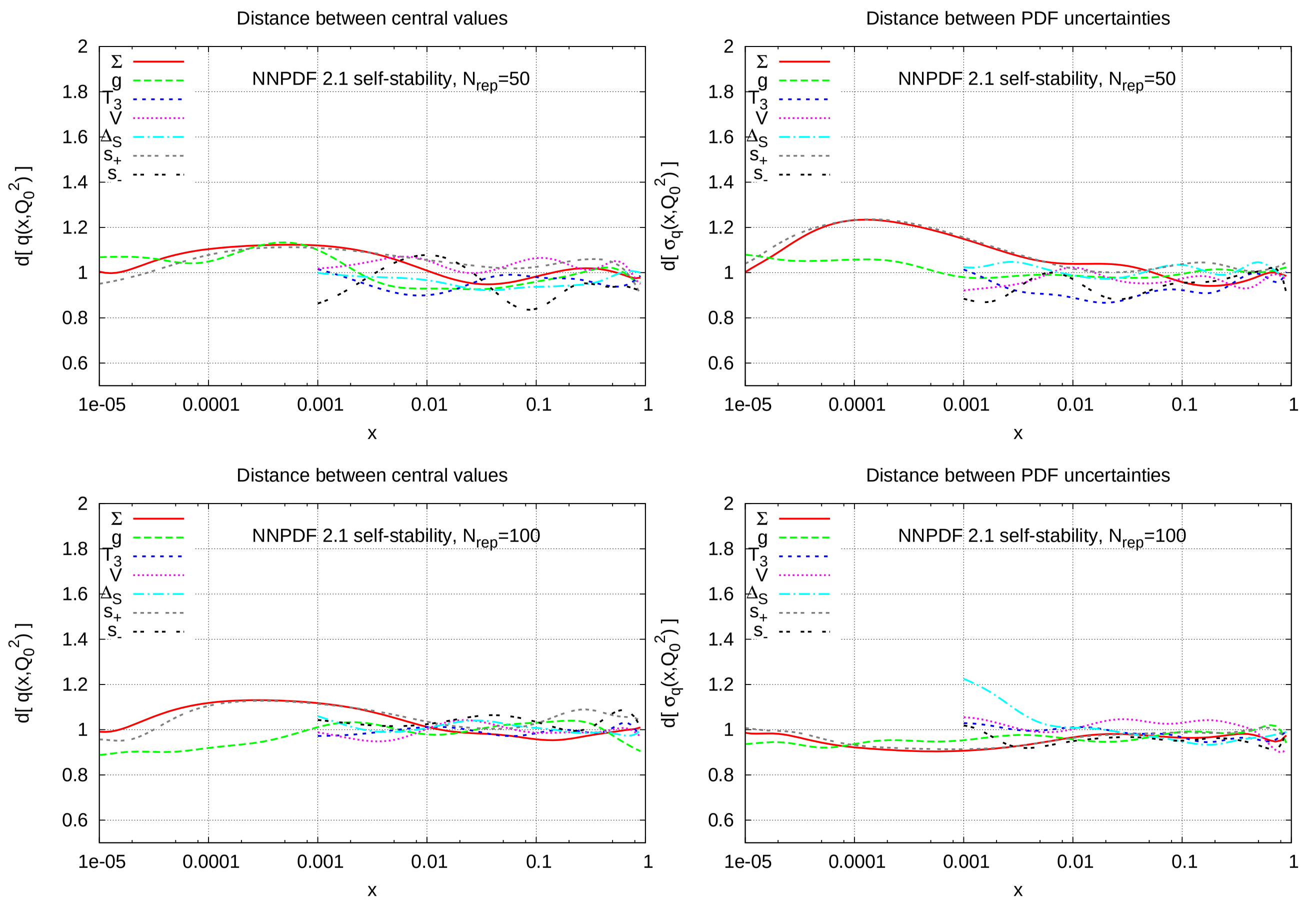
SCALING & STABILITY

- COMPARE RESULTS BETWEEN DIFFERENT SETS OF REPLICAS $\Rightarrow$ STATISTICALLY EQUIVALENT
- REPEAT COMPARISON FOR DIFFERENT $N_{\rm rep}$ $\Rightarrow$ FLUCTUATIONS SCALE WITH $N_{\rm rep}$!

DISTANCE

$$d^2\left(\langle q^{(1)}\rangle, \langle q^{(2)}\rangle\right) = \frac{\left(\langle q^{(1)}\rangle_{(1)} - \langle q^{(2)}\rangle_{(2)}\right)^2}{\sigma^2_{(1)}[\langle q^{(1)}\rangle] + \sigma^2_{(2)}[\langle q^{(2)}\rangle]} \quad \text{WITH } \sigma^2_{(i)}[\langle q^{(i)}\rangle] = \frac{1}{N^{(i)}_{\rm rep}}\sigma^2_{(i)}[q^{(i)}]$$

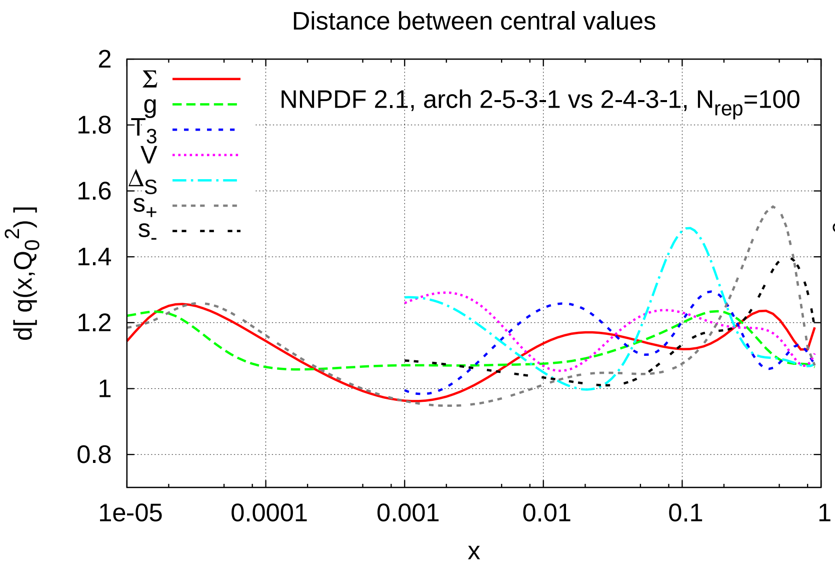
& similarly for uncertainties

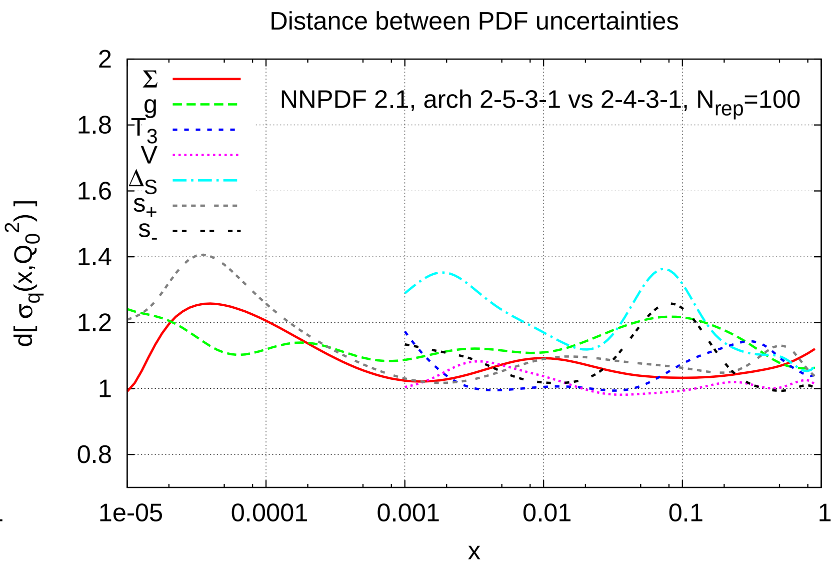


PARAMETRIZATION INDEPENDENCE

COMPARE RESULTS OBTAINED WITH DIFFERENT ARCHITECTURE

OF NEURAL NETWORK: 2-4-3-1 VS 2-5-3-1 (31 PARAMETERS VS 37)





STATISTICALLY EQUIVALENT!

DESPITE USING $6 \times 7 = 42$ LESS PARAMETERS

CONSISTENT INFORMATION PROCESSING

NEW DATA $\Rightarrow$ BAYES' THEOREM

$$\langle \mathcal{O} \rangle_{\rm new} = \int \mathcal{O}[f]\, \mathcal{P}_{\rm new}(f)\, Df, = \mathcal{N}_\chi \int \mathcal{O}[f]\, \mathcal{P}(\chi^2|f)\mathcal{P}_{\rm old}(f)\, Df,$$

IN A MONTE CARLO APPROACH...

$$\langle \mathcal{O} \rangle_{\rm new} = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{N}_\chi \mathcal{P}(\chi^2|f_k)\mathcal{O}[f_k] = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} w_k\, \mathcal{O}[f_k], \quad w_k = \mathcal{N}(\chi^2_k)^{n/2-1} e^{-\frac{1}{2}\chi^2_k}$$

$\Rightarrow$ EFFECT OF NEW DATA IS ACCOUNTED FOR BY

REWEIGHTING MONTE CARLO AVERAGES

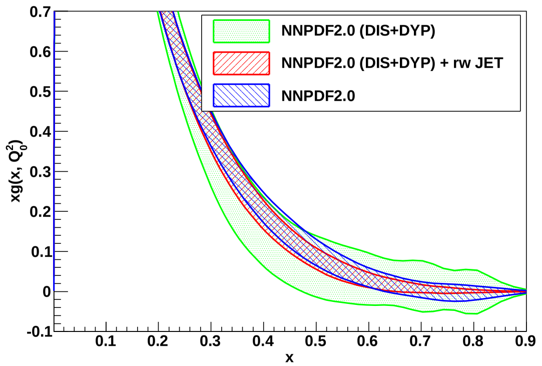
- DETERMINE PDFS INCLUDING SOME DATA BY BAYES' THEOREM (REWEIGHTING)
- DETERMINE PDFS BY ENLARGING THE DATASET TO THE NEW DATA (REFITTING)
- COMPARE RESULTS $\Rightarrow$ STRONG CONSISTENCY CHECK

