WORKSHOP ON PRECISION MEASUREMENTS OF α_s

Max-Planck-Institute for Physics
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alpha_s from JLQCD

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@ alpha_s workshop, Feb 11, 2011.

This work is from …

JLQCD collaboration

- running a project of dynamical overlap fermion (since 2006). Unique applications with exact chiral symmetry
  - Dirac operator spectrum (and chiral condensate), topological susceptibility, VV-AA correlator, nucleon strange quark content,
  - and alpha_s.

- E. Shintani et al. [JLQCD and TWQCD collaborations], arXiv: 0807.0556 [hep-lat], Phys. Rev. D79, 074510 (2009).
  - Attempt of the method on two-flavor configs.
- E. Shintani et al. [JLQCD collaboration], arXiv:1002.0371 [hep-lat], Phys. Rev. D80, 074505 (2010).
  - Improved method with the conserved current.
  - Physical 2+1-flavor result.

Extraction of α_s from lattice calc

- Basic inputs:
  1. Lattice scale
     - meson/baryon masses, decay constants (any physical quantity with non-zero mass dimension; there are practical (dis)advantages, though)
     - after tuning quark masses to their physical ones.
  2. Perturbative quantity calculated non-perturbatively on the lattice
     - Perturbative expansion needed to be known at least to α_s^2.
     - ex: heavy quark potential at short distances, small Wilson loops, ...
     - For better convergence, recommended NOT to use the lattice bare coupling. Better to use some renormalized coupling (Lapage-Mackenzie).

Vacuum polarization

- As the "perturbative" quantity, we chose

$$\int d^4x e^{iQx} \langle 0 | J^a_\mu(x) J^{b\dagger}_\nu(0) | 0 \rangle = \delta^{ab} \left[(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi^{(1)}_J(Q) - Q_\mu Q_\nu \Pi^{(0)}_J(Q) \right]$$

  - *a*, *b* : isospin indices (we consider flavor non-singlet)
  - *J* : vector (V) or axial-vector (A) current
  - $\Pi_J^{(1)}(Q)$: vacuum polarization function (transverse part)
  - $\Pi_J^{(0)}(Q)$: vacuum polarization function (longitudinal part), vanish for V, proportional to *m* for A

  - *Q*: Euclidean (= space-like) momentum
  - Perturbative expansion (+ OPE) known to α_s^4 in the continuum theory.

Euclidean?

- Ground state
  - Typically, we (lattice theorists) use the asymptotic behavior

$$C(t,\mathbf{p}) = \int dp_0 e^{ipx} \frac{Z}{p_0^2 + \mathbf{p}^2 + m^2} \sim \frac{Z}{2E}\exp(-Et), \quad E = \sqrt{m^2 + \mathbf{p}^2}$$

    to extract the ground state. Not directly looking at the pole.

- Direct look at the correlator
  - In principle, the same info can be obtained by directly analyzing the correlator

$$\frac{Z_1}{p^2 + m_1^2} + \frac{Z_2}{p^2 + m_2^2} + \dots$$

    in the Euclidean ($p^2>0$) region. Here, we consider the large p^2 region (above 1 GeV^2), which is perturbative.

Perturbative expansion

- Vacuum polarization function at high Q^2:
  - expandend using OPE

$$\Pi_J(Q) = \Pi_J^{(1)}(Q) + \Pi_J^{(0)}(Q)$$
$$= c + C_0(Q^2,\mu^2,\alpha_s) + C_m^J(Q^2,\mu^2,\alpha_s)\frac{\bar{m}^2(Q)}{Q^2}$$
$$+ \sum_{q=u,d,s} C_{\bar{q}q}^J(Q^2,\alpha_s)\frac{\langle m_q \bar{q}q\rangle}{Q^4} + C_{GG}(Q^2,\alpha_s)\frac{\langle(\alpha_s/\pi)GG\rangle}{Q^4} + O\left(\frac{1}{Q^6}\right)$$

