The QCD Coupling from Lattice QCD

G. Peter Lepage
(for HPQCD)
Cornell University

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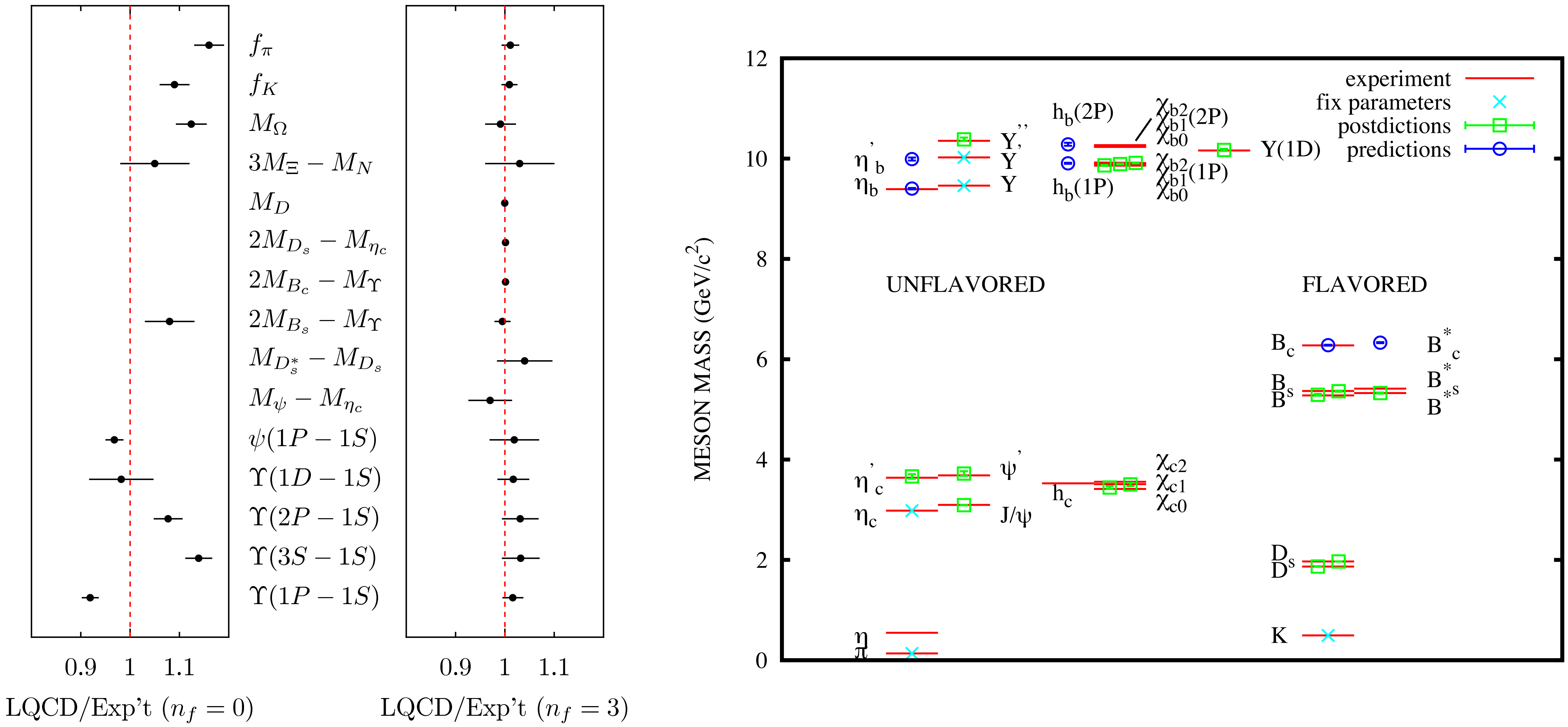
Lattice QCD

LQCD = QCD ± couple % (or better)



Plus extensive theoretical/numerical studies: e.g., spectrum of lattice Dirac operator (rooting, zero modes, chirality...)

α_s from Lattice QCD

1. Tune five free parameters – bare m_u=m_d , m_s , m_c , m_b and α_s – using $m(\pi)$, $m(K)$, $m(\eta_c)$, $m(\Upsilon)$ and $2f_K$–f_π (or $\Delta E_\Upsilon(2S\text{-}1S)$...)
2. Evaluate vacuum expectation values of numerous operators, using Monte Carlo path-integration, for multiple values of lattice spacing a and $m_{u,d}$ (and lattice volume). Extrapolate to physical values so that LQCD = QCD. (No free parameters!)
3. "Measure" short-distance quantities $Y^{(i)}$ (nonperturbatively) in simulation. Extract coupling α_s by comparing with perturbative expansions:

$$Y^{(i)} = \sum_{n=1}^{\infty} c_n^{(i)} \alpha_s^n(q^{*(i)})$$

Optimize Operators — Why so accurate?