CONSISTENT INFORMATION PROCESSING II

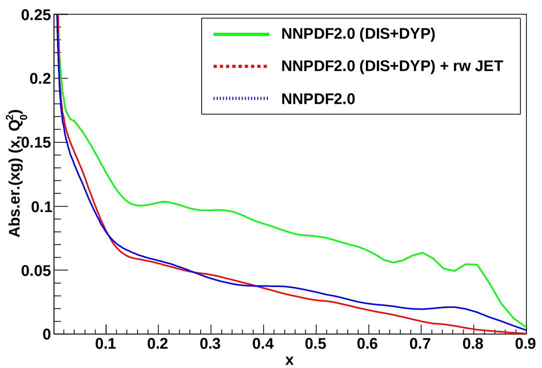
INCLUSION OF JET DATA: REWEIGHTING VS. REFITTING

NNPDF2.0DIS+DY VS. NNPDF2.0FULL

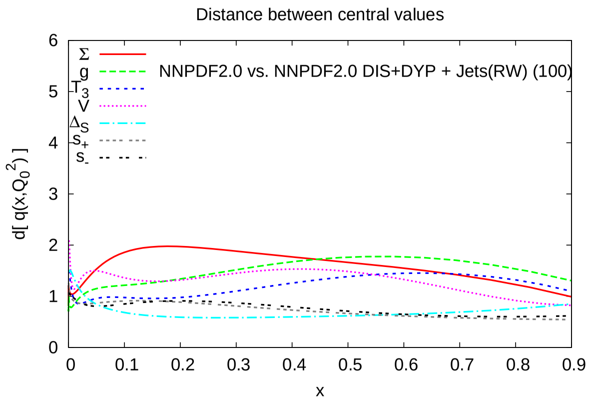
GLUON

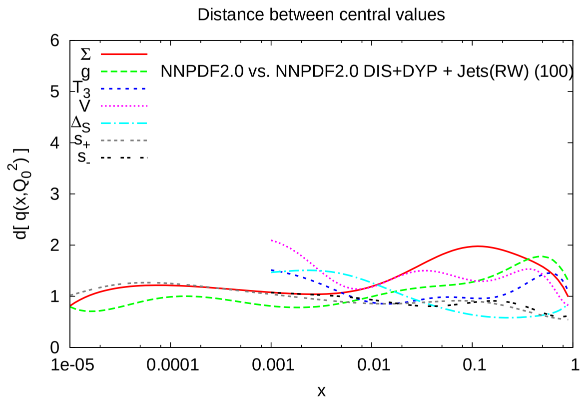


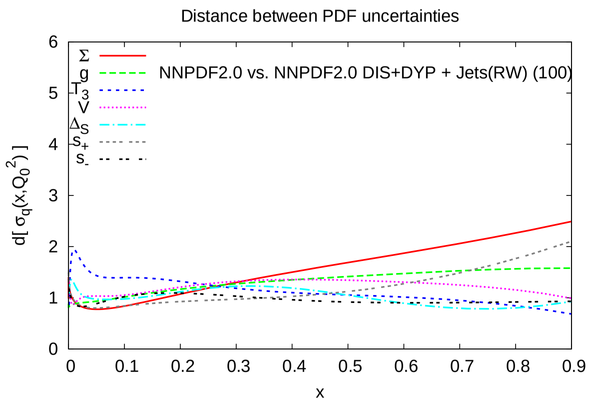
GLUON UNCERTAINTY

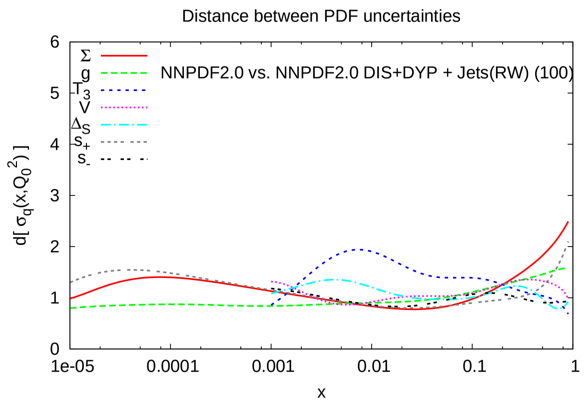


DISTANCES





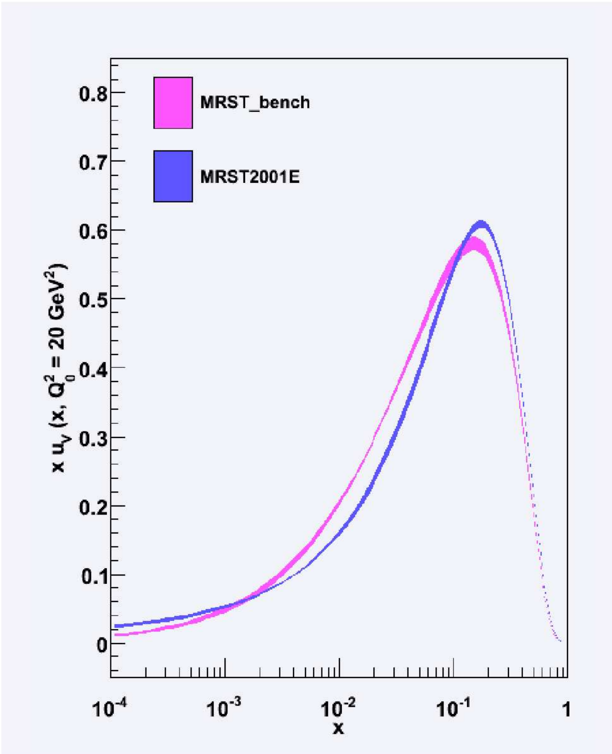


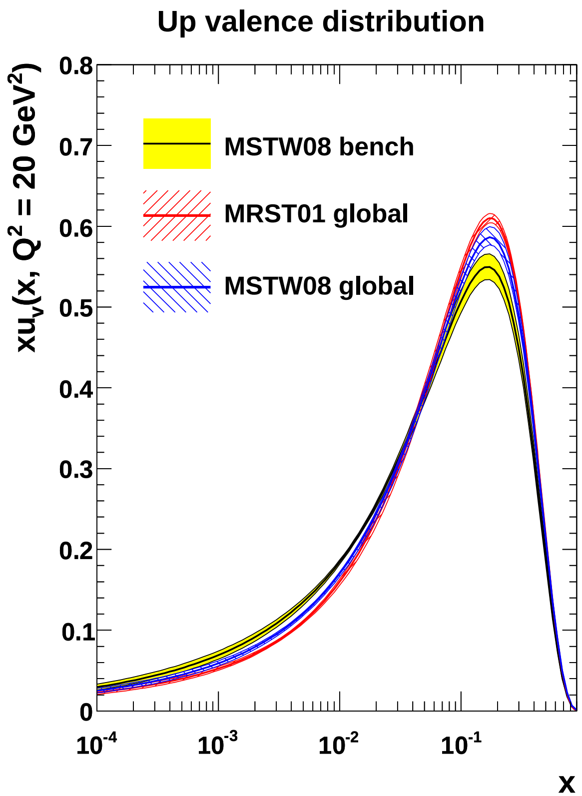


EXCELLENT CONSISTENCY!

THE PROBLEM OF BENCHMARK FITS... (HERALHC 2005-2008)

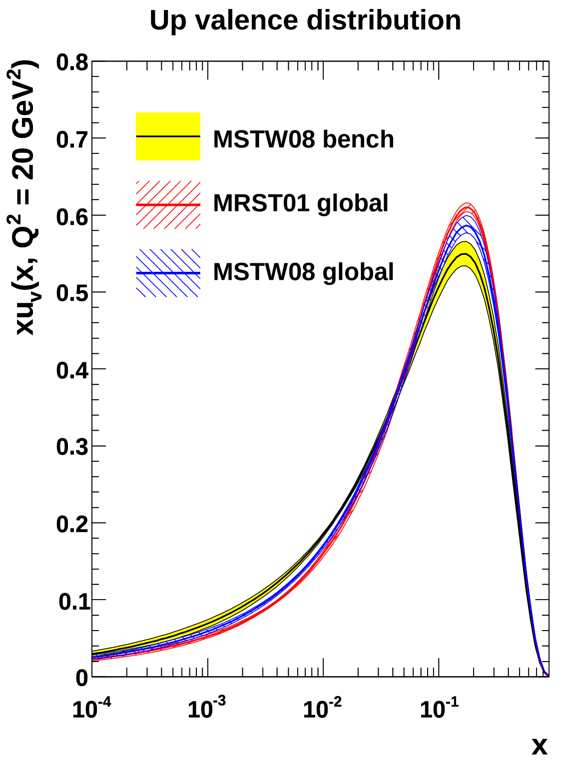
- PERFORM A MRST (MRSTBENCH) FIT TO A CONSISTENT SUBSET OF DATA, USE $\Delta\chi^2 = 1$
  $\Rightarrow$ RESULTS NOT CONSISTENT, UNCERTAINTY DOES NOT GROW AS DATASET DECREASES
- ...BUT MRST WAS DONE WITH TOLERANCE 50: REPEAT WITH DYNAMICAL TOLERANCE (MSTW08BENCH)
- IMPROVEMENT, BUT PROBLEM NOT SOLVED
  $\Rightarrow$ MUST TUNE PARAMETRIZATION AND STATISTICAL TREATMENT TO DATASET



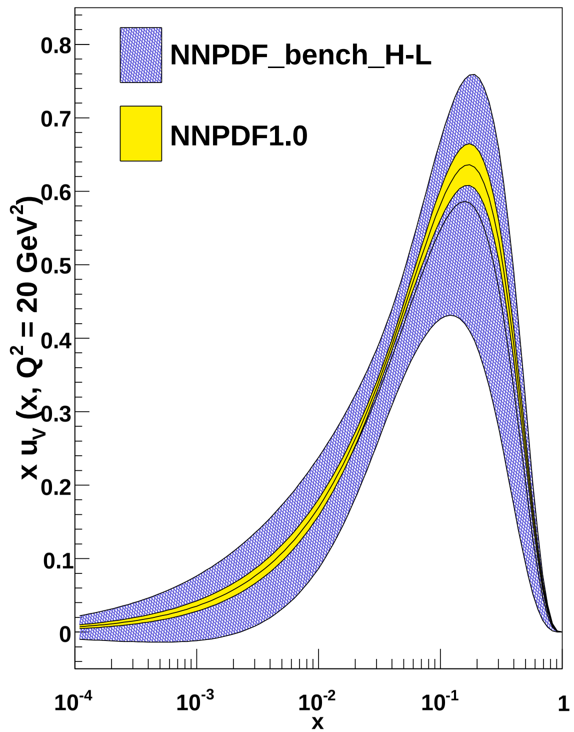


...AND THE NNPDF SOLUTION (HERALHC 2008)

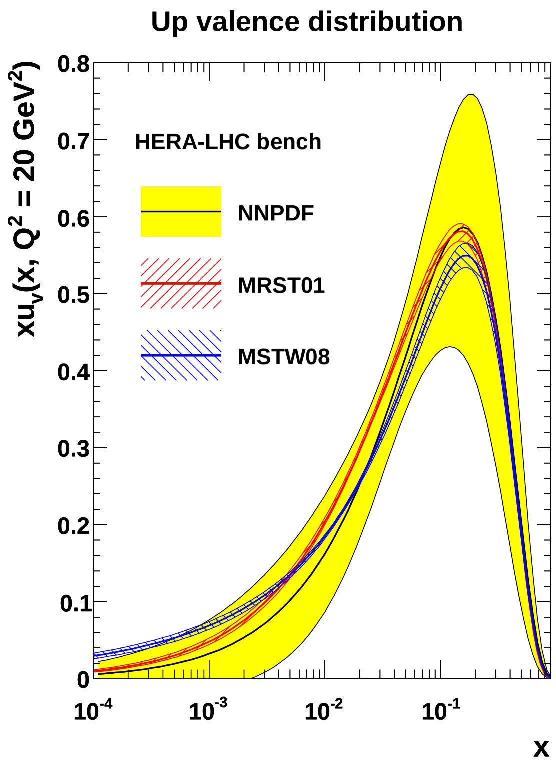
MRST/MSTW: BENCH VS REF



NNPDF: BENCH VS REF



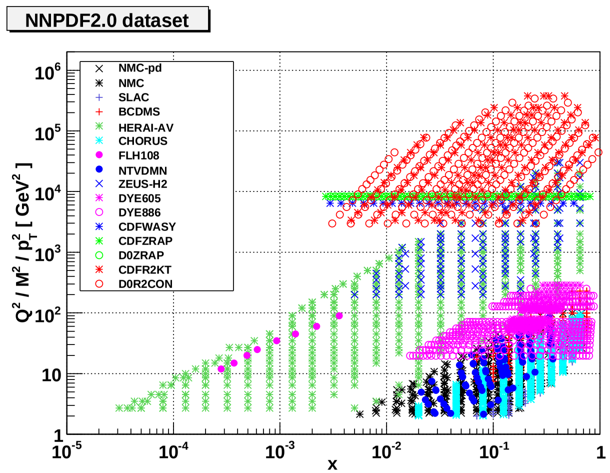
NNPDF BENCH VS MRST/MSTW BENCH



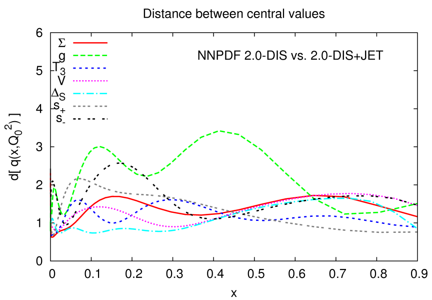
- SINGLE PARAMETRIZATION AND STAT. TREATMENT CAN ACCOMMODATE DIFFERENT DATASETS
- IMPACT OF DATA CAN BE STUDIED INDEPENDENT OF THEORETICAL FRAMEWORK

