$\alpha_{S}$ from Bottomonium

NORA BRAMBILLA

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N. B., Xavier Garcia, Joan Soto, Antonio Vairo Phys. Rev. D75, 074014 (2007)

## $\alpha_{s}$ from PDG06



From W.-M. Yao et al., J. Phys. G 33, 1 (2006)


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Notice that this is an important determination: one of the few at relatively low energy with a relatively small error

We have obtained a new extraction of $\alpha_{s}$ from

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R_{\gamma} \equiv \frac{\Gamma(\Upsilon(1 S) \rightarrow \gamma X)}{\Gamma(\Upsilon(1 S) \rightarrow X)}
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-> make possible a QCD description of the photon spectrum X. Garcia, J. SOTO 04, 05

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- New data from CLEO (Heprex/0512061)
- Combined use of NRQCD, pNRQCD and SCET
-> make possible a QCD description of the photon spectrum X. GARCIA, J. SOTO O4, 05
- Accurate estimates of the octet contributions from the lattice (boowin, lee, sinclair os) and from continuum (GARCIA, SOTO 05)
- Recent CLEO measurement of the photon spectrum


[^0]
# Two types of contributions to the photon spectrum: 

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## Fragmentation Contribution

$$
\frac{d \Gamma^{f r a g}}{d z}=\sum_{a} C_{a} \otimes D_{a \rightarrow \gamma} \quad-\infty>\infty \quad 1 \quad+\infty
$$




- GARCIA, SOTO 05



Two types of contributions to the photon spectrum:

Fragmentation Contributions Direct Contributions


- GARCIA, SOTO 05
Two types of contributions to the photon spectrum:


## Fragmentation Contributions

## Direct Contributions

it is described by NRQCD

$$
\frac{d \Gamma}{d z}=\sum_{i} C_{i}(M, z)\langle\Upsilon| \mathcal{O}_{i}|\Upsilon\rangle
$$

and valid in the central region

-GARCIA, SOTO 05

# Two types of contributions to the photon spectrum: 

## Fragmentation Contributions

- For $z \rightarrow 1$ NRQCD expansion breaks down. Collinear degrees of freedom become relevant
- Large $\log (1-z)$ need to be resummed

Photiadis '85; Bauer et al. '01; Fleming and Leibovich '02 '04

- Shape functions must be introduced. Rothstein and Wise '97 Can be calculated assuming Coulombic state x.g.t. and Soto '04

Taking into account all these contributions the complete spectrum can be well described Garcia, soto o5


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- from the data plus the theory description a value for $\Gamma_{\gamma g g}^{\text {direct }}$ can be obtained

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- from the data plus the theory description a value for $\Gamma_{\gamma g g}^{\text {direct }}$ can be obtained
- $\Gamma_{g g g}$ is determined experimentally
in this way $\quad R_{\gamma}=\frac{\Gamma_{g g \gamma}^{\text {direct }}}{\Gamma_{g g g}}=\frac{N_{g g \gamma}^{\text {direct }}}{N_{g g g}}$ is measured ${ }_{R_{\gamma}} \sim \frac{\alpha}{\alpha_{\mathrm{s}}}$
$R_{\gamma}^{e x p}=0.0245 \pm 0.0001 \pm 0.0013$
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Now we need to obtain a precise theoretical calculation of $R_{\gamma}$
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Now we need to obtain a precise theoretical calculation of $R_{\gamma}$