- Euclidean short-distance: No need for KLN/Weinberg theorem or analytic continuation; no hadronization corrections (no on-shell hadrons); no collinear logs or Sudakov form factors (single scale q^*).
- Easy to measure: negligible statistical errors.
- Easy to analyze: *e.g.*, modeling non-perturbative effects (always included in fits).
- Many quantities to test modeling of non-perturbative and other systematic effects.
- Vary q^* over large range, varying $\alpha_s(q^*)$, to fit for 4th and higher order coefficients. (Needs high precision at all q^*s.)

Wilson Loops

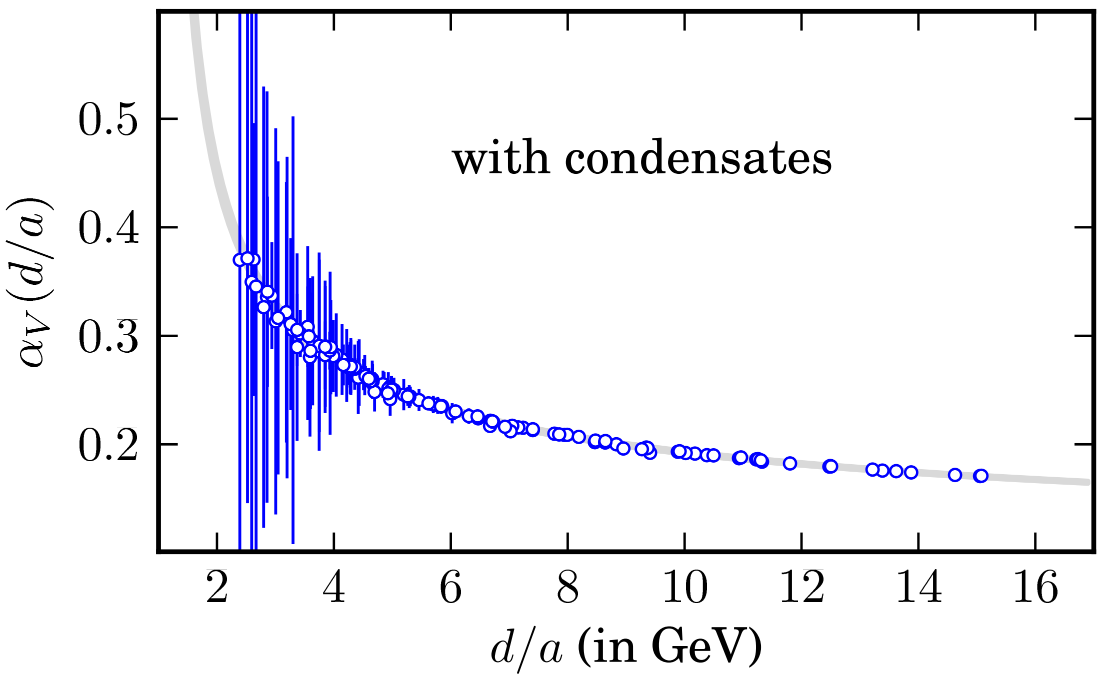
Small Wilson Loops

Simplest short-distance quantities to simulate are *ma* x *na* Wilson loops for small integer values of *m* and *n*:

$$\log W_{mn} \equiv \log \tfrac{1}{3}\langle 0|\mathrm{Re\,Tr\,P\,e}^{-ig\oint_{mn} A\cdot dx}|0\rangle$$