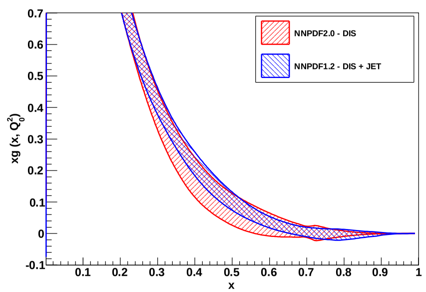
THE IMPACT OF NEW DATA:
ADDING JET DATA TO A DIS FIT

	DIS	DIS+JET	NNPDF2.0
χ^2_{tot}	1.20	1.18	1.21
NMC-pd	0.85	0.86	0.99
NMC	1.69	1.66	1.69
SLAC	1.37	1.31	1.34
BCDMS	1.26	1.27	1.27
HERAI	1.13	1.13	1.14
CHORUS	1.13	1.11	1.18
FLH108	1.51	1.49	1.49
NTVDMN	0.71	0.75	0.67
ZEUS-H2	1.50	1.49	1.51
CDFR2KT	**0.91**	**0.79**	0.80
D0R2CON	**1.00**	**0.93**	0.93
DYE605	7.32	10.35	0.88
DYE866	2.24	2.59	1.28
CDFWASY	13.06	14.13	1.85
CDFZRAP	3.12	3.31	2.02
D0ZRAP	0.65	0.68	0.47



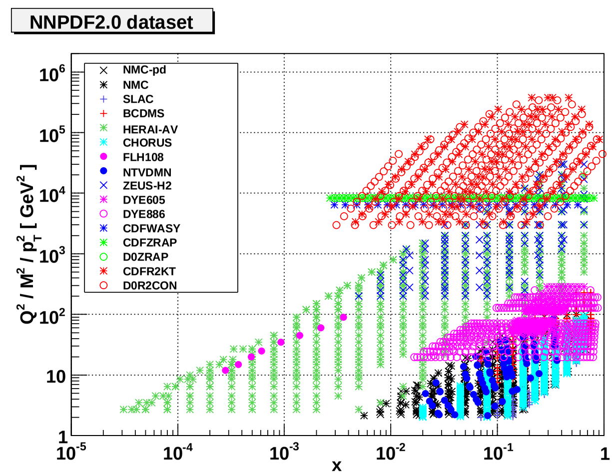
- HIGH E_T JET DATA WELL REPRODUCED EVEN WHEN NOT FITTED $\Rightarrow$ LARGE x GLUON WELL DETERMINED BY SCALING VIOLATIONS!
- SIGNIFICANT IMPROVEMENT IN LARGE x GLUON ACCURACY
- OTHER PDFs UNCHANGED



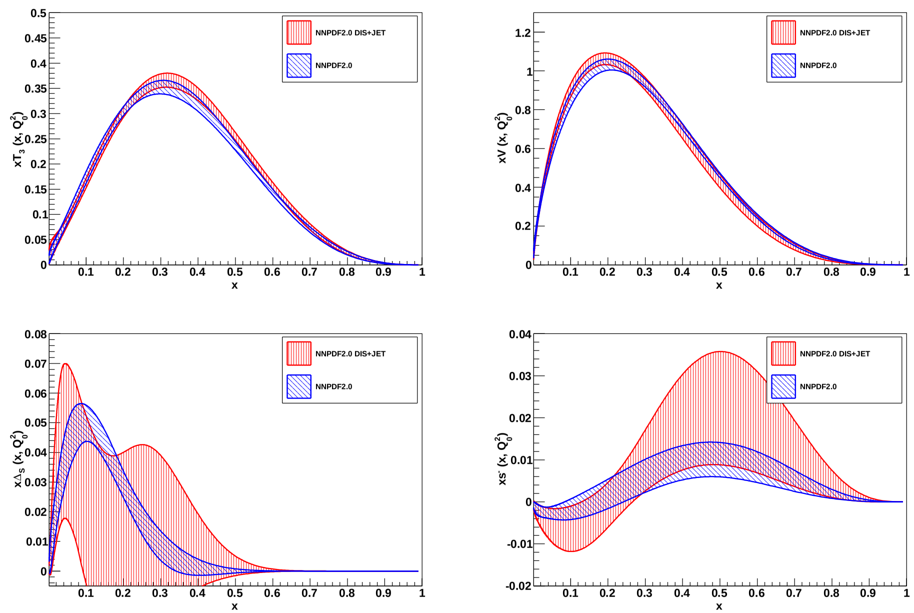


THE IMPACT OF NEW DATA: ADDING DRELL-YAN (AND W, Z) TO DIS+JETS

	DIS	DIS+JET	NNPDF2.0
χ^2_{tot}	1.20	1.18	1.21
NMC-pd	0.85	0.86	0.99
NMC	1.69	1.66	1.69
SLAC	1.37	1.31	1.34
BCDMS	1.26	1.27	1.27
HERAI	1.13	1.13	1.14
CHORUS	1.13	1.11	1.18
FLH108	1.51	1.49	1.49
NTVDMN	0.71	0.75	0.67
ZEUS-H2	1.50	1.49	1.51
CDFR2KT	0.91	0.79	0.80
D0R2CON	1.00	0.93	0.93
DYE605	7.32	**10.35**	**0.88**
DYE866	2.24	**2.59**	**1.28**
CDFWASY	13.06	**14.13**	**1.85**
CDFZRAP	3.12	**3.31**	**2.02**
D0ZRAP	0.65	**0.68**	**0.47**



- VERY SUBSTANTIAL IMPROVEMENT IN FIT QUALITY WHEN DATA INCLUDED $\Rightarrow$ SOME PDF COMBINATIONS POORLY DETERMINED WITHOUT THESE DATA
- HUGE IMPROVEMENT IN SEA ASYM $\bar{u} - \bar{d}$ & STRANGENESS $s - \bar{s}$
- SIGNIFICANT IMPROVEMENT IN TOTAL VALENCE $(\sum_i (q_i - \bar{q}_i))$ & ISOTRIPLET $(u + \bar{u} - (d + \bar{d}))$



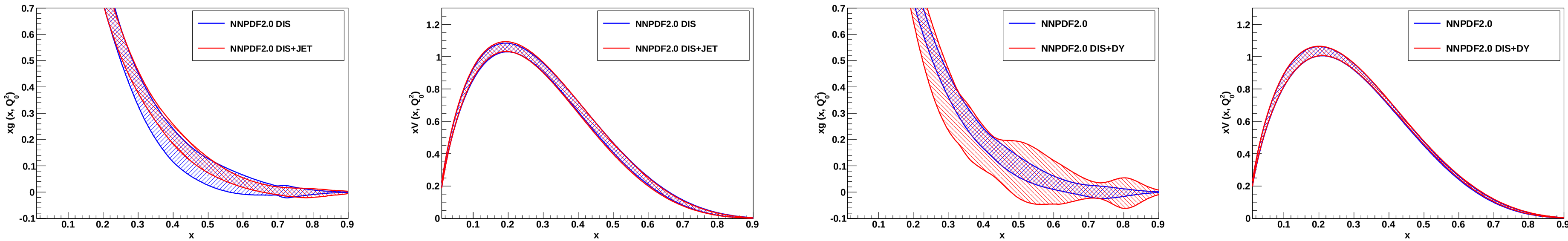
DATA COMPATIBILITY

DIS VS. HADRONIC DATA

A SENSITIVE TEST: IS THE IMPACT OF A DATASET INDEP. OF THE DATA IT IS ADDED TO?

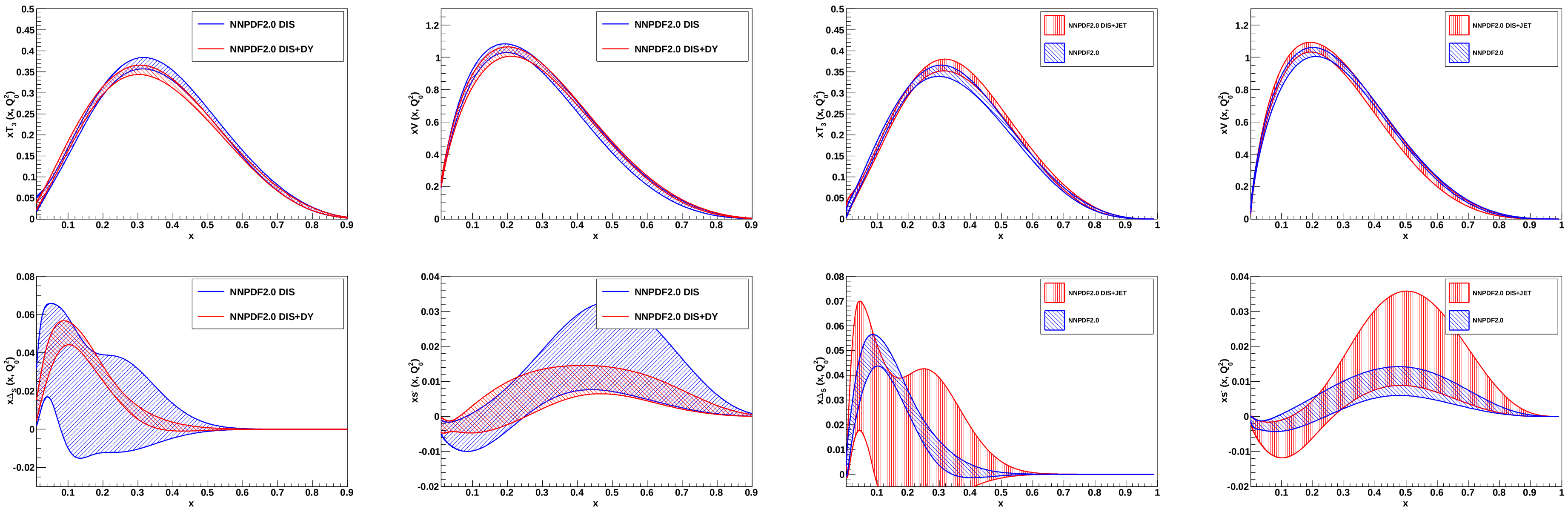
ADDING JET DATA. . .

. . . TO DIS DATA . . . TO DIS+DY DATA



ADDING DRELL-YAN DATA. . .

. . . TO DIS DATA . . . TO DIS+JET DATA

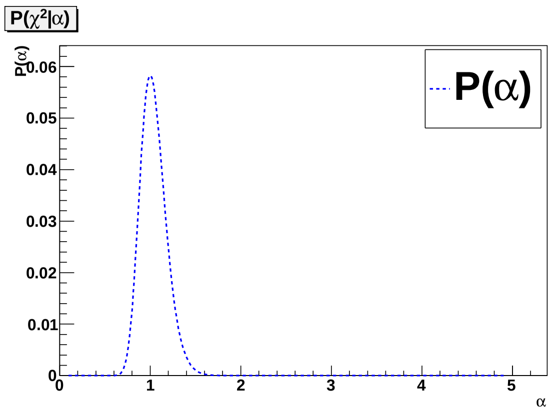


FITS "COMMUTE"

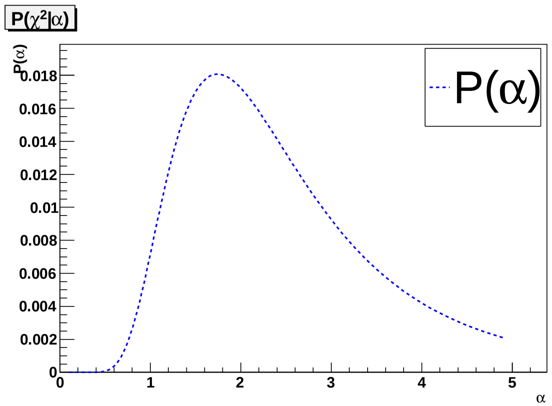
DATA COMPATIBILITY

- INCONSISTENT DATA $\Leftrightarrow$ UNDERESTIMATED UNCERTAINTIES
- RESCALE ALL UNCERTAINTIES IN A GIVEN EXPERIMENT BY SOME FACTOR α: $\chi^2_\alpha = \chi^2/\alpha$ (TOLERANCE)
- DETERMINE PROBABILITY DISTRIBUTION OF α VALUES BY BAYES' THEOREM $\Rightarrow$ REWEIGHTING: $\mathcal{P}(\alpha) = \frac{\mathcal{N}}{\alpha} \sum_{k=1}^{N} w_k w_k(\alpha)$.

