On the Other Side of Asymptotic Freedom

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Colloquium

Quantum Chromodynamics

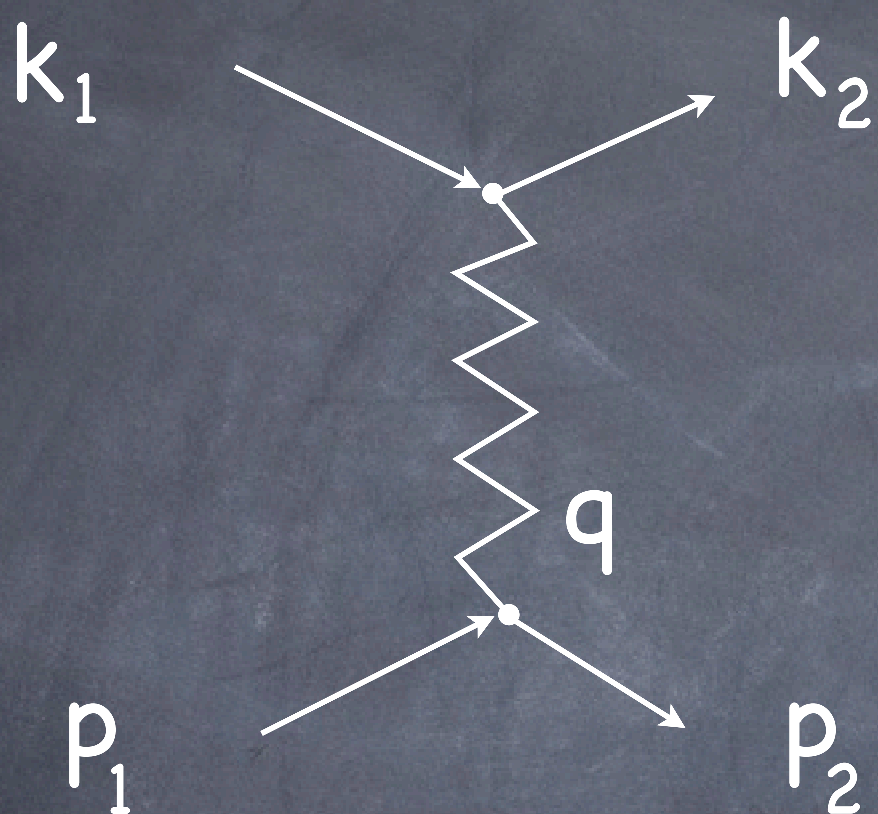
QCD

Harald Fritzsch & Murray Gell-Mann (1973)

is a relativistic
Quantum
Field
Theory

General Principles of a relativistic QFT
imply serious restrictions upon physical processes

scattering
annihilation



relativistic crossing

$s=(p_1+k_1)^2$ invariant energy

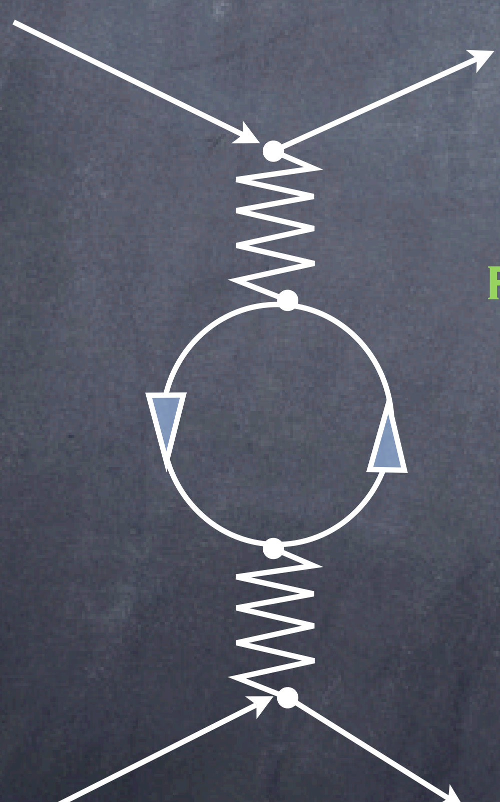
$t=(p_1-p_2)^2$ momentum transfer

one and the same amplitude as a function of its *invariants* A(s,t) describes three physically different processes related by *crossing*

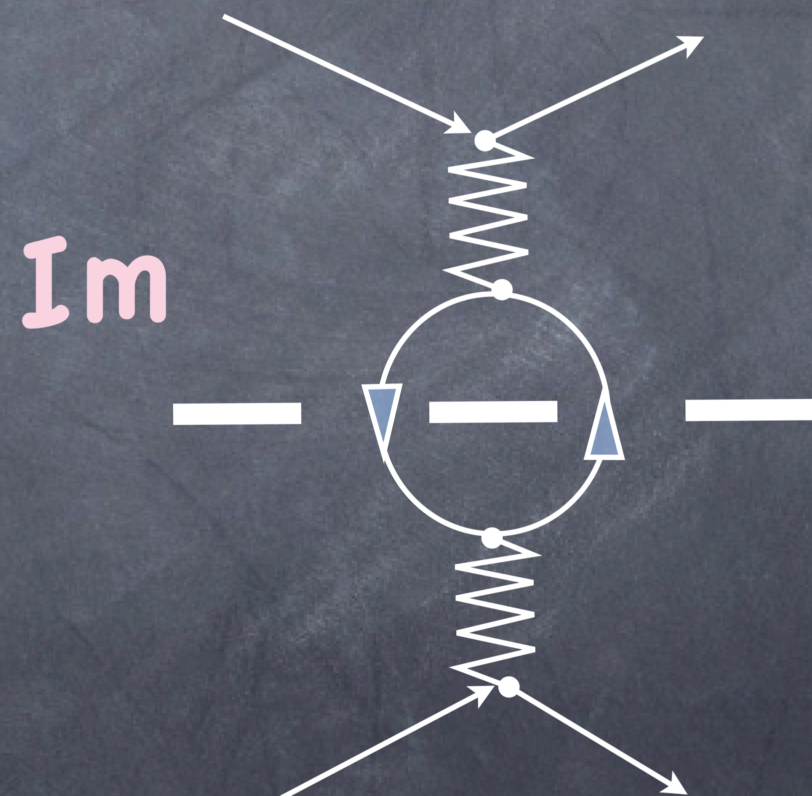
A(s,t) is an **analytic function** of energy s (causality) and of the momentum transfer t (crossing) whose **singularities** are determined by the unitarity as any symmetry,

the **crossing symmetry** has many a powerful, and sometimes dramatic, consequences

in particular, it is *crossing* and *unitarity* that made one think that the "*asymptotically free*" behavior of the effective coupling (***QCD***) is impossible



VACUUM POLARIZATION



Im A = BB*>0

a brief history of Asymptotic Freedom

1955

The polarization of **QED** vacuum makes the coupling run with virtuality $\alpha \to \alpha(\mathbf{k^2})$

Initial calculation of the fermion loop produced *a wrong sign* - a ***QCD**-ish* β-function

This ***error*** was not a ***mistake*** : it was worth making***!***

mistake:	smth. done wrongly, or smth. that should not have been done.

Longman Dictionary of Contemporary English

The time spanned *before B.Ioffe and A.Galanin have pointed at the error* proved to be enough for *L.Landau* and *I.Pomeranchuk* to develop and enthusiastically discuss with their pupils a beautiful physical picture of what we know now under the name of "***asymptotic freedom***".

Having corrected the ***error*** and having understood the physical origin of the *sign of the beta-function,* seemed to have been signing the death sentence for ***QFT*** in general...

Looked as general, inevitable property of any QFT ... (*Pomeranchuk*, 1955-58)

1958 Dyson : " *the correct meson theory will not be found in the next hundred years*"

1960 Landau : " *the Hamiltonian method for strong interactions is dead and must be buried, although of course with deserved honour* "

The fact that the vacuum fluctuations have to *screen* the external charge, seems to follows from first principles: *positiveness of probability in the cross-channel* (unitarity + Lorentz invariance + causality)

So, one expected the effective interaction strength to *increase at small distances* (large momenta) and *decrease at large distances* in any QFT...

Why then - and how - did this argument fail in the non-Abelian gauge field theory ?

Autopsy of Asymptotic Freedom

1969

I. Khriplovich : *the **SU(2)** Yang–Mills gauge theory coupling **disrespects** this wisdom !*

- To address a question starting from *what* or *why* we better talk *physical degrees of freedom*; use the Hamiltonian language

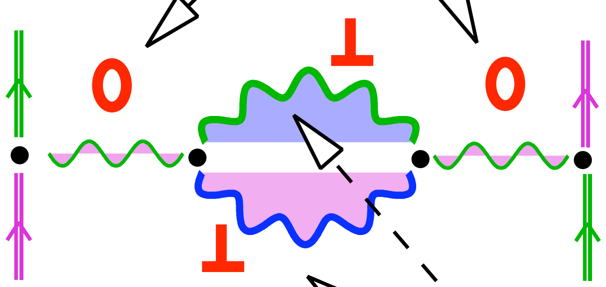
- Then, we have gluons of *two sorts*:

two "physical" transversely polarized gluons and

Coulomb gluon field - the mediator of the *instantaneous interaction* between colour charges.

Consider Coulomb interaction between two colour charges

Instantaneous Coulomb interaction

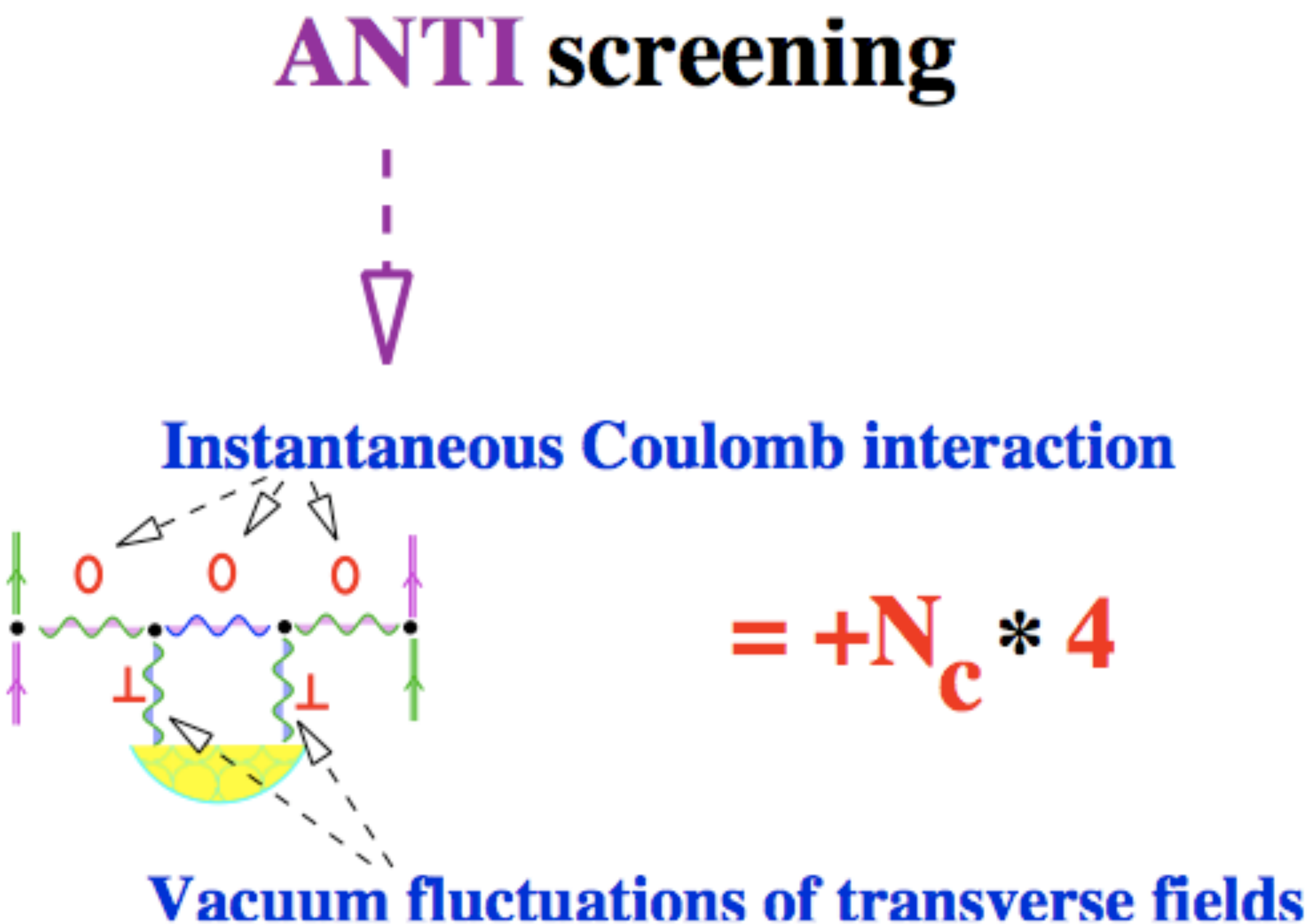


$$= -N_c * \frac{1}{3} \;\; -n_f * \frac{2}{3}$$

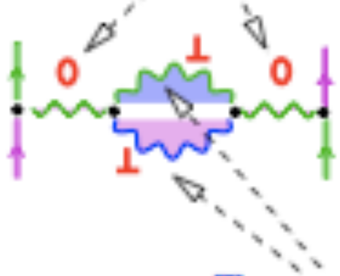
Transverse gluons **(and quarks)**

screening

Consider Coulomb interaction between two colour charges



putting together :



$$= -N_c * \frac{1}{3} \quad -n_f * \frac{2}{3}$$

ANTI screening

Khriplovich (1969)
Gribov (1976)

$$= +N_c * 4$$

Combine into the QCD β–function:

$$\beta(\alpha_s) = \frac{\mathrm{d}}{\mathrm{d}\ln Q^2} 4\pi\alpha_s^{-1}(Q^2)$$
$$= \left[4 - \frac{1}{3}\right] * N_c - \frac{2}{3} * n_f$$