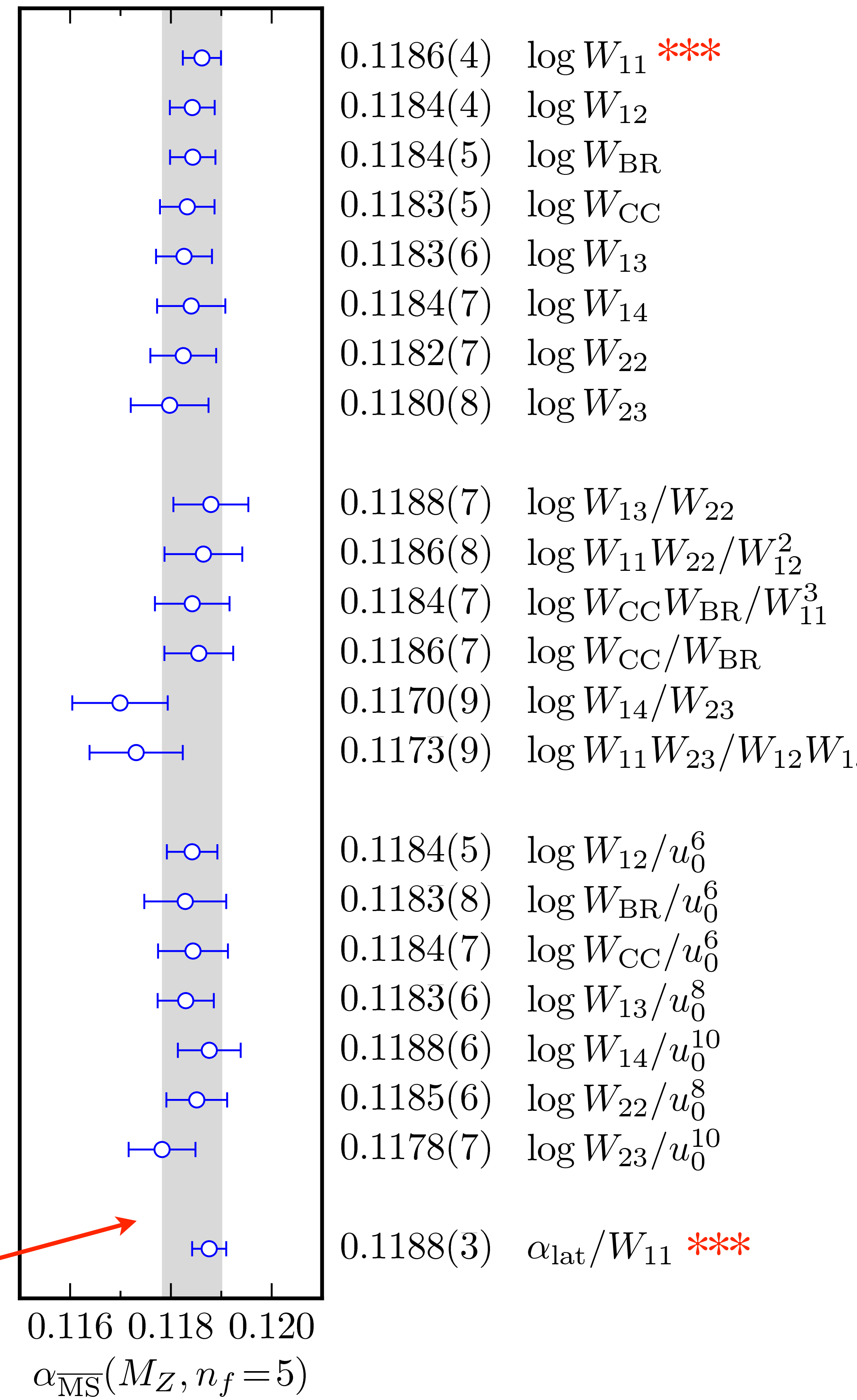
Note:

- Extremely accurate to simulate (5-digits): examined 22 quantities, each independently.
- UV divergent, so most perturbative quantities (with smallest α_s) on lattice: *q**s proportional to 1/*a* for fixed *m*,*n* and of order π/*a*.
- Computed perturbation theory using lattice QCD regulator so lattice artifacts included to all orders in *a*. Known to 3rd order in α_s (hard!).
- Varied π/*a* — six values from 3.5 to 14 GeV — to fit 4th and higher orders. (Possible because extremely accurate.)



- 264 different α_Vs.
- Know 3 terms in pert'n theory; allow for 10 in all (only 4 needed).
- Use BLM/LM scale $q^*=d/a$ with α_V.
- n_f=3. Convert to MS-bar and evolve to M_Z n_f=5 using continuum pert'n theory.
- Nonperturbative corrections: chiral (measured) and gluon condensates (sensitivity varies by 100s).

$$\alpha_{\overline{\rm MS}}(M_Z, n_f = 5) \;=\; 0.1184\,(6)$$



Error Budget

	$\log W_{11}$	$\log W_{12}$	$\log W_{22}$	$\log W_{11}W_{22}/W_{12}^2$	$\log W_{12}/u_0^6$	$\log W_{22}/u_0^8$	$\alpha_{\text{lat}}/W_{11}$
$c_1 \ldots c_3$	0.1%	0.1%	0.1%	0.3%	0.1%	0.1%	0.1%
c_n for $n \geq 4$	0.1	0.1	0.2	0.3	0.2	0.3	0.1
am_q, $r_1 m_q$ extrapolation	0.0	0.0	0.1	0.1	0.1	0.1	0.0
$(a/r_1)^2$ extrapolation	0.0	0.0	0.2	0.3	0.1	0.2	0.0
$(r_1/a)_i$ errors	0.2	0.2	0.2	0.2	0.2	0.2	0.2
r_1 errors	0.1	0.1	0.1	0.1	0.1	0.1	0.1
gluon condensate	0.1	0.2	0.2	0.2	0.2	0.1	0.1
statistical errors	0.0	0.0	0.0	0.1	0.1	0.1	0.0
$V \to \overline{\text{MS}} \to M_Z$	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Total	0.3%	0.4%	0.5%	0.6%	0.4%	0.5%	0.3%