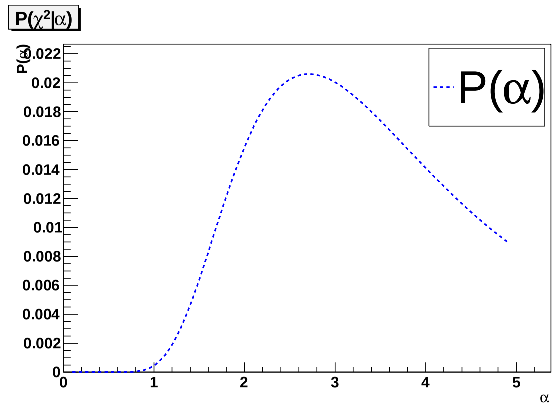
JETS



D0 e INCLUSIVE



D0 e HIGH E_T



- JETS: $\Rightarrow$ CONSISTENT DATA
- $W^\pm$ CHARGE ASYMMETRIES, D0 INCLUSIVE e DATA $\Rightarrow$ UNCERTAINTIES UNDERESTIMATED BY $\sim 30\%$ (PROB. PEAKS AT $\alpha \sim 1.7$)
- $W^\pm$ CHARGE ASYMMETRIES, D0 e DATA WITH $E_T > 35$ GEV $\Rightarrow$ INCONSISTENT DATA

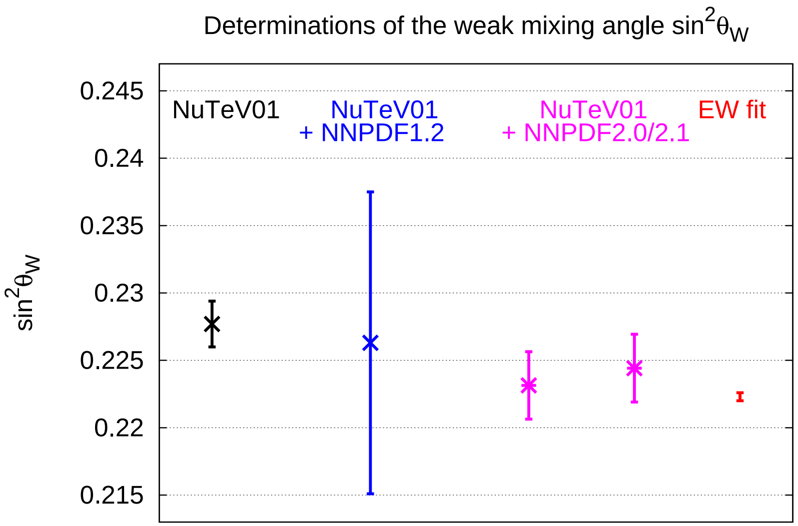
A DISCOVERY THAT WASN'T
THE NUTEV ANOMALY...

$$R_{\rm PW} \equiv \frac{\sigma(\nu\mathcal{N}\to\nu X)-\sigma(\bar\nu\mathcal{N}\to\bar\nu X)}{\sigma(\nu\mathcal{N}\to\ell X)-\sigma(\bar\nu\mathcal{N}\to\bar\ell X)} = \frac{1}{2}-\sin^2\theta_{\rm W}+\left(\frac{(U^- - D^-)+(C^- - S^-)}{Q^-}\frac{1}{6}\left(3-7\sin^2\theta_{\rm W}\right)\right),$$

- PASCHOS-WOLFENSTEIN RATIO CAN BE MEASURED IN NEUTRINO DIS
- RESULT DEPENDS ON EW MIXING ANGLE, VALENCE ISOSPIN BREAKING (WITH ISOSINGLET TARGET), STRANGENESS VALENCE MOMENTUM ASYMMETRY
- STRANGENESS VALENCE MOMENTUM ASSUMED BY NUTEV TO VANISH $\Rightarrow$ THREE σ DISCREPANCY WITH GLOBAL FIT

...IS GONE

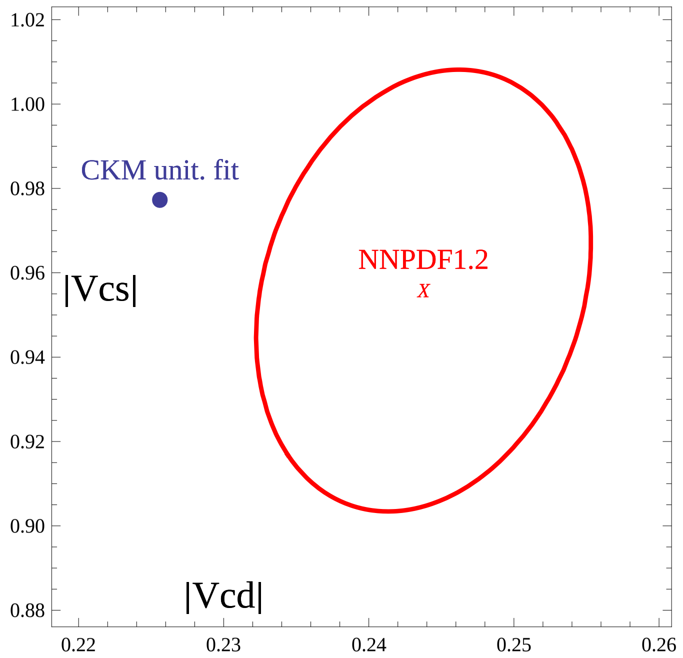
- NNPDF: s, $\bar s$ LIKE ANY OTHER PDF $\to$ 37 PARMS. EACH (CTEQ6.6: $s=\bar s$, TWO PARMS; MSTW08, S4S4, $\bar s$ TWO PARMS EACH)
- IF STRANGENESS UNCERTAINTY KEPT INTO ACCOUNT (DIS ONLY FIT: NNPDF1.2) $\Rightarrow$ EFFECT LOSES STAT. SIGNIFICANCE
- IF HADRONIC DATA INCLUDED (NNPDF2.0 GLOBAL FIT) $\Rightarrow$ STRANGENESS ASYMMETRY DETERMINED QUITE ACCURATELY $\to$ CORRECTED RESULT IN IMPRESSIVE AGREEMENT WITH SM GLOBAL FIT



(NNPDF2.1, 2010)

CKM MATRIX ELEMENTS

CAN BE DETERMINED WITH SURPRIZING ACCURACY

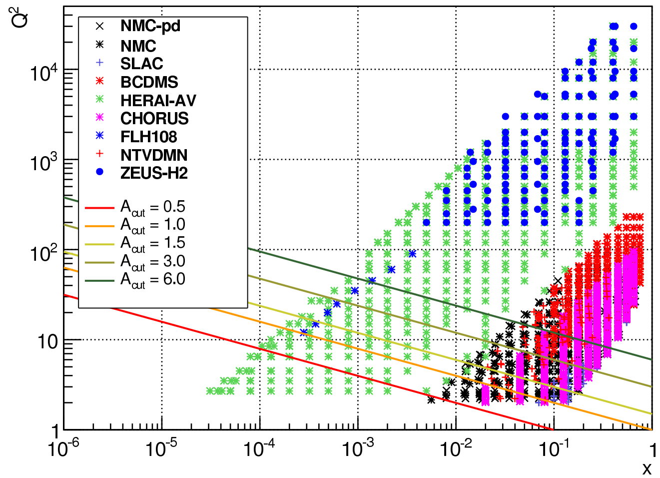


ANALYSIS	DETERMINATION	$\lvert V_{cs}\rvert$
NNPDF1.2	DIRECT FROM GLOBAL PDF ANALYSIS	$0.96 \pm 0.07^{\text{tot}}$
CDHS	LO FROM $\nu N \to \mu^+\mu^- X$	≥ 0.59 (90% C.L.)
CCFR	NLO FROM $\nu N \to \mu^+\mu^- X$	≥ 0.74 (90% C.L.)
PDG08	AVERAGE FROM D DECAYS	1.04 ± 0.06
HOCKER	AVERAGE FROM $\nu N \to \mu^+\mu^- X$	1.04 ± 0.16
DELPHI	DIRECT FROM $W^+ \to c\bar{s}$ DECAYS	$0.94^{+0.32}_{-0.26} \pm 0.13$
PDG08	CKM UNITARITY FIT	0.97334 ± 0.00023

ANALYSIS	DETERMINATION	$\lvert V_{cd}\rvert$
NNPDF1.2	DIRECT FROM GLOBAL PDF ANALYSIS	$0.244 \pm 0.019^{\text{tot}}$
CDHS	LO FROM $\nu N \to \mu^+\mu^- X$	0.24 ± 0.03
CCFR	NLO FROM $\nu N \to \mu^+\mu^- X$	$0.232^{+0.017}_{-0.019}$
PDG08	AVERAGE FROM $\nu N \to \mu^+\mu^- X$	0.230 ± 0.011
PDG08	AVERAGE FROM $D \to K/\pi l \nu$ DECAYS	0.218 ± 0.023
PDG08	CKM UNITARITY FIT	0.2256 ± 0.0010

- FIRST DETERMINATION OF CKM MATRIX ELEMENTS FROM DIS
- NNPDF1.2: ONLY DIS DATA
- MORE ACCURATE (V_{cs}) OR COMPETITIVE (V_{cd}) THAN OTHER DIRECT DETERMINATIONS

BEYOND DGLAP?
DISCOVERING A NEW QCD EFFECT IN HERA DATA

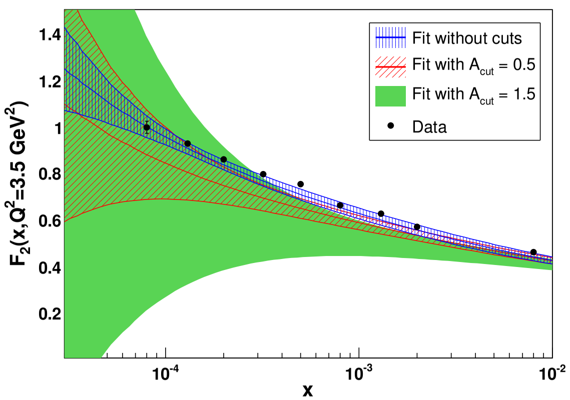


IDEA: (Gélis, 2008, $\Rightarrow$ Caola, s.f. ,Rojo 2010)

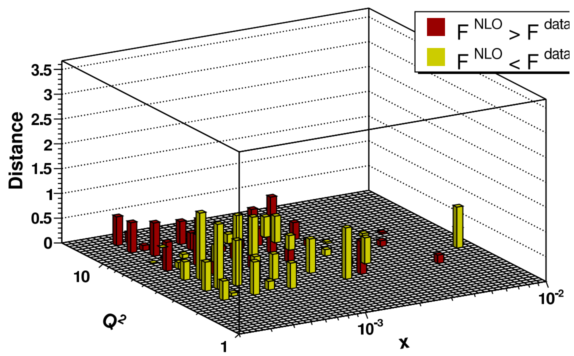
- CUT OUT DATA IN THE "DANGEROUS" (SMALL x) REGION
- DETERMINE PDFs IN THE "SAFE" (LARGE x AND Q^2) REGION
- EVOLVE BACKWARDS AND COMPARE TO DATA

OLD HERA DATA

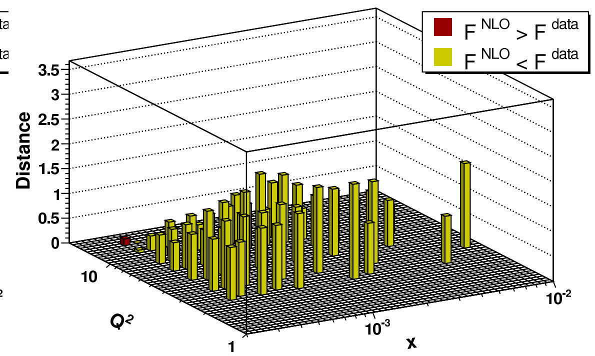
BACKWARD EV. VS DATA



DAT/TH DIST: NO CUT



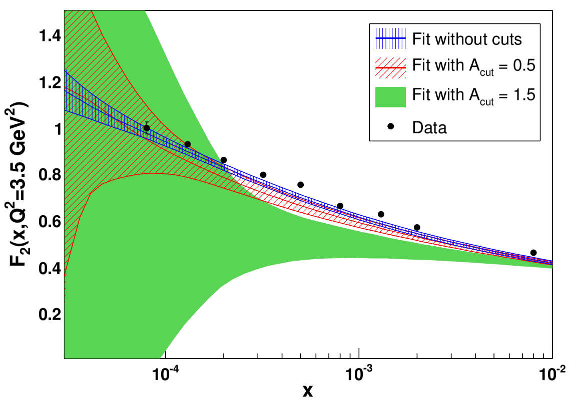
DAT/TH DIST: CUT



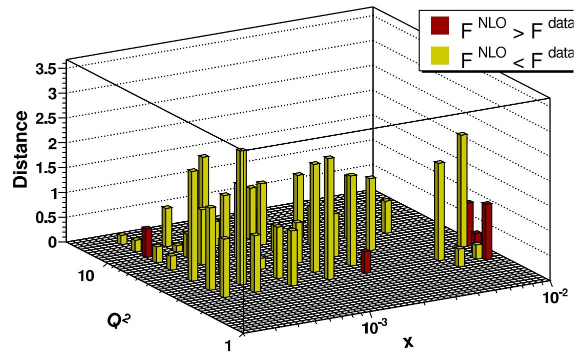
- BACKWARD EVOLVED FIT LIES SYSTEMATICALLY BELOW DATA
- DATA AT LOW x AND Q^2 SHOW LESS EVOLUTION THAN PREDICTED BY NLO DGLAP
- IF LOW x AND Q^2 DATA INCLUDED, THE FIT COMPENSATES READJUSTING PDFs

