Lattice QCD Calculations and α_s

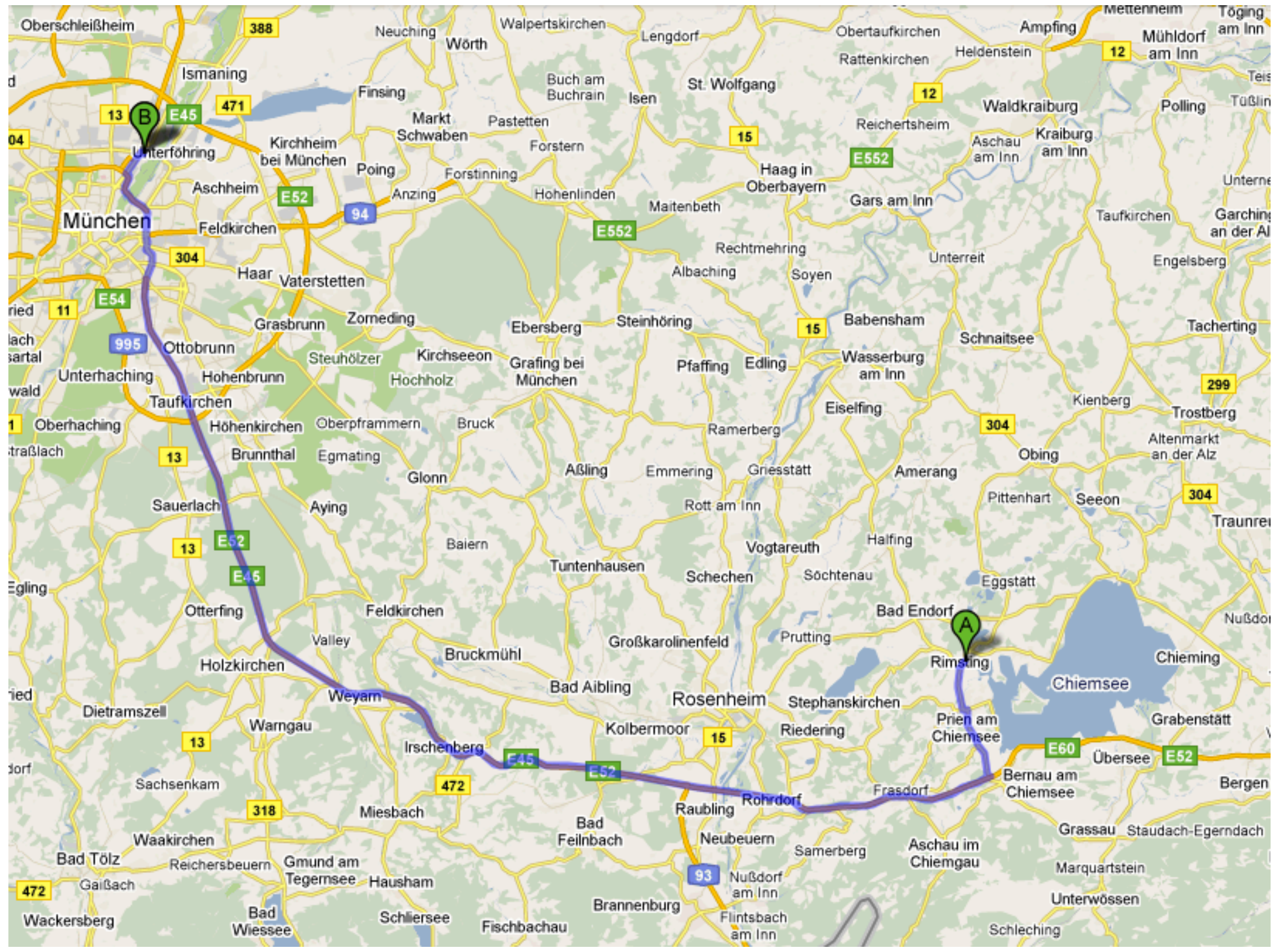
Andreas S. Kronfeld

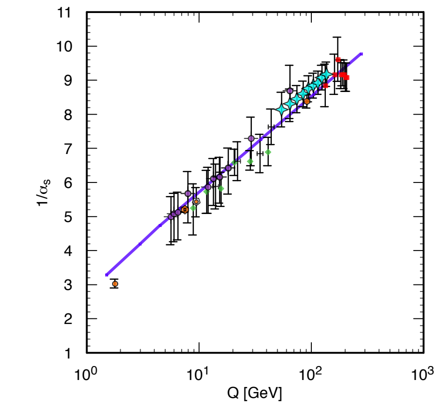
Fermilab

11 February 2011

Workshop on Precision Measurements of α_s

Max-Planck-Insitut für Physik, München





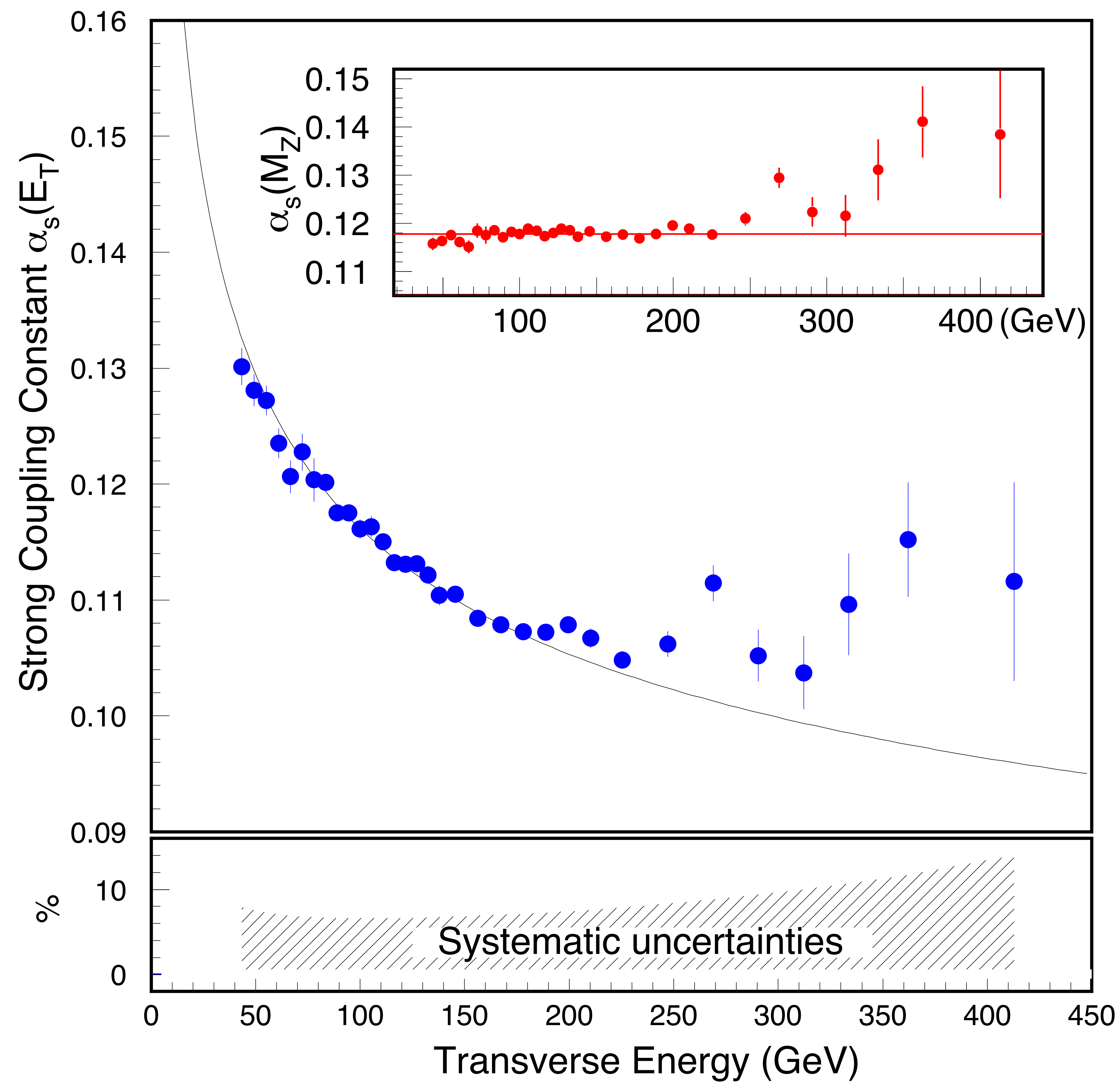
Requirements for α_s Determinations

- A dimensionless quantity, R, sensitive to QCD at a (range of) short distance(s), Q^{-1};
  - if not dimensionless, use Q to make it so; then $R = \mathcal{R}(\alpha_s(Q_s)) + o((\Lambda_{\text{QCD}}/Q)^r)$.
- A theoretical framework—or at least a notion—to separate short-distance scales from Λ_{QCD} (and other long-distance scales).
- An N^{n}LO/N^{n}LL calculation of $\mathcal{R}(\alpha_s(Q_s))$; recall $\alpha_s^{-1}(Q) \approx 2\beta_0 \ln(Q/\Lambda_s)$.
- Measurements of R over a range of Q large enough to control power-law effects, $(\Lambda_{\text{QCD}}/Q)^r$.
- Control of non-QCD physics at scales probed by Q (*e.g.*, electroweak or new physics).
- A measurement of Q—usually a *calibration*, in contrast to *event counting* for R.

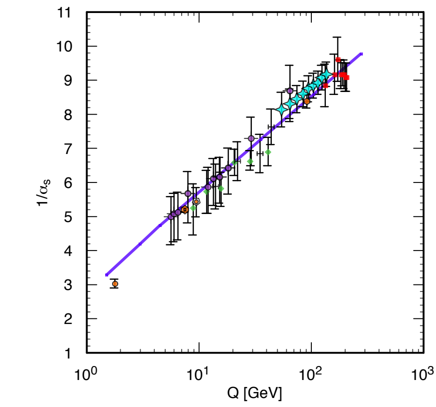
Tevatron α_s from Inclusive-Single-Jet Production

CDF, arXiv:hep-ex/0108034

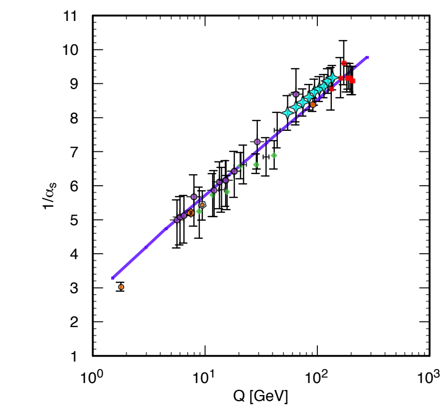
- $Q = E_T$, $R = E_T^3 d\sigma/dE_T \propto \alpha_s^2$ @ LO.
- Factorization; $\mathcal{R}$ to NLO [Ellis & Sexton].
- 33 independent E_T bins; range ×8.
- $\alpha_s = 0.1178 \pm 0.0022 \pm 0.0082 \pm 0.0085$:
  - errors from R (stats+norm), $\mathcal{R}$, Q.
- Calibration measurement of Q!!!
- Quark substructure for $E_T > 250$ GeV?



α_s from Lattice QCD



- How is R "measured"?
- Compute R from QCD functional integral.
- What's Q?
- $Q = a^{-1}$ (lattice), Q, $2m_Q$, L^{-1} (box).
- Frameworks for scale separation.
- Symanzik EFT, OPE, duality….
- Perturbative calculations of $\mathcal{R}(\alpha_s(Q_s))$ at NNLO or N^{3}LO.
- Lattice perturbation theory ($Q = a^{-1}$); continuum perturbation theory (else).
- How many "measurements"?
- Several to numerous.
- Extrapolate from computer to QCD.
- Discretization; unphysical quark masses.
- How is Q calibrated?
- Ultimately hadron masses: $Q = (Qa/Ma)M$.



Outline

- Introduction: How to Determine α_s
- Lattice Gauge Theory in a Nutshell
- From Computer to QCD
- Overview of Lattice QCD α_s Methods
- Summary & Outlook

Lattice Gauge Theory in a Nutshell

Lattice Gauge Theory

K. Wilson, *PRD* **10** (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].

- Gauge symmetry on a spacetime lattice:

  - mathematically rigorous definition of QCD functional integrals;

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) \left[\bullet\right]$$

  - enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.