Davies *et al* (HPQCD), Phys. Rev. D**78**, 114507 (2008) [arXiv:0807.1687]

McNeile *et al* (HPQCD) Phys. Rev. D**82**, 034512 (2010) [arXiv:1004.4285]

Current-Current Correlators

Heavy-Quark *hh* Pseudoscalar Correlator

Compute

$\overline{\psi}_h \gamma_5 \psi_h$

$$G(t) \equiv a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle$$

euclidean t

- Mass factors imply UV finite (PCAC because HISQ)
- Implies:

$$\boxed{G_{\rm cont}(t) = G_{\rm lat}(t) + \mathcal{O}(a^2) \quad \text{for all } t}$$

Moments

Low n moments perturbative ($E_{\text{threshold}} = 2m_h$):

$$G_n = \sum_t (t/a)^n G(t)$$
$$\rightarrow \frac{\partial^n}{\partial E^n} \Pi(E=0)$$

Implies:

from lattice simulations

$$G_n = \frac{g_n(\alpha_{\overline{\text{MS}}}, \mu/m_h)}{(am_h(\mu))^{n-4}}$$

from continuum perturbation th. — gives coupling

gives m_h (m_h only scale)

Reduced Moments — Reduced Systematic Errors

- Reduce finite-*a* errors (3x), lattice-spacing uncertainties (3x) by analyzing dimensionless reduced moments:

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4, \\ \dfrac{am_{\eta_h}}{2am_{0h}} \left(G_n/G_n^{(0)}\right)^{1/(n-4)} & \text{for } n \geq 6, \end{cases}$$

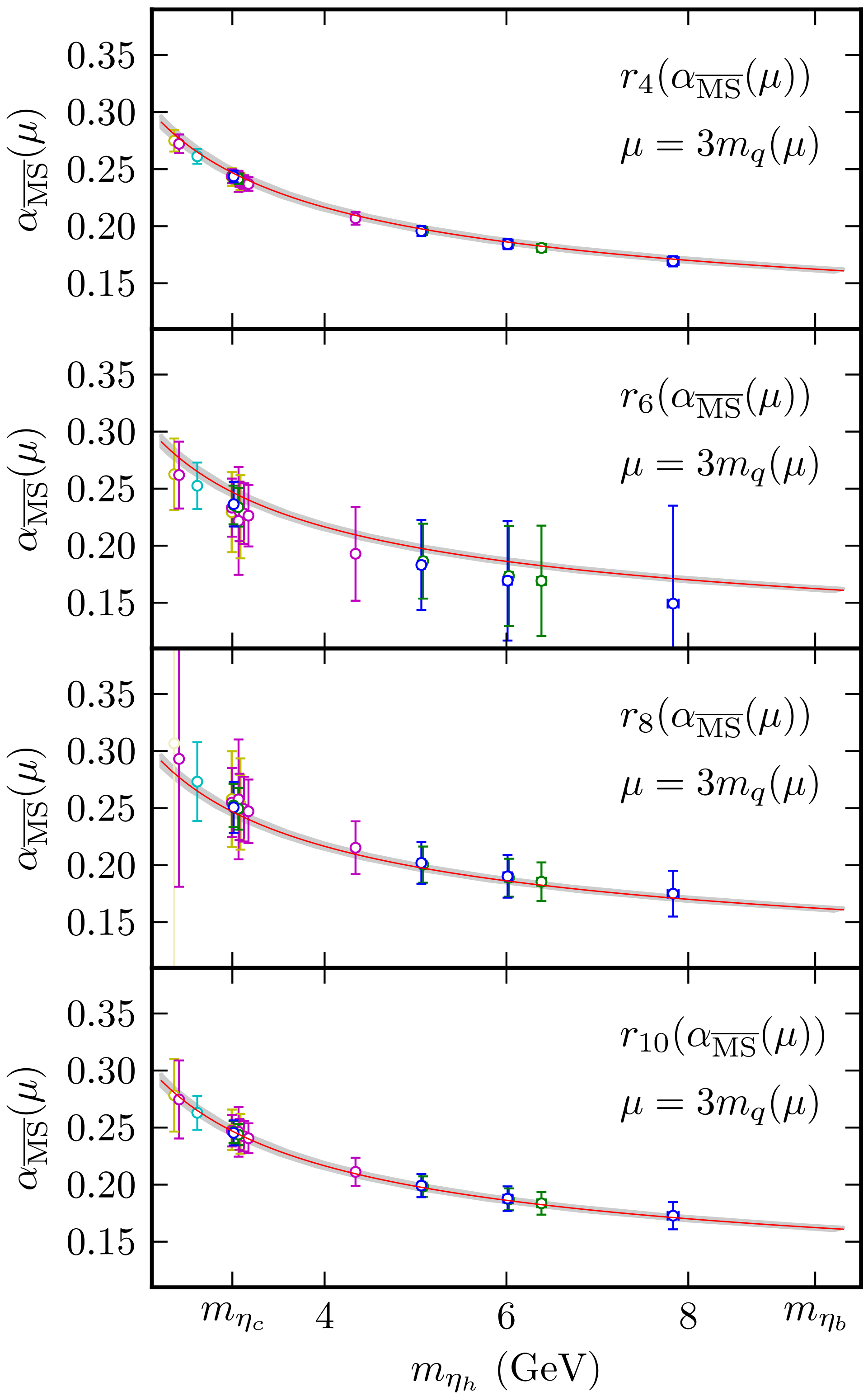
- Fix μ/m_h (=3). Then, in continuum, all moments depend upon two universal functions of $m_{\eta h}$ + perturbation theory:

$$R_n \equiv \begin{cases} r_4(\alpha_{\overline{\mathrm{MS}}}, \mu/m_h) & \text{for } n = 4, \\ z(\mu/m_h, m_{\eta_h})\, r_n(\alpha_{\overline{\mathrm{MS}}}, \mu/m_h) & \text{for } n \geq 6, \end{cases}$$

z ↑ $\dfrac{m_{\eta_h}}{2m_h(\mu)}$ $\qquad$ $\alpha_{\overline{\mathrm{MS}}}$ ↑ $\alpha_{\overline{\mathrm{MS}}}(\mu)$

Results — α_s

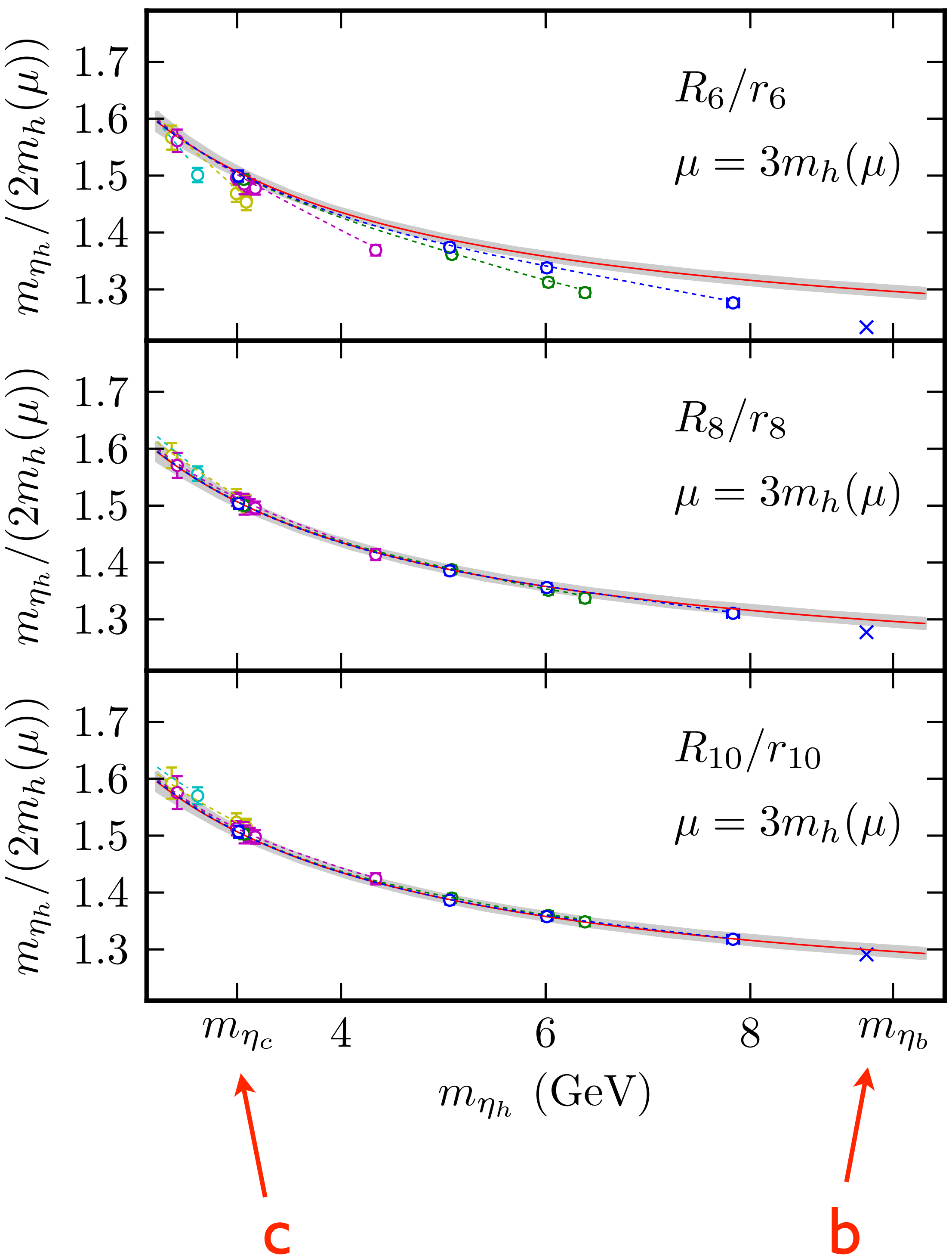
$$\alpha_{\overline{\rm MS}}(M_Z, n_f = 5) = 0.1183(7)$$