From the comparison of the two we get $\alpha_{s}$
we use NRQCD to calculate the decay

$$
\Gamma(\Upsilon(1 S) \rightarrow X)=\sum_{n} \frac{C_{n}}{m^{d_{n}-4}}\langle\Upsilon(1 S)| O_{n}^{4-\text { fermions }}|\Upsilon(1 S)\rangle
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short distance matching coefficients
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- $m_{b} v \gg \Lambda_{Q C D}$
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- $\frac{\alpha_{s}\left(M_{\Upsilon(1 S)}\right)}{\pi} \sim v^{2} \sim \alpha_{s}^{2}\left(m_{b} v\right)$
- $v^{2}=0.08$

$$
R_{\gamma} \equiv \frac{\Gamma(\Upsilon(1 S) \rightarrow \gamma X)}{\Gamma(\Upsilon(1 S) \rightarrow X)}=\frac{36}{5} \frac{e_{b}^{2} \alpha}{\alpha_{s}} \frac{N}{D}
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\begin{aligned}
& N=1+C_{g g \gamma} \frac{\alpha_{s}}{\pi}+C_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)} \mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}+ \\
& +\frac{\pi}{\alpha_{s}} C_{\gamma O_{8}\left({ }^{1} S_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)}+\frac{\pi}{\alpha_{s}} C_{\gamma O_{8}\left({ }^{3} P_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{3} P_{0}\right)}+\mathcal{O}_{N}\left(v^{3}\right)
\end{aligned}
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$$
\left[\begin{array}{l}
D=1+C_{g g g} \frac{\alpha_{\mathrm{s}}}{\pi}+C_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)} \mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}+\frac{\pi}{\alpha_{\mathrm{s}}} C_{O_{8}\left({ }^{3} S_{1}\right)} \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)} \\
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## WITH

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\mathcal{R}_{O}=\frac{\langle\Upsilon(1 S)| O|\Upsilon(1 S)\rangle}{\left(m_{b}^{\Delta_{d}}\langle\Upsilon(1 S)| O_{1}\left({ }^{3} S_{1}\right)|\Upsilon(1 S)\rangle\right)}
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$\mathcal{O}_{1}\left({ }^{3} S_{1}\right)=\psi^{\dagger} \boldsymbol{\sigma} \chi \cdot \chi^{\dagger} \boldsymbol{\sigma} \psi, \quad \mathcal{O}_{8}\left({ }^{1} S_{0}\right)=\psi^{\dagger} T^{a} \chi \chi^{\dagger} T^{a} \psi$,
$\mathcal{O}_{8}\left({ }^{3} S_{1}\right)=\psi^{\dagger} \boldsymbol{\sigma} T^{a} \chi \cdot \chi^{\dagger} \boldsymbol{\sigma} T^{a} \psi . \quad \mathcal{P}_{1}\left({ }^{3} S_{1}\right)=\frac{1}{2}\left[\psi^{\dagger} \boldsymbol{\sigma} \chi \cdot \chi^{\dagger} \boldsymbol{\sigma}\left(-\frac{i}{2} \stackrel{\leftrightarrow}{\mathbf{D}}\right)^{2} \psi+\right.$ h.c. $]$,
$\mathcal{O}_{8}\left({ }^{3} P_{0}\right)=\frac{1}{3} \psi^{\dagger}\left(-\frac{i}{2} \overrightarrow{\mathbf{D}} \cdot \sigma\right) T^{a} \chi \chi^{\dagger}\left(-\frac{i}{2} \overrightarrow{\mathbf{D}} \cdot \sigma\right) T^{a} \psi$

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The matching coefficients are known in the literature:

$$
\begin{array}{cl}
C_{O_{8}\left({ }^{1} S_{0}\right)}=81 /\left(8 \pi^{2}-72\right) & C_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}=-\left(19 \pi^{2}-132\right) /\left(12 \pi^{2}-108\right) \\
C_{O_{8}\left({ }^{3} S_{1}\right)}=81 n_{f} /\left(20 \pi^{2}-180\right) & C_{O_{8}\left({ }^{3} P_{0}\right)}=567 /\left(8 \pi^{2}-72\right) \\
C_{\gamma O_{8}\left({ }^{1} S_{0}\right)}=27 /\left(4 \pi^{2}-36\right) & C_{\gamma O_{8}\left({ }^{3} P_{0}\right)}=189 /\left(4 \pi^{2}-36\right) \\
C_{g g \gamma}=-1.71\left(\text { for } n_{f}=4\right) & C_{g g g}=3.79 \pm 0.54\left(\text { for } n_{f}=4\right)
\end{array}
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## Octet matrix elements:

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## Octet matrix elements:

- $\mathcal{O}_{8}\left({ }^{1} S_{0}\right)$ and $\mathcal{O}_{8}\left({ }^{3} P_{0}\right)$ have been estimated in the continuum (weak coupling)

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- $\mathcal{O}_{8}\left({ }^{3} S_{1}\right)$ and $\mathcal{O}_{8}\left({ }^{1} S_{0}\right)$ have been calculated on the lattice

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$$
\left.+\frac{\pi}{\alpha_{s}} C_{O_{8}\left({ }^{3} S_{1}\right)} \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)}+\frac{\pi}{\alpha_{s}} C_{O_{8}\left({ }^{1} S_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)}+\frac{\pi}{\alpha_{s}} C_{O_{8}\left({ }^{3} P_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{3} P_{0}\right)}+\mathcal{O}_{D}\left(v^{3}\right)\right]
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The continuum and lattice calculation of $\mathcal{O}_{8}\left({ }^{1} S_{0}\right)$ are compatible

$$
R_{\gamma} \equiv \frac{\Gamma(\Upsilon(1 S) \rightarrow \gamma X)}{\Gamma(\Upsilon(1 S) \rightarrow X)}=\frac{36}{5} \frac{e_{b}^{2} \alpha}{\alpha_{s}} \frac{N}{D}
$$

$$
\begin{aligned}
& N=1+C_{g g \gamma} \frac{\alpha_{s}}{\pi}+C_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)} \mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}+ \\
& +\frac{\pi}{\alpha_{s}} C_{\gamma O_{8}\left({ }^{1} S_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)}+\frac{\pi}{\alpha_{s}} C_{\gamma O_{8}\left({ }^{3} P_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{3} P_{0}\right)}+\mathcal{O}_{N}\left(v^{3}\right)
\end{aligned}
$$

$$
\left(\begin{array}{l}
D=1+C_{g g g} \frac{\alpha_{\mathrm{s}}}{\pi}+C_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)} \mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}+\frac{\pi}{\alpha_{\mathrm{s}}} C_{O_{8}\left({ }^{3} S_{1}\right)} \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)} \\
+\frac{\pi}{\alpha_{s}} C_{O_{8}\left({ }^{3} S_{1}\right)} \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)}+\frac{\pi}{\alpha_{s}} C_{O_{8}\left({ }^{1} S_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)}+\frac{\pi}{\alpha_{s}} C_{O_{8}\left({ }^{3} P_{0}\right)} \mathcal{R}_{O_{8}\left({ }^{3} P_{0}\right)}+\mathcal{O}_{D}\left(v^{3}\right)
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& +\frac{\pi}{\alpha_{s}} C_{o_{8}\left(3 S_{1}\right)} \mathcal{R}_{O_{s}\left({ }^{3} S_{1}\right)}^{\pi}+\frac{\pi}{\alpha_{s}} C_{o_{s}\left(1 S_{0}\right)} \mathcal{R}_{o_{s}\left(1 S_{0}\right)}+\frac{\pi}{\alpha_{s}} C_{o_{\mathbf{s}}\left(3 P_{0}\right)} \mathcal{R}_{o_{8}\left(3 P_{0}\right)}+\mathcal{O}_{D}\left(v^{3}\right)
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- we include all contributions up to $O\left(v^{2}\right)$ in our counting: radiative, relativistic, octet $-->$ accurate at NLO in $v^{2}$ and $\alpha_{s}$
- the nonperturbative contributions are the same in $N$ and $D$ apart from $\mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)}$ that turns out to be small

Two different extractions

- C (for continuum)
- L (for lattice)