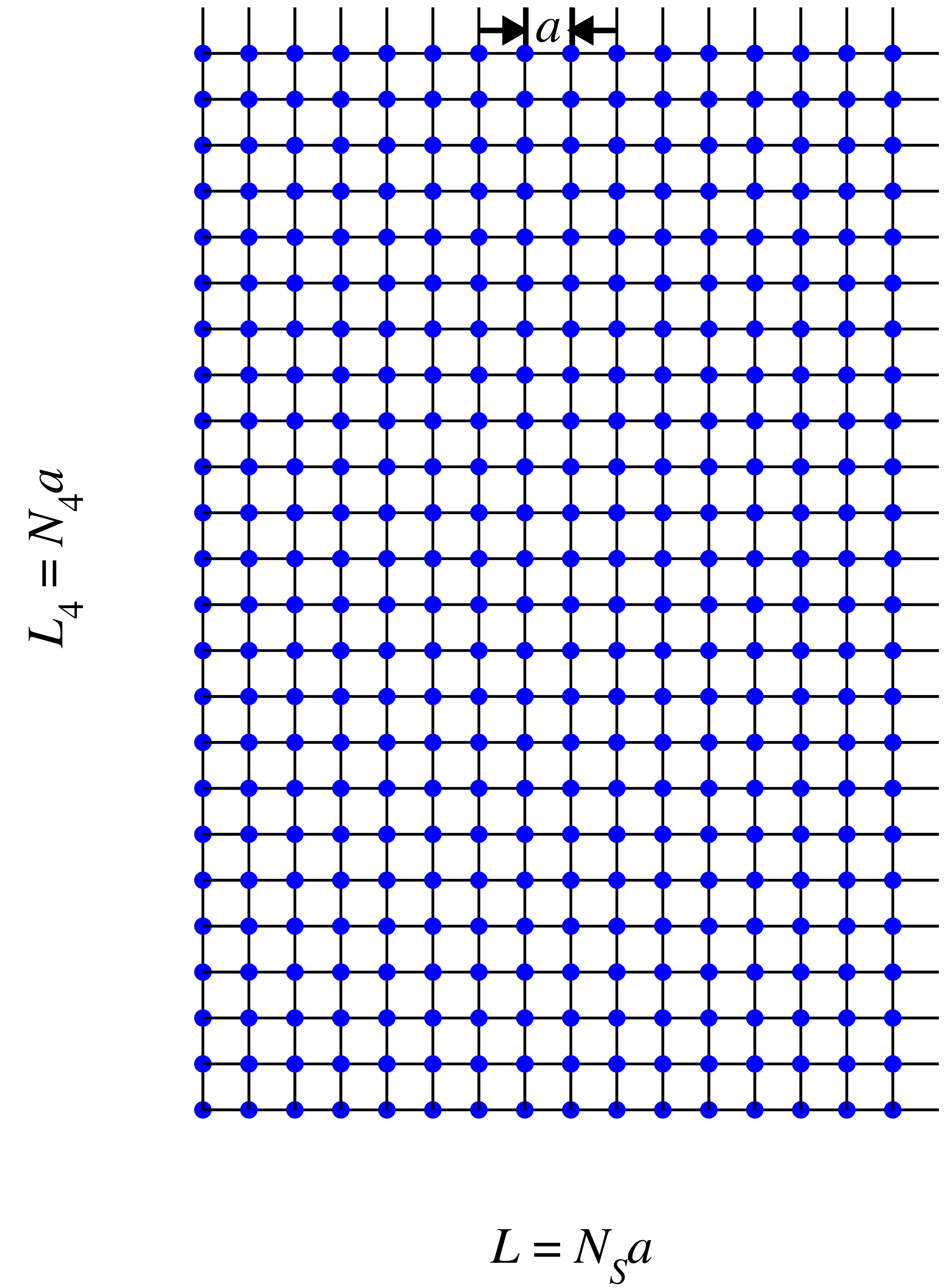
- Lowest-order strong coupling expansion demonstrates confinement.

Numerical Lattice QCD

- Nowadays "lattice QCD" usually implies a numerical technique.
- Integrate the functional integral on a $N_S^3 \times N_4$ lattice (spacing a) numerically:

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

- Finite lattice: can evaluate integrals on a computer; dimension ~ 10^8, using *importance sampling*.
- Healthy research field to devise MC algorithms.



Numerical Lattice QCD

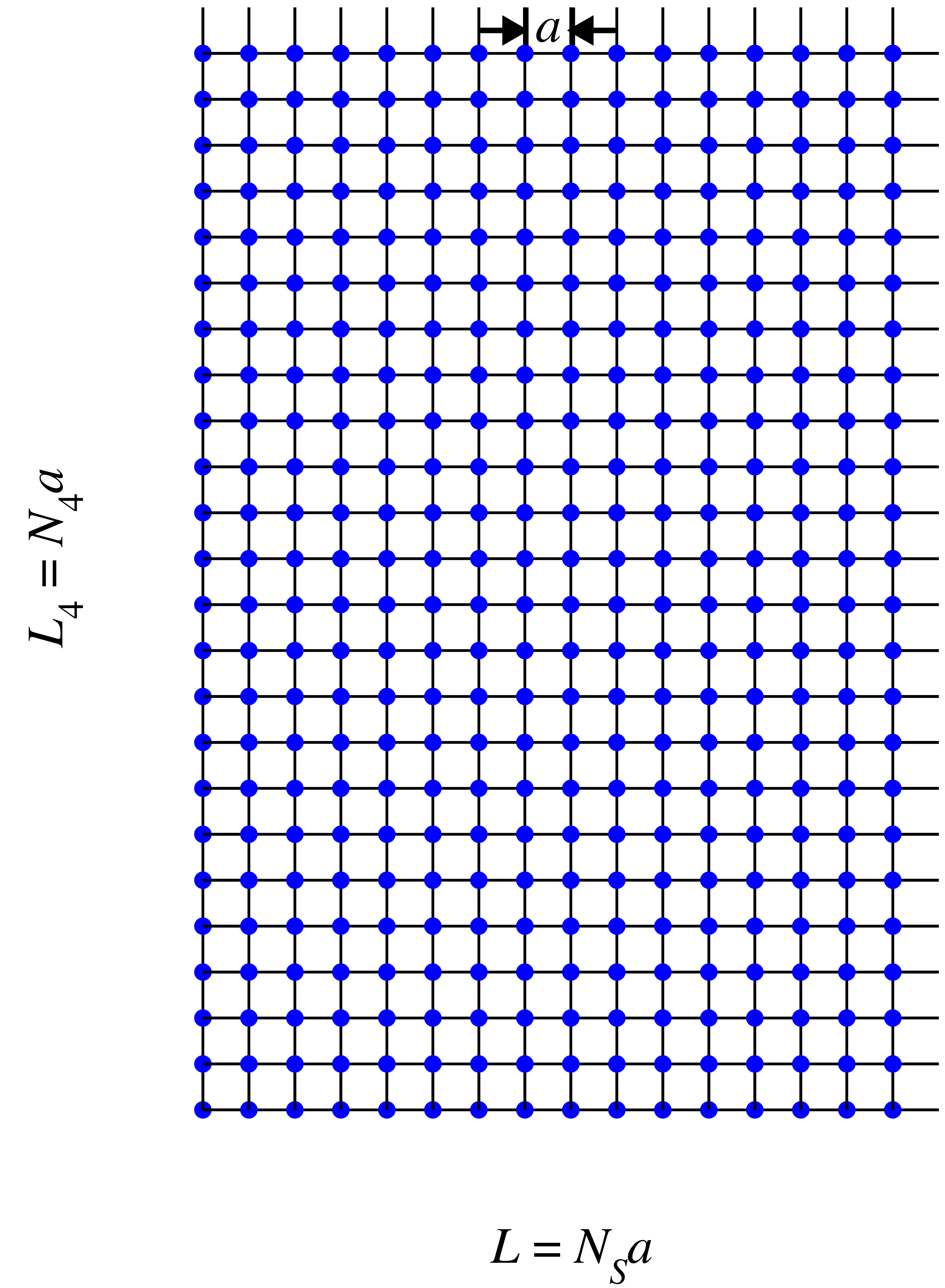
- Nowadays “lattice QCD” usually implies a numerical technique.

- Integrate the functional integral on a $N_S^3 \times N_4$ lattice (spacing a) numerically:

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \boxed{\underset{\text{hand}}{\mathcal{D}\psi\mathcal{D}\bar{\psi}}} \exp\left(-S\right) [\bullet]$$

- Finite lattice: can evaluate integrals on a computer; dimension ~ 10^8, using *importance sampling*.

- Healthy research field to devise MC algorithms.

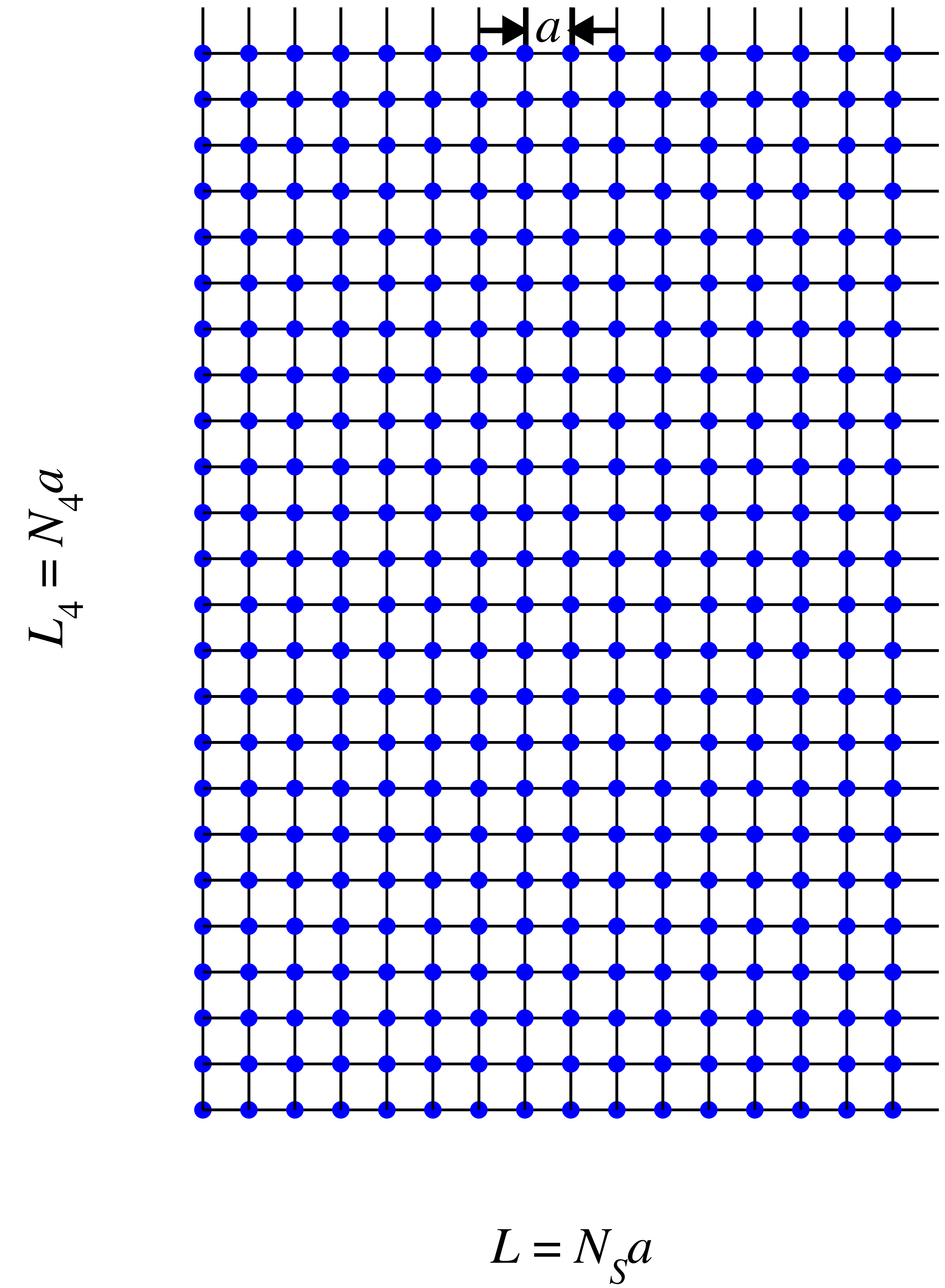


Numerical Lattice QCD

- Nowadays "lattice QCD" usually implies a numerical technique.
- Integrate the functional integral on a $N_S^3 \times N_4$ lattice (spacing a) numerically:

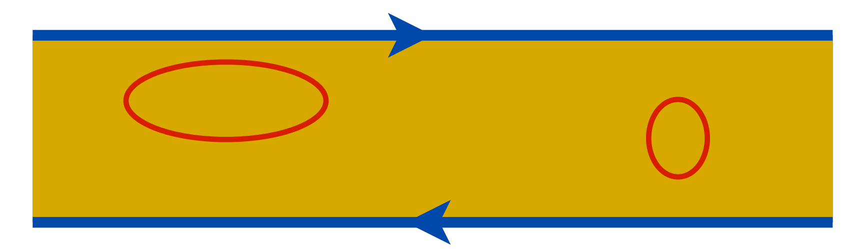
$$\langle \bullet \rangle = \frac{1}{Z} \int \underbrace{\mathcal{D}U}_{\text{MC}} \underbrace{\mathcal{D}\psi \mathcal{D}\bar{\psi}}_{\text{hand}} \exp(-S) [\bullet]$$

- Finite lattice: can evaluate integrals on a computer; dimension ~ 10^8, using *importance sampling*.
- Healthy research field to devise MC algorithms.