Note:

- Statistical errors in 5th decimal.
- Fit n=4,6,8,10 and approximately eight values of m_h from m_c to $0.9m_b$ simultaneously. (88 data points.)
- Non-perturbative contributions: chiral, gluon condensates.
- Fit $(am_h)^n$ and $m_{q,\rm sea}$ dependence. Highly improved quark action.
- Perturbation theory through 3rd order known; fit through 6th order. (Assume n>3 uncorrelated at μ=m_h.)
- Little change for $\mu = m_h \rightarrow 9m_h$

Results — Heavy-Quark Masses



From $m_{\eta h}/(2m_h(\mu))$ ... for n=6,8,10:

m_c(3 GeV, n_f=4) = 0.985(6) GeV
m_b(m_b,n_f=5) = 4.155(23) GeV
$m_b(\mu,n_f)/m_c(\mu,n_f)$ = 4.52(4)

Compare:

- Continuum result from vector current + $R(e^+e^-)$:

  m_c(3GeV) = 0.986(13) GeV
  m_b(m_b) = 4.163(16) GeV

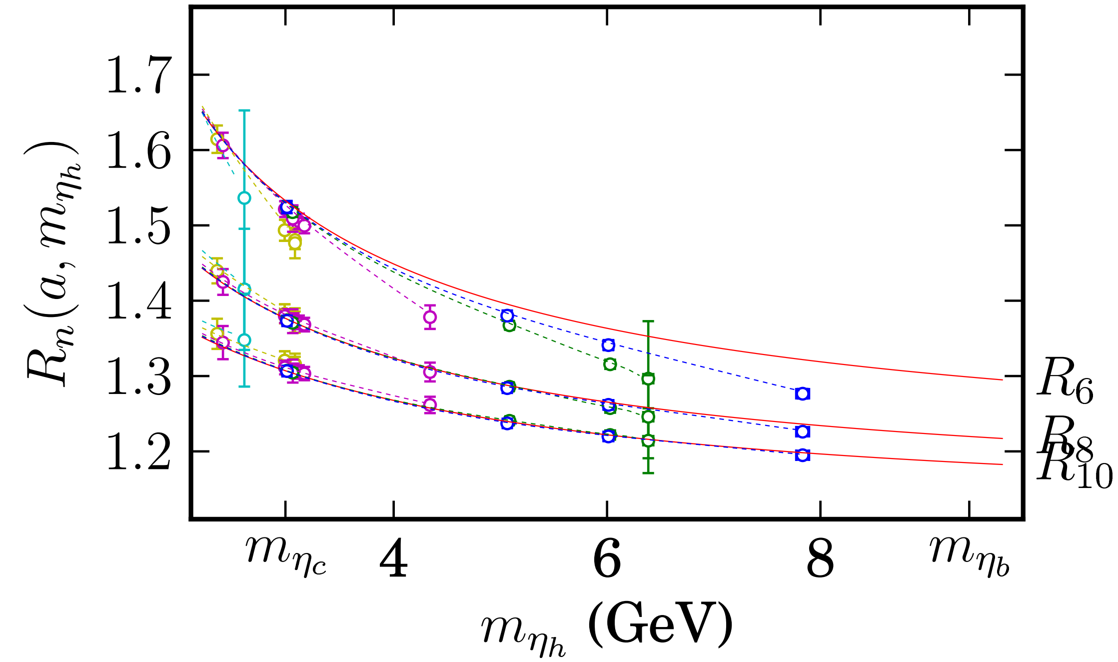
  Chetyrkin et al, PRD 80:074010 (2009).