NEW COMBINED HERA DATA

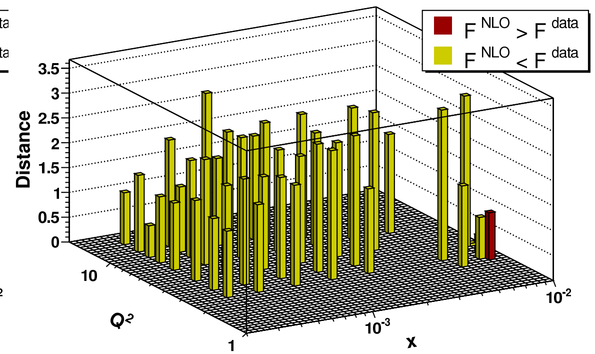
BACKWARD EV. VS DATA



DAT/TH DIST: NO CUT

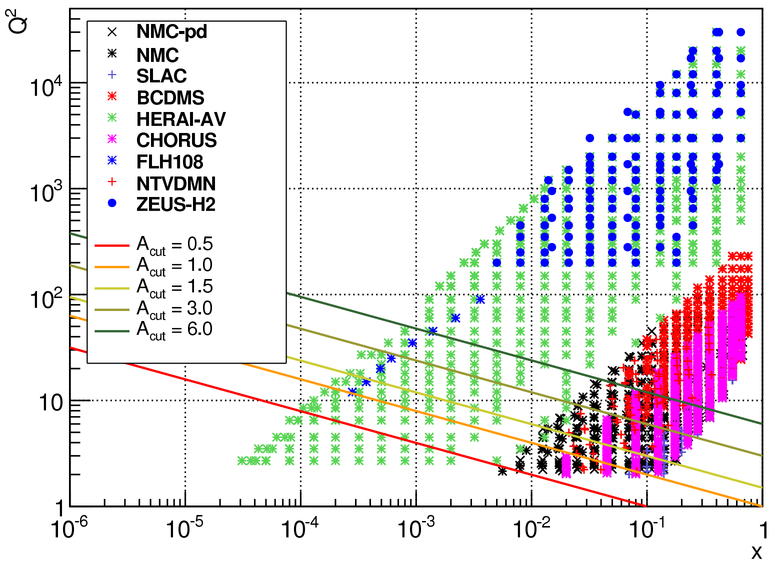


DAT/TH DIST: CUT

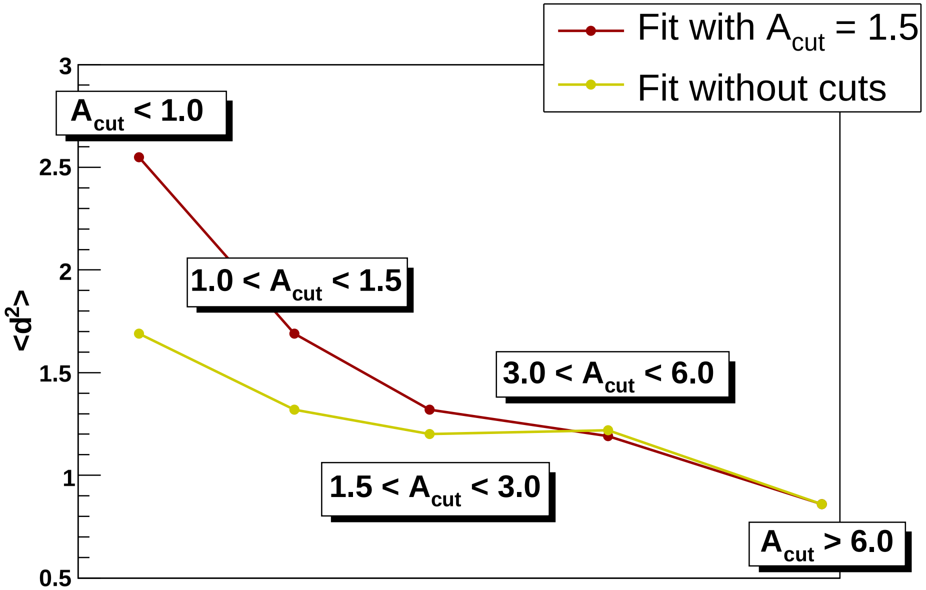


- DATA AT LOW x AND Q^2 SHOW LESS EVOLUTION THAN PREDICTED BY NLO DGLAP
- BACKWARD EVOLVED FIT LIES SYSTEMATICALLY BELOW DATA
- WITH MORE PRECISE DATA, THE FIT NO LONGER MANAGES TO COMPENSATE BY READJUSTING THE PDFS: EVEN FULL FIT LIES BELOW DATA

DETERIORATION IN FIT QUALITY:

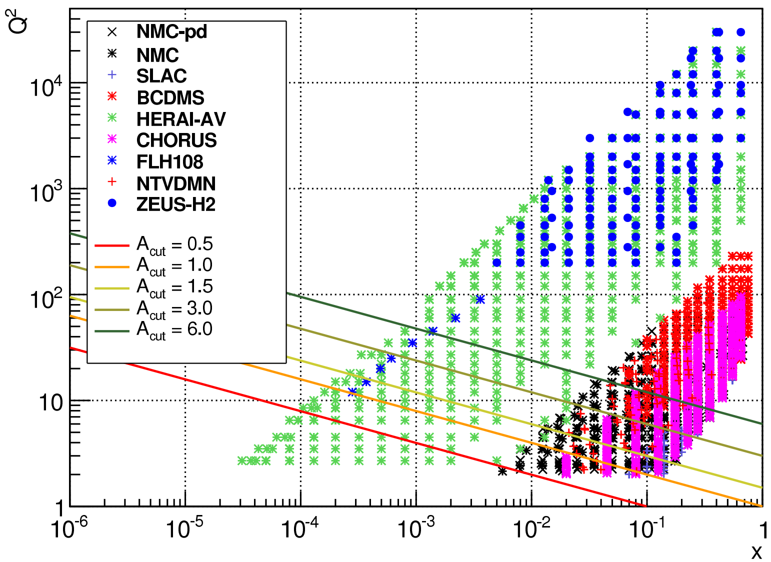


$\langle d^2 \rangle$ VS x SLICES

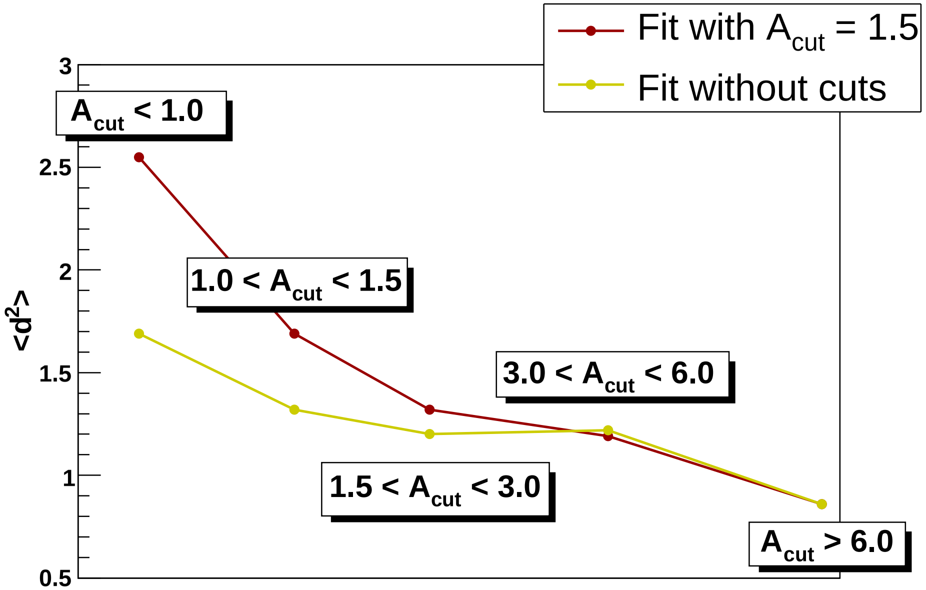


- QUALITY OF UNCUT FIT DETERIORATES IN LOW x REGIONS
- QUALITY OF CUT FIT INCREASINGLY POOR AS x DECREASES
- DISTANCE RISES DESPITE HUGE INCREASE IN UNCERTAINTY

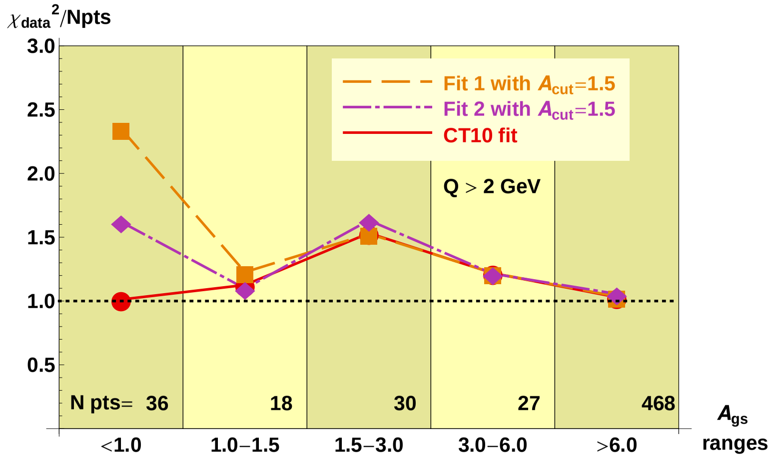
DETERIORATION IN FIT QUALITY:



$\langle d^2 \rangle$ VS x SLICES



χ^2 VS x SLICES FOR CT10



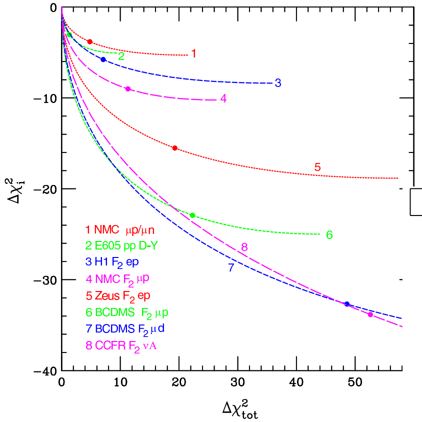
- QUALITY OF UNCUT FIT DETERIORATES IN LOW x REGIONS
- QUALITY OF CUT FIT INCREASINGLY POOR AS x DECREASES
- DISTANCE RISES DESPITE HUGE INCREASE IN UNCERTAINTY
- IN HESSIAN FIT (CTEQ) RESULTS DEPEND ON PARAMETRIZATION $\Rightarrow$ EVIDENCE INCONCLUSIVE

THE TOLERANCE PROBLEM

MSTW/CTEQ: ONE σ IS DEFINED UP TO A "TOLERANCE"

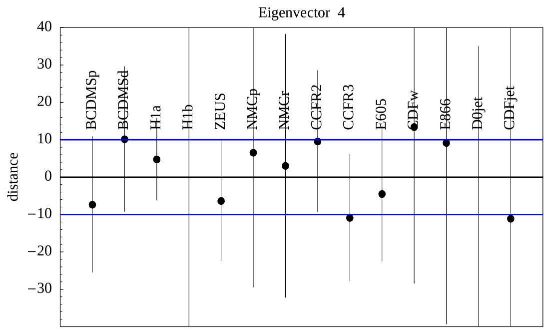
- STANDARD $\Delta\chi^2 = 1$ BANDS TOO NARROW $\Rightarrow$ LARGE DISCREPANCIES FOR INDIVIDUAL EXPERIMENTS
- TOLERANCE $\Rightarrow$ ENVELOPE OF UNCERTAINTIES OF EXPERIMENTS
- DYNAMICAL $\Rightarrow$ SEPARATELY DETERMINED FOR EACH HESSIAN EIGENVECTOR