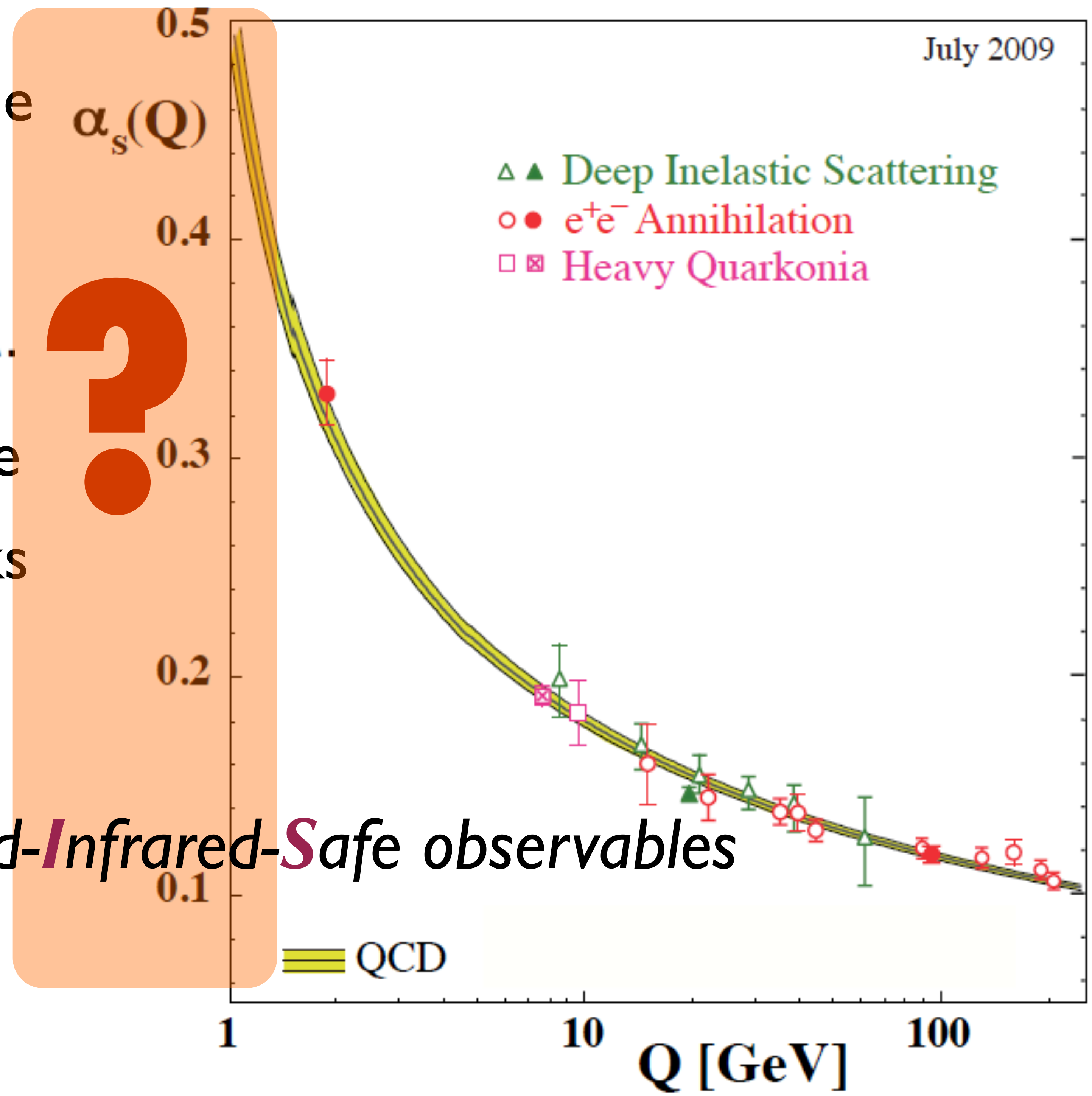
The origin of *antiscreening* — deepening of the ground state under the 2nd order perturbation in NQM:

$$\Delta E_0 = \sum_n \frac{|\langle 0|\delta V|n\rangle|^2}{E_0 - E_n} < 0.$$

Running QCD coupling

$$Q^2\frac{\partial\alpha_s}{\partial Q^2} = -b_0\alpha_s^2 \quad\Rightarrow\quad \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1+b_0\alpha_s(Q_0^2)\ln\frac{Q^2}{Q_0^2}} = \frac{1}{b_0\ln\frac{Q^2}{\Lambda^2}}$$

- Λ_{QCD} - the fundamental scale at which coupling *blows up*
- Perturbative calculations are valid for large scales $Q \gg \Lambda$.
- Not an obvious statement: we deal with hadrons in nature, while applying QCD to quarks and gluons
- *Animalistic Ideology* : some observables are more equal than the other : ***C****ollinear-and-****I****nfrared-****S****afe observables*



What becomes of the QCD coupling "on the other side" of AF ?..

It is not even clear whether this question makes sense, to start with...

*Can one make meaning of the **quark-gluon interaction strength** α_s at the distances where quarks and gluons are believed to seize to exist ?!..*

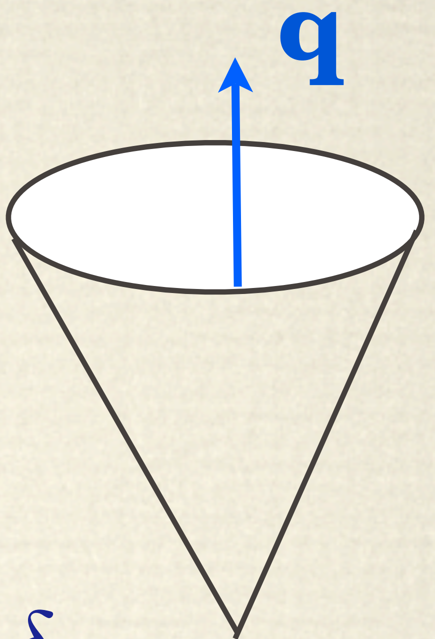
There is no known way to rigorously define the coupling beyond the PT-domain.

However, it is worth trying.

*For that we have to look into **Non-Perturbative physics**.*

*Not in the full-swing NP physics of hadrons and their interactions, but, timidly, into manifestation of **NP** effects in **PT**-calculable quantities - **CIS observables**.*

Collinear-and-Infrared-Safe observables



G.Sterman & S.Weinberg *(1979)*

Probability that all but a fixed (small) fraction ϵ of the energy of a quark-initiated jet flows into the cone of a fixed (small) opening angle δ.

$$w(\epsilon, \delta) \quad \simeq \; \exp\left\{-\frac{2C_F\alpha_s}{\pi}\ln\epsilon\ln\delta\right\}$$

CIS ideology :

if a cross section can be calculated, talking quarks and gluons, without encountering either collinear or infrared divergencies, the result of such PT calculation should be directly comparable with the corresponding cross section measurable experimentally (hadrons)

Screwing Non-Perturbative QCD with Perturbative Tools

About 15 years ago first theoretical attempts have been made to quantify genuine non-perturbative effects in perturbatively calculable (CIS) observables

The test case : the total cross section of e^+e^- annihilation into hadrons.

To predict $\sigma_{tot} \to$ hadrons one calculates instead the cross sections of ***quark*** and ***gluon*** production, $(e^+ e^- \to q\,\bar{q}) + (e^+ e^- \to q\,\bar{q} + g)$ + etc., where quarks and gluons are being treated ***perturbatively*** as real (un-confined, flying) objects.

The *completeness* argument provides an apology for such a brave substitution :

Once instantaneously produced by the electromagnetic (electroweak) current, the quarks (and secondary gluons) have nowhere else to go but to convert, ***with unit probability***, into hadrons in the end of the day.

This ***guess*** looks rather solid and sounds convincing,

but relies on ***two hidden assumptions*** *:*

1. The allowed hadron states should be ***numerous*** as to provide the quark-gluon system the means for "**regrouping**", "**blanching**", "**fitting**" into hadrons.

2. It implies that the "**production**" and "**hadronization**" stages of the process can be separated and treated independently.

1. To comply with the first assumption, the annihilation energy has to be large enough, $s \equiv Q^2 \gg s_0$. In particular, it fails miserably in the resonance region $Q^2 < s_0 \sim 2M^2_{res}$.

Thus, the point-by-point correspondence between hadron and quark cross sections, $\sigma^{tot}_{hadr}(Q^2)$? = $\sigma^{tot}_{qq}(Q^2)$, cannot be sustained except at very high energies.

It can be traded, however, for something more manageable.

Invoking the ***dispersion relation for the photon propagator*** (causality = analyticity) one can relate the *energy integrals* of $\sigma^{tot}(s)$ with the correlator of electromagnetic currents in a deeply Euclidean region of large *negative* Q^2.

The latter corresponds to ***small space-like distances*** between the interaction points, where the perturbative approach is definitely valid.

Expanding the answer in a formal series of ***local operators***, one arrives at the structure in which
a) the corrections to the trivial unit operator generate the usual perturbative series in powers of α_s (*logarithmic corrections*), whereas
b) the vacuum expectation values of dimension-full (Lorentz- and colour-invariant) QCD operators provide non-perturbative ***corrections suppressed as powers*** of Q.

This is the realm of the famous "**ITEP sum rules**" which proved to be successful in linking the parameters of the low-lying resonances in the ***Minkowski space*** with expectation values characterising a non-trivial structure of the QCD vacuum in the ***Euclidean space***.

ITEP "NP physics" **=** ***non-singular long range gluon fields***

Shifman, Vainstein & Zakharov *Nucl Phys B*. (1979)

The leaders among them are the gluon condensate $\langle\alpha_s G_{\mu\nu}G^{\mu\nu}\rangle$ and the quark condensate $\langle\psi\bar\psi\rangle$ $\langle\psi\bar\psi\rangle$ which contribute to the total annihilation cross section, symbolically, as

$$\sigma^{\rm tot}_{\rm hadr}(Q^2) - \sigma^{\rm tot}_{q\bar q+X}(Q^2) = c_1\frac{\langle\alpha_s G^2\rangle}{Q^4} + c_2\frac{\langle\psi\bar\psi\rangle^2}{Q^6} + \ldots$$

2. Validating the second assumption also calls for large $\mathbf{Q^2}$. To be able to separate the two stages of the process, it is *necessary* to have the production time of the quark pair $\mathbf{Q^{-1}}$ to be much smaller than the time $\mathbf{t_1 \sim \mu^{-1} \sim 1\ fm/c}$ when the first hadron appears in the system.

Whether this condition is *sufficient*, is another valid question. And a tricky one.

As we know, due to the ***gluon bremsstrahlung*** the perturbative production of secondary gluons and quark pairs spans an immense interval of time, ranging from a very short time, $\mathbf{t_{form} \sim Q^{-1} \ll t_1}$, all the way up to a macroscopically large time $\mathbf{t_{form} \sim Q/\mu^2 \gg t_1}$.

This accompanying radiation is responsible for formation of ***hadron jets***.

It does not, however, affect the ***total cross section***.
It is the rare hard gluons with large energies and transverse momenta, $\mathbf{\sim Q}$, that only matter.

This statement follows from the celebrated Bloch-Nordsieck theorem which states that the logarithmically enhanced (divergent) contributions due to real production of ***collinear*** and ***soft*** quanta cancel against the corresponding virtual corrections :

$$\sigma^{\text{tot}}_{q\bar{q}+X} = \sigma_{Born}\left(1 + \frac{\alpha_s}{\pi}\left[\infty_{\text{real}} - \infty_{\text{virtual}}\right] + \ldots\right) = \sigma_{Born}\left(1 + \frac{3}{4}\frac{C_F\alpha_s(Q^2)}{\pi} + \ldots\right)$$

Can the Bloch-Nordsieck result hold ***beyond*** perturbation theory?

Looking into this problem produced an extremely interesting result that has laid a foundation for the development of perturbative techniques aimed at analysing non-perturbative effects.

V. Braun, M. Beneke and V. Zakharov have demonstrated that the real-virtual cancellation actually proceeds ***much deeper*** than was originally expected.

[*Phys.Rev.Lett.* ***73*** *(1994) 3058*]

- Introduce into the calculation of the radiative correction ***gluon mass*** $\boldsymbol{m}$ as an IR cutoff.

- Study the dependence of the answer on $\boldsymbol{m}$.

  A CIS quantity, by definition, remains finite in the limit $\boldsymbol{m=0}$. This does not mean, however, that it is totally insensitive to the modification of the gluon propagation.

  In fact, the $\boldsymbol{m}$-dependence provides a handle for probing the *small transverse momenta* inside Feynman integrals. It is this region of integration over parton momenta where the QCD coupling gets out of control and the ***genuine NP physics*** comes onto the stage.

- Then, the sensitivity of a given CIS observable to the ***infrared domain*** is determined by the first non-vanishing term ***non-analytic*** in $\boldsymbol{m^2}$ at $\boldsymbol{m=0}$.

Bloch-Nordsieck theorem extended

In the case of one-loop analysis of the total annihilation cross section that we are discussing, one finds that in the sum of real and virtual contributions not only the terms singular at ***m=0***,

$$\ln^2 m^2 \quad \text{and} \quad \ln m^2 ,$$

cancel, *as required by the Bloch-Nordsieck theorem*,

but that the cancellation extends also to the whole tower of ***finite terms :***

$$m^2 \ln^2 m^2, \; m^2 \ln m^2, \; m^2, \; m^4 \ln^2 m^2, \; m^4 \ln m^2 .$$

The first ***non-analytic term*** appears at the level of m^6 :

$$\frac{3}{4}\frac{C_F \alpha_s}{\pi}\left(1 + 2\frac{m^6}{Q^6}\ln\frac{m^2}{Q^2} + \mathcal{O}\left(m^8\right)\right)$$

It signals the presence of the non-perturbative Q^{-6} correction, which is equivalent to that of the ITEP ***quark condensate***.

The ***gluon condensate*** contribution emerges in the next order in α_s

Why ***"gluon mass"*** ? What is it and why/how does it serve as a large-distance probe ?

wave function renormalization $\Longrightarrow$ ***running coupling***

$$G(k^2) = \frac{1}{-k^2 - i0} \Longrightarrow \frac{Z(k^2)}{-k^2 - i0} = \frac{\alpha_s(-k^2)}{-k^2 - i0} \qquad \frac{Z(-k^2)}{Z(-\mu_R^2)} = \frac{\alpha(k^2)}{\alpha(\mu_R^2)}$$

We want this identification to make sense in the entire k^2 plane

We know sufficiently well how α_s**($-k^2$)** behaves in the Euclidean region, at large negative **k^2** while we know next to nothing about the small-**k^2** region.

However, whatever the function α_s is, it had better respect ***causality***.

Therefore we suppress the formal PT *tachion* ("Landau singularity") and choose the "***physical cut***" alone, **$0<k^2$**, as a support for the **dispersive relation** :

$$\boxed{\alpha_s(q^2) = \int_0^\infty \frac{dm^2\, q^2}{(m^2+q^2)^2}\,\alpha_{\text{eff}}(m^2)}$$

$$\frac{d}{d\ln\mu^2}\alpha_{\text{eff}}(\mu^2) = -\frac{1}{2\pi i}\,\text{Disc}\left\{\alpha_s(-\mu^2)\right\}$$

It can be formally inverted as an operator relation :

$$\alpha_{\text{eff}}(\mu^2) = \frac{\sin(\pi\mathcal{P})}{\pi\mathcal{P}}\,\alpha_s(\mu^2)\,, \quad \mathcal{P} = \mu^2\frac{d}{d\mu^2}$$

We are ready now for a "***heavy gluon***" :

$$\boxed{\frac{\alpha_s(-k^2)}{-k^2-i0} = \int_0^\infty \frac{dm^2}{m^2}\,\alpha_{\text{eff}}(m^2)\cdot\frac{-d}{d\ln m^2}\frac{1}{m^2-k^2-i0}}$$

Born propagator of a "***massive gluon***"

Substitute into the Feynman diagram, and integrate first over the gluon 4-momentum ***K*** :

$$\boxed{V(Q^2,x) = \int_0^\infty \frac{dm^2}{m^2}\,\alpha_{\text{eff}}(m^2)\,\dot{\mathcal{F}}_V(\epsilon,x)}$$

$$\epsilon = m^2/Q^2 \qquad \dot{\mathcal{F}} \equiv \frac{-d\mathcal{F}}{d\ln\epsilon}$$

$\boldsymbol{\mathcal{F}_V}$ - a "***Characteristic Function***" for the observable ***V***

At this point there is no difference with the usual PT answer.