- Some compromises:
    - finite human lifetime ⇒ Wick rotate to Euclidean time: $x^4 = ix^0$;
    - finite memory ⇒ finite space volume & finite time extent; nonzero lattice spacing;
    - finite CPU power ⇒ light quarks heavier than up and down; nonzero lattice spacing.
- The first introduces no error, but can be an obstacle (*e.g.*, fragmentation functions).
- Finite volume unimportant for stable hadrons.
- *Continuum* and *chiral* limits are crucial when calibrating physical units (*i.e.*, computing mass spectrum), but for α_s recall $\delta M \Rightarrow \delta\alpha_s = \frac{1}{2}\beta_0\alpha_s^2\,\delta M$.

Some Jargon



- QCD observables (quark integrals by hand):

$$\langle \bullet \rangle = \frac{1}{Z}\int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D}+m_f)\exp\left(-S_{\text{gauge}}\right)[\bullet]$$

sea

valence: $(\not{D} + m)^{-1}$

- *Quenched* means replace $\det$ with 1. (Obsolete.)

- *Un*quenched means not to do that.

- *Partially* quenched (usually) doesn't mean "n_f too small", but $m_{\text{val}} \neq m_{\text{sea}}$, or $\not{D}_{\text{val}} \neq \not{D}_{\text{sea}}$ ("mixed action").

Sea Quarks

- Staggered quarks, with rooted determinant, $\mathrm{O}(a^2)$.
- Wilson quarks, $\mathrm{O}(a)$:
  - twisted mass term—auto $\mathrm{O}(a)$ improvement $\Rightarrow \mathrm{O}(a^2)$;
  - tree or nonperturbatively $\mathrm{O}(a)$ improved $\Rightarrow \mathrm{O}(a^2)$.
- Ginsparg-Wilson (domain wall or overlap), $\mathrm{O}(a^2)$:
  - $\not{D}\gamma_5 + \gamma_5\not{D} = 2a\not{D}^2$ implemented w/ $\mathrm{sign}(\not{D}_\mathrm{W})$.

Sea Quarks

- Staggered quarks, with rooted determinant, $\mathrm{O}(a^2)$.
- Wilson quarks, $\mathrm{O}(a)$:
    - twisted mass term—auto $\mathrm{O}(a)$ improvement $\Rightarrow$ $\mathrm{O}(a^2)$;
    - tree or nonperturbatively $\mathrm{O}(a)$ improved $\Rightarrow$ $\mathrm{O}(a^2)$.
- Ginsparg-Wilson (domain wall or overlap), $\mathrm{O}(a^2)$:
    - $D\!\!\!/\,\gamma_5 + \gamma_5 D\!\!\!/ = 2aD\!\!\!/^2$ implemented w/ $\mathrm{sign}(D\!\!\!/_{\mathrm{W}})$.

fast

clean

- Many numerical simulations with sea quarks are called (perhaps misleadingly) “full QCD.”
    - $n_f = 2$: with same mass, omitting strange sea;
    - $n_f = 3$: may (or may not) imply 3 of same mass;
    - $n_f = 2{+}1$: strange sea + 2 as light as possible for up and down;
    - $n_f = 2{+}1{+}1$: add charmed sea to $2{+}1$.
- “Full QCD” can also just mean the *unitary* setup with $m_{\text{val}} = m_{\text{sea}}$ and $\not{D}_{\text{val}} = \not{D}_{\text{sea}}$.

- Many numerical simulations with sea quarks are called (perhaps misleadingly) "full QCD."
  - $n_f = 2$: with same mass, omitting strange sea;
  - $n_f = 3$: may (or may not) imply 3 of same mass;
  - $n_f = 2{+}1$: strange sea + 2 as light as possible for up and down;
  - $n_f = 2{+}1{+}1$: add charmed sea to $2{+}1$.
- "Full QCD" can also just mean the *unitary* setup with $m_{\mathrm{val}} = m_{\mathrm{sea}}$ and $\not{D}_{\mathrm{val}} = \not{D}_{\mathrm{sea}}$.

From the Computer to QCD

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \\
&- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \\
&+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}]
\end{aligned}
$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
    - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$.

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \\ &- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \\ &+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}] \end{aligned}$$

m_Ω or $\Upsilon(2S\text{-}1S)$ or f_π or r_1 or ....

m_π, m_K, m_{Ds} or $m_{J/\psi}$, m_{Bs} or m_Υ, ...

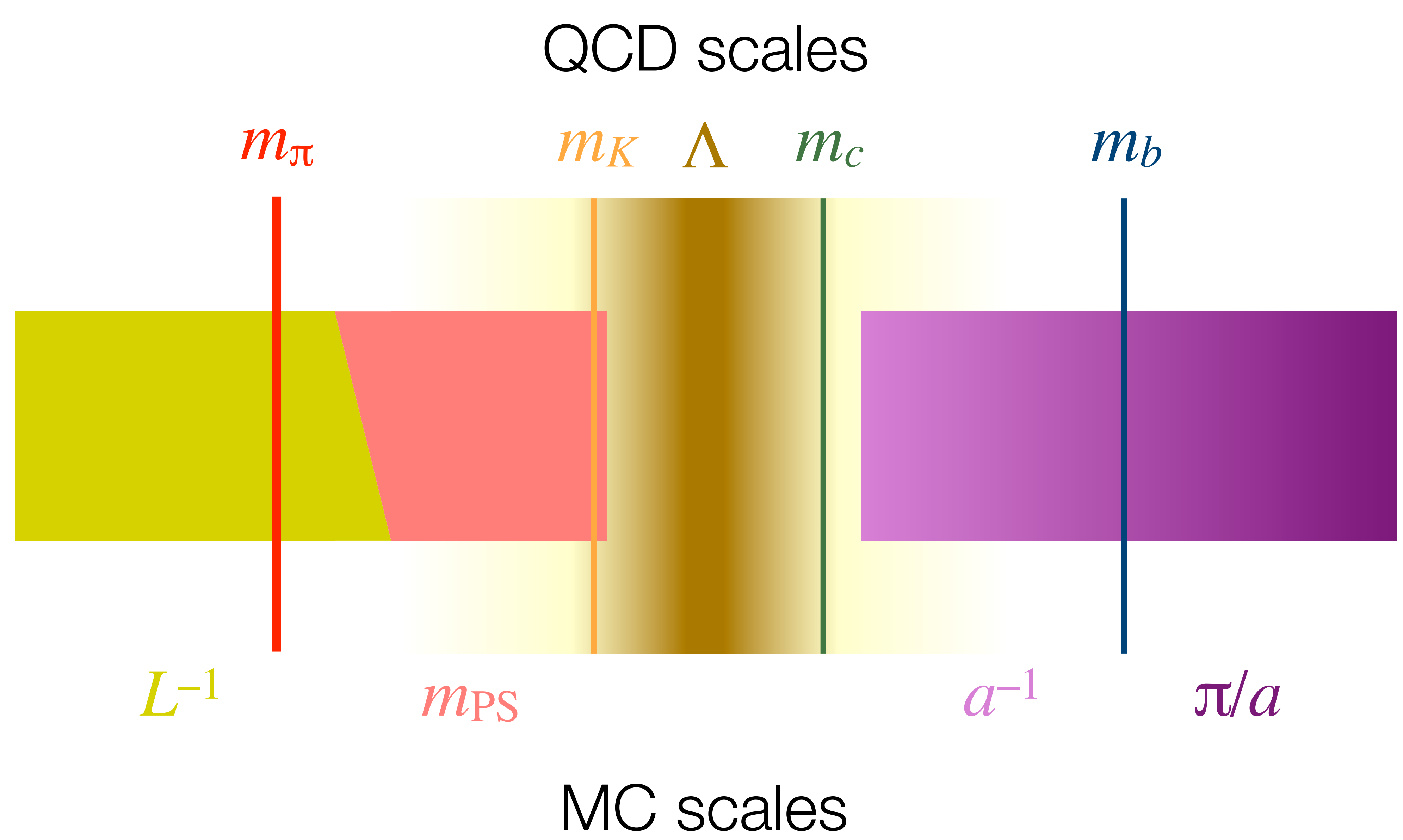
$\theta = 0.$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
  - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$.

Systematic Uncertainties and Effective Field Theories

ASK, hep-lat/0205021

- Multi-scale problem: m_u, m_s, m_π, m_K, Λ_QCD, m_c, m_b, m_t; Q^2; a^{-1}; L^{-1}.



- Most conveniently handled with effective field theories.

Scale	Effective field theory	Small parameter	Physics
a^{-1}	Symanzik EFT	Λa	Discretization errors: o($(\Lambda a)^n$)
m_Q	HQET	Λ/m_Q	Nearly static heavy quark
	NRQCD	$p/m_Q \sim v$	Slowly orbiting $\bar{Q}Q$
m_q, m_{PS}	Chiral PT (χPT)	$(m_{PS}/4\pi f_\pi)^2 \sim m_q/\Lambda$	Chiral symmetry constrains pion cloud
L^{-1}	Lüscher EFT, χPT	$\exp(-m_\pi L)$, $(\Lambda L)^{-1}$	Self-energy "around the world"

- Symanzik EFT (outgrowth of Callan-Symanzik equation) provides a framework for
  - "improving" the action;
  - guiding the continuum limit.
- Chiral perturbation theory removes the (too massive, too compact) pion cloud of the computer, and replaces it with the real one: $m_q \to m_u$.
  - Some recent work (PACS-CS, BMW) has m_q so close to m_d that χPT isn't needed.

Symanzik Effective Field Theory

K. Symanzik, NPB **226** (1983) 187, ibid. 205; P. Weisz, NPB **212** (1983) 1; NPB **236** (1984) 397

- Illustrate with pure gauge theory. Classically,

$$\mathcal{L}_{\mathrm{LGT}} \approx \frac{1}{g_0^2}\left\{\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + a^2\,\mathcal{K}_1\,\mathrm{tr}[D_\mu F_{\mu\nu} D^\mu F^{\mu\nu}] + a^2\,\mathcal{K}_2\,\mathrm{tr}[F_\mu^{\ \nu} F_\nu^{\ \rho} F_\rho^{\ \mu}] + a^2\,\mathcal{K}_3\,\mathrm{tr}[D^\mu F_{\mu\nu} D_\rho F^{\rho\nu}] + \cdots\right\}$$

- Symanzik formalizes this idea to include radiative corrections:

$$\mathcal{L}_{\mathrm{LGT}} \doteq \frac{1}{g^2}\left\{\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + a^2\,\mathcal{K}_1(g^2)\,\mathrm{tr}[D_\mu F_{\mu\nu} D^\mu F^{\mu\nu}] + a^2\,\mathcal{K}_2(g^2)\,\mathrm{tr}[F_\mu^{\ \nu} F_\nu^{\ \rho} F_\rho^{\ \mu}] + \text{redundant} + \cdots\right\}$$

- Consider these operators to be renormalized, *i.e.*, with power divergences subtracted off.

- Structure established in perturbation theory and believed to hold nonperturbatively.

- Symanzik provides a “continuum-QCD description” of lattice QCD’s discretization effects.

Fermions and Chiral Symmetry

- Lattice fermions coexist uneasily with axial symmetries:

Discretization	$U_V(1)$	$SU_V(n_f)$	$SU_A(n_f)$	$U_A(1)$
Ginsparg-Wilson*	√	√	√	magic
Wilson	√	√	broken, o(a)	broken, a/a
Staggered†	√	Γ_4	$U_A(n_r)$	broken, a/a
Rooted staggered	√	$SU(n_f) \subset SU(4n_f)$		broken, a/a

* symmetries exact for overlap; domain-wall fermions break axial symmetries at o($\exp(-mL_5)$)

† with n_r staggered fermion fields, the number of physical species $n_f = 4n_r$

- If the lattice action is ultra-local, then there is either a 16-fold replication of species (naive), the axial symmetries are explicitly broken (Wilson), or a compromise (staggered/Susskind).

- Lattice-artifact chiral-symmetry breaking still treatable with chiral perturbation theory.

- Broad consensus that (unrooted) staggered, Wilson, domain-wall, & overlap all yield QCD.

- To reduce the 4-fold replication of staggered species (aka "tastes"), one can replace sea

$$\det_4 \left(D_{\text{stag}} + m\right) \to \left[\det_4 \left(D_{\text{stag}} + m\right)\right]^{1/4}$$

- Weak coupling suggests, and numerical simulation corroborates, that staggered fermions have a "taste" basis in which

$$D_{\text{stag}} + m \doteq \begin{pmatrix} \not{D} + m & & a\Delta & \\ & \not{D} + m & & \\ & & \not{D} + m & \\ & a\Delta & & \not{D} + m \end{pmatrix},$$

where $a\Delta$ leads to discretization errors of o(a^2).

- This structure suggests that the fourth root yields one species in the continuum limit.

- Rooting violates unitarity at nonzero $a \neq 0$, but if SU(4) emerges as a "phantom" symmetry, it provides a "safe house" for these effects, *cf.* scalar and longitudinal gluon polarizations.