## The Analysis

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- We do not expand the $\mathrm{O}(\mathrm{v} \wedge 2)$ terms in the denominator-
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## - L (for lattice)

$$
\begin{aligned}
& 0 \leq \mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)} \leq 4.8 \times 10^{-3} \\
& 0 \leq \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)} \leq 1.6 \times 10^{-4} \\
&-2.4 \times 10^{-4} \leq \mathcal{R}_{O_{8}\left({ }^{3} P_{0}\right)} \leq 2.4 \times 10^{-4} \\
&-0.052 \leq \mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)} \leq-0.035
\end{aligned}
$$

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$2.4 \times 10^{-4} \leq \mathcal{R}_{O_{8}\left({ }^{3} P_{0}\right)} \leq 2.4 \times 10^{-4}$
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$0 \leq \mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)} \leq 4.8 \times 10^{-3} \quad$ lattice data plus $100 \%$
$0 \leq \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)} \leq 1.6 \times 10^{-4}$

Gremm Kapustin $\mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}=\frac{E_{\text {bin }}}{m_{b}}=-\frac{\left(C_{F} \alpha_{\mathrm{s}}\right)^{2}}{4}$,
The uncertainty on $\alpha_{\mathrm{s}}$ induced by a given parameter is evaluated by varying it in the range and keeping all other parameters at their central values. We obtain
$\delta_{C_{g g 9}} \alpha_{\mathrm{s}}=0.0026$,
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estimate higher order corr $-0.04 \leq \mathcal{O}_{N}\left(v^{3}\right), \mathcal{O}_{D}\left(v^{3}\right) \leq 0.04$

We sum up linearly the errors $\delta_{\mathcal{R}_{O_{8}\left({ }^{1} S_{0}\right)}}$ and $\delta_{\mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)}}$, which are correlated, : sum the rest quadratically

$$
\begin{gathered}
\alpha_{\mathrm{s}}\left(M_{\Upsilon(1 S)}\right)=0.183 \pm 0.013 . \\
\alpha_{\mathrm{s}}\left(M_{Z}\right)=0.119 \pm 0.005 .
\end{gathered}
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- $C$ (for continuum $)$

In weak coupling the matrix elements can be calculated, the same central value used to fit the photon spectrum in Garcia-Soto 04 is used
$\langle\Upsilon(1 S)| O_{1}\left({ }^{3} S_{1}\right)|\Upsilon(1 S)\rangle$ is needed at NNLO
$\mathcal{R}_{\mathcal{P}_{1}\left({ }^{3} S_{1}\right)}=-0.015$,

$$
\mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)}=8 \times 10^{-5} .
$$

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\mathcal{R}_{O_{8}\left({ }^{( } S_{0}\right)}
\end{array}
$$

To get the errors we make the variation

$$
\begin{aligned}
& 0.18 \leq \alpha_{\mathrm{s}}\left(m_{b} v\right) \leq 0.38 \\
& 0.32 \leq \alpha_{\mathrm{S}}\left(m_{b} v^{2}\right) \leq 1.3 \\
& 0 \leq \mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)} \leq 1.6 \times 10^{-4}
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this large variation expected to account for uncertainties
$O\left(\Lambda_{Q C D}\right)$
in the weak coupling estimate of the octets

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\end{array} \quad \begin{array}{ll}
\text { in the weak coupling } \\
\text { the upper limit of } O_{8}\left({ }^{3} S_{1}\right) & \text { estimate of the octets }
\end{array}
$$

corresponds to twice the lattice value

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The uncertainty on $\alpha_{\mathrm{s}}$ induced by a given parameter is evaluated by varying it in the range and keeping all other parameters at their central values. We obtain