MINIMUM χ_i^2 VS GLOBAL χ^2

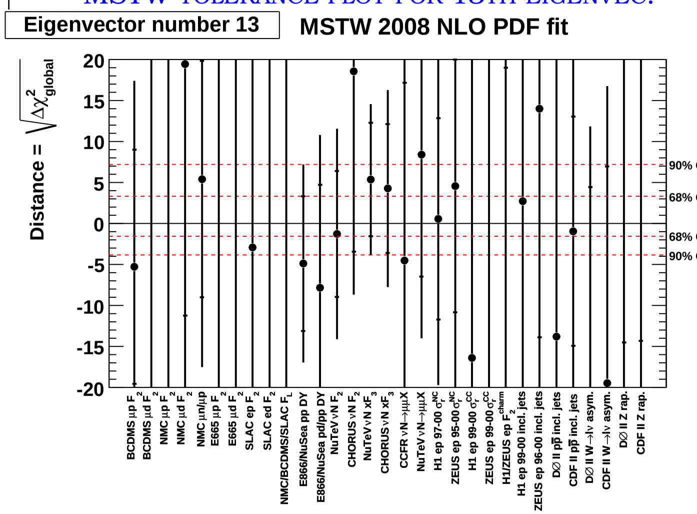


Collins, Pumplin 2001

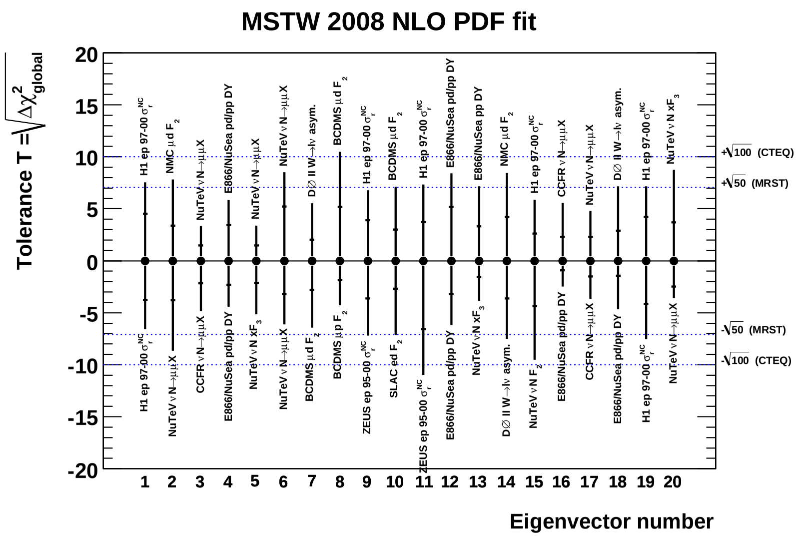
CTEQ TOLERANCE PLOT FOR 4TH EIGENVEC.



MSTW TOLERANCE PLOT FOR 13TH EIGENVEC.



GLOBAL MSTW TOLERANCE

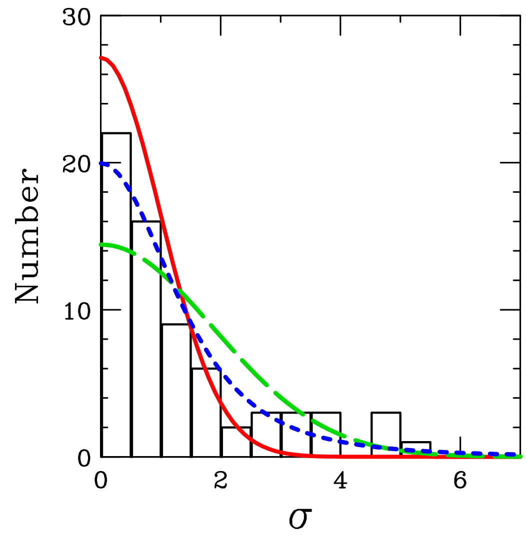


WHERE IS THE UNCERTAINTY COMING FROM?

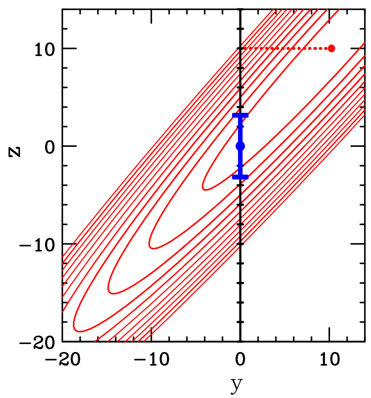
WHY DOES ONE NEED LARGE TOLERANCES?

DATA INCOMPATIBILITY(Pumplin, 2009)

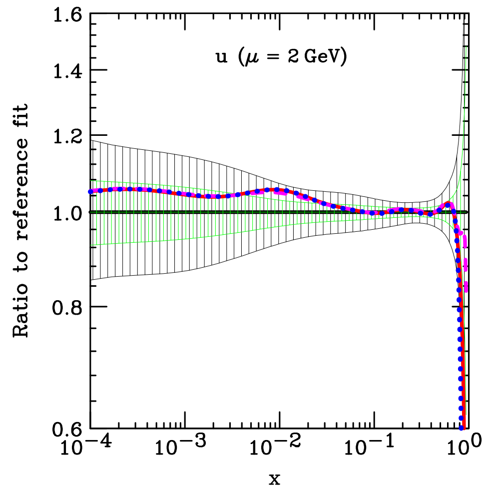
- CAN "REDIAGONALIZE": DIAGONALIZE SIMULTANEOUSLY χ^2 FOR TOTAL AND i–TH EXPT $\Rightarrow$ COMPATIBILITY OF EACH EXPT WITH GLOBAL FIT
- STUDY DISTRIBUTION OF DISCREPANCIES
- APPROX. GAUSSIAN WITH UNCERTAINTIES RESCALED BY 2 $\Rightarrow$ $\Delta\chi^2 \sim 10$ FOR 90%C.L.



FUNCTIONAL BIAS (Pumplin, 2010)

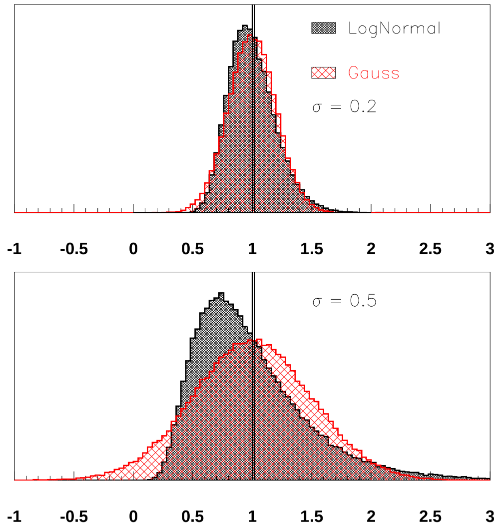


- IF PARM. NOT GENERAL ENOUGH, GLOBAL MIN. IS NOT TRUE MIN.
- ONE-σ VARIATION ABOUT FAKE MIN CORRESP. TO LARGE χ^2 VARIATION
- USE OF CHEBYSHEV POLYNOMIALS SUGGESTS "MOST GENERAL" PARM. COMPATIBLE WITH $\Delta\chi^2 = 100$ RANGE OF CT10 PARM.



PARAMETRIZATION UNCERTAINTIES?
NONGAUSSIAN BEHAVIOUR?

LOGNORMAL VS. GAUSSIAN

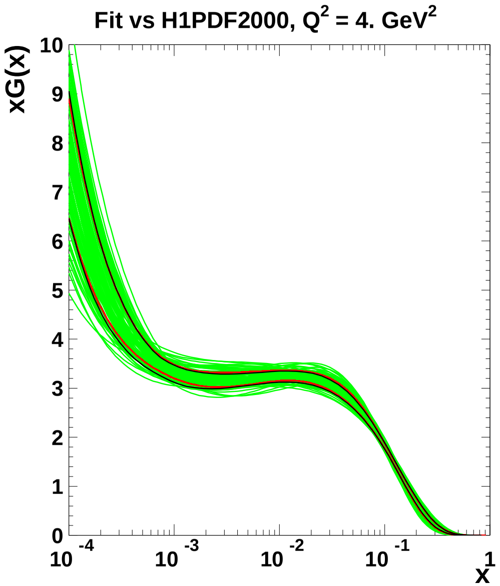


THE HERALHC BENCHMARK

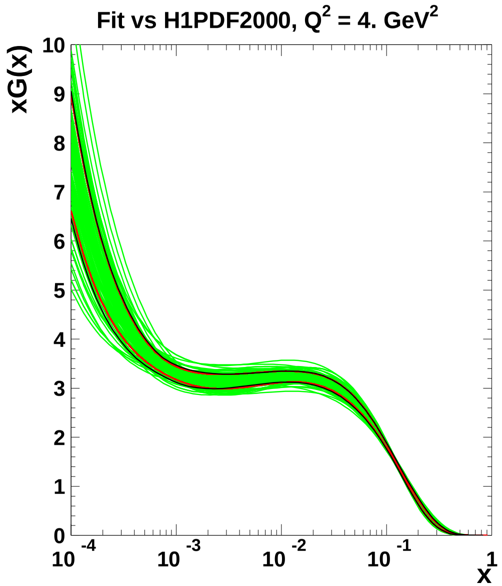
(Feltesse, Glazov, Radescu + NNPDF 2008)

- TRY EXPERIMENTAL SYSTEMATICS GIVEN BY EITHER GAUSSIAN OR LOGNORMAL DISTRIBUTION
- REPEAT (BENCHMARK) HERAPDF, WITH MONTECARLO LOGNORMAL OR GAUSSIAN, IN EITHER CASE DETERMINE UNCERTAINTY EITHER WITH HESSIAN OR MONTECARLO

LOGNORMAL: HESS. VS MC



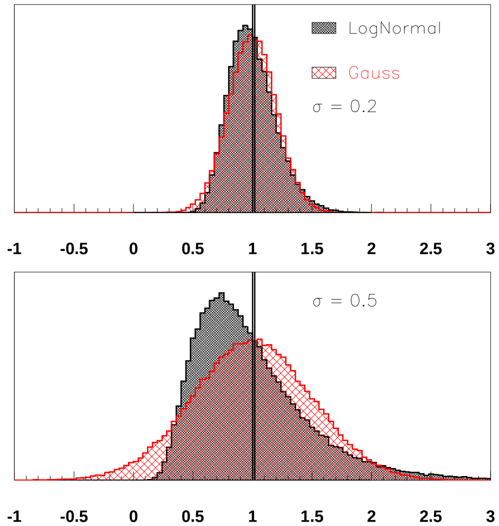
GAUSSIAN: HESS. VS MC



- NO DIFFERENCE BETWEEN LOGNORMAL, GAUSSIAN, MC, HESSIAN

PARAMETRIZATION UNCERTAINTIES?
NONGAUSSIAN BEHAVIOUR?