  - c: scheme dependent divergence.
    - derivative is a physical quantity (Adler function) $D(Q^2) = -Q^2\frac{d\Pi(Q^2)}{dQ^2}$
  - Other terms are finite:
    - Perturbative expansion in the continuum scheme (MSbar) is directly applicable to analyze the lattice data. α_s in MSbar appears.
    - C_0: known to $O(\alpha_s^4)$

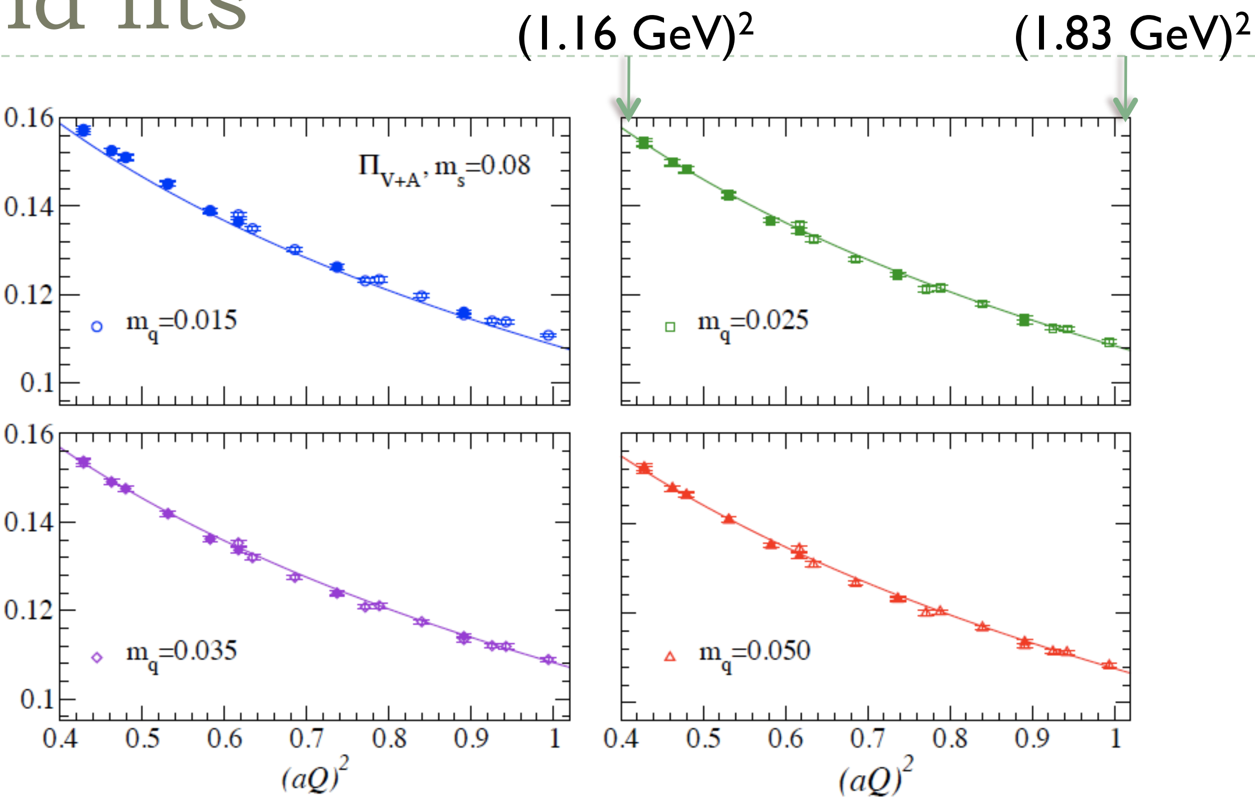
Strategy

- Simple, in principle
  1. Calculate the V and A two-point functions on the lattice
  2. Fit the data at high Q^2 with the continuum perturbative formula.
     - Fit parameters: c, $\alpha_s(\mu)$, and condensates.
  3. Determine the scale $1/a$ from other quantities. Then, $\alpha_s(\mu)$ is obtained.
- Need to be careful about
  - Discretization effects? : more important at high Q^2. how are they estimated?
  - Window? : can we find the region where the pert formula safely applies while disc error is small enough?
  - Enough sensitivity? : can we get enough precision for $\alpha_s(\mu)$ to be interesting?