The CIS nature of the observable V guarantees convergence of the m^2 integration :

$\dot{\mathcal{F}}$ vanishes as a power of $\epsilon\ (\epsilon^{-1})$ in the $\epsilon \to 0\ (\epsilon \to \infty)$ limit.

Therefore the distribution $\dot{\mathcal{F}}$ ***has a maximum*** at some $\epsilon = C_V(x) = \mathcal{O}(1)$, and the integral is dominated by the large-momentum region $m^2 \sim Q^2$.

Approximating $\alpha_{\mathrm{eff}}(\mathbf{m^2}) \simeq \alpha_{\mathbf{s}}(\mathbf{Q^2})$ we reproduce the one-loop PT answer :

$$V(Q^2, x) = \int_0^\infty \frac{dm^2}{m^2}\ \alpha_{\mathrm{eff}}(m^2)\ \dot{\mathcal{F}}_V(\epsilon, x) \ \simeq\ \alpha_s(Q^2)\mathcal{F}_V(0, x) \qquad \dot{\mathcal{F}} \equiv \frac{-d\mathcal{F}}{d\ln\epsilon}$$

Using the *observable-dependent position of the maximum* of the m^2-distribution as the scale for the coupling, $\alpha_{\mathbf{s}}(\mathbf{C_V(x)\cdot Q^2})$, does a ***better job*** since it minimizes higher order effects.

The dispersive technology in this respect is close to the idea of "*commensurate scales*" (*Brodsky et al*).

"BLM scale fixing"

Can we account for higher order PT corrections ?

At the one loop level we may substitute $\alpha_{\rm eff}(\mathbf{m^2}) = \alpha_{\rm s}(\mathbf{m^2})$, develop the geometric series

$$\alpha_{\rm eff}(m^2) \simeq \alpha_s \sum_{k=0}^{\infty} \left(\frac{\beta_0 \alpha_s}{4\pi} \ln \frac{Q^2}{m^2} \right)^k, \quad \alpha_s \equiv \alpha_s(Q^2)$$

and look for higher order perturbative corrections to our observable :

$$V(Q^2, x) - \alpha_s(Q^2) \mathcal{F}_V(0, x) \simeq \alpha_s \sum_{k=1}^{\infty} \left(\frac{\beta_0 \alpha_s}{4\pi} \right)^k R_k \quad \text{with} \quad \boxed{R_k = \int_0^{\infty} \frac{d\epsilon}{\epsilon} \left(\ln \frac{1}{\epsilon} \right)^k \dot{\mathcal{F}}_V(\epsilon)}$$

In the IR region :

$$\dot{\mathcal{F}}_V(\epsilon) \simeq \epsilon^p \, f_V(\ln \epsilon) \Rightarrow R_k^{\rm IR} = \int_0^1 \frac{d\epsilon}{\epsilon} \left(\ln \frac{1}{\epsilon} \right)^k \epsilon^p \, f(\ln \epsilon) \sim \boxed{p^{-k}\, k!}$$

non-Borel--summable series !

Attempts to ascribe meaning to such a nasty series give rise to unphysical complex contributions at the level of $\mathbf{Q}^{-2p}$ terms : ***INFRARED RENORMALON*** problem.

This is generally interpreted as an intrinsic uncertainty of summing the perturbative series.

In fact, ***infrared renormalons*** are a purely ***perturbative*** phenomenon and have no direct relation to the presence of the "***Landau singularity***" in the running coupling **!**

The problem is of physical nature and cannot be resolved by formal mathematical manipulations alone.

It requires genuinely new physical input to obtain a sensible answer.

introducing the coupling "in the infrared"

Let's invent the representation

$$\alpha_s(k^2) = \alpha^{\mathrm{PT}}(k^2) + \alpha^{\mathrm{NP}}(k^2)$$

It should be made clear that such a splitting is symbolic :

it represents the coupling not in terms of two ***functions*** but rather of two ***procedures***.

Having met α^{PT} under the integral we are advised to calculate it ***perturbatively***, that is in terms of (*not too long*) a series at the point ***$k^2 \sim Q^2$*** that our integral is "sitting" around.
At the same time we are supposed not to worry about the PT-coupling being sick in the IR region.

On the contrary, integrals with α^{NP} are determined by that very same **IR** region and ***converge*** :

$$\int_0^\infty \frac{dk^2}{k^2}\,\alpha_s^{\mathrm{NP}}(k^2)\cdot k^{2p} \;=\; (\text{few 100s MeV})^{2p}$$

The ***ITEF picture***
of non-singular long-range
NP fields *(vacuum condensates)*

"smooth" NP fields and non-analyticity in m^2

$$\alpha_s^{\mathrm{NP}}(q^2) = \int_0^\infty \frac{dm^2\, q^2}{(m^2+q^2)^2}\, \alpha_{\mathrm{eff}}^{\mathrm{NP}}(m^2)$$

Convergence of the integrals of the NP coupling translates into vanishing of (*first few*) integer moments of α_{eff} :

$$\lim_{k\to\infty} k^{2p}\alpha^{\mathrm{NP}}(k^2) = 0 \iff \int_0^\infty \frac{dm^2}{m^2} m^{2p}\alpha_{\mathrm{eff}}^{\mathrm{NP}}(m^2) = 0$$

Within the *ITEF picture* this is the case at least for

$$\mathbf{p} \le \beta_0 \sim \mathbf{9}$$

(*small size **instantons***)

Does this imply that the new non-perturbative dimensional parameters won't emerge until $[m]^{18}$?..

No.

In order to get a nonzero answer, it suffices the ***NP part of the effective coupling*** to be integrated over m^2 with the weight (**characteristic function**) ***non-analytic*** in m^2 !

This explains the "mystery" of ***non-analyticity*** in m^2 *necessary to trigger on **large distances***

The non-analyticity of ϵ, necessary to generate NP power correction, is typically of two kinds.

- In the first case an ***integer*** power $\epsilon^{\mathbf{p}}$ is accompanied by ***logarithm*(s)** of ϵ .

  This is the case of ***DIS*** structure functions, the Drell-Yan "***K-factor***",
  the width of hadronic ***tau-lepton*** decay, the total ***e+e- annihilation*** cross section :

$$\lim_{m\to 0} \dot{\mathcal{F}}_{\mathrm{DIS}} = a(x) \cdot \frac{m^2}{Q^2} \ln \frac{Q^2}{m^2} \quad + \ldots$$

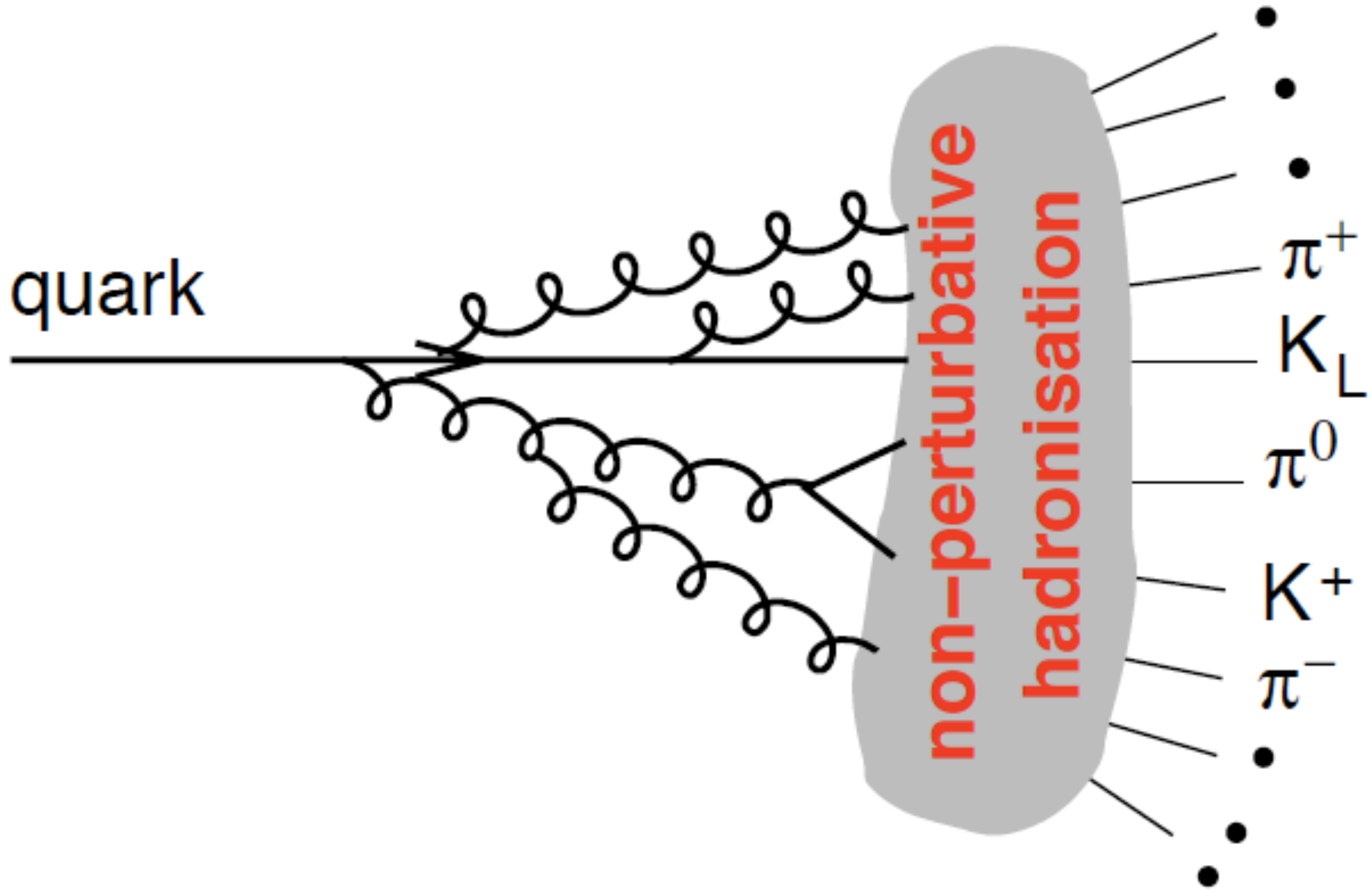
- Secondly, one may have a ***half-integer*** **p**.

  This is the case for many so-called ***jet-shape observables*** that characterise,
  in a CIS manner, the structure of final states produced in hard processes.

Thrust (***T***), invariant jet masses, ***C***-parameter, jet broadening (***B***), energy-energy correlation (***EEC***) etc. belong to the **p=1/2** class : they embody ***1/Q*** power effects due to confinement physics.

$$\lim_{m\to 0} \dot{\mathcal{F}}_{V} = a_V \cdot \sqrt{\frac{m^2}{Q^2}} \quad + \ldots$$

Parton fragmentation



Gluon emission:

$$\int \alpha_s \frac{dE}{E}\frac{d\theta}{\theta} \gg 1$$

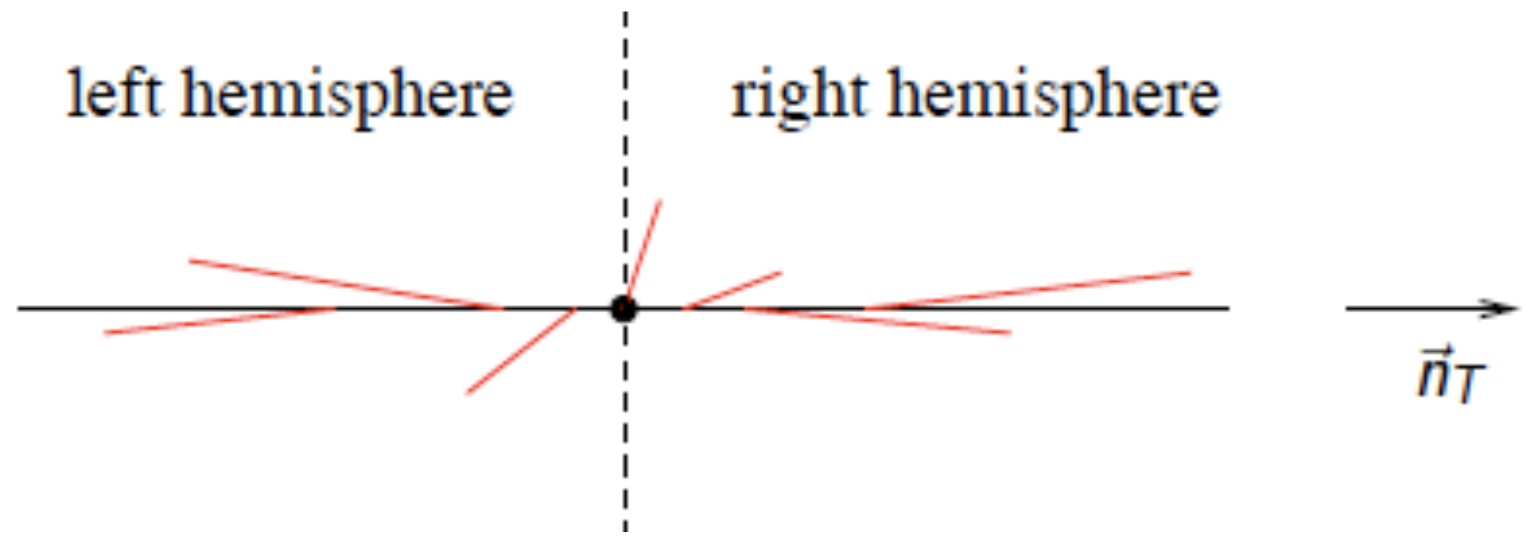
At low scales:

$$\alpha_s \to 1$$

This is a jet

(better be resilient to parton showering and hadronization)

jet shape observables



thrust $\quad T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$

jet-mass $\quad \rho = \frac{\left(\sum_{i\in\text{hemisphere}} p_i\right)^2}{\left(\sum_i E_i\right)^2}$

C-param. $\quad C = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$

broadening $\quad B_T = \frac{\sum_i p_{ti}}{\sum_i |\vec{p}_i|}$

All these are formally calculable in pQCD (**being collinear and infrared safe**)

but possess ***large non-perturbative 1/Q–suppressed corrections*** !

thrust

thrust $$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i . \vec{n}|}{\sum_i |\vec{p}_i|}$$

Two back-to-back particles produced in the c.m.s. of ***e+e-*** annihilation correspond to ***T=1***.