LOGNORMAL VS. GAUSSIAN



THE HERALHC BENCHMARK
(Feltesse, Glazov, Radescu + NNPDF 2008)

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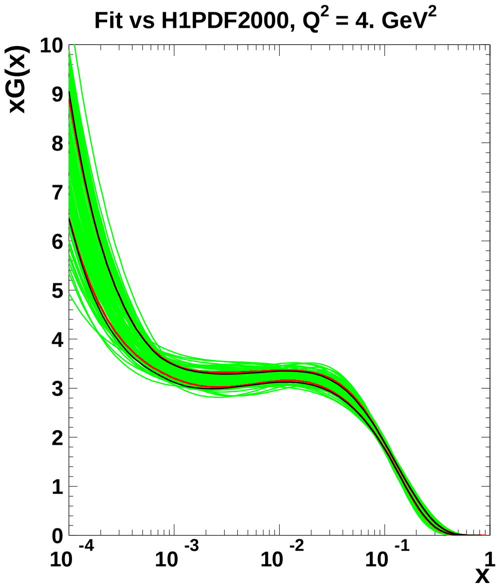
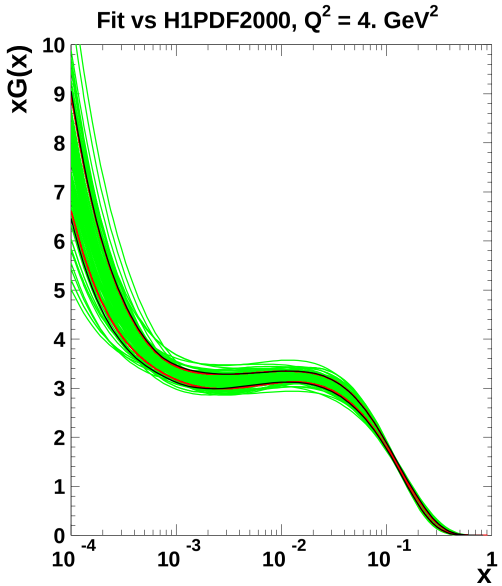
- REPEAT (BENCHMARK) HERAPDF, WITH MONTECARLO LOGNORMAL OR GAUSSIAN, IN EITHER CASE DETERMINE UNCERTAINTY EITHER WITH HESSIAN OR MONTECARLO
- COMPARE TO NNPDF FIT TO SAME DATA

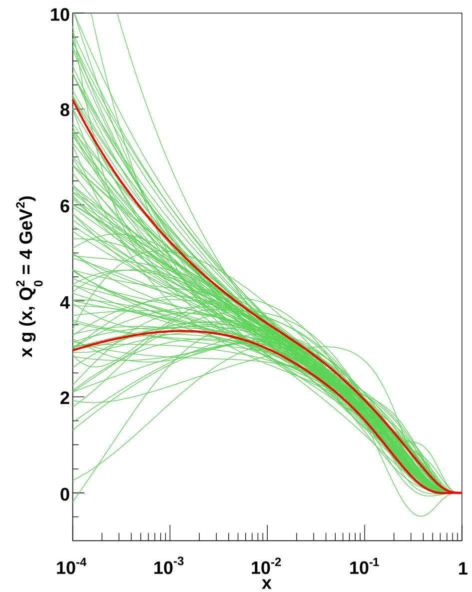
LOGNORMAL: HESS. VS MC



GAUSSIAN: HESS. VS MC



NNPDF

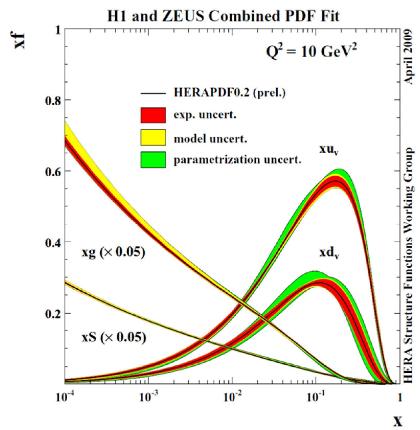


- NO DIFFERENCE BETWEEN LOGNORMAL, GAUSSIAN, MC, HESSIAN
- SIZABLE DIFFERENCE WR TO FLEXIBLE NNPDF PARAMETRIZATION

PARAMETRIZATION UNCERTAINTIES?

EXPLORING THE SPACE OF PARAMETERS: HESSIAN APPROACH

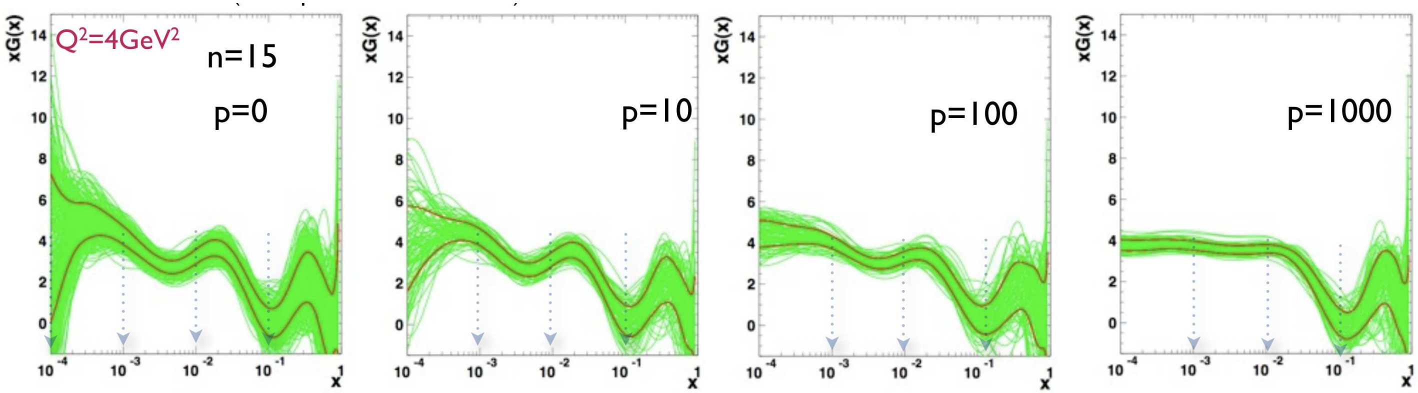
- IN HESSIAN APPROACH CAN VARY THE FUNCTIONAL FORM, ASSUMPTIONS, STARTING SCALE
- DONE IN THE HERAPDF1.0 FIT: VARIATION OF STRANGENESS FRACTION, LARGE x BEHAVIOUR, HIGHER ORDER POLYNOMIAL TERMS
- NO TOLERANCE ($\Delta\chi^2 = 1$), UNCERTAINTY DOUBLED



- Red: experimental uncertainties
- Yellow: model uncertainties
- Green: pdf parametrization uncertainties

ORTHOGONAL POLYNOMIALS

- OLD IDEA (PARISI, SOURLAS, 1978; ZOMER 1996): EXPAND PDFS OVER BASIS OF ORTHOGONAL POLYNOMIALS
- GLAZOV, RADESCU, 2009: COUPLED TO MONTE CARLO METHOD
- LENGTH PENALTY TO STABILIZE THE FIT



(Glazov, Radescu, 2009)

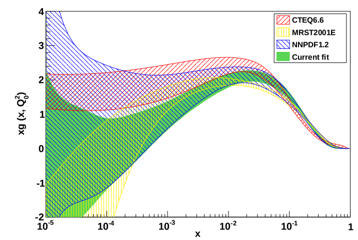
WHERE IS THE UNCERTAINTY COMING FROM?

CENTRAL VALUES: VARYING PARTITION VS FIXED PARTITION

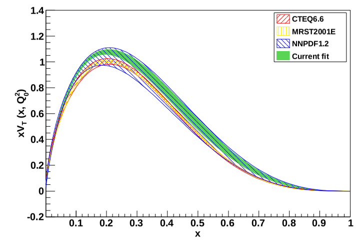
	REPLICAS	CENTRAL VALUE	FIXED PARTITION
χ^2	1.32	1.32	$\sim$1.3
$\langle\chi^2\rangle_{\rm rep}$	2.79 ± 0.24	1.65 ± 0.20	$\sim 1.6 \pm 0.2$
$\langle\sigma^{\rm dat}\rangle$	0.039	0.035	$\sim$0.03

fixed partition results obtained averaging over 5 different choices of partition (100 replicas each); more partitions needed for accurate results

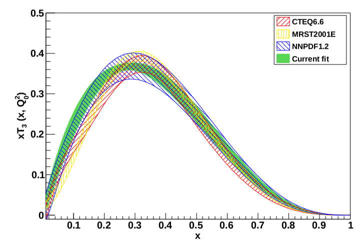
GLUE



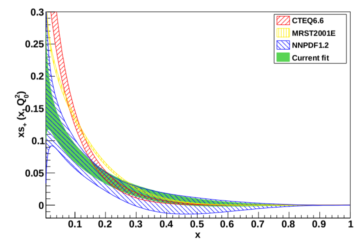
VALENCE



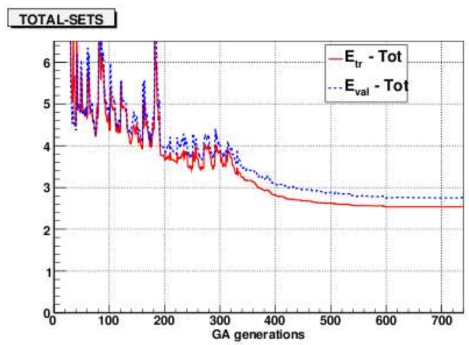
TRIPLET

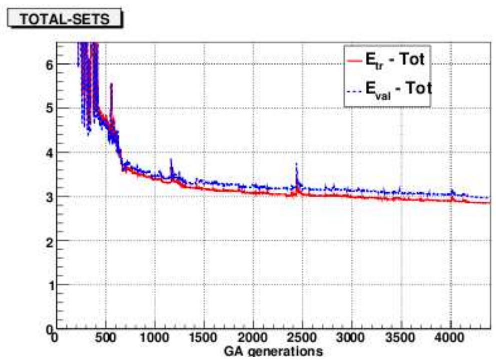


STRANGE



- QUALITY OF FIT UNCHANGED
- $\langle\chi^2\rangle_{\rm rep}$ UNCHANGED $\Rightarrow$ CENTRAL FIT UNCHANGED
- UNCERTAINTY ON PREDICTION (I.E. ON PDFS) REDUCED





FUNCTIONAL UNCERTAINTY

- MORE THAN HALF OF UNCERTAINTY DUE TO "FUNCTIONAL FORM": $\langle\sigma^{\rm dat}\rangle =\sim 0.03$ SMALLER FOR HERA DATA
- REMAINING UNCERTAINTY ROUGHLY SCALES WITH DATA UNCERTAINTY: $\langle\sigma^{\rm dat}\rangle =\sim 0.005$ CENT.; $\langle\sigma^{\rm dat}\rangle =\sim 0.009$ REP.

NORMALIZATION UNCERTAINTIES

- NORMALIZATION UNCERTAINTIES IN COVARIANCE MATRIX
  $(\mathrm{cov}_{t_0})_{IJ} = \sigma_{I,n}\sigma_{J,n}F_I F_J$
  $\Rightarrow$ MAXIMUM-LIKELIHOOD RESULT BIASED (d'Agostini, 1994)

- "PENALTY TRICK": RESCALE BY λ & ADD $\frac{(\lambda-1)^2}{\sigma_n}^2$ TO χ^2
  SOMETIMES ALSO HIGHER ORDER POWERS
  $\Rightarrow$ ALSO BIASED, THOUGH BIAS DOES NOT GROW WITH N_{dat}

- BIAS DUE TO χ^2 NOT QUADRATIC IN MEASURED QUANTITY
  (NONGAUSSIAN AND IMPROPERLY NORMALIZED LIKELIHOOD)

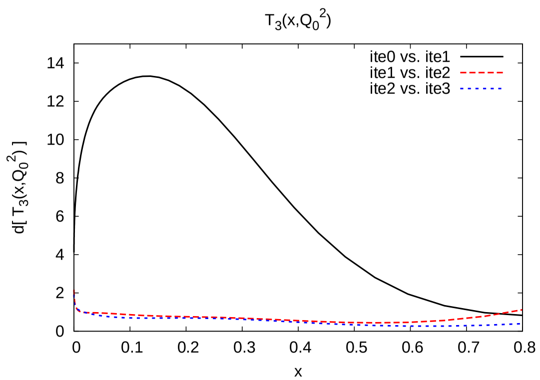
THE t_0 METHOD (R.D. Ball et al., 2010)

- NORMALIZATION UNCERTAINTIES IN COVARIANCE MATRIX, BUT COMPUTED AS FUNCTION OF RESULT OF PREVIOUS FIT $F_I^{(0)}$: $(\mathrm{cov}_{t_0})_{IJ} = \sigma_{I,n}\sigma_{J,n}F_I^{(0)}F_J^{(0)}$