Lattice

- JLQCD simulations
  - Dynamical overlap fermion: exact chiral symmetry at m=0, continuum-like Ward-Takahashi identities.
  - Lattice is modest, at single *a*
    - β=2.30 (Iwasaki), a=0.11 fm (1/a=1.83 GeV), 16^3x48
    - N_f=2+1: 5 ud quark masses, covering $m_s/6 \sim m_s$ (x2) s quark masses
  - Advantage:
    - chiral condensate $\langle m_q qq \rangle$ is guaranteed to vanish at m_q=0. Additional terms such as $1/(aQ)^2$ or $1/(aQ)^4$ are expected without chiral symmetry.
    - $\langle qq \rangle$ calculated from other sources = not a fit parameter, but an input.
  - Disadvantage:
    - significantly more costly. Continuum extrapolation is hard (now).

Data and fits

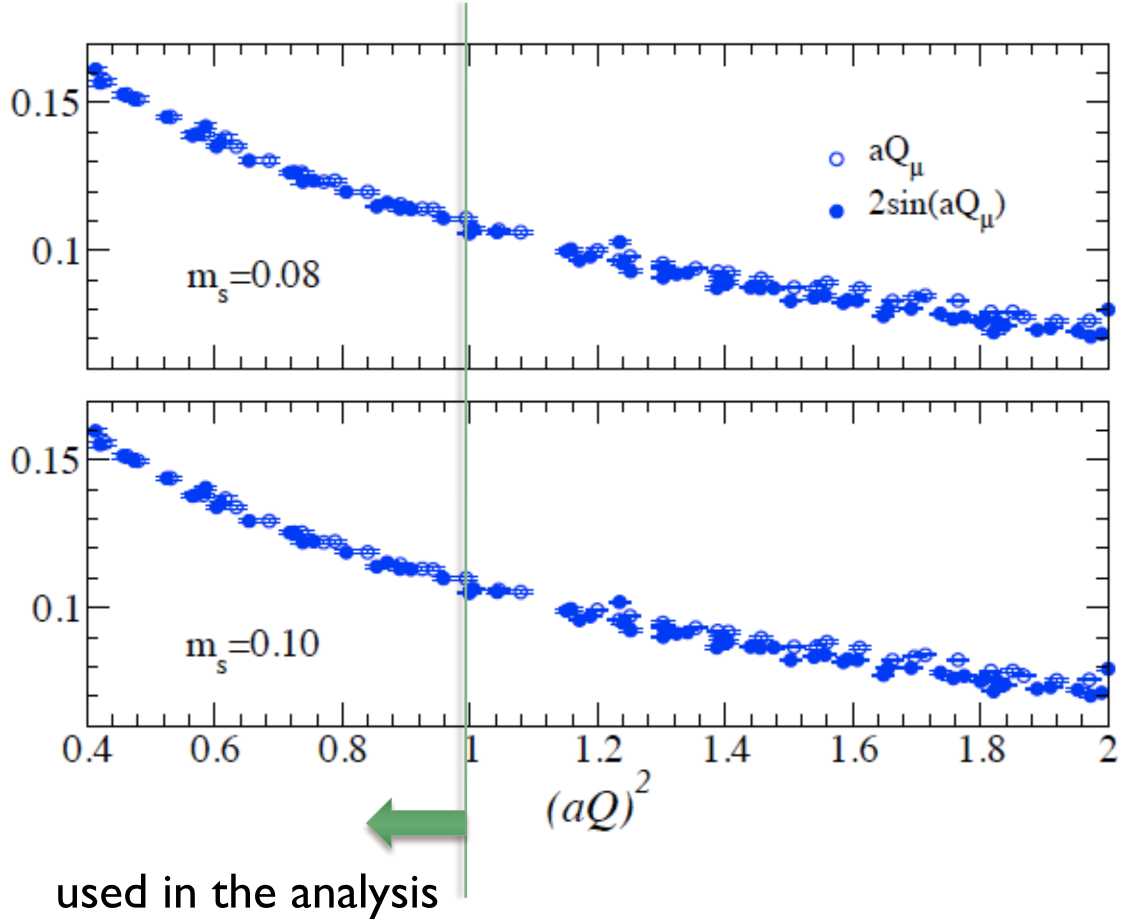


- Filled symbols: $Q_\mu \leq 2\pi/L$, open symbols: $Q_\mu < 4\pi/L$
- Different Q^2 points are (highly) correlated. Our fit gives χ^2/dof=1.7 for filled symbols or O(100) including open symbols.

(Lattice artifact is not included in the fit function.)