- Numerical evidence that these effects can be handled with "rooted staggered chiral perturbation theory"; for example, $n_r = 0.28(2)(3)$ [MILC, arXiv:0710.1118 [hep-lat]].

- Concerns remain along two lines of reasoning:

    - UV: perhaps rooted sea leads to anomalous dimension -1 for $a\Delta$;

    - IR: symmetries of the ’t Hooft vertex incompatible with rooting.

- Assuming correctness, uncertainties from rooting are part of chiral extrapolation error bar.

- Everyone agrees that calculations with rooted staggered sea should be repeated with other formulations of sea quarks.

- Rooting violates unitarity at nonzero $a \neq 0$, but if SU(4) emerges as a "phantom" symmetry, it provides a "safe house" for these effects, *cf.* scalar and longitudinal gluon polarizations.

- Numerical evidence that these effects can be handled with "rooted staggered chiral perturbation theory"; for example, $n_r = 0.28(2)(3)$ [MILC, arXiv:0710.1118 [hep-lat]].

- Concerns remain along two lines of reasoning:

  - UV: perhaps rooted sea leads to anomalous dimension -1 for $a\Delta$; no evidence

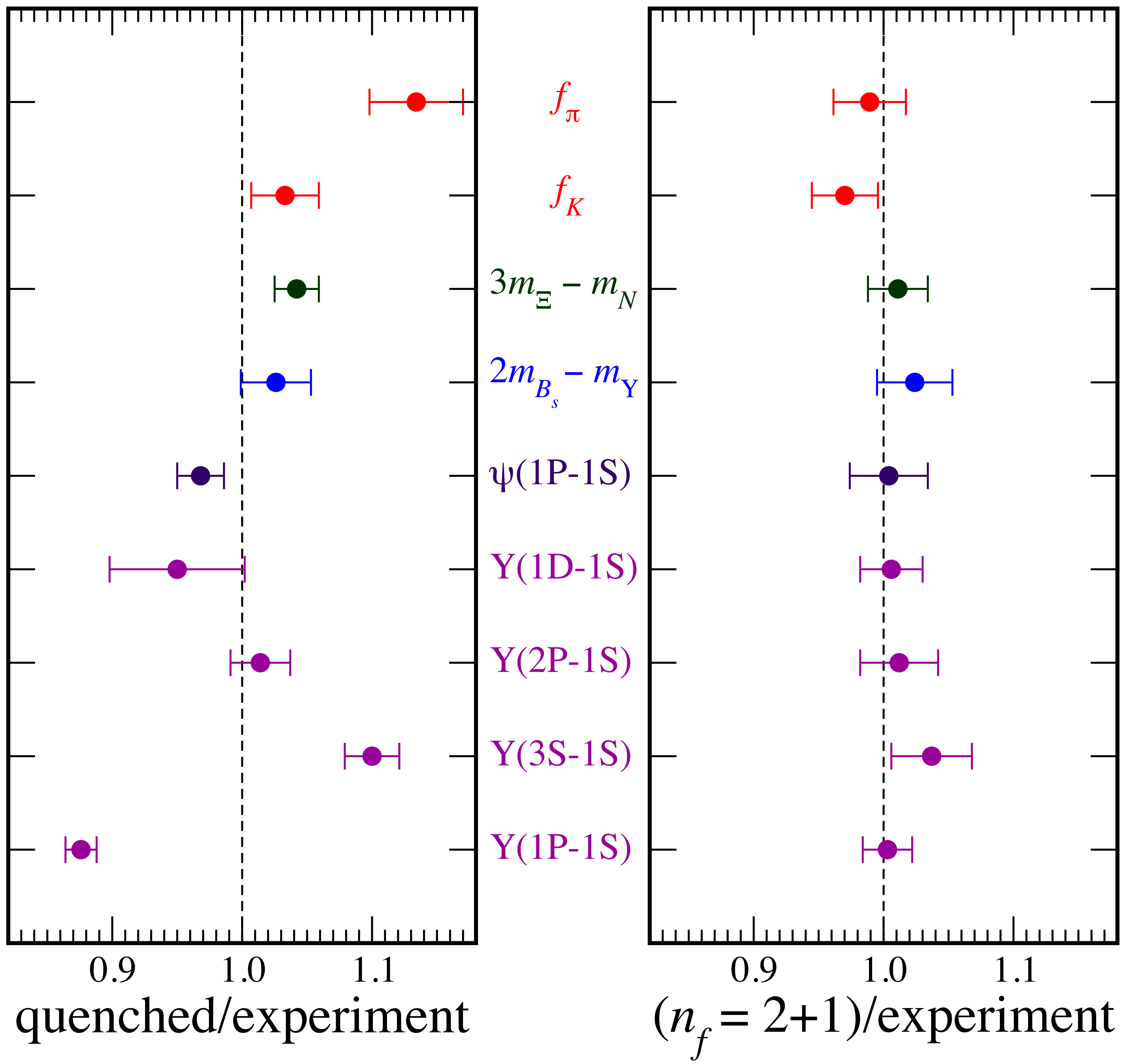
  - IR: symmetries of the 't Hooft vertex incompatible with rooting. wrong derivation

- Assuming correctness, uncertainties from rooting are part of chiral extrapolation error bar.

- Everyone agrees that calculations with rooted staggered sea should be repeated with other formulations of sea quarks.

2+1 Sea Quarks!

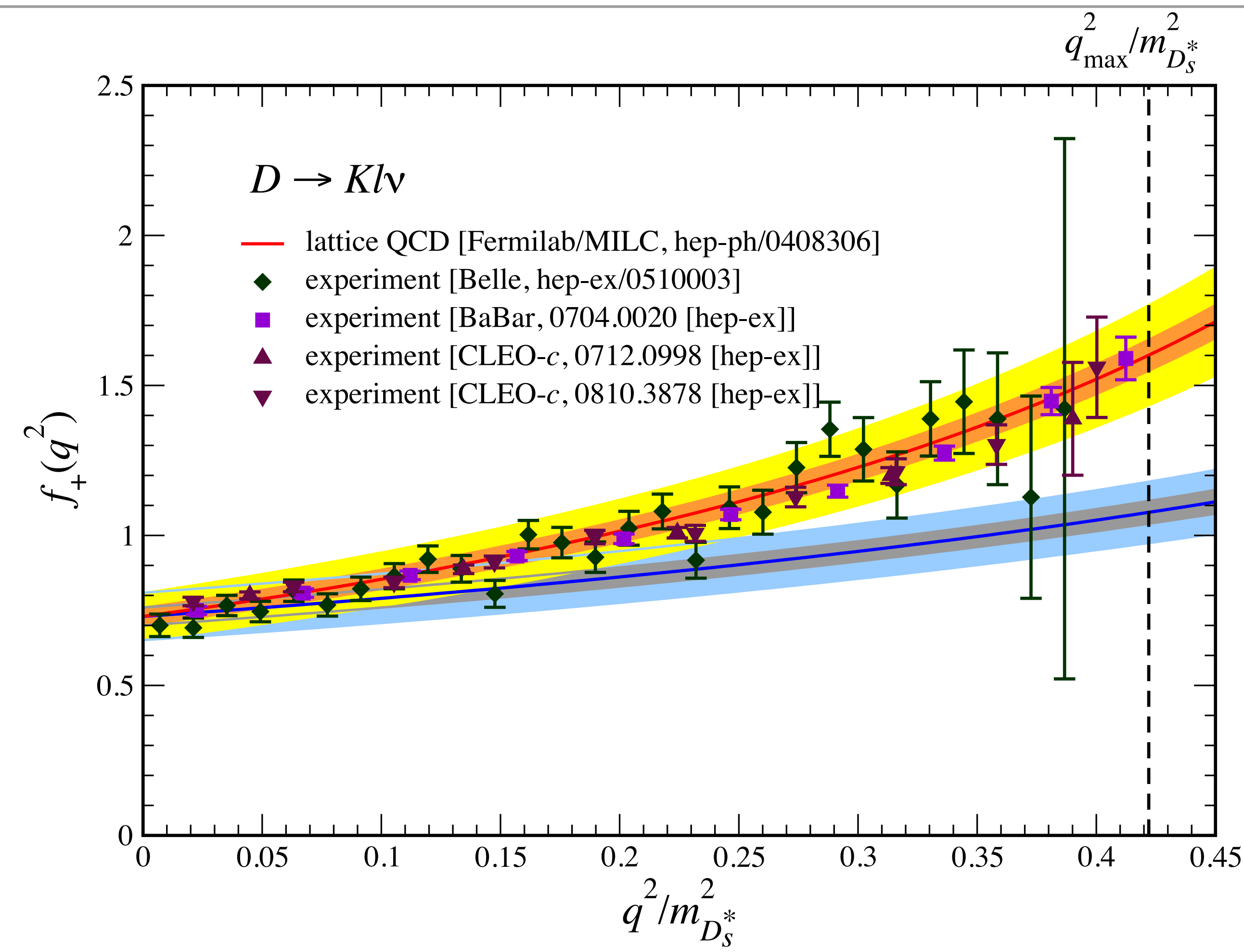
HPQCD, MILC, Fermilab Lattice, hep-lat/0304004



- $a = 0.12$ & 0.09 fm;
- $O(a^2)$ improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, Υ(2S-1S) input.

Predictions

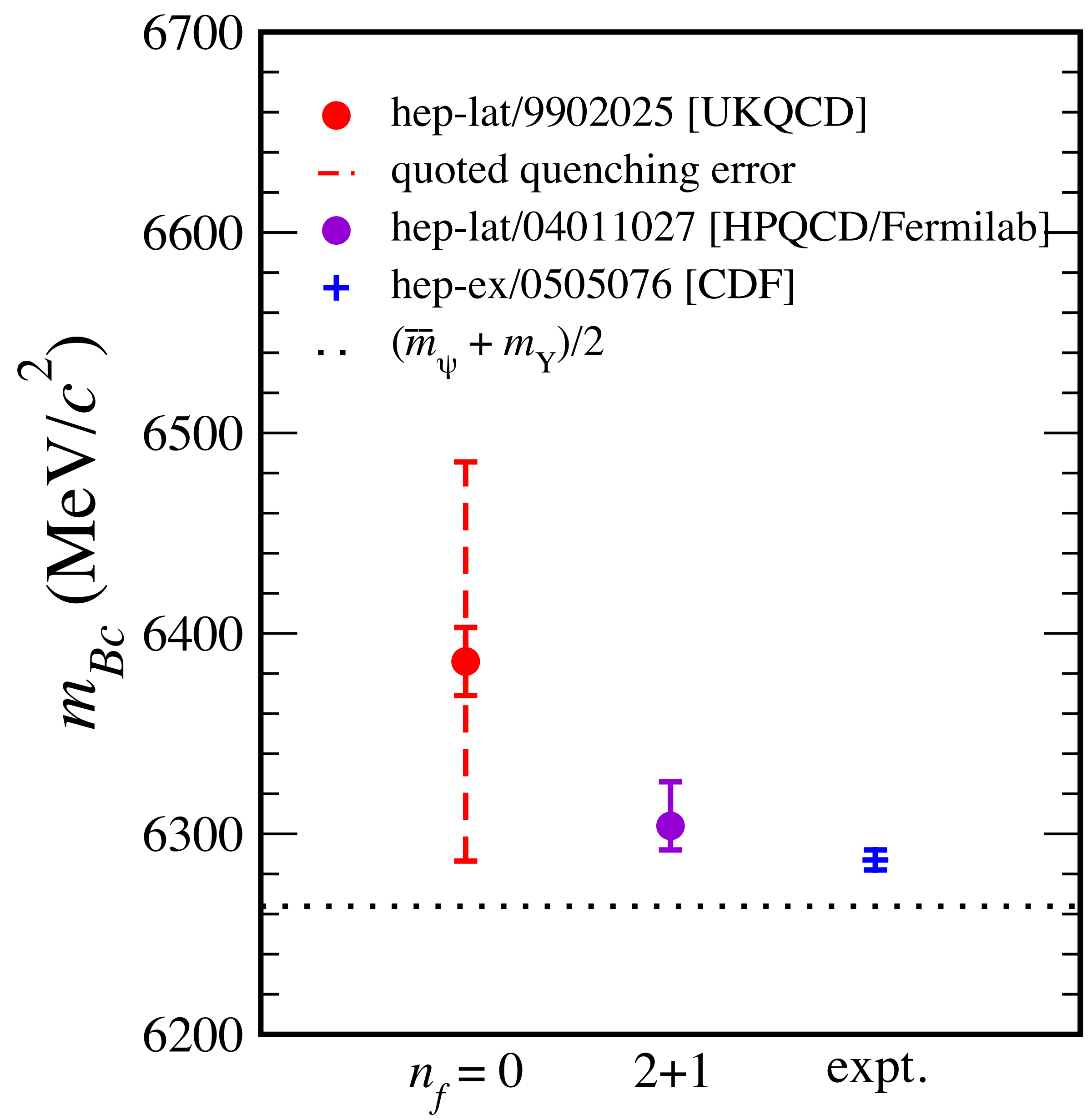
Fermilab Lattice, MILC, HPQCD, hep-ph/0408306, hep-lat/0411027, hep-lat/0506030