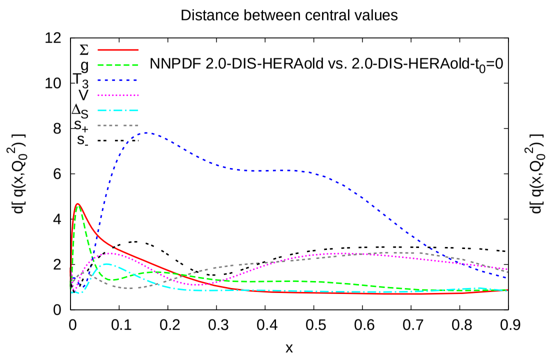
- ITERATE UNTIL CONVERGENCE

DISTANCE BETWEEN ITERATIONS

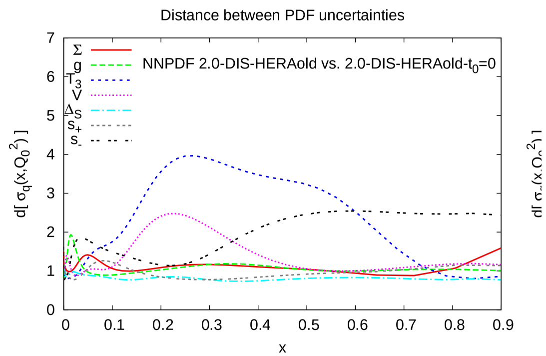
(TRIPLET)



t_0: IMPACT ON CENTRAL VALS



t_0: IMPACT ON UNCERTAINTIES



PREPROCESSING

- PDFS PARAMETRIZED AS $f(x,Q_0^2) = A_f\,(1-x)^{m_f}\,x^{-n_f}\mathrm{N}N_f(x)$, WITH $\mathrm{N}N_f(x)$ A NEURAL NETWORK
- PREFACTOR NECESSARY IF ONE WANTS $f(x)$ UNBOUNDED IN $0,1$ BECAUSE OUTPUT OF NEURAL NET BOUNDED
- WITH ARBITRARY LONG TRAINING, ANY ADMISSIBLE BEHAVIOUR CAN BE LEARNT, BUT TRAINING VERY LONG IF NN HAS TO OFFSET SHARP RISE OR DROP
- EXPONENTS m_f, n_f CHOSEN AT RANDOM WITH FLAT DISTRIBUTION IN WIDE RANGE
- RANGE DETERMINED BY WIDENING UNTIL FIT QUALITY DETERIORATES
- TEST FOR UNIFORM FIT QUALITY BY DETERMINING CORRELATION BETWEEN χ^2 AND PREPROCESSING EXPONENTS IN GIVEN RANGE

PREPROCESSING EXPONENTS: RANGE AND CORRELATION

PDF	$[m_{\min}, m_{\max}]$	$[n_{\min}, n_{\max}]$	$r\left[\chi^2, m\right]$	$r\left[\chi^2, n\right]$
$\Sigma(x,Q_0^2)$	$[2.55, 3.45]$	$[1.05, 1.35]$	-0.018	0.131
$g(x,Q_0^2)$	$[1.05, 1.35]$	$[1.05, 1.35]$	-0.002	0.050
$T_3(x,Q_0^2)$	$[2.55, 3.45]$	$[0, 0.5]$	-0.023	-0.130
$V_T(x,Q_0^2)$	$[2.55, 3.45]$	$[0, 0.5]$	0.003	-0.068
$\Delta_S(x,Q_0^2)$	$[12, 14]$	$[-0.95, -0.65]$	0.000	-0.069
$s^+(x,Q_0^2)$	$[2.55, 3.45]$	$[1.05, 1.35]$	0.021	-0.055
$s^-(x,Q_0^2)$	$[2.55, 3.45]$	$[0, 0.5]$	-0.027	-0.015

THE “HESSIAN MONTE CARLO”

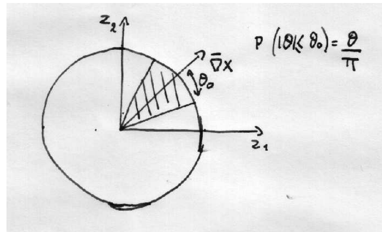
Q: IF ONE PICKS REPLICAS AT RANDOM ON THE ONE-SIGMA CONTOUR WHAT IS THE CHANCE OF “FILLING” THE ENVELOPE?

A: DETERMINE THE PROBABILITY FOR AT LEAST ONE REPLICA TO BE WITHIN ANGLE θ OF DIRECTION $\vec{\nabla} X$ OF MAX

TWO PARAMETERS: ONE REPLICA WITH $\theta < \theta_0 \Rightarrow P(2,1:\theta_0) = \frac{\theta_0}{\pi}$

PROBABILITY OF MAX(ENVELOPE)=$\sigma_X \cos\theta_0$

$\Rightarrow$ ALL n REPLICAS HAVE $\theta > \theta_0 \Rightarrow P(2,n;\theta_0) = \left(1 - \frac{\theta_0}{\pi}\right)^n$

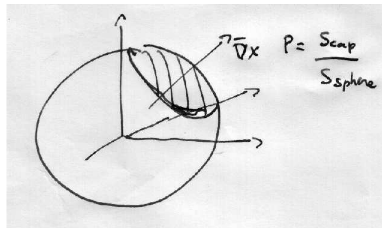


d PARAMETERS: ONE REPLICA WITH $\theta < \theta_0$

$$\Rightarrow P(d,1:\theta_0) = \frac{\Gamma\left(\frac{d}{2}\right)}{(d-1)\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)}\theta_0^{d-1}(1+O(\theta_0)) \approx \frac{\theta_0^{d-1}}{\sqrt{2\pi d}}$$

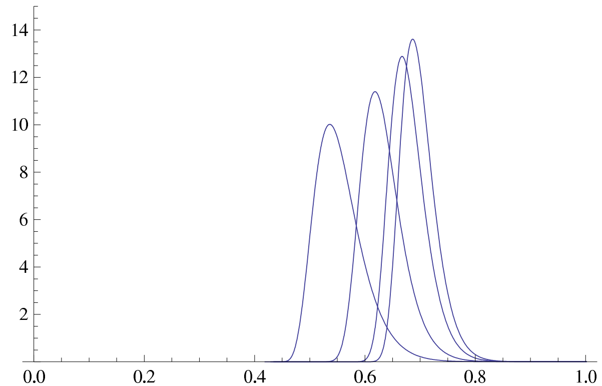
PROBABILITY OF MAX(ENVELOPE)=$\sigma_X \cos\theta_0$

$$\Rightarrow P(d,n;\theta_0) = \left(1 - \frac{\theta_0^{d-1}}{\sqrt{2\pi d}}\right)^n$$



PROBABILITY FOR THE WIDTH OF THE ENVELOPE TO BE SMALLER BY A FACTOR R THAN THE STANDARD DEVIATION σ_X PLOTTED VS R FOR

$d = 23$ PARAMETERS AND $n = 10,\ 100,\ 500,\ 1000$ REPLICAS



MONTE CARLO ERROR ESTIMATES

PARAMETER SPACE: NOT ADVISABLE

OBSERVABLE X DEPENDS ON PARAMETERS $\vec{z}$
VARIANCE: $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$
AVERAGES: $\langle X \rangle = \int d^d z X(\vec{z}) P(\vec{z})$, WITH
$P(\vec{z}) \Rightarrow$ PROBABILITY DISTN. OF PARAMETER VALUES
& INTEGRAL PERFORMED BY MONTE CARLO SAMPLING
HOW MANY REPLICAS DOES ONE NEED? THREE BINS PER PARM $\Rightarrow 3^d$ BINS
FOR 23 PARMS., NEED $> 10^{11}$ REPLICAS

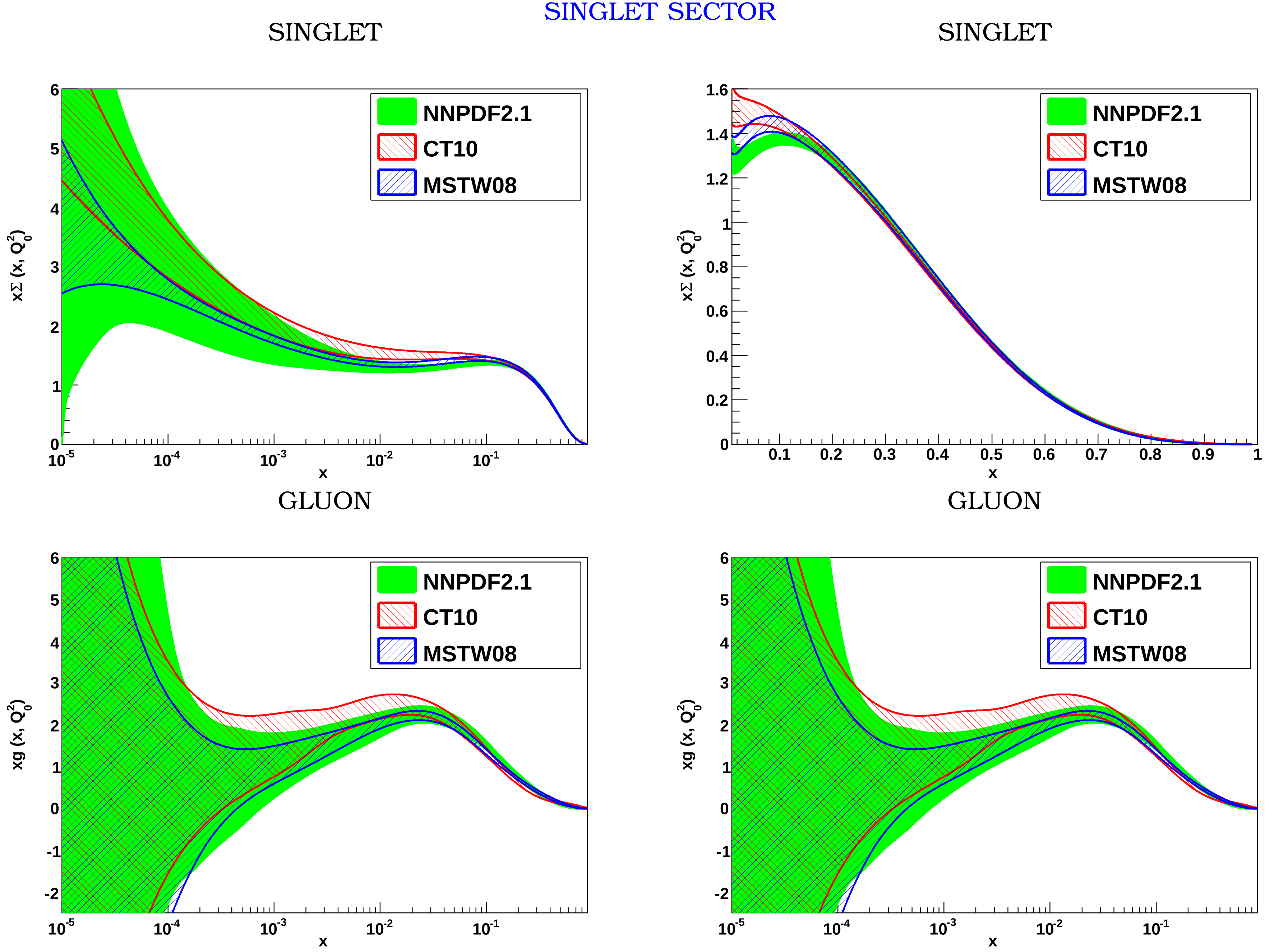
DATA SPACE

DIAGONALIZATION: CHOOSE PARM z_1 ALONG $\vec{\nabla} X$
ALL OTHER PARMS $\Rightarrow$ FLAT DIRECTIONS
AVERAGES: $\langle X \rangle = \int dz_1 X(\vec{z}) P(z_1)$
HOW MANY REPLICAS DOES ONE NEED? ONE–DIMENSIONAL AVERAGE OF n REPLICAS
CONVERGES TO TRUE AVERAGE WITH STANDARD DEV. $\frac{\sigma}{\sqrt{n}}$
10 REPLICAS ENOUGH FOR $\frac{\sigma}{3}$ ACCURACY
Q: HOW IS IT DONE IN PRACTICE?
A: CHOOSE REPLICAS OF THE DATA, DISTRIBUTED AS THE DATA

NNPDF2.1 PDFs

COMPARED TO OTHER GLOBAL PDF SETS (MSTW08, CT10)

SINGLET SECTOR



NNPDF2.1 PDFs

COMPARED TO OTHER GLOBAL PDF SETS (MSTW08, CT10)

NONSINGLET SECTOR

TOTAL VALENCE ISOSPIN TRIPLET