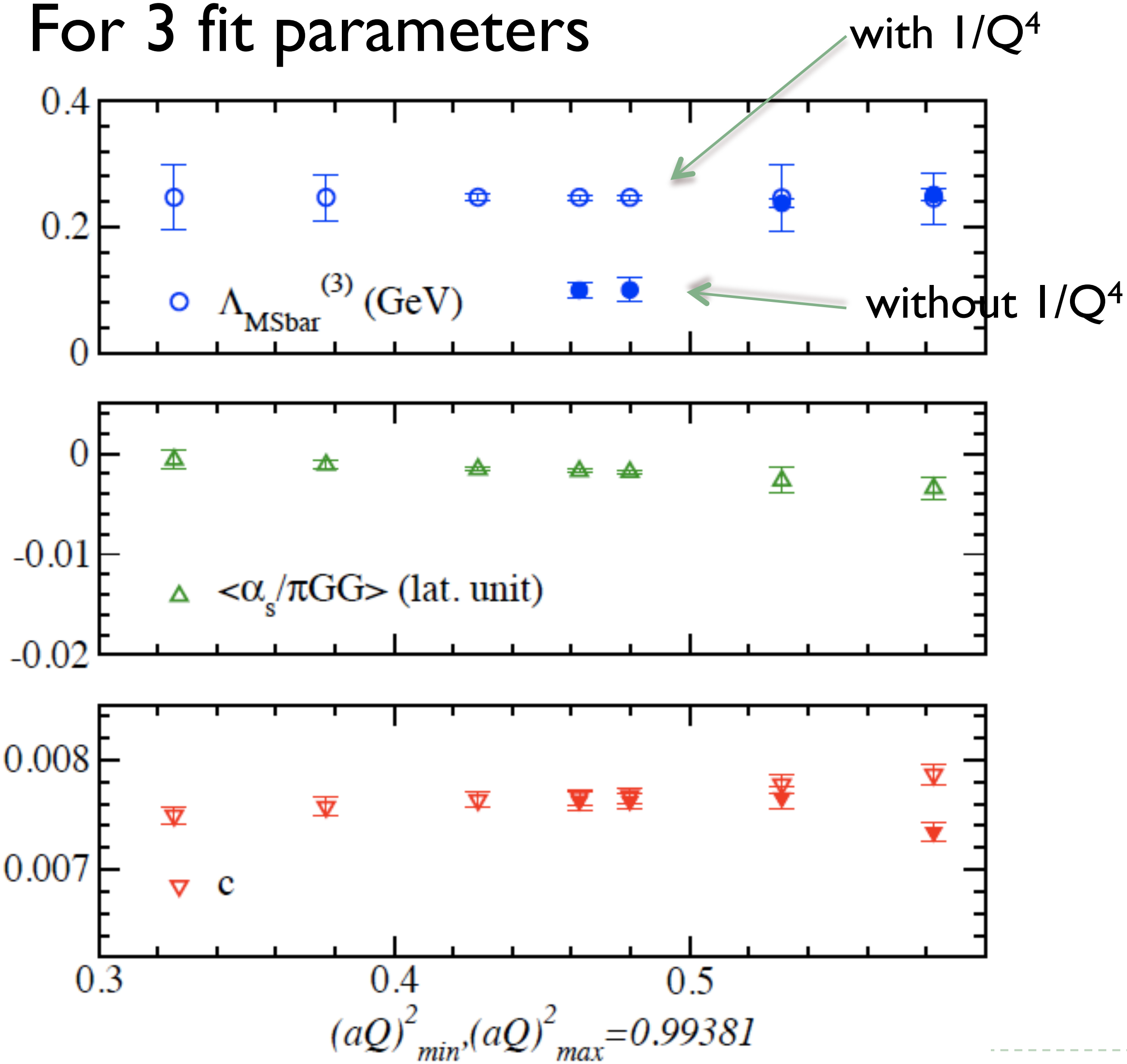
High end of Q^2

- With different momentum definitions



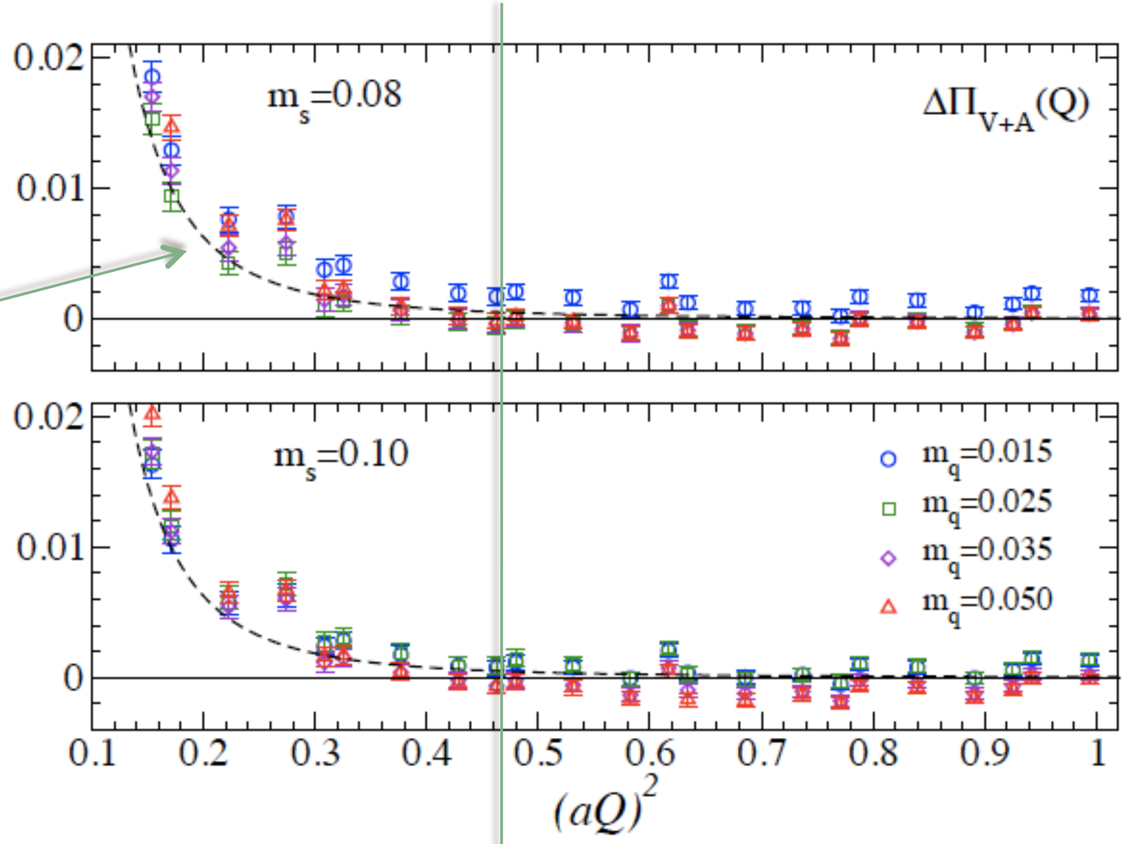
Low end of Q^2

- For 3 fit parameters



Low end of Q^2

- Lattice - Pert

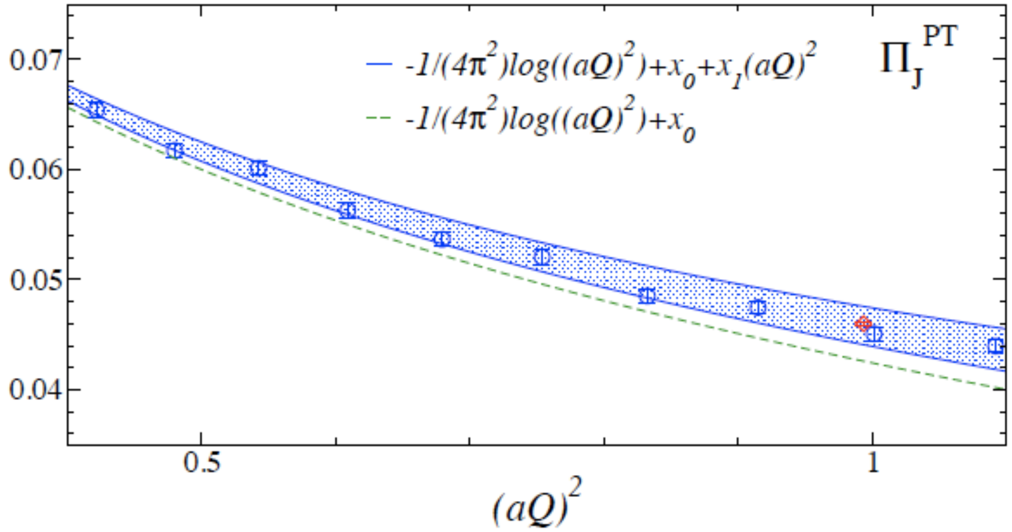


$1/Q^6$ curve significant below (1 GeV)^2