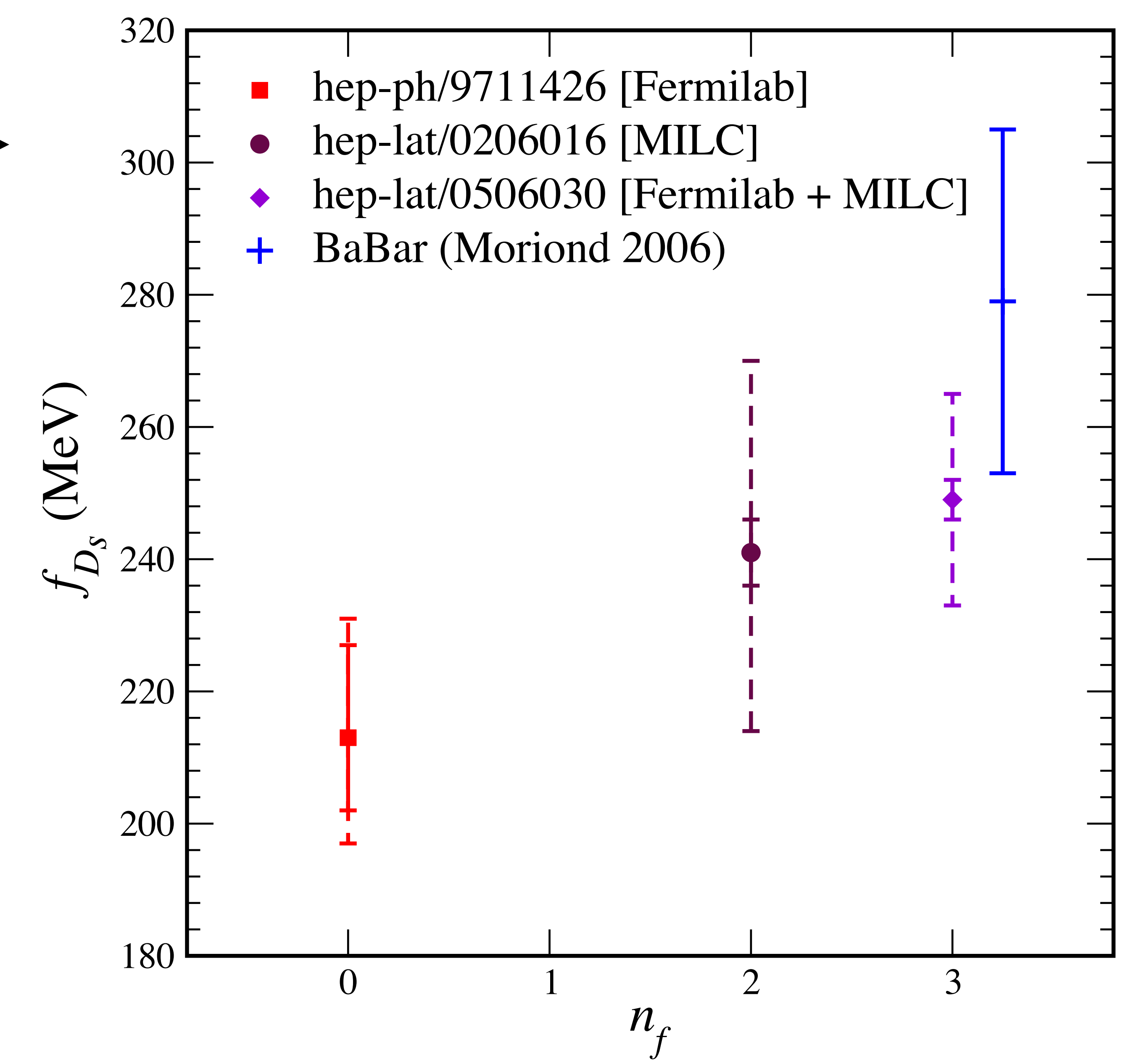


- Semileptonic form factor for $D \to Kl\nu$
- Mass of B_c meson
- Charmed-meson decay constants

2004

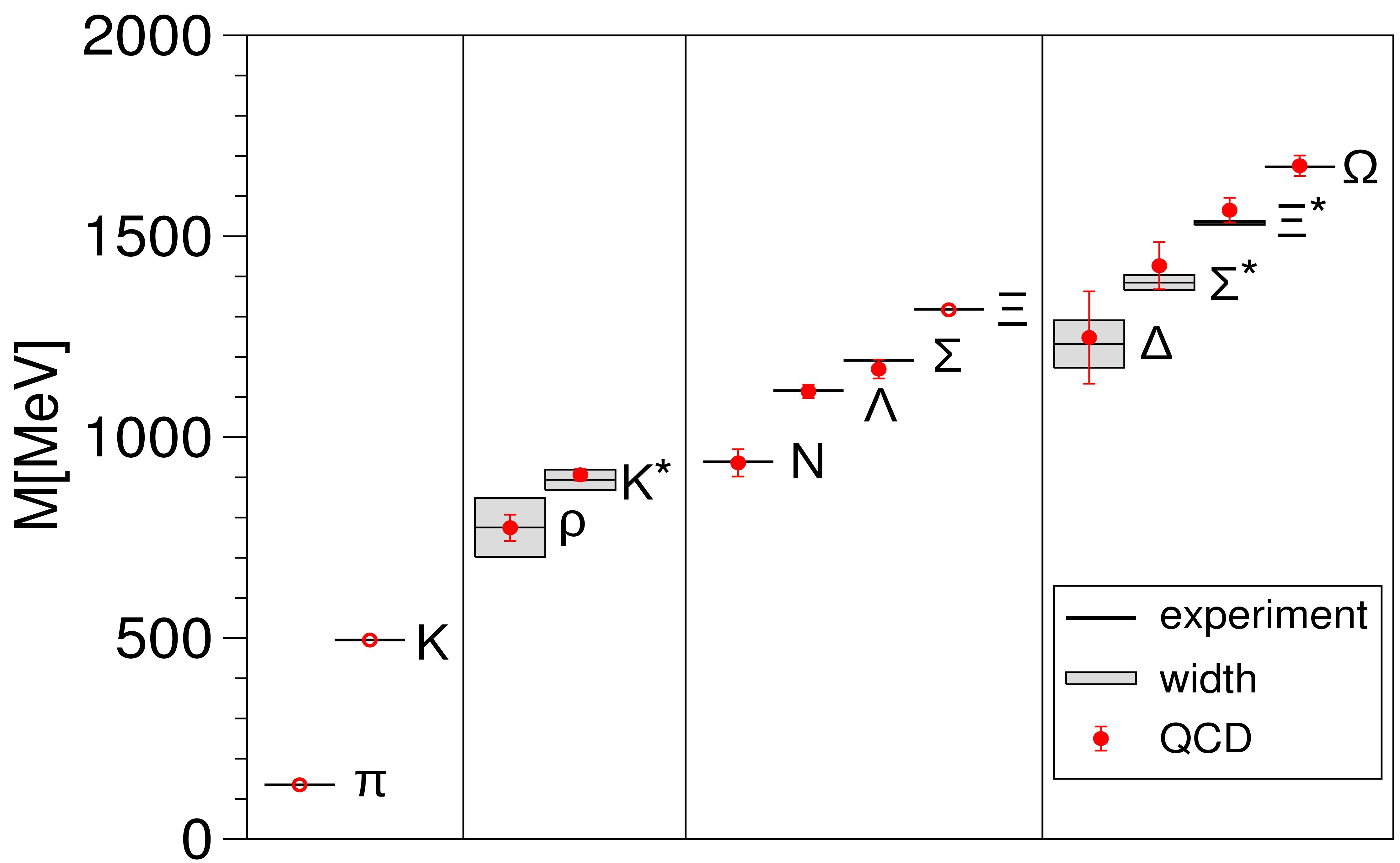
2005





Hadron Spectrum

e.g., BMW Collaboration: *Science* **322** (2008) 1224



- $a = 0.125, 0.085, \& \ 0.065$ fm;
- tree $\mathrm{O}(a)$ Wilson;
- 6× stout smearing;
- $2m_l < m_q < 1.7m_s$;
- π, K, Ξ input.

QCD postdicts the low-lying hadron masses

Light Quark Masses

- The nonzero pion (kaon) mass is very sensitive to the light (strange) masses.
- Chiral perturbation theory predicts *ratios* of masses, but not the overall scale.

Lattice QCD	**MILC**	**RBC**	**BMW**	**HPQCD**
$\overline{m}_u(2\ \mathrm{MeV})$	1.9 ± 0.2	2.24 ± 0.35	2.15 ± 0.11	
$\overline{m}_u(2\ \mathrm{MeV})$	4.6 ± 0.3	4.65 ± 0.35	4.79 ± 0.14	
$\overline{m}_s(2\ \mathrm{MeV})$	88 ± 5	97.6 ± 6.2	95.5 ± 1.9	92.4 ± 1.5
$\overline{m}_c(\overline{m}_c)$				1268 ± 9

- Competitive results for m_s from elsewhere?

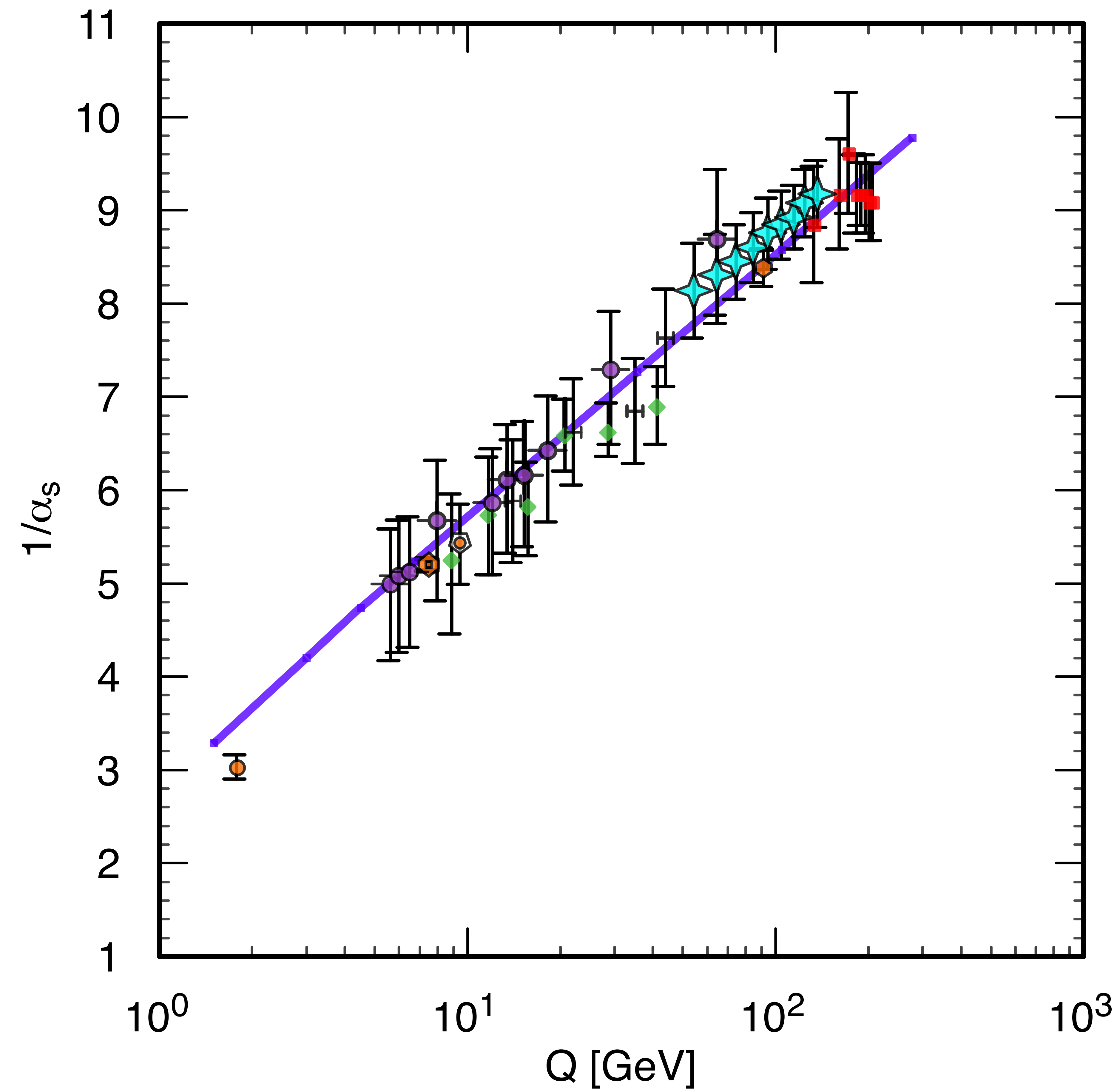
Pertinent Synopsis

- The spectrum results suggest that the calibration step is understood:
  - Continuum limit under control: 3–5 different lattice spacings—up to ×3;
  - Chiral extrapolation under control;
  - Finite-volume effects small (as expected for masses of stable particles);
  - Several groups (MILC, PACS-CS, BMW) with 2+1 spectrum and few % errors.