Thrust deviates from unity for two reasons.

One is PT gluon bremsstrahlung : $$\langle 1 - T \rangle^{(\mathrm{PT})} = \mathcal{O}(\alpha_s)$$

Another reason is pure hadronisation physics :

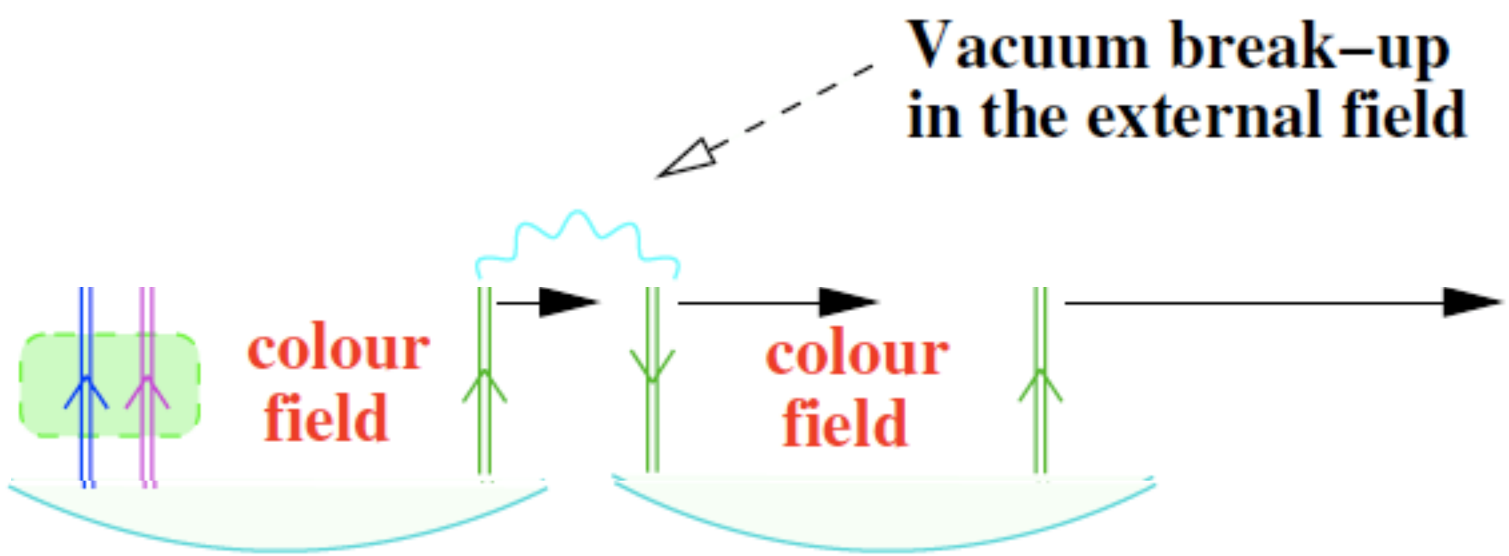
PT radiation switched off, two outgoing quarks are believed to produce two narrow jets of ***hadrons***

Lund model

Kogut–Susskind vacuum breaking picture

- In a DIS a *green* quark in the proton is hit by a virtual photon
- The quark leaves the stage and the colour field starts to build up

- A *green–anti-green* quark pair pops up from the vacuum, splitting the system into two globally blanched sub-systems.



"One" hadron per $\frac{\Delta\omega}{\omega}$; Hadron multiplicity $\propto \ln Q$.

thrust

Hadrons are uniformly distributed in rapidity and have limited transverse momenta with respect to the jet axis (Field-Feynman hot-dog, or "Lund string").

Take a simplified "tube model" with an exponential inclusive distribution of hadrons :

$$\frac{dN}{d\eta dk_\perp} = \mu^{-1}\,\vartheta(\eta_m - |\eta|)\,e^{-k_\perp/\mu}\,, \quad \mu = \langle k_\perp \rangle$$

Total energy :

$$\sum_i |\vec{p}_i| = 2\int d\eta\, dk_\perp \frac{dN}{d\eta dk_\perp} k_\perp \cosh\eta = 2\mu \sinh\eta_m = Q$$

z-momentum projections :

$$\sum_i |p_{zi}| = 2\int d\eta\, dk_\perp \frac{dN}{d\eta dk_\perp} k_\perp \sinh\eta = 2\mu(\cosh\eta_m - 1)$$

Constructing the ratio,

$$T = \frac{\cosh\eta_m - 1}{\sinh\eta_m} = 1 \boxed{- \frac{2\mu}{Q}} + \mathcal{O}\left(\frac{\mu^2}{Q^2}\right)$$

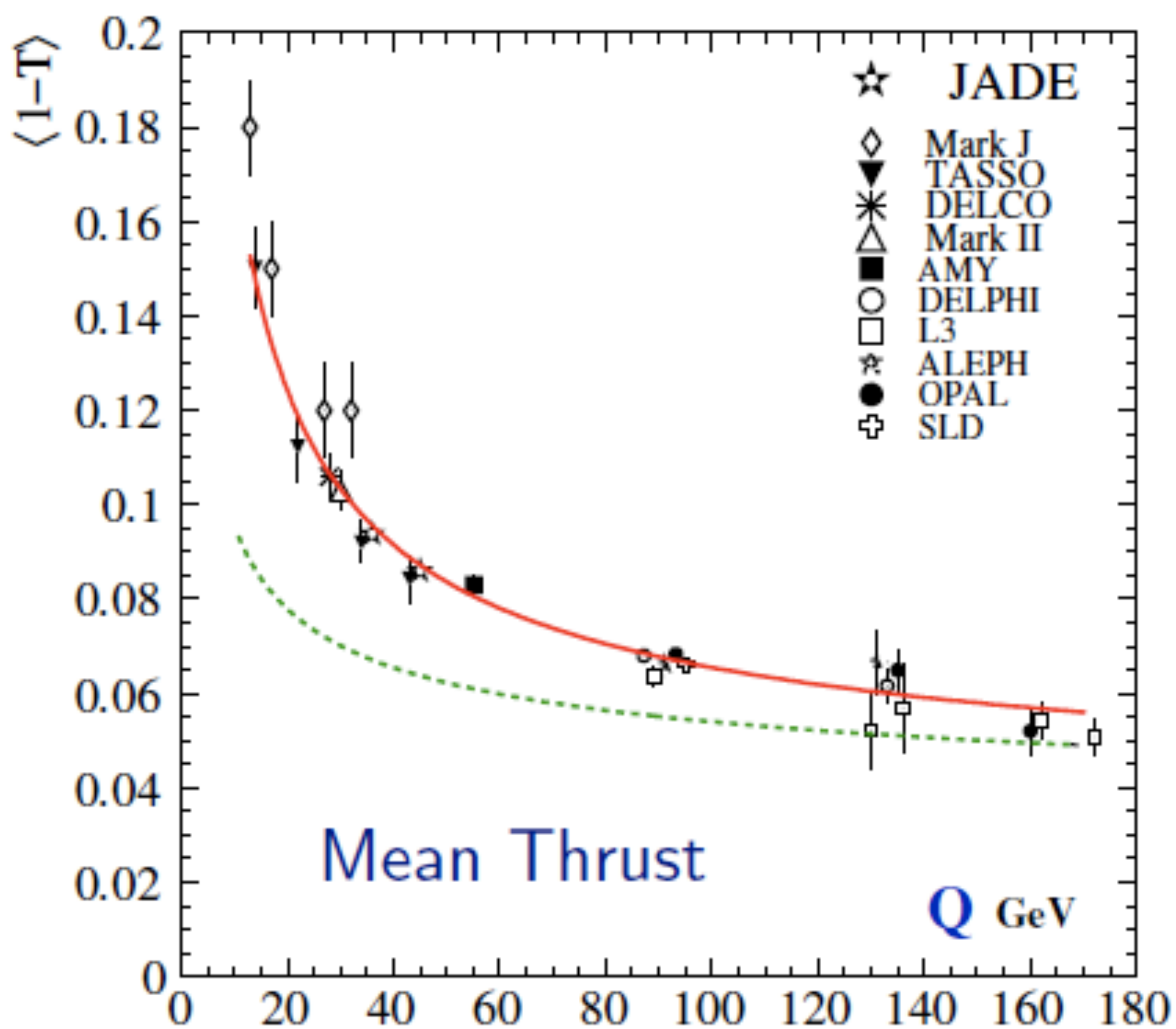
$$\boxed{\langle 1 - T\rangle^{(\mathrm{NP})} \simeq \frac{2\,\langle k_\perp\rangle}{Q}}$$

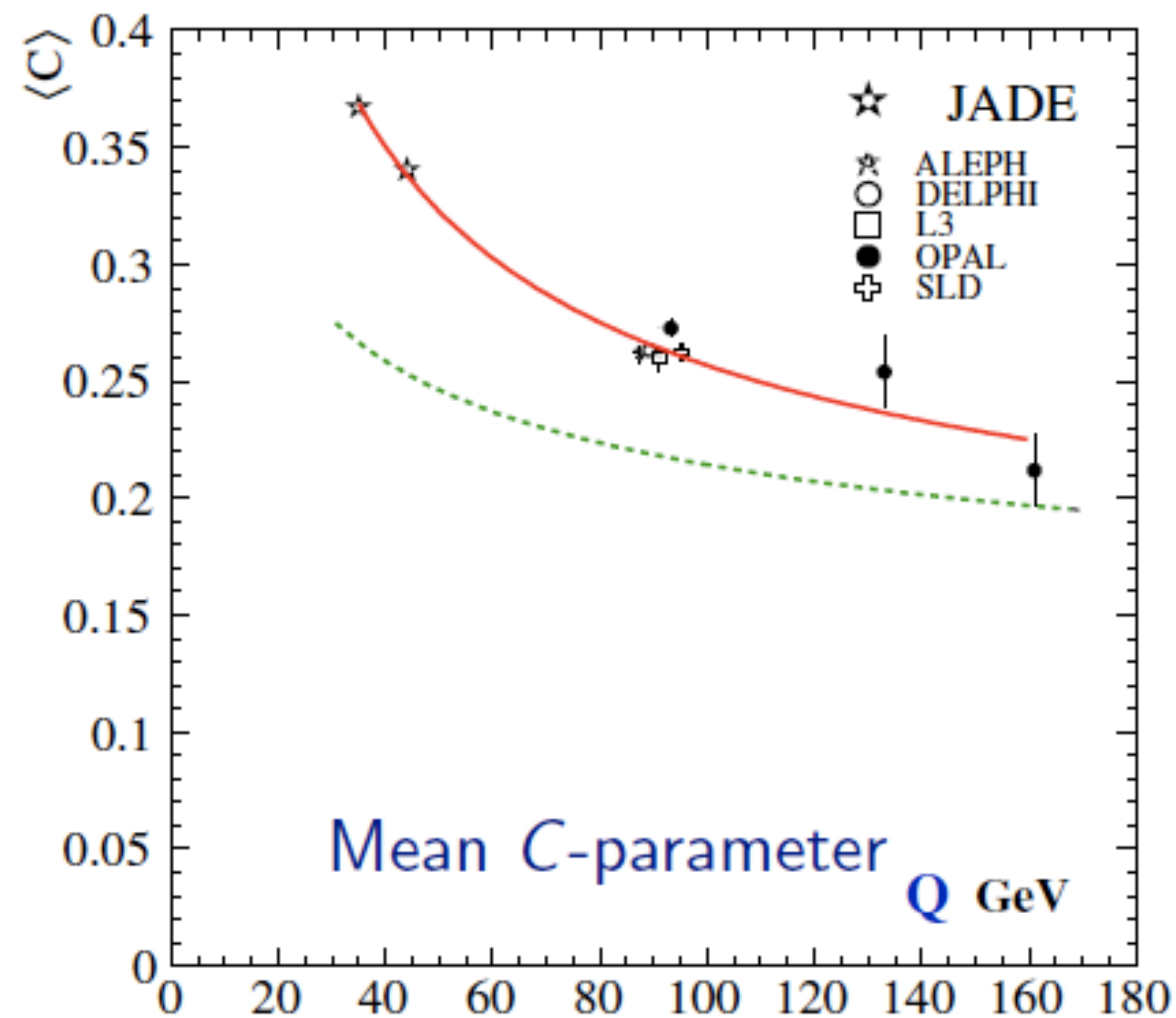
A negative ***1/Q*** ***hadronization correction !***

It is from the study of hadronization models that the ***1/Q*** effects first came into focus (Webber, *1995*)

Wise Dispersive Method B.R.Webber, et al, G.Marchesini (*1996*)

NP effects





$$\langle 1-T\rangle_{\mathrm{hadron}} \approx \langle 1-T\rangle_{\mathrm{parton}} + 1\ \mathrm{GeV}/Q$$

$$\langle C\rangle_{\mathrm{hadron}} \approx \langle C\rangle_{\mathrm{parton}} + 4\ \mathrm{GeV}/Q$$

WDM "theory" :

$$\left\{ \frac{4}{1} \Longrightarrow \frac{3\pi}{2} \right.$$

The PT approach normally would not provide us with such a ***dimensional*** parameter: gluon transverse momenta are broadly (*logarithmically*) distributed which results in the mean $\langle k_\perp \rangle \propto \alpha_s Q$

However now we have a PT-handle on the large-distance physics : the "***gluon-mass***" trigger.

Contribution to ***thrust*** from a single gluon with momentum ***k*** reads, in terms of Sudakov variables,

$$\delta(1-T) \;=\; \min\{\alpha,\beta\} \qquad\qquad k = \alpha p + \beta \bar{p} + \mathbf{k}_\perp ,$$

$$\mathcal{F}_T \simeq \frac{C_F}{\pi}\int \frac{d\alpha}{\alpha}\frac{d\beta}{\beta} dk_\perp^2\, \delta(\alpha\beta Q^2 - k_\perp^2 - m^2)\cdot \min\{\alpha,\beta\} \;=\; \frac{2C_F}{\pi Q}\int_0^{Q^2} \frac{dk_\perp^2}{\sqrt{k_\perp^2+m^2}}$$

Evaluating the logarithmic derivative of the characteristic function,

$$\dot{\mathcal{F}} \equiv -m^2 \frac{d\mathcal{F}}{dm^2} \simeq \frac{2C_F}{\pi}\frac{m}{Q}$$

Thus, for non-perturbative correction to ***mean values*** of jet shape observables we get the integral

$$\langle V \rangle^{\mathrm{NP}} = \frac{a_V}{Q}\cdot\frac{C_F}{2\pi}\int_0^\infty \frac{dm^2}{m^2}\sqrt{m^2}\,\alpha_{\mathrm{eff}}^{\mathrm{NP}}(m^2)$$

where the coefficients $\mathbf{a_V}$ are simple numbers having a clear geometric origin.