used in the fit with $1/Q^4$

Discretization effect

- One-loop calculation with the lattice (overlap) fermion



- Deviation is seen from the continuum form:

$$\Pi_{V+A}(Q) = c - \frac{1}{2\pi^2}\ln(aQ)^2 + 0.0062(40)\cdot(aQ)^2$$

  - included in the error estimate.

Discretization effect

- Currents used on the lattice is partly not conserved.

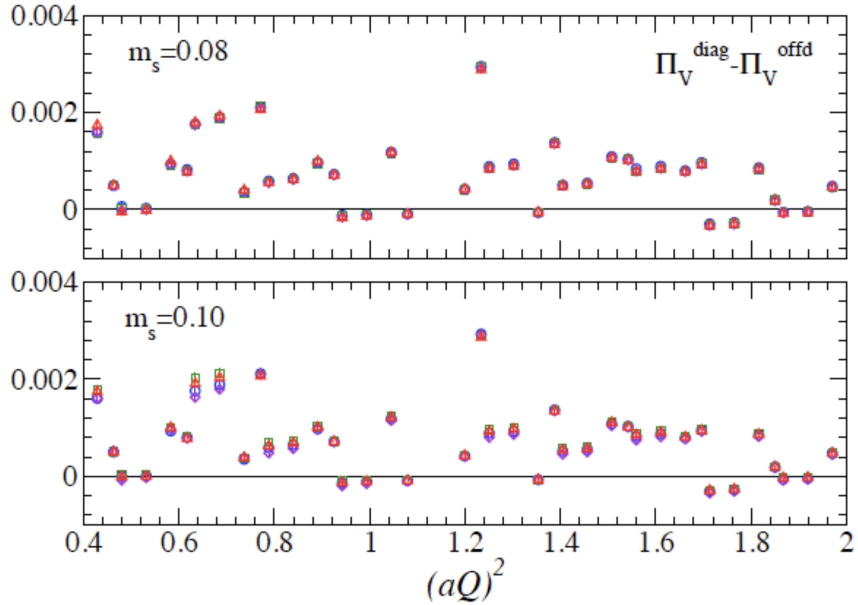
$$\overline{\left\langle J_\mu^{(cv)} J_\nu^{(loc)\dagger} \right\rangle}(Q) = (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu)\Pi_J^{(1)}(Q) - Q_\mu Q_\nu \Pi_J^{(0)}(Q) + \Delta_{\mu\nu}^J(Q)$$