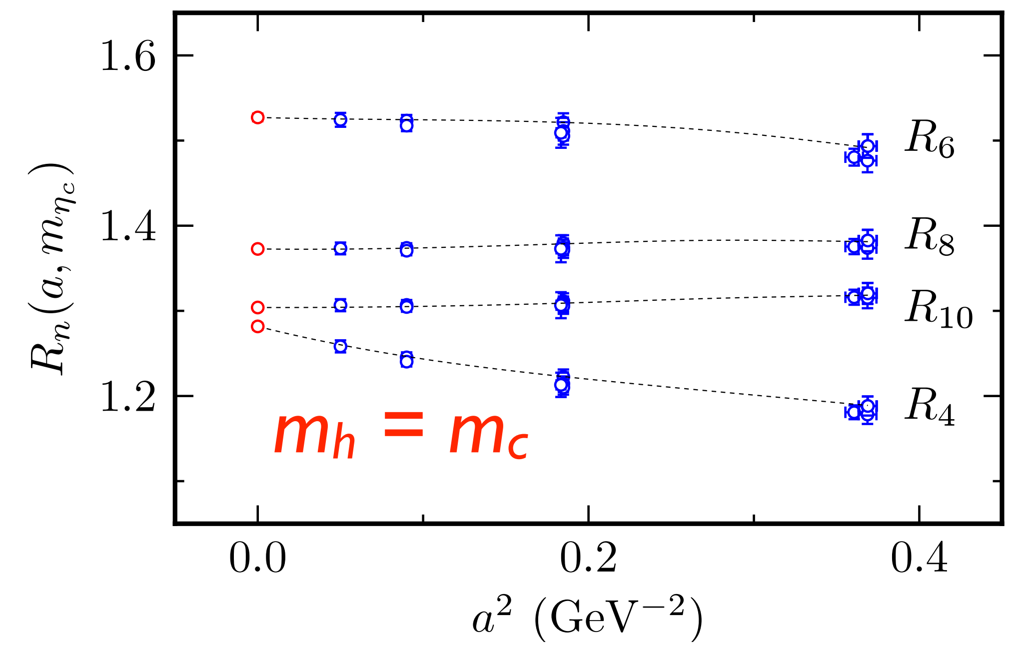
- Nonperturbative LQCD result:

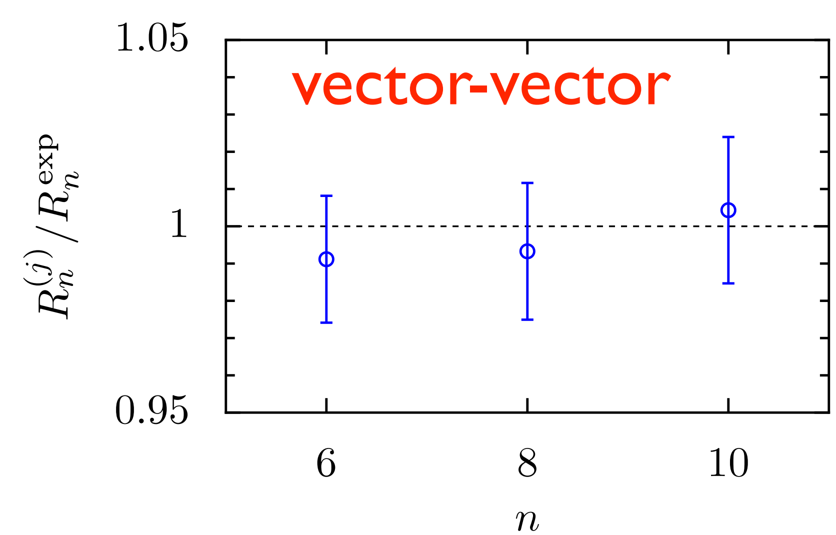
  $m_b(\mu,n_f)/m_c(\mu,n_f)$ = 4.49(4)

Moments Different but Agree

- Agree with each other on *z*-function and coupling for eight different heavy-quark masses (not just m_c and m_b).
- Agree with Wilson loops on coupling to within 0.6%.
- $n = 4 - 18$ moments for pseudoscalar, axial-vector and vector currents agree to within few – several percent (including continuum).
- *b*/*c* mass ratio agrees with non-perturbative LQCD result to <1%.
- Vector-vector moments from lattice agree with continuum results from $R(e^+e^-)$ to better than 2%.