Parametrization of the answer in terms of the ***full coupling*** :

$$\int_0^{\mu_I} dk\, \alpha^{\mathrm{NP}}(k^2) = \int_0^{\mu_I} dk\, \alpha_s(k^2) - \int_0^{\mu_I} dk\, \alpha^{\mathrm{PT}}(k^2) = \mu_{\mathbf{I}}\cdot\left[\alpha_{\mathbf{0}} - \left(\alpha_{\mathbf{s}}(\mathbf{Q^2}) + \beta_{\mathbf{0}}\frac{\alpha_{\mathbf{s}}^{\mathbf{2}}(\mathbf{Q^2})}{\mathbf{2}\pi}\ln\frac{\mathbf{Q}}{\mu_{\mathbf{I}}} + \ldots\right)\right]$$

The characteristic non-perturbative parameter - the average of the coupling over the IR region :

$$\boxed{\alpha_0(\mu_I) \equiv \frac{1}{\mu_I}\int_0^{\mu_I} dk\, \alpha_s(k^2)}$$

Non-perturbative corrections to ***mean values*** of jet shapes $\boxed{\langle V\rangle = \langle V\rangle^{\mathrm{PT}}(\alpha_s) + a_V \cdot \mathcal{P}}$

with $$\mathcal{P} = \frac{4C_F\mathcal{M}}{\pi^2}\frac{\mu_I}{Q}\cdot\left\{\alpha_0(\mu_I) - \left[\alpha_s + \beta_0\frac{\alpha_s^2}{2\pi}\left(\ln\frac{Q}{\mu_I} + 1 + \frac{K}{\beta_0}\right) + \ldots\right]\right\}$$

the so-called "*Milan factor*" takes care of next-to-leading PT effects in the leading NP power correction

Interestingly, the same NP parameter enters the ***differential distributions*** of jet shapes :

$$\boxed{\frac{d\sigma}{dV}(V) = \frac{d\sigma^{(\mathrm{PT})}}{dV}(V - a_V\mathcal{P})}$$

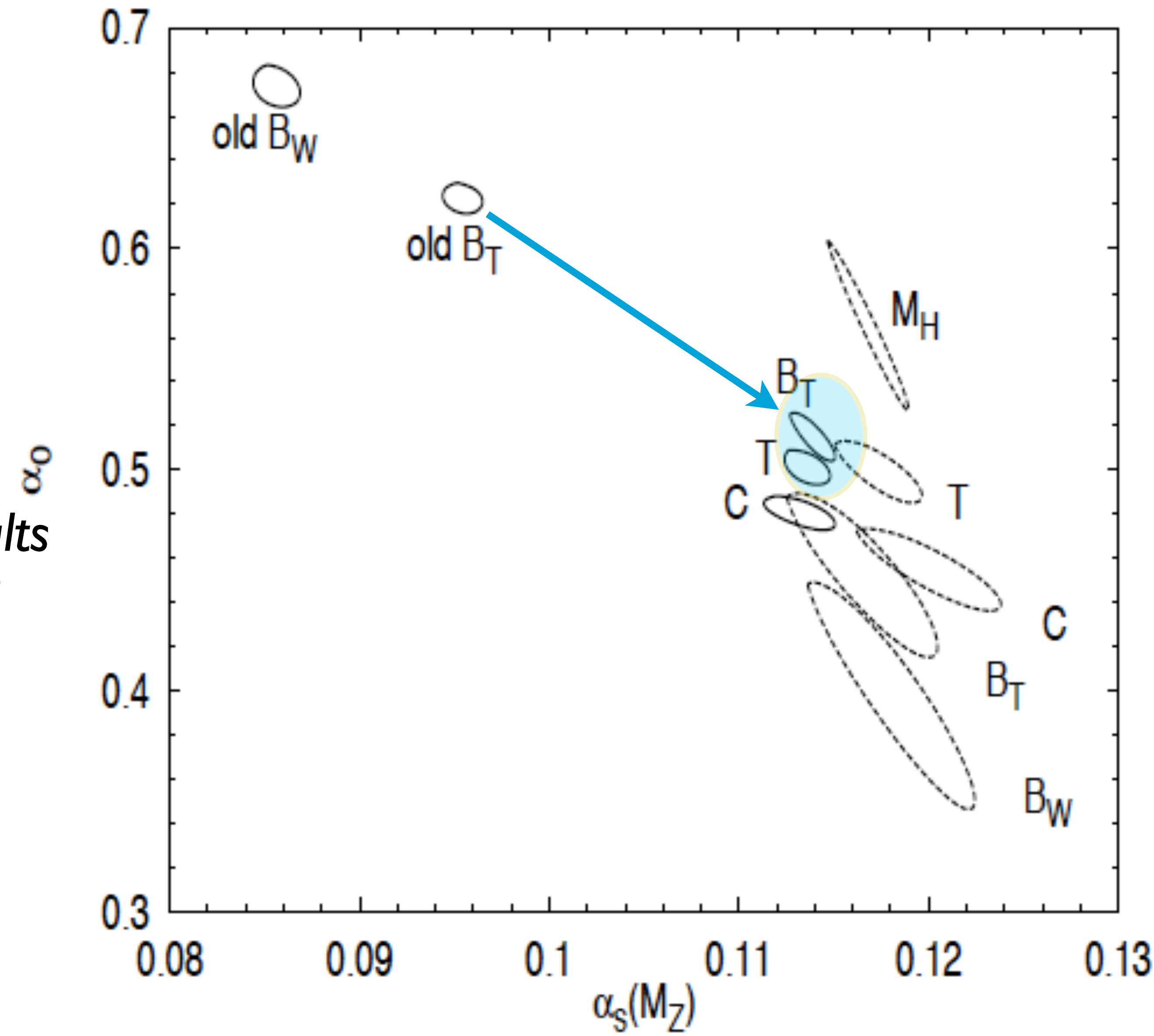
"*Power Corrections to Event Shape Distributions*"
B.R. Webber et al (*1997*)

Perturbatively calculable "*geometrical*" ***coefficients*** entering the jet shapes :

$V =$	$1-T$	C	M_T^2	M_H^2
$a_V =$	2	3π	2	1

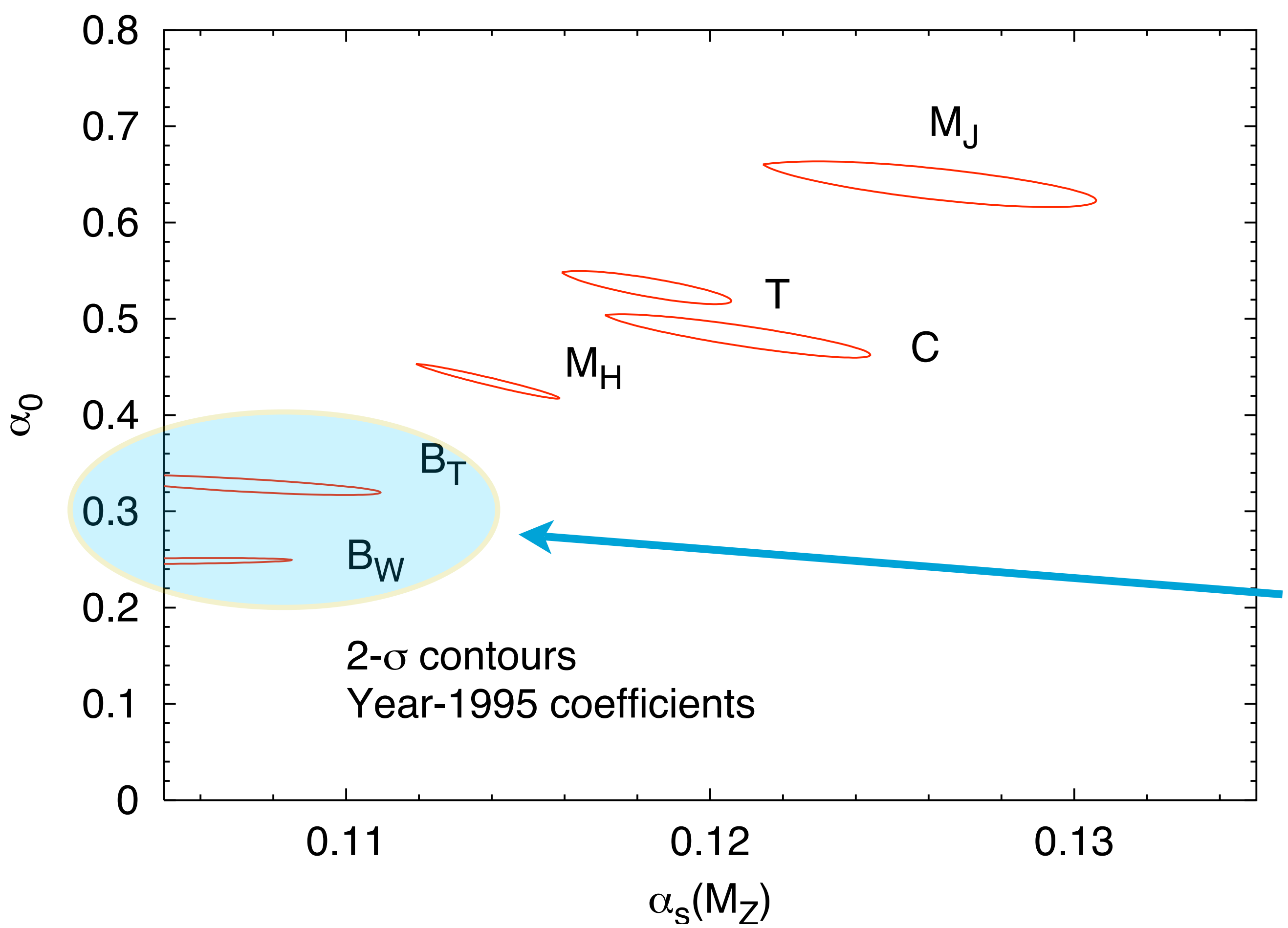
The phenomenology of power-suppressed contributions to jet shapes had a troubled childhood.
Only thrust and C-parameter remained unaffected by theoretical misconceptions...

By 1997 a ball-park value of α_0 ~0.5 was repeatedly emerging from the analyses of jet shapes in e+e- and DIS current jets in the Breit frame.



A typical resume ran like : "*The concept of a universal Power Correction parameter* α_0 *in DIS* ***ep*** *scattering and* ***e+e-*** *annihilation is supported*".

Montpellier 1998 QCD conference : "*inconsistent results for the total and wide-jet* ***broadening distributions***"



Correspondingly, the ***mean values*** of Broadenings (***T***otal and ***W***ide-jet) were found off-mark too...

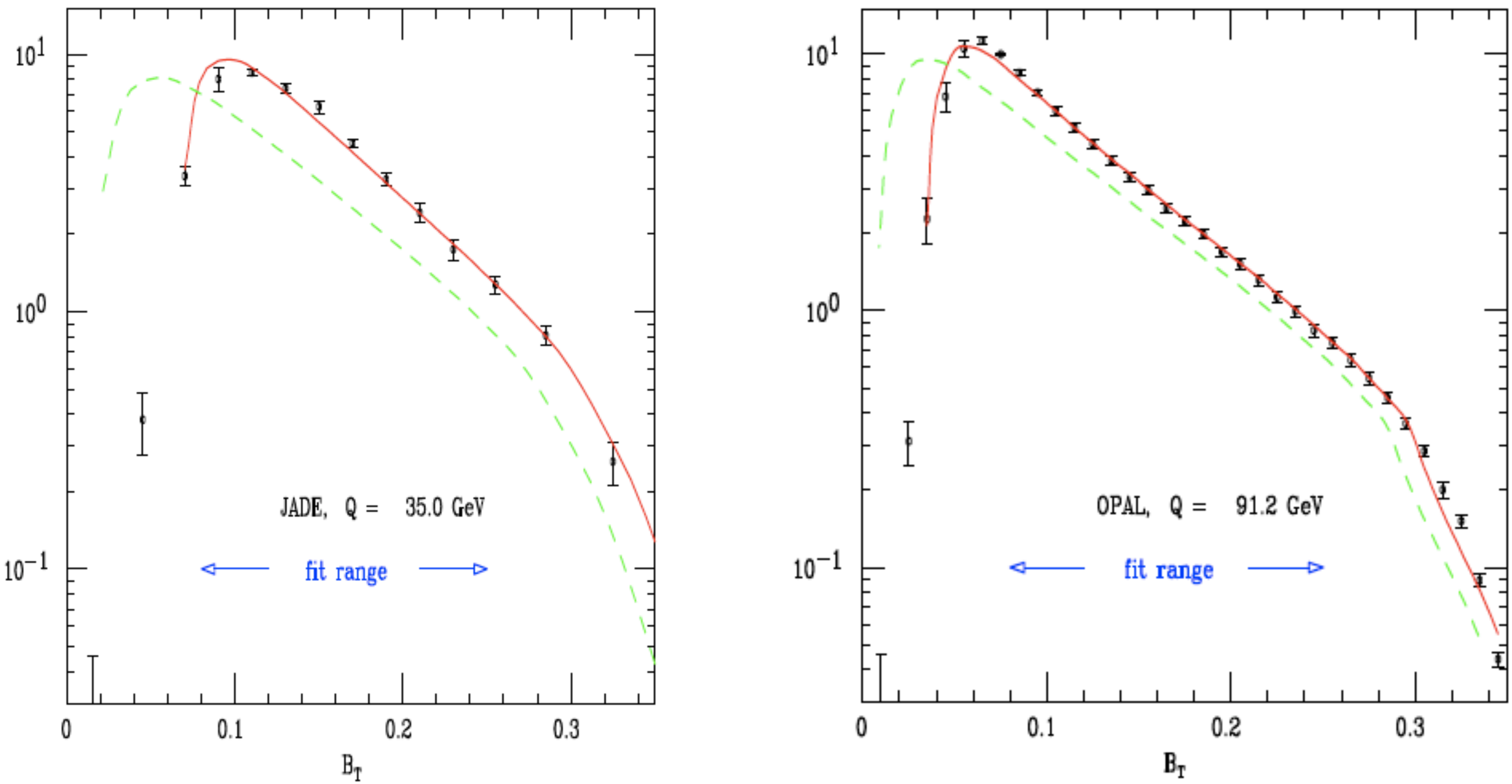
Theoretical revisiting of the Broadening measure taught an ***important lesson*** and has finally calmed the game ...

The broadening was put under scrutiny by the ***resurrected* JADE** collaboration.

Not only have they observed the discrepancy, but also have clarified what was going on !

They showed that hadronization effects in broadening not only ***shift*** the distribution to larger ***B*** values (as it is the case for ***1-T*** and ***C***) but also ***squeeze*** it. A bizarre observation !..

How can it be that when you ***smear*** the distribution
(moving from ***partons*** to ***hadrons***) it actually becomes ***sharper*** !?