  - can be proved by looking at diagonal (μ=ν) and off-diagonal (μ≠ν) determinations:

$$\Pi_J^{\rm diag}(Q) = \langle J_\mu^{\rm cv} J_\mu^{\rm loc} \rangle(Q)/(\hat{Q}^2 - \hat{Q}_\mu \hat{Q}_\mu),$$
$$\Pi_J^{\rm offd}(Q) = \langle J_\mu^{\rm cv} J_\nu^{\rm loc} \rangle(Q)/(-\hat{Q}_\mu \hat{Q}_\nu),$$

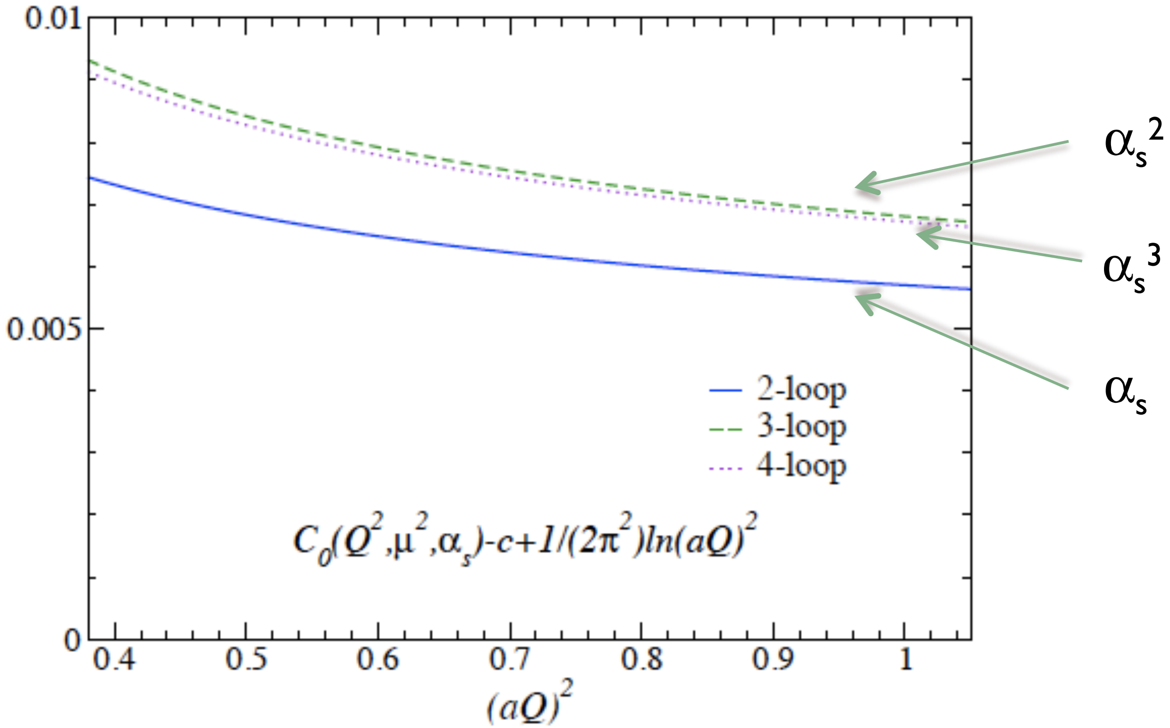
    - Difference is smaller than the estimate given in the previous page.

diff: diag-off-diag



Perturbative expansion

- Convergence



  - The contribution of $O(\alpha_s^3)$ is not substantial. The correction of $O(\alpha_s^4)$ is about the same size.