$$
\begin{aligned}
& \delta_{C_{g g g}} \alpha_{\mathrm{s}}=0.0009, \\
& \delta_{\alpha_{\mathrm{s}}\left(m_{b} v\right)} \alpha_{\mathrm{s}}={ }_{-0.0064}^{+0.0006}, \\
& \delta_{\alpha_{\mathrm{s}}\left(m_{b} v^{2}\right)} \alpha_{\mathrm{s}}={ }_{-0.0076}^{+0.0083}, \\
& \delta_{\mathcal{R}_{O_{8}\left({ }^{3} S_{1}\right)}} \alpha_{\mathrm{s}}=0.0016, \\
& \delta_{\mathcal{R}_{\mathcal{O}_{N}\left(v^{3}\right)}} \alpha_{\mathrm{s}}={ }_{-0.0034}^{+0.0035} \\
& \delta_{\mathcal{R}_{\mathcal{O}_{D}\left(v^{3}\right)}} \alpha_{\mathrm{s}}={ }_{-0.0025}^{+0.0026} \\
& \delta_{R_{\gamma}^{\exp } \alpha_{\mathrm{s}}}=0.01 .
\end{aligned}
$$

We assume these errors to be independent and sum them up quadratically, obtaining

$$
\begin{aligned}
& \alpha_{\mathrm{s}}\left(M_{\Upsilon(1 S)}\right)=0.185_{-0.015}^{+0.014} \\
& \alpha_{\mathrm{S}}\left(M_{Z}\right)=0.120_{-0.006}^{+0.005}
\end{aligned}
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Errors Discussion

- Results are rather insensitive to the values of $O_{8}\left({ }^{1} S_{0}\right) \quad O_{8}\left({ }^{3} P_{0}\right)$ they would be sensitive to $O_{8}\left({ }^{3} S_{1}\right)$ that however is very small from the lattice evaluation (smaller than NRQCD power counting)
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They are expected to be small (no new qualitative feautures), they have been calculated partially (in v) for O_D (Bodwin, Petrelli 02) -> if we estimate them we get

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0.02 \text { compatible with }-0.04 \leq \mathcal{O}_{N}\left(v^{3}\right), \mathcal{O}_{D}\left(v^{3}\right) \leq 0.04 \text {, }
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with respect to PDG06: Hinchllefe, manohar

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between $30 \%$ to $100 \%$; in $\Gamma(\Upsilon(1 S) \rightarrow X)$ they are underestimated
- Older less precise data are used and a model extrapolation to low z
- The extraction is done from $\frac{\Gamma(\Upsilon(1 S) \rightarrow \gamma X)}{\Gamma\left(\Upsilon(1 S) \rightarrow l^{+} l^{-}\right)}$
claiming it is cleaner. But the uncertainty increases due to the an increased sensibility to the octets. Moreover the leptonic width suffer from large corrections in alphas.

We obtain the final result

$$
\alpha_{\mathrm{S}}\left(M_{\Upsilon(1 S)}\right)=0.184_{-0.014}^{+0.015}
$$

## corresponding to



We obtain the final result

$$
\alpha_{\mathrm{s}}\left(M_{\Upsilon(1 S)}\right)=0.184_{-0.014}^{+0.015}
$$

corresponding to

$$
\alpha_{\mathrm{s}}\left(M_{Z}\right)=0.119_{-0.005}^{+0.006}
$$



## Future Prospects for $\alpha_{s}$ determinations from quarkonia

- More precise data for $R_{\gamma}$ for $\Upsilon(1 S)$ would allow a more precise extraction of $\alpha_{s}\left(M_{\Upsilon(1 S)}\right)$
- A new measurement of the inclusive photon spectrum at BESIII would provide the possibility to extract $\alpha_{s}\left(M_{J / \psi}\right)$

GARCIA, SOTO 2007

- Previous to its discovery the $\eta_{b}$ was indicated as the place where a very precise determination of $\alpha_{s}$ a could be obtained from the hyperfine separation

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KNIEHL,PENIN. PINEDA,SMIRNOV,STEINHAUSER 2007
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## BACKUP


[^0]:    [D. Besson et al. [CLEO Collaborationl. Phvs. Rev. D 74 (2006) 012003 (hep-ex/0512061)]