NP-PT interplay

It was soon realized that one essential phenomenon was overlooked in the original NP-treatment of broadening, namely an ***interplay*** between ***NP*** and ***PT*** phenomena.

The effects produced by "gluers" in the presence of normal PT gluons are different from the effects of NP-radiation inferred from a pure first-order analysis, when the PT-radiation is "switched off".

The ***B*** distribution was found to have a rich structure exhibiting ***lnB/Q*** and **lnQ/Q** NP effects...

The simplest example : the story of the ***Jet Mass*** observables.

To trigger the NP-contribution we are advised to add to the parton system a ***soft gluer***.

When we do so at the Born level - add a ***gluer*** to the quark-antiquark system as the 3rd and only secondary parton - we find a 1/Q confinement contribution to the squared mass of the ***quark-gluer*** system : the "***heavy jet***". Meanwhile, the opposite "***lighter***" jet containing a lonely quark gets none :

$$\boxed{a_T = a_{M_T^2} = a_{M_H^2}\,, \quad a_{M_L^2} = 0}$$

$$\boxed{a_T = a_{M_T^2} = 2a_{M_H^2} = 2a_{M_L^2}}$$

There are always normal PT gluons in the game which are responsible for the bulk of the jet mass : it's not gluer's business to decide which jet is going to be *heavier*. Confinement effects are shared ***equally***.

Now we are ready to address the ***squeezed broadening*** issue.

The feature that ***1-T*** and ***C*** have in common is that the dominant **NP**-contribution is determined by radiation of gluers at ***large angles***. This radiation is insensitive to the tiny mismatch $\Theta_q = \mathcal{O}(\alpha_s)$ between the ***quark*** and ***thrust axis*** directions which is due to omnipresent **PT** gluon radiation.

Therefore the ***quark momentum direction*** can be identified with the ***thrust axis***.

the broadening escape

The broadening, on the contrary, accumulates contributions that do not depend on rapidity, so that the *mismatch* between the ***quark*** and the ***thrust*** axis matters both in the B-means and distributions.

Having naively assumed that the quark direction coincides with that of the thrust axis, ***B*** accumulated NP-contributions from ***gluers*** with rapidities up to $\eta_i \le \eta_{\max} \simeq \ln(Q/k_{ti})$.

In this case the shift in the B-spectrum would be logarithmically enhanced, $\boxed{\Delta_B = a_B \mathcal{P} \cdot \ln \frac{Q}{Q_B}}$

High-energy gluers are collinear to the ***quark*** rather than to the thrust axis and do not contribute to B.

$$\delta B_1^{(\mathrm{NP})} \simeq a_1 \mathcal{P} \left\langle \ln \frac{1}{\Theta_q} \right\rangle$$

As a result, the NP correction to ***B*** comes out proportional to the ***quark rapidity*** !

It is the quark Sudakov form factor that describes the distribution of relative *quark* - *jet axis* angles.

For ***mean values*** of ***B*** observables this yields

$$\left\langle \ln \frac{1}{\Theta_q} \right\rangle \simeq \frac{\pi}{2\sqrt{C_F \alpha_s(Q)}}$$

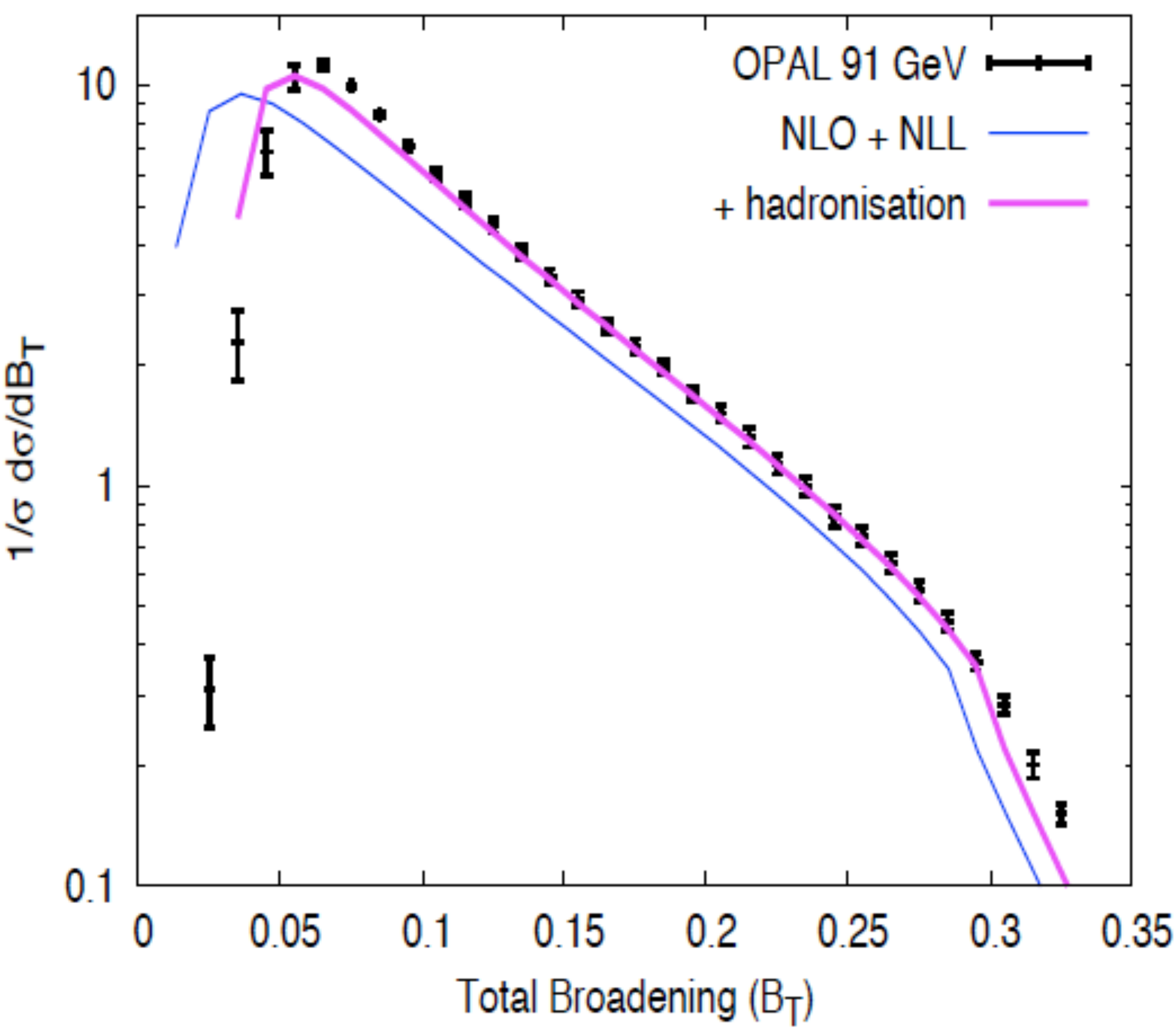
How about the ***distributions*** in ***B*** ***?***

The shift in the ***single jet*** (*wide jet*) broadening is evaluated by averaging over the perturbative distribution in the ***quark angle*** while keeping the value of ***B*** fixed.

Since Θ_q is kinematically proportional to ***B*** $\Rightarrow$ $\boxed{\Delta_1(B) \simeq a_1 \mathcal{P} \cdot \ln \frac{B_0}{B}}$

(the B_T distribution has a somewhat more intricate structure ...)

The smaller is ***B*** , the larger the non-perturbative shift : ***squeezing***

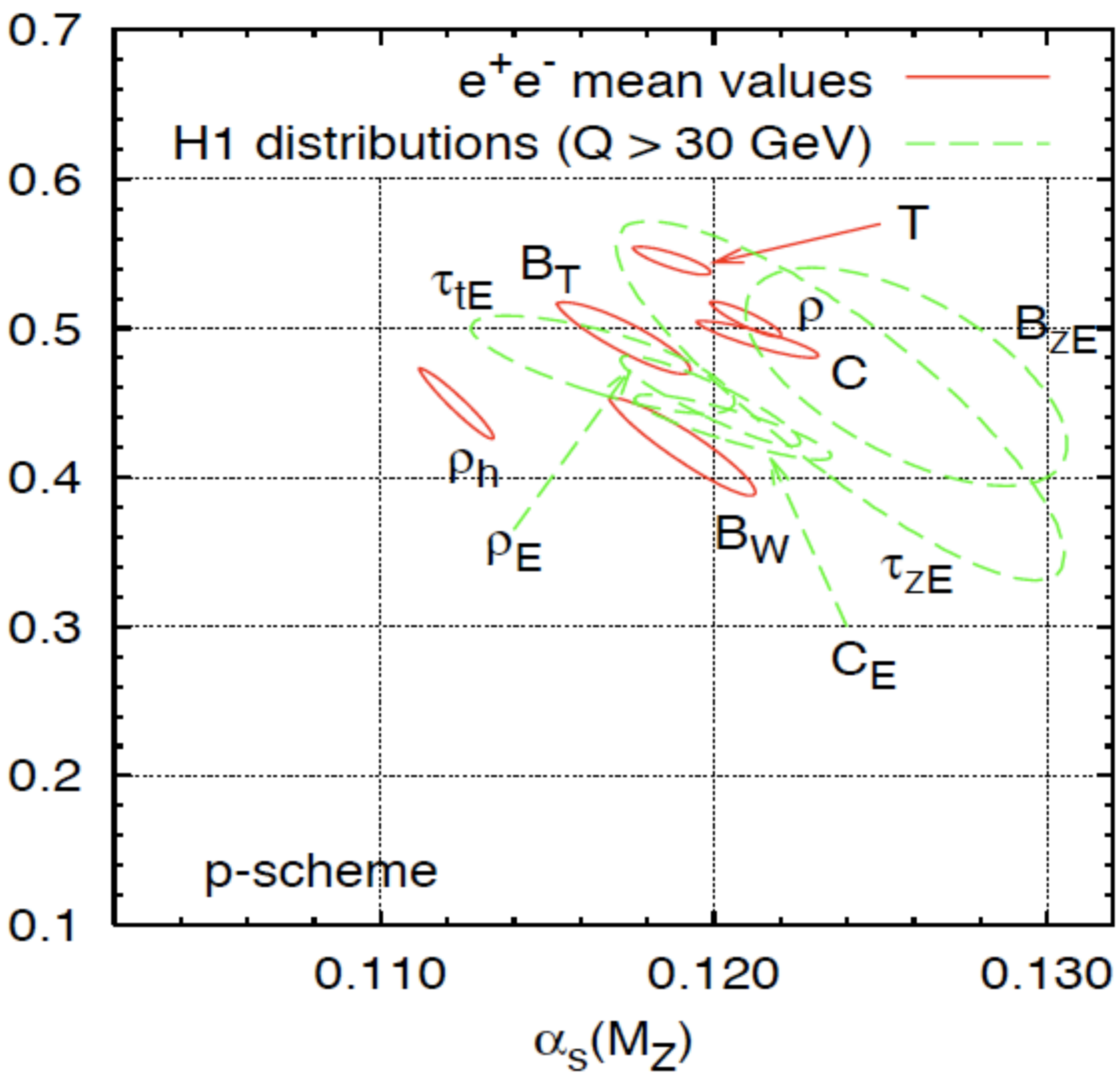


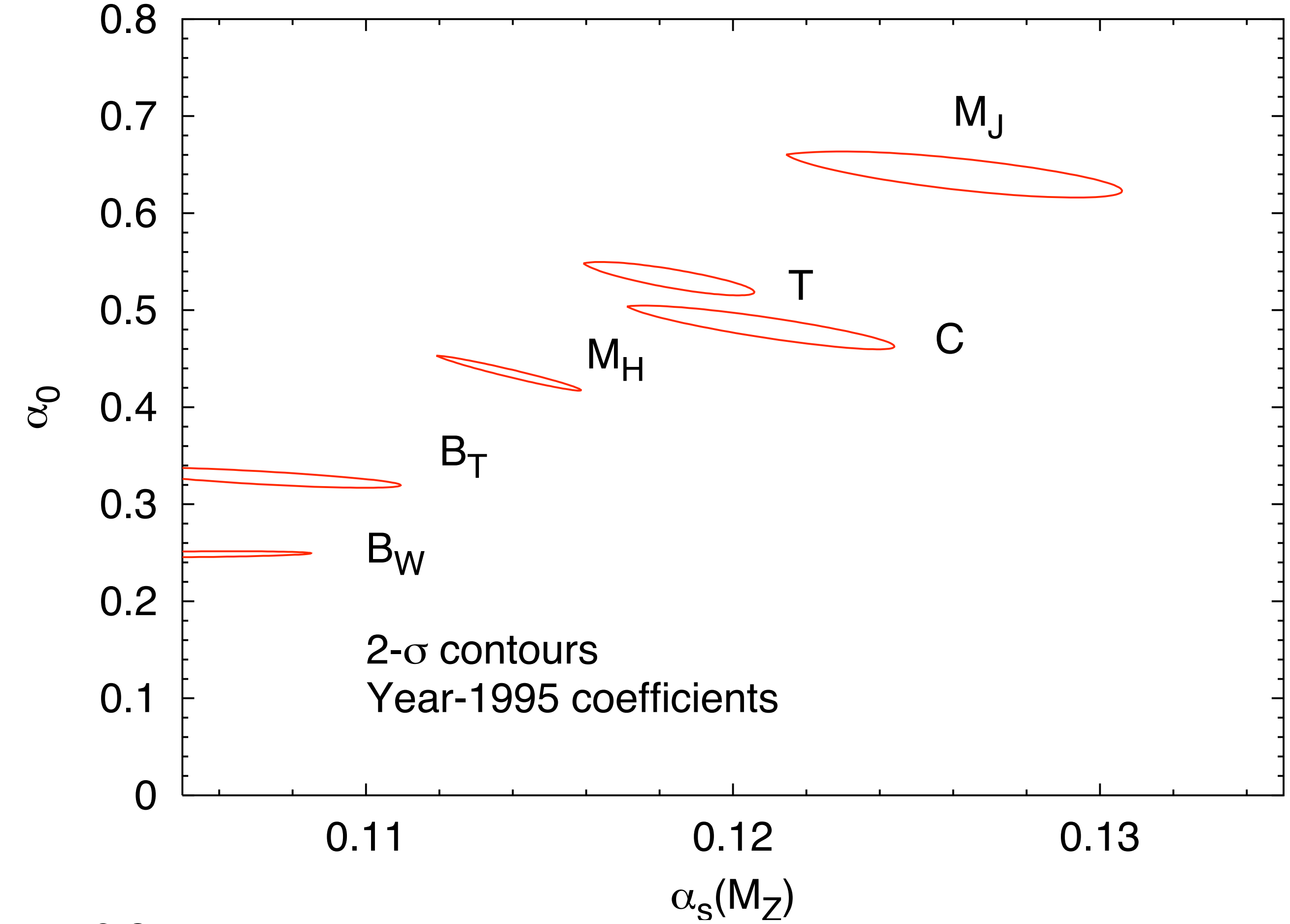
OPAL 91 GeV
NLO + NLL
+ hadronisation
$1/\sigma\ d\sigma/dB_T$
Total Broadening (B_T)
10
1
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