gold-plated

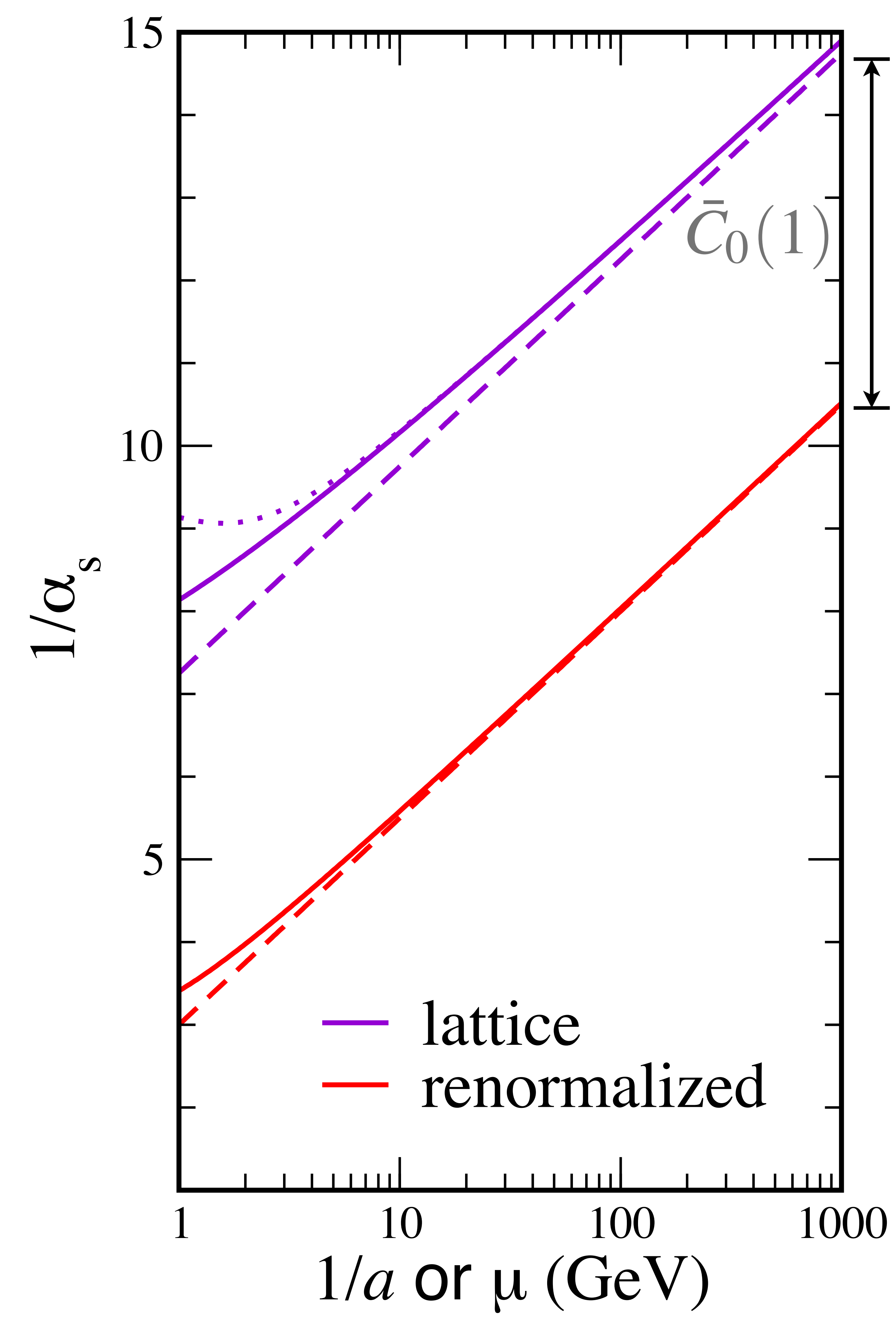
- Influential results: matrix elements for flavor physics (aids search for NP in *B* decays and mixing), thermodynamics (early universe, heavy ions), chiral condensate (Nambu's pion), nucleon sigma term (relevant to DM search), nucleon structure (parton densities), nucleon interactions (neutron stars) ….

Methods for α_s



Matching (Obsolete)

- Lattice perturbation theory is completely well-defined.
- Compute a renormalized (*e.g.*, BPHZ) coupling α_R ($= R$) with lattice and dimensional regulators.
- Equate $\alpha_\mathrm{R} = \alpha_\mathrm{R}$, obtaining $\bar{\alpha}^{-1}(\mu) = \alpha_0^{-1} - \bar{C}_0(\mu a) + \cdots$; asymptotically, $\Lambda_{\overline{\mathrm{MS}}} = \Lambda_0 \exp\left[\bar{C}_0(1)/2\beta_0\right]$.
- Theory of power effects is Symanzik EFT.
- Range of $Q = a^{-1}$ limited.
- With standard lattice actions, asymptopia is too far away and convergence is too slow to be useful.



Lattice Perturbation Theory

Lepage and Mackenzie, *PRD* **48** (1992) 2250

- The lattice gauge field is $U_\mu(x) = \mathrm{P}\exp\left[ag_0 \int_x^{x+ae_\mu} \mathrm{d}s\, A_\mu(x+sae_\mu)\right]$.
- In the perturbation expansion, the second-order part of U_μ leads to an extra-gluon vertex, suppressed by a.
- These vertices lead to tadpole diagrams. Pre-factors of a are cancelled by inverse powers from UV divergent loop integrals.
- Tadpoles lead to large coefficients of α_s and explain the bad behavior of the previous slide.
- These cancel in various “tadpole-improved” combinations of short-distance coefficients.
- A further key to making lattice perturbation theory viable is to eliminate g_0, for example, by re-expanding short-distance quantities in a (quasi-)renormalized coupling.

Small Wilson Loops

HPQCD, arXiv:0807.1687; Maltman, arXiv:0807.2020

- Wilson loop $W(\mathcal{P}) = \mathrm{Re}\langle \mathrm{tr} \exp \oint_{\mathcal{P}} dz \cdot A \rangle$ has UV singularities.

- Creutz ratios $\chi(N_1, M_1, N_2, M_2) = \dfrac{W(N_1 \times M_1) W(N_2 \times M_2)}{W(N_2 \times M_1) W(N_1 \times M_2)}$

  cancels these, but still has UV behavior.

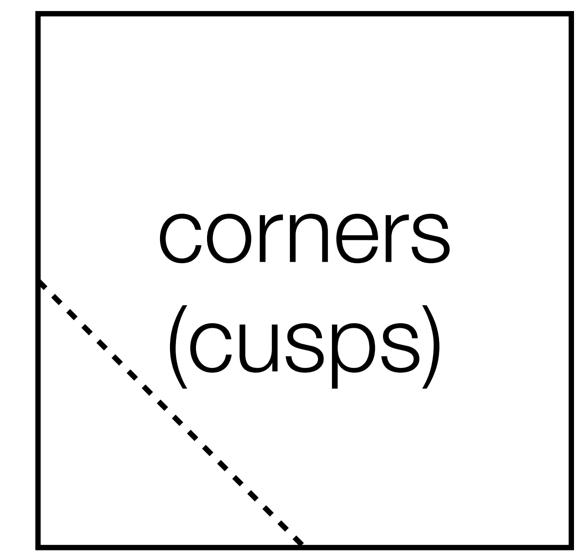
- Tadpole-improved Wilson loops: cancel sides.

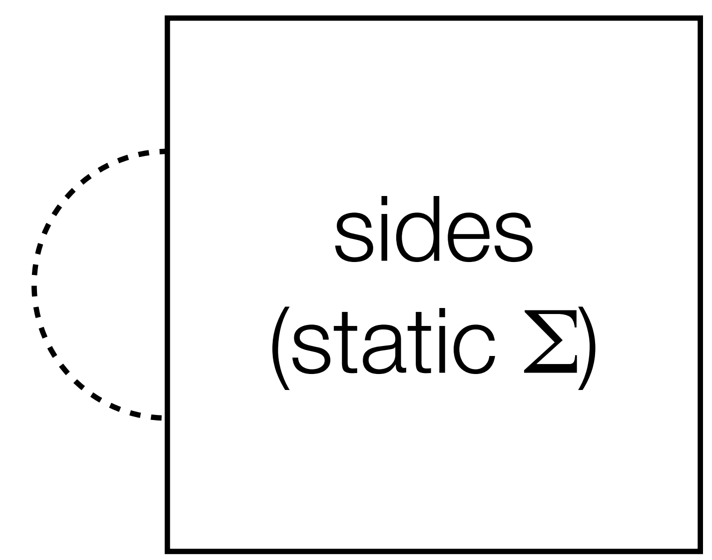
- OPE/Symanzik: $W_{N\times M} = Z_{N\times M}(\alpha_s) + a^4 \mathcal{K}_{N\times M}(\alpha_s)\alpha_s \,\mathrm{tr}\, G_{\mu\nu}G^{\mu\nu}$,
  so some condensate information must be accounted for.

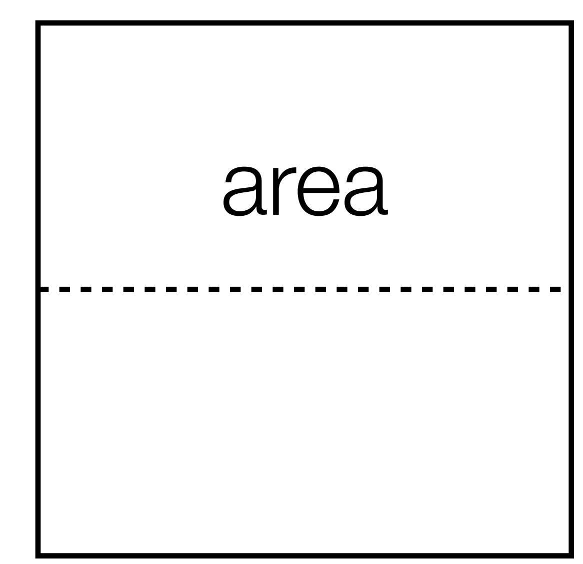
- $R = -\ln W$, $-\ln\chi$; $Q = d/a$, with d estimated via BLM.

- Range of Q at present is ~$6 = 1.8/0.3$.

- $\mathcal{R}$ to NNLO.





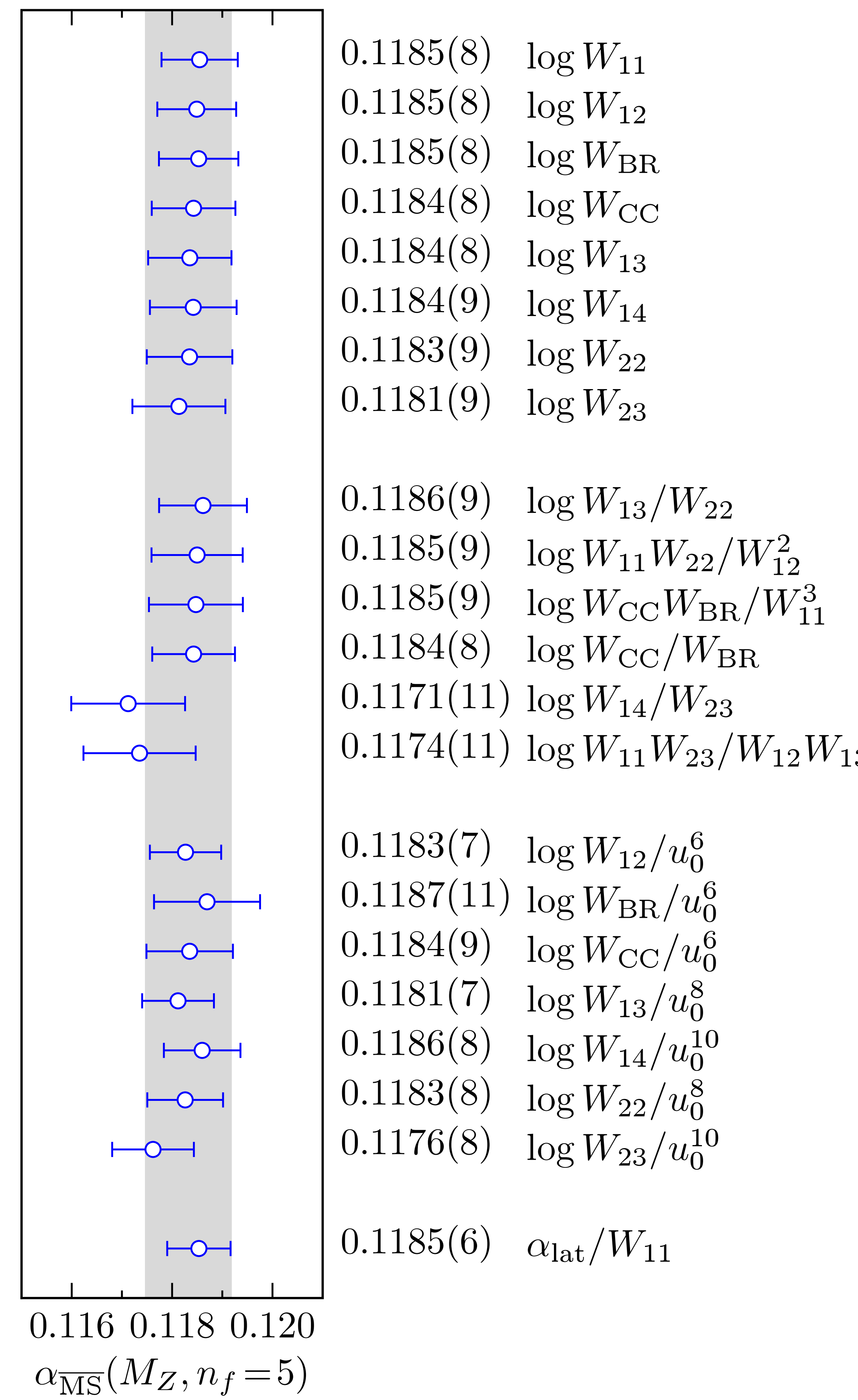


Small Wilson Loops

HPQCD, arXiv:0807.1687; Maltman, arXiv:0807.2020

- Wilson loop $W(\mathcal{P}) = \mathrm{Re}\langle \mathrm{tr} \exp \oint_{\mathcal{P}} dz \cdot A \rangle$ has UV singularities.
- Creutz ratios $\chi(N_1, M_1, N_2, M_2) = \dfrac{W(N_1 \times M_1) W(N_2 \times M_2)}{W(N_2 \times M_1) W(N_1 \times M_2)}$