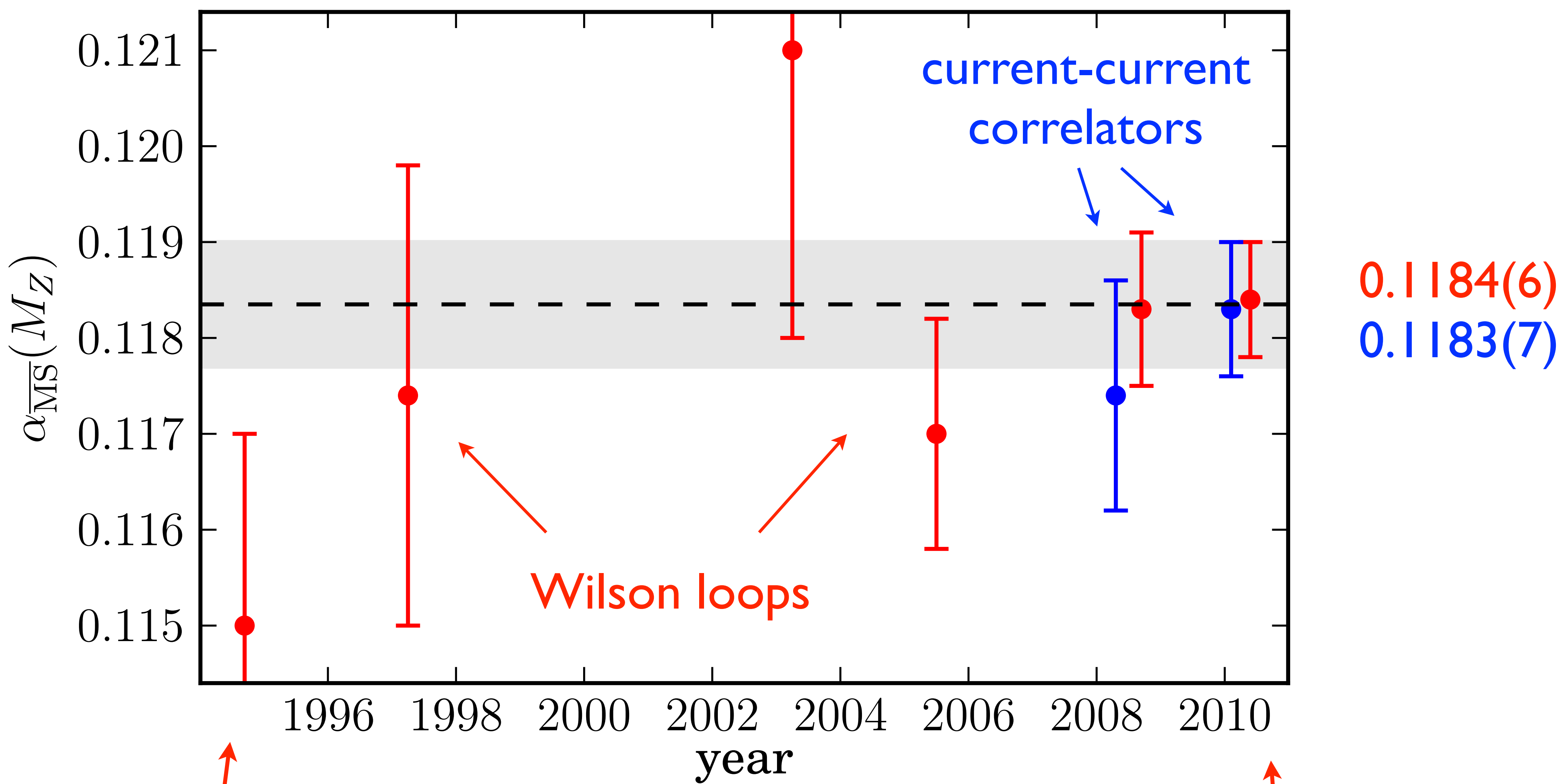
Error Budget

	$m_c(3)$	$m_b(10)$	m_b/m_c	$\alpha_{\overline{\rm MS}}(M_Z)$
a^2 extrapolation	0.2%	0.6%	0.5%	0.2%
Perturbation theory	0.5	0.1	0.5	0.4
Statistical errors	0.1	0.3	0.3	0.2
m_h extrapolation	0.1	0.1	0.2	0.0
Errors in r_1	0.2	0.1	0.1	0.1
Errors in r_1/a	0.1	0.3	0.2	0.1
Errors in m_{η_c}, m_{η_b}	0.2	0.1	0.2	0.0
α_0 prior	0.1	0.1	0.1	0.1
Gluon condensate	0.0	0.0	0.0	0.2
Total	0.6%	0.7%	0.8%	0.6%

Allison *et al* (HPQCD+Karlsruhe), Phys.Rev. D**78**, 054513 (2008) [arXiv:0805.2999]
McNeile *et al* (HPQCD) Phys. Rev. D**82**, 034512 (2010) [arXiv:1004.4285]

Conclusion

QCD Coupling — HPQCD History



π/a = 4.8 GeV n_f = 0, 2→3
simple discretization
2nd order in α_s
(4 data points)

π/a = 4–14 GeV n_f = 3
highly improved discretization
3rd + approx 4th order in α_s
(352 data points)