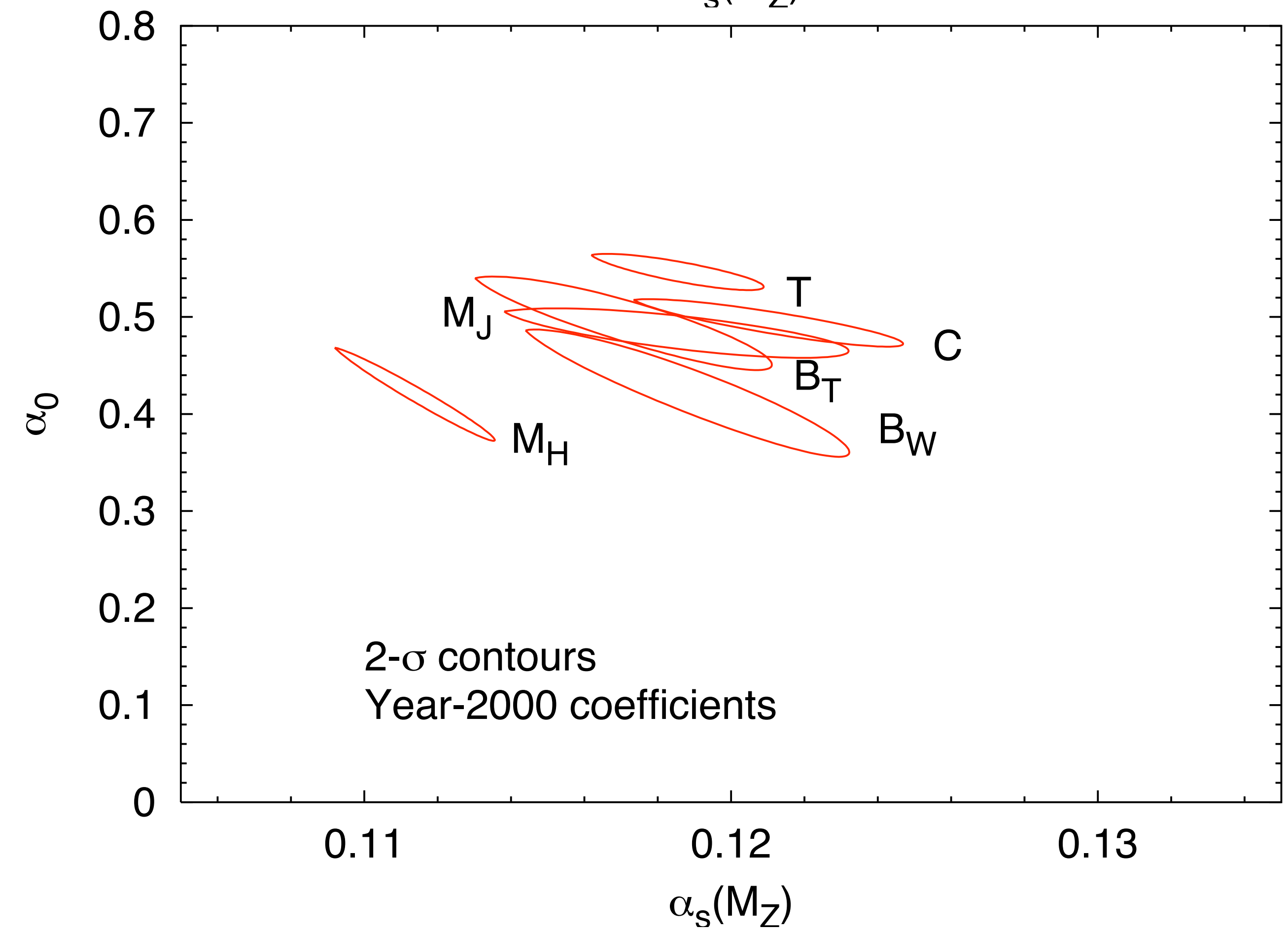
NP effects in jet shapes

After the mis-concepts and errors have been fixed, the ensemble of jet shape measurements clustered :

Moreover, both ***LEP*** (+ JADE) and ***HERA*** have contributed to the scrutiny and to the convergence







Theory + Phenomenology of 1/Q effects in event shape observables, both in e^+e^- annihilation and **DIS** systematically pointed at the ***average value*** *of the* ***infrared coupling***

$$\alpha_0 \equiv \frac{1}{2\ \mathrm{GeV}} \int_0^{2\ \mathrm{GeV}} dk\, \alpha_s(k^2) \sim 0.5$$

$\alpha_s = 0.1153 \pm 0.0017(\mathrm{exp}) \pm 0.0023(\mathrm{th})$
$\alpha_0 = 0.5132 \pm 0.0115(\mathrm{exp}) \pm 0.0381(\mathrm{th})$

T.Ghermann, M.Jaquier, G.Luisoni

The main features of this result are as follows : the average IR coupling is

- Universal — holds to within $\pm 15\%$

  If not for the ***universality***,
  the whole game would made no sense : it would have meant just trading ***one unknown*** - non-perturbative "smearing" effects in a given observable (like in MC event generators) - for ***another unknown*** function - the shape of the coupling in the infrared...

- Reasonably small (which opens intriguing possibilities . . .)
- Comfortably above the Gribov's critical value $(\pi \cdot 0.137 \simeq 0.4)$

QCD faith transition

?

Can we use the quark-gluon language at "large distances" ?

***Strong interaction* = *Gluon exchange* ?**

Two examples :

"**Hard interactions**" in "**Soft**" kinematical domain

"quark counting rules"

"**Soft**" physics

minimum bias (soft) hadron scattering cross sections

hadron production in hadron scattering and in jets

"Hard" Physics: Scaling in exclusive reactions

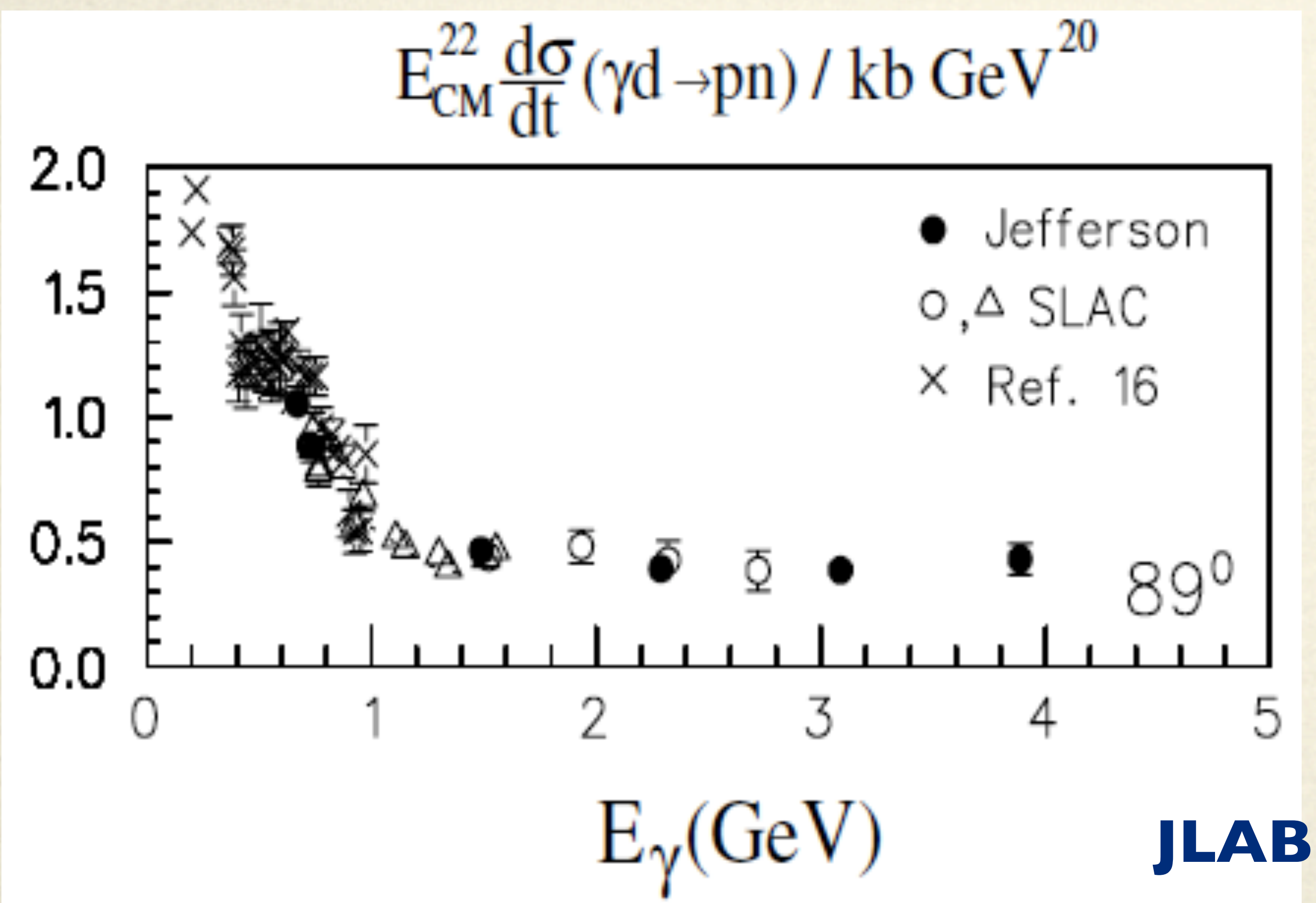
Dimensional counting *("quark counting rules")*

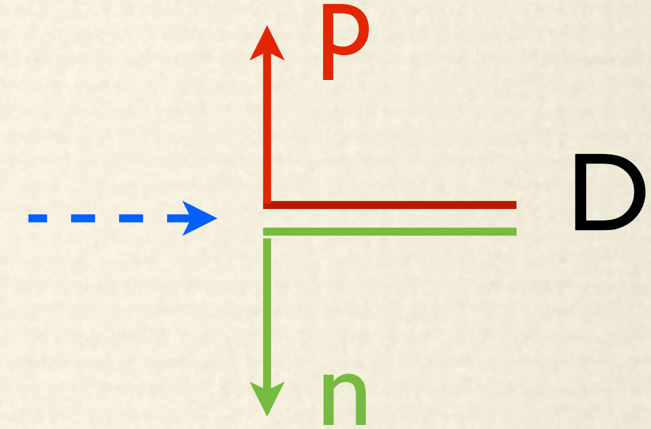
large angle scattering in the high energy / momentum transfer regime

$$\boxed{\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \qquad \frac{t}{s} = \text{const}}$$

***K** the number of participating elementary fields (quarks, leptons, intermediate bosons, etc)*

Example *: deuteron break-up by a photon,* $\gamma + D \longrightarrow p + n$





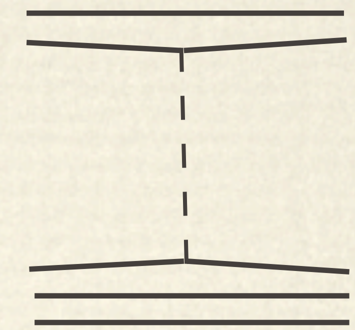
***K** = 1+6 + 6 = 13*

it is very difficult to digest how the naive asymptotic regime settles that early !..

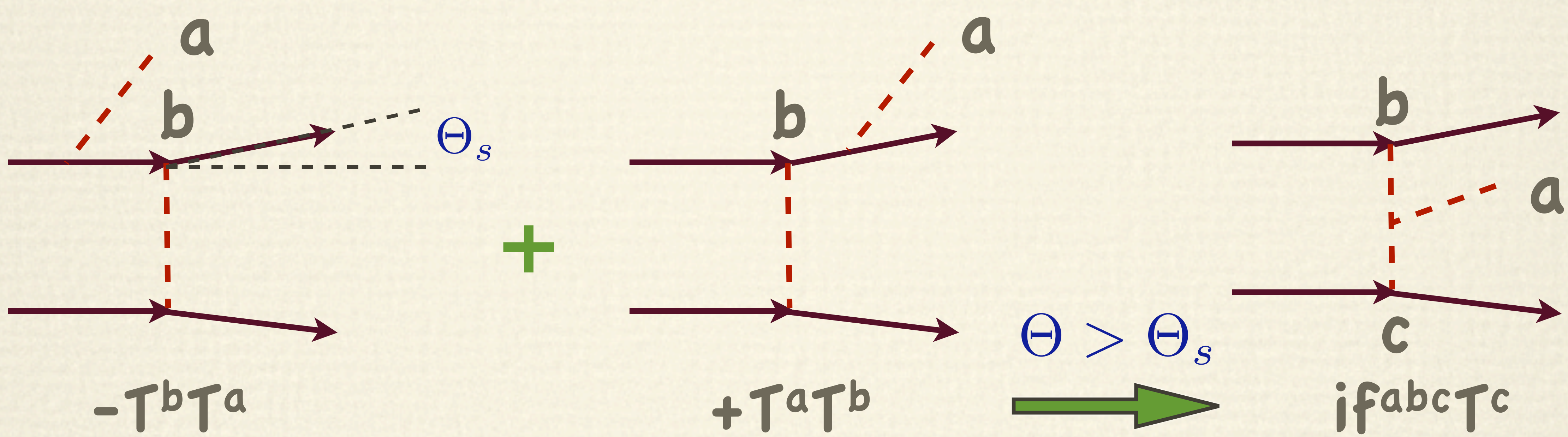
$$d\sigma \sim \alpha_s^{10}(q^2/N)$$

Soft Physics: Hadron high energy scattering

Cross section $\sigma_{tot} \propto s^{2(J-1)}$



Particle production



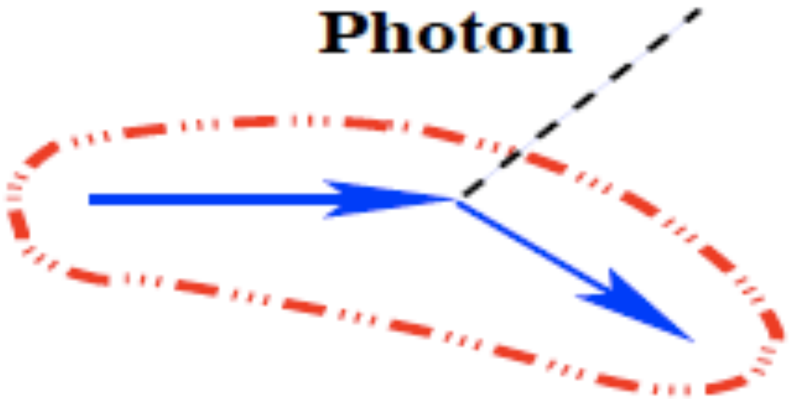
Accompanying radiation depends on the t-channel color exchange but not on the nature of colliding objects !