  cancels these, but still has UV behavior.
- Tadpole-improved Wilson loops: cancel sides.
- OPE/Symanzik: $W_{N\times M} = Z_{N\times M}(\alpha_s) + a^4 \mathcal{K}_{N\times M}(\alpha_s)\alpha_s \,\mathrm{tr}\, G_{\mu\nu} G^{\mu\nu}$, so some condensate information must be accounted for.
- $R = -\ln W, -\ln \chi$; $Q = d/a$, with d estimated via BLM.
- Range of Q at present is ~6 = $1.8/0.3$.
- $\mathcal{R}$ to NNLO.



Adler Function

JLQCD, arXiv:1002.0371

- Vacuum polarization at space-like momentum transfer—ideal for Euclidean field theory:

$$\mathrm{FT}\langle J_\mu(x) J_\nu(0)\rangle = \left(\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu\right)\Pi_J^{(1)}(Q^2) + Q_\mu Q_\nu\, \Pi_J^{(0)}(Q^2)$$

- Let $\Pi = \Pi_V^{(0)} + \Pi_V^{(1)} + \Pi_A^{(0)} + \Pi_A^{(1)}$ and $R(Q^2) = D(Q^2) = -Q^2\, \mathrm{d}\Pi/\mathrm{d}Q^2$, which has a continuum limit. Use nonsinglet currents.

- Operator product expansion (OPE):

$$R(Q^2) = \mathcal{R}\left(\alpha_s(Q^2)\right) + \frac{\bar{m}^2}{Q^2}\mathcal{R}_m\left(\alpha_s(Q^2)\right) + \text{condensates}$$

known to N^{3}LO in continuum pQCD. (Cited JLQCD publication uses NNLO.)

- Compute $R(Q^2)$ for $a \ll Q^{-1} \ll \Lambda_{\mathrm{QCD}}$, take continuum limit, extract α_s.

Moments of the Charmonium Correlator

Bochkarev & de Forcrand, hep-lat/9505025; HPQCD+KIT, arXiv:0805.2999

- Similar idea, but now J is a $\bar{c}c$ current; in particular $m_c\bar{c}\gamma^5 c$. Take moments:

$$G_n = a\sum_t t^n a^3 \sum_{\boldsymbol{x}} m_c^2 \langle \bar{c}\gamma^5 c(\boldsymbol{x},t)\bar{c}\gamma^5 c(0)\rangle = \frac{\mathcal{R}_n(\alpha_s(2m_c))}{[\bar{m}_c(2m_c)]^{n-4}} + \mathrm{o}\left((m_c a)^2\right) + \mathrm{o}\left((\Lambda/2m_c)^2\right)$$

  where $\mathcal{R}_n$ is computed in continuum PT; known to NNLO.

- Use $G_4 = \mathcal{R}_4$ for α_s; others for m_c (and cross checks).

- Moments correspond to derivatives of the Fourier transformed correlator at $q^2 = 0$, where the charmed quarks are far off shell, so the relevant short distance is $Q = (2m_c)^{-1}$.

- Virtuality depends on n.

- Continuum power effects from OPE.

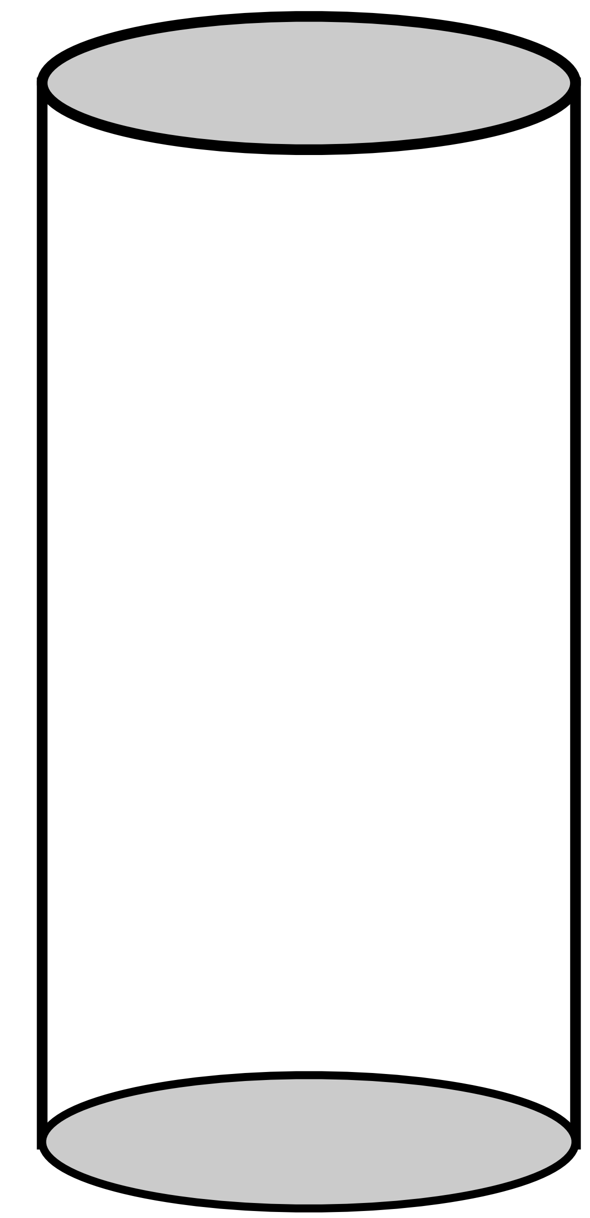
Other Such Quantities

- The theme of the previous two methods can be generalized.
- Other examples include the static-quark potential (or force).
- In general, they require $a^{-1} \gg Q \gg \Lambda_{\mathrm{QCD}}$, or at least $a^{-1} \gg Q$ & $2Q \gg \Lambda_{\mathrm{QCD}}$.
- A key advantage is that *you* can take the CL data from the lattice paper, and carry out your own α_s analysis [Maltman, arXiv:0807.2020].
- A disadvantage is that Q will be limited, though not more so than many determinations from high-energy scattering.

Schrödinger Functional

Wolff, *NPB* **265** (1986) 506, 567; Lüscher, Narayanan, Weisz, Wolff, *NPB* **384** (1992) 168

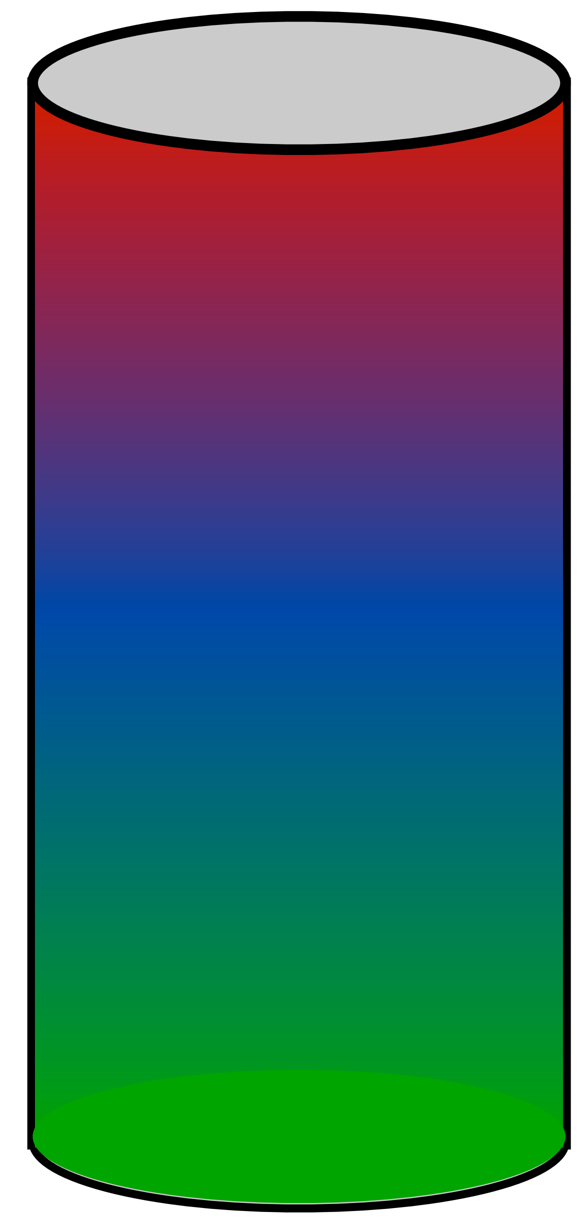
- QCD in a can (well, on a 3-torus), typically $L^3 \times 2L$, $Q = L^{-1}$.
- Apply boundary conditions at caps, filling the can with some sort of chromodynamic muck. Femtoscale hadronization?!
- Parton-hadron duality says energy in can, for $Q \gg \Lambda_{\mathrm{QCD}}$, can be computed with partons, *i.e.*, with perturbation theory.
- Actually, $R^{-1} = -L^{-1}\, \mathrm{d} \ln Z(A_\Omega)/\mathrm{d}A_\Omega$ (removing an additive UV divergence; A_Ω = boundary potential).
- Vary $Q = L^{-1}$ over potentially *enormous* range: $\times 10^3$ [*Alpha*].
- No theory of effects suppressed by of $(L\Lambda_{\mathrm{QCD}})^s$.



Schrödinger Functional

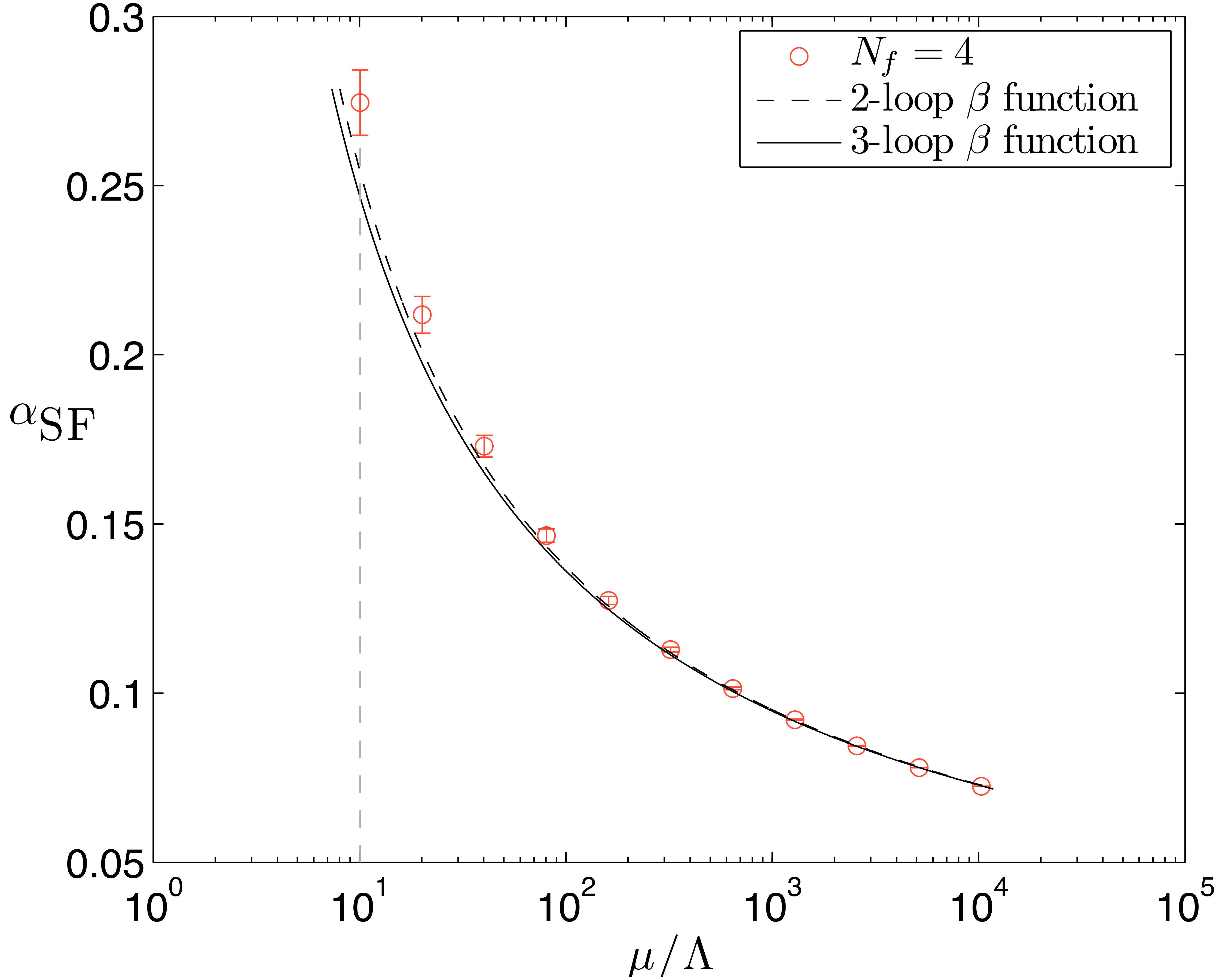
Wolff, *NPB* **265** (1986) 506, 567; Lüscher, Narayanan, Weisz, Wolff, *NPB* **384** (1992) 168