Baikov, Chetyrkin, Kuhn (2008)

Systematic errors

- Error to $\alpha_s^{(5)}(M_Z)$

Sources	Estimated error in $\alpha_s^{(5)}(M_z)$
Uncorrelated fit	± 0.0003
Lattice artifact ($\mathcal{O}(a^2)$ effect)	$+0.0003$
$\Delta_{\mu\nu}^{V+A}$	± 0.0002
Quark condensate	± 0.0001
Z_m	± 0.0001
Perturbative expansion	± 0.0003
$1/Q^2$ expansion	< 0.0001
$m_{c,b}$	$^{+0.0001}_{-0.0003}$
Lattice spacing	$^{+0.0013}_{-0.0010}$
Total (in quadrature)	$^{+0.0014}_{-0.0012}$

Dominant error:
$1/a$=
1.83(1) GeV r_0=0.49 fm
1.97(4) GeV f_π
1.76(8) GeV m_Ω

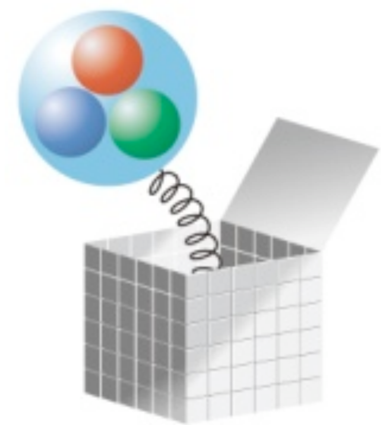
Result and conclusion

- Our result: Shintani et al., Phys. Rev. D82, 074505 (2010).

$$\alpha_s^{(5)}(M_Z) = 0.1181(3)\left({}^{+14}_{-12}\right)$$

  - consistent with other lattice groups
  - similar in precision

- Final remarks:
  - vacuum polarization function:
    - much more useful than I initially thought. (There is the window.)
    - room for improvement (to the level of ±0.0005)
    - every lattice groups would calculate this anyway. Analytic formulae are available. Should try!



Thank you for your attention.

Backup slides

Overlap fermion

Neuberger, Narayanan (1998)

$$D = \frac{1}{a}\left[1 + \frac{X}{\sqrt{X^\dagger X}}\right], X = aD_W - 1$$
$$= \frac{1}{a}\left[1 + \gamma_5 \,\mathrm{sgn}(aH_W)\right], aH_W = \gamma_5(aD_W - 1)$$

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

- Exact chiral symmetry via the Ginsparg-Wilson relation.

$$\delta\bar\psi = i\alpha\bar\psi\left(1 - \frac{a}{2}D\right)\gamma_5, \delta\psi = i\alpha\gamma_5\left(1 - \frac{a}{2}D\right)\psi$$

  - Continuum-like Ward-Takahashi identities hold.
  - Index theorem (relation to topology) satisfied.
  - Topology change is costly; large-scale simulation is feasible only at fixed topology
    - induces O(1/V) effects in general; can be accounted for in the spectral function analysis

Parameters

N_f = 2 runs

- β=2.30 (Iwasaki), a=0.12 fm, 16^3x32
- 6 sea quark masses covering $m_s/6$~m_s
- Q=0 sector only, except for Q=-2, -4 runs at m_q=0.050
- ε-regime run at m=0.002 (m_q~ 3 MeV), β=2.30

N_f = 2+1 runs

- β=2.30 (Iwasaki), a=0.11 fm, 16^3x48
- 5 ud quark masses, covering $m_s/6$~m_s
  - x 2 s quark masses
- Q=0 sector only, except for Q=1 at m_{ud}=0.015
- Larger volume lattice 24^3x48 running at m_{ud}=0.015, 0.025.
- ε-regime run at m=0.002 (m_q ~ 3 MeV)