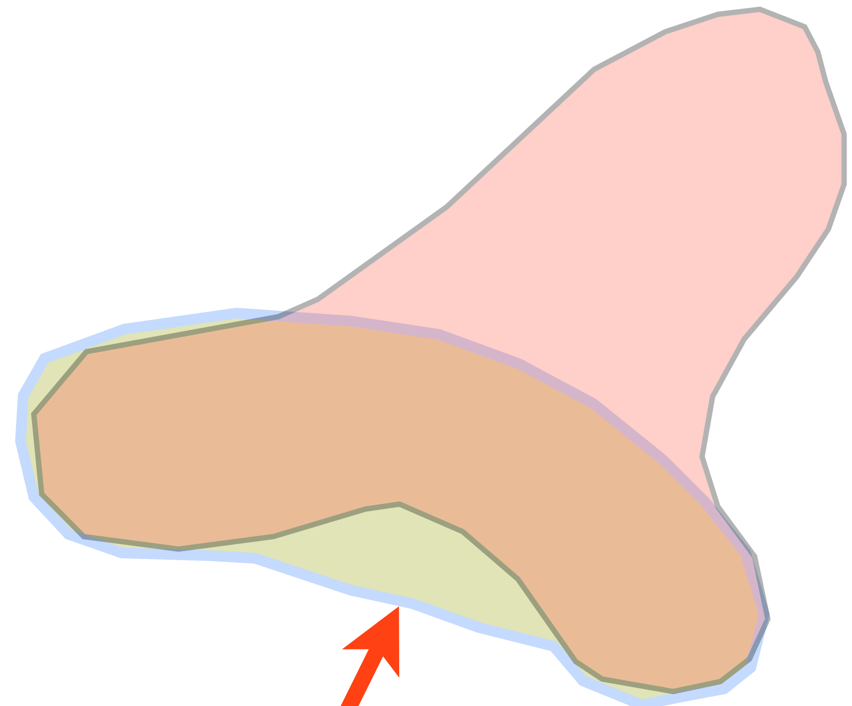
$$dn \propto \frac{d\omega}{\omega}$$

hadron "plateau"

Such universality - in the language of the *Gribov-Regge theory of high energy hadron interactions* - is known under the name of ***Pomeron***

Soft Physics: hadron production *in-between* jets





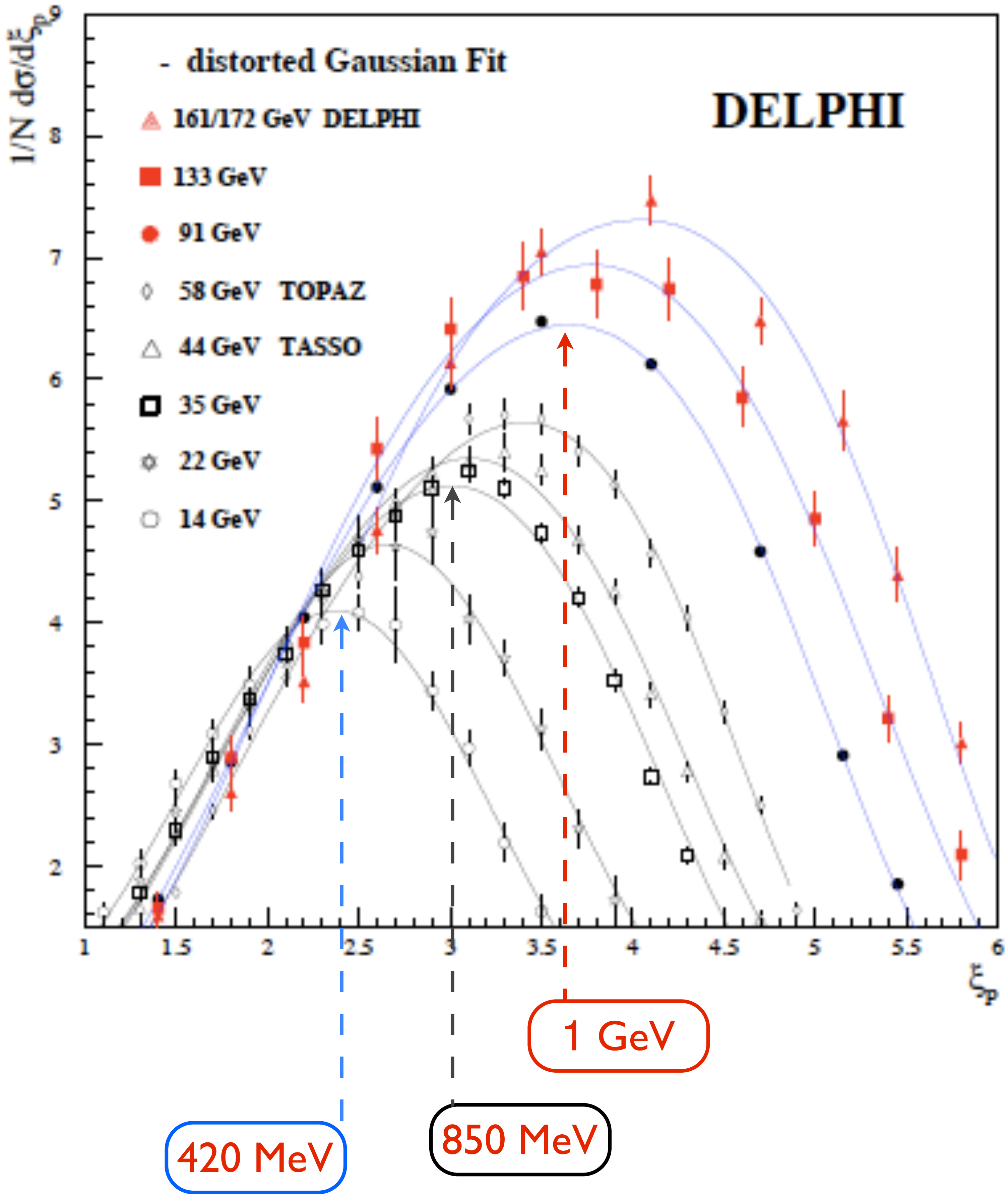
(experiment: 2.3 ± 0.2)

The overlay results in a magnificent "*String effect*"
depletion of particle production in the $q\bar{q}$ valley
Destructive interference from the QCD point of view

$$\frac{dN_{q\bar{q}}^{(q\bar{q}\gamma)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{2(N_c^2-1)}{N_c^2-2} = \frac{16}{7}$$

On one hand, a robust pQCD prediction - asymptotic prediction (!)
In reality - sheer madness : particle flows = pions below 1 GeV

Soft Physics: hadron production *inside* jets



$$\ln \frac{1}{x}$$

punchline

- pQCD, talking *quarks* and *gluons*, did the job it has been asked to perform
  - to measure quark and gluon spins
  - to establish $SU_c(3)$ as the true QCD gauge group
  - to verify Asymptotic Freedom.

- Moreover, comparing theoretical predictions concerning multiplication of partons, with production of hadrons in jets,
  - inclusive energy spectra of (relatively soft) hadrons INSIDE Jets, and
  - soft hadron multiplicity flows IN-BETWEEN Jets

  taught us an important lesson, or rather are sending us a hint, about non-violent nature of hadronization – "*Soft Confinement*".

- First semi-quantitative understanding of the geniune Non-Perturbative physics of the Hard–Soft Interface has been gained.

QCD *is about to undergo a* faith transition

QCD practitioners prepare themselves - slowly but steadily - to start using, in earnest, the language of ***quarks*** and ***gluons*** down into the region of small characteristic momenta - "large distances"

Unusual analytic properties of ***quark*** and ***gluon*** Green functions will take responsibility for what we refer to as "***colour confinement***".

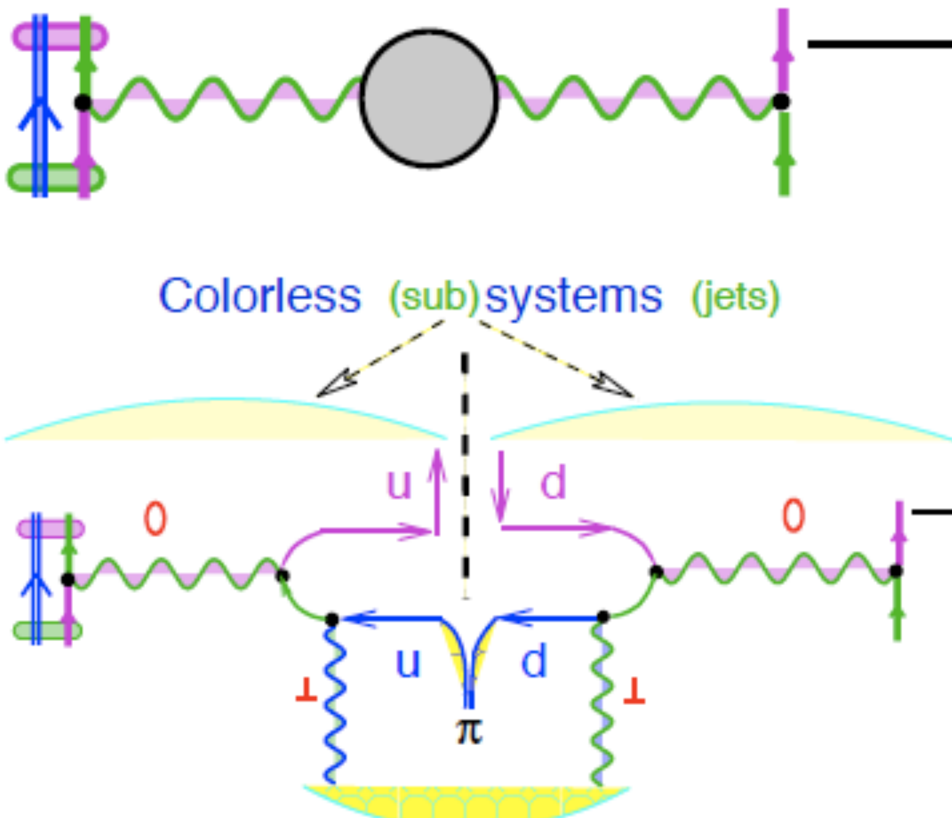
Gribov supercritical quark confinement scenario implies all above and demands the QCD coupling in the infrared to exceed

$$\frac{\alpha_{\mathrm{crit}}}{\pi} = C_F^{-1}\left[1-\sqrt{\frac{2}{3}}\right] \simeq 0.137$$

One can well expect that in ***n*** years from now (*with* $n = \mathcal{O}(1)$) participants of Munich alpha_s meetings will be discussing the accuracy of α_s determination at scales of **1** GeV and below

EXTRAS

What happens with the Coulomb field when the sources move apart?



Bearing in mind that virtual quarks live in the background of gluons (zero fluctuations of $A_\perp$ gluon fields) what we look for is a mechanism for binding (negative energy) *vacuum quarks* into *colorless hadrons* (positive energy physical states of the theory)

V.Gribov suggested such a mechanism — the supercritical binding of light fermions subject to a Coulomb-like interaction. It develops when the coupling constant hits a definite "critical value" (Gribov 1990)

$$\boxed{\frac{\alpha}{\pi} > \frac{\alpha_{\rm crit}}{\pi} = C_F^{-1}\left[1-\sqrt{\frac{2}{3}}\right] \simeq 0.137}$$

$$\left(C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}\right)$$

An amazing success of the relativistic theory of electron and photon fields — quantum electrodynamics (QED) — has produced a long-lasting negative impact: it taught the generations of physicists that came into the business in/after the 70's to "not to worry".

Indeed, today one takes a lot of things for granted :

- One rarely questions whether the alternative roads to constructing QFT — *secondary quantization*, *functional integral* and the *Feynman diagram* approach — really lead to the ***same quantum theory*** of interacting fields

- One feels ashamed to doubt an elegant powerful, *but potentially deceiving,* technology of translating the dynamics of quantum fields into that of statistical systems

- One takes the original concept of the "***Dirac sea***" — the picture of the fermionic content of the vacuum — as an anachronistic model

- One was taught to look upon the problems that arise with field-theoretical description of point-like objects and their interactions at very small distances (*ultraviolet divergences*) as purely technical : ***renormalize it and forget it***.

QED : physical objets — ***electrons*** and ***photons*** — are in *one-to-one correspondence* with the fundamental fields that one puts into the local Lagrangian of the theory.

The role of the QED Vacuum is "trivial": it makes e.m. charge (and the electron mass operator) run, but does not affect the nature of the interacting fields.

QCD : the Vacuum changes the bare fields ***beyond recognition*** ...

- The question of interest is

  the confinement in real world (with 2 very light ***u*** and ***d*** quarks), rather than ***a*** confinement.

- No mechanism for binding massless bosons (gluons) seems to exist in Quantum Field Theory (QFT), while the Pauli exclusion principle may provide means for binding together massless fermions (light quarks).

- The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently ***infrared-unstable dynamics*** : the ***ultraviolet*** and ***infrared*** regimes of the theory may be tightly linked.

- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects.

  Feynman's famous $i\epsilon$ prescription was designed for (*and applies only to*) quantum field theories with ***stable perturbative vacua***.

- To understand and describe a physical process in a ***confining theory***, it is necessary to take into consideration the ***response of the vacuum***, which leads to essential modifications of the quark and gluon Green functions.

A known QFT example of such a violent response of the vacuum — screening of ***super-charged ions*** with ***Z > 137***.

The expression for Dirac energy levels of an electron in a field created by the point-like electric charge Z contains $\epsilon \propto \sqrt{1-(\alpha_{\mathrm{e.m.}}Z)^2}$.

For $Z > 137$ the energy becomes *complex*. This means instability.

- Classically, the electron "*falls onto the centre*".
- Quantum-mechanically, it also "*falls*", but into the Dirac sea.

$$A_Z \Longrightarrow A_{Z-1} + e^+ , \qquad \text{for } Z > Z_{\text{crit.}} \qquad \text{(Pomeranchuk \& Smorodinsky 1945)}$$

In the QCD context, the increase of the running quark-gluon coupling at large distances replaces the large Z of the QED problem.

Gribov generalized the problem of supercritical binding in the field of an ***infinitely heavy source*** to the case of two ***massless fermions*** interacting via Coulomb-like exchange.

He found that in this case the supercritical phenomenon develops much earlier.

Namely, a pair of light fermions develops supercritical behaviour if the coupling hits a definite critical value

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = 1 - \sqrt{\frac{2}{3}}$$

With account of the QCD colour Casimir operator, the value of the coupling above which restructuring of the perturbative vacuum leads to ***chiral symmetry breaking*** and, *likely,* to ***confinement*** , translates into

$$\boxed{\frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1}\left[1 - \sqrt{\frac{2}{3}}\right] \simeq 0.137}$$