- QCD in a can (well, on a 3-torus), typically $L^3 \times 2L$, $Q = L^{-1}$.
- Apply boundary conditions at caps, filling the can with some sort of chromodynamic muck. Femtoscale hadronization?!
- Parton-hadron duality says energy in can, for $Q \gg \Lambda_{\mathrm{QCD}}$, can be computed with partons, *i.e.*, with perturbation theory.
- Actually, $R^{-1} = -L^{-1}\, \mathrm{d} \ln Z(A_\Omega)/\mathrm{d}A_\Omega$ (removing an additive UV divergence; A_Ω = boundary potential).
- Vary $Q = L^{-1}$ over potentially *enormous* range: $\times 10^3$ [*Alpha*].
- No theory of effects suppressed by of $(L\Lambda_{\mathrm{QCD}})^s$.



Running with 4 Flavors

Sommer, Tekin, Wolff [Alpha], arXiv:011.2332



- Reminiscent of the CDF plot, but:
  - no pdf uncertainties
  - no question of non-QCD contributions
  - energy calibration will be good and crosschecked (eventually)

Summary & Outlook

Selected $\alpha_s(M_Z)$ Results from Lattice QCD

$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z)$	R	Q range	$\mathcal{R}$	sea	collab	when
0.1170(12)	Wilson loops	3	NNLO	2+1 $\sqrt{}$stag	HPQCD	2005
0.1183(8)		7				2008
0.1192(11)					Maltman ...	2008
0.1174(12)	$\overline{Q}Q$ correlator	1–2	NNLO	2+1 $\sqrt{}$stag	HPQCD + KIT	2008
0.1183(7)		3–6				2010
0.1181(3)(+14/–12)	Adler	5	NNLO	2+1 overlap	JLQCD	2010
0.1205(8)(5)(+0/–17)	Schrödinger	80	asymptote	2+1 Wilson	PACS-CS	2009
$\Lambda^{(2)}_{\overline{\mathrm{MS}}} = 245(23)\frac{0.5\text{ fm}}{r_0}$ MeV	Schrödinger	270	asymptote	2 Wilson	*Alpha*	2004
0.1xxx(y)		1000	asymptote	2+1+1 Wilson		2012

- Superseded; re-analysis.

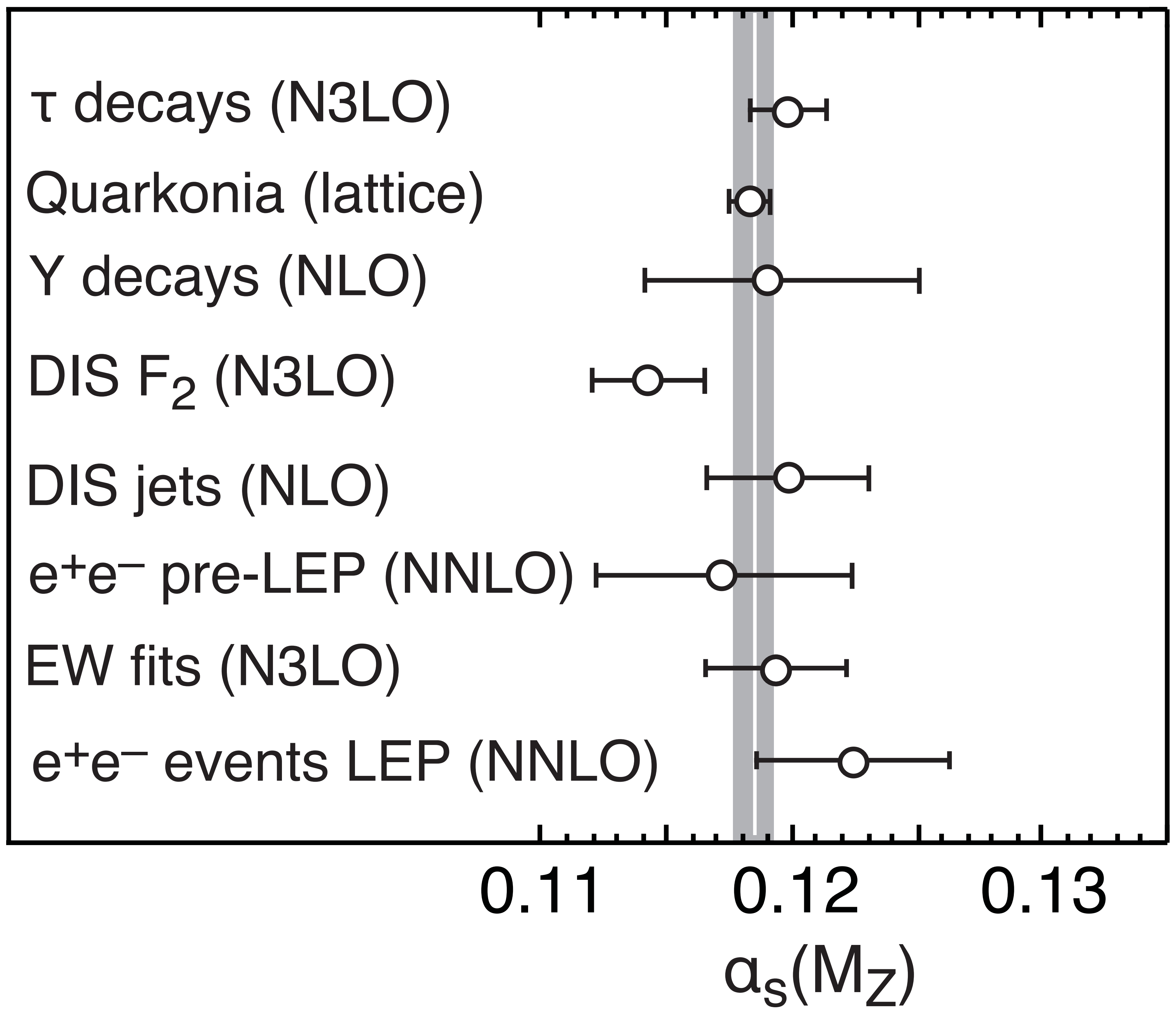
Click on year to find paper.

Has $\alpha_s(M_Z)$ from Lattice QCD Changed?

mea culpa

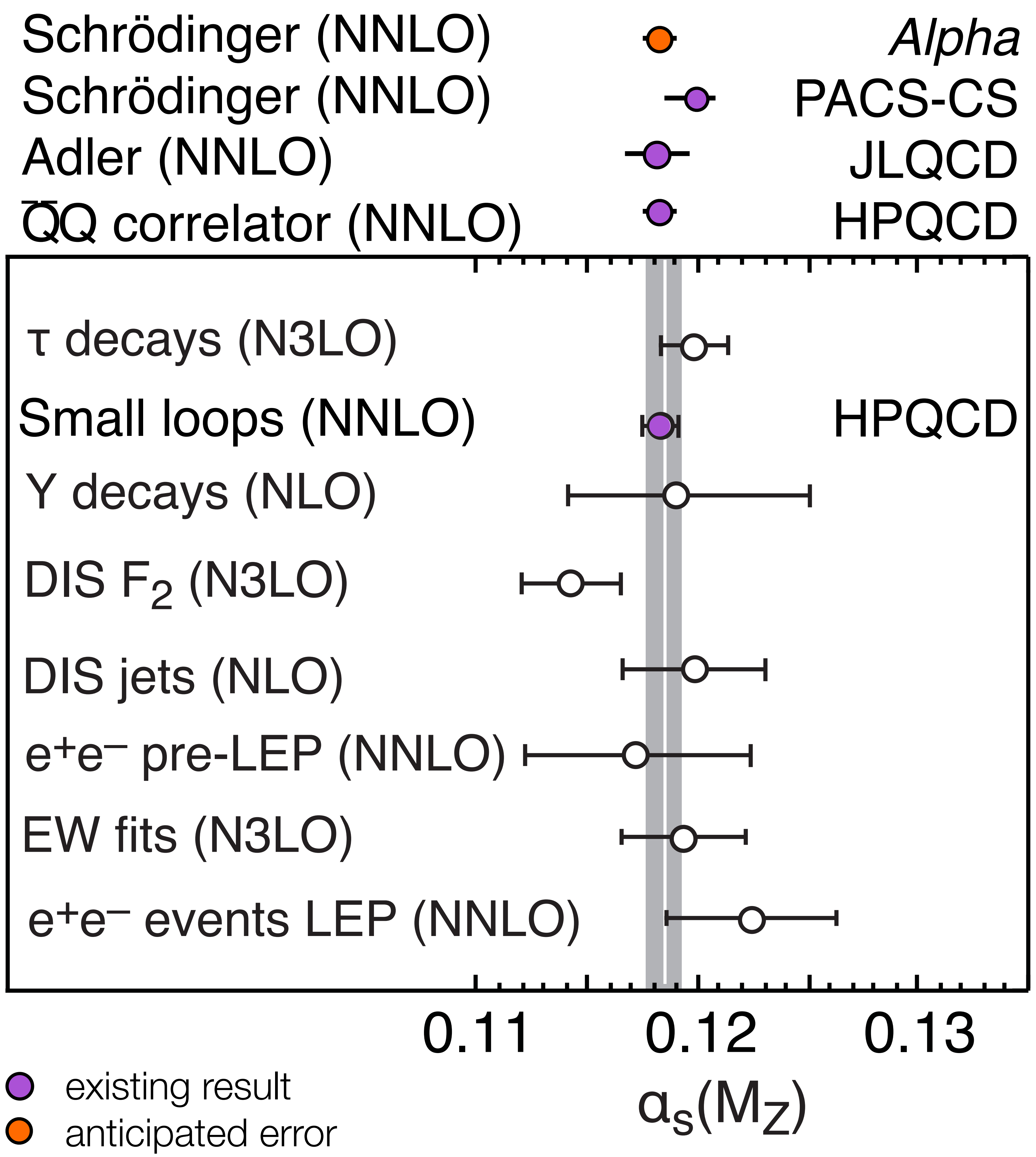
- In 1992, one of the first lattice QCD results reported $\alpha_s(M_Z) = 0.107 \pm 0.004$.
- Attracted some interest.
- One-loop matching $\alpha_0/W_{1\times1}$ to $\alpha^{(0)}_{\overline{\text{MS}}}(5\ \text{MeV})$, two-loop running to M_Z.
- Quenched approximation, so scale not clear.
- Focus on charmonium splittings for Q, argued insensitive to quenching.
- Potential-model argument to justify a correction for $n_f = 0$ running between 0.75–5.0 GeV.
- NNLO PT & restoring sea quarks could both easily explain 2σ shift.

Future of α 



- Many determinations of α_s from lattice QCD are & will be available.
- No important error from experiment.
- Transparent theory.
- Small uncertainties.
- Tempting to use lattice QCD α_s as input for high-energy scattering to:
  - understand QCD;
  - search for new physics.

Future of α 



- Many determinations of α_s from lattice QCD are & will be available.
- No important error from experiment.
- Transparent theory.
- Small uncertainties.
- Tempting to use lattice QCD α_s as input for high-energy scattering to:
  - understand QCD;
  - search for new physics.

Homework

- MILC, ETMC, others are generating ensembles of gauge fields with 2+1+1 sea quarks:
  - α_s calculations will be repeated here, tested perturbativity of charmed threshold.
- Other methods that can exploit continuum PT will appear.
- Your homework:
  - study and familiarize yourself with lattice QCD error budgets;
  - ask why unfamiliar items are included;
  - ask why expected items are left out.