





Qs from Bottomonium







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N. B., Xavier Garcia, Joan Soto, Antonio Vairo Phys. Rev. D75, 074014 (2007)

α_s from PDG06



From W.-M. Yao et al., J. Phys. G 33, 1 (2006)

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Notice that this is an important determination: one of the few at relatively low energy with a relatively small error

$$R_{\gamma} \equiv \frac{\Gamma(\Upsilon(1S) \to \gamma X)}{\Gamma(\Upsilon(1S) \to X)}$$

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- New data from CLEO (HEP-EX/0512061)
- Combined use of NRQCD, pNRQCD and SCET
 -> make possible a QCD description of the photon spectrum
 X. GARCIA, J. SOTO 04, 05
- Accurate estimates of the octet contributions from the lattice (BODWIN, LEE, SINCLAIR 05) and from Continuum (Garcia, Soto 05)

New data from CLEO

The photon spectrum ($\Upsilon \to X\gamma$)



[D. Besson et al. [CLEO Collaboration]. Phys. Rev. D 74 (2006) 012003 (hep-ex/0512061)]



• GARCIA, SOTO 05

 $(z = 2E_{\gamma}/M_{\Upsilon})$

Two types of contributions to the photon spectrum:

Fragmentation Contribution





• GARCIA, SOTO 05

 N_{γ}

8000

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Two types of contributions to the photon spectrum:

Fragmentation Contribution

important in the low energy region of the spectrum



1

1.2



0.2

 N_{γ}

8000

6000

4000

2000

0.4

0.6

Data from CLEO III, 2005

0.8



• GARCIA, SOTO 05

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Two types of contributions to the photon spectrum:

Fragmentation Contributions Direct Contributions



• GARCIA, SOTO 05

 N_{γ}

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• GARCIA, SOTO 05



- For $z \to 1$ NRQCD expansion breaks down. Collinear degrees of freedom become relevant
 - Large log(1-z) need to be resummed

Photiadis '85; Bauer et al. '01; Fleming and Leibovich '02 '04

Shape functions must be introduced. Rothstein and Wise '97 Can be calculated assuming Coulombic state X.G.T. and Soto '04

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• Γ_{ggg} is determined experimentally



$R_{\gamma}^{exp} = 0.0245 \pm 0.0001 \pm 0.0013$







$$\Gamma(\Upsilon(1S) \to X) = \sum_{n} \frac{C_n}{m^{d_n - 4}} \langle \Upsilon(1S) | O_n^{4 - fermions} | \Upsilon(1S) \rangle$$

 $\Gamma(\Upsilon(1S) \to X) = \sum \underbrace{C_n}_{m^{d_n-4}} \langle \Upsilon(1S) | O_n^{4-fermions} | \Upsilon(1S) \rangle$ short distance matching coefficients

 C_n fermions $\Gamma(\Upsilon(1S) \to X) = \sum_{n=1}^{\infty}$ $\Upsilon(1S)$ $\Upsilon(1S)$

short distance matching coefficients

nonperturbative matrix elements: contain singlet and octet operators

 C_n -fermions $\Gamma(\Upsilon(1S) \to X) = \sum_{i=1}^{N}$ $\Upsilon(1S)$ $\Upsilon(1S)$ short distance matching nonperturbative matrix coefficients elements: contain singlet and octet operators

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 $\Gamma(\Upsilon(1S) \to X) = \sum \frac{C_n}{Z_n d_n - 4}$ $O_{\pi}^{4-fermions}|\Upsilon(1S)|$ $\langle \Upsilon(1S) |$ short distance matching nonperturbative matrix coefficients elements: contain singlet

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• $v^2 = 0.08$

$$R_{\gamma} \equiv \frac{\Gamma(\Upsilon(1S) \to \gamma X)}{\Gamma(\Upsilon(1S) \to X)} = \frac{36}{5} \frac{e_b^2 \alpha}{\alpha_s} \frac{N}{D}$$

$$\begin{pmatrix}
N = 1 + C_{gg\gamma} \frac{\alpha_s}{\pi} + C_{\mathcal{P}_1(^3S_1)} \mathcal{R}_{\mathcal{P}_1(^3S_1)} + \\
+ \frac{\pi}{\alpha_s} C_{\gamma O_8(^1S_0)} \mathcal{R}_{O_8(^1S_0)} + \frac{\pi}{\alpha_s} C_{\gamma O_8(^3P_0)} \mathcal{R}_{O_8(^3P_0)} + \mathcal{O}_N(v^3)
\end{pmatrix}$$

$$\frac{D = 1 + C_{ggg} \frac{\alpha_s}{\pi} + C_{\mathcal{P}_1(^3S_1)} \mathcal{R}_{\mathcal{P}_1(^3S_1)} + \frac{\pi}{\alpha_s} C_{O_8(^3S_1)} \mathcal{R}_{O_8(^3S_1)} \\
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WITH

$$\mathcal{R}_{O} = \frac{\langle \Upsilon(1S) | O | \Upsilon(1S) \rangle}{(m_{b}^{\Delta_{d}} \langle \Upsilon(1S) | O_{1}({}^{3}S_{1}) | \Upsilon(1S) \rangle)}$$

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 $\mathcal{O}_{1}(^{3}S_{1}) = \psi^{\dagger}\boldsymbol{\sigma}\chi \cdot \chi^{\dagger}\boldsymbol{\sigma}\psi, \qquad \mathcal{O}_{8}(^{1}S_{0}) = \psi^{\dagger}T^{a}\chi \chi^{\dagger}T^{a}\psi,$ $\mathcal{O}_{8}(^{3}S_{1}) = \psi^{\dagger}\boldsymbol{\sigma}T^{a}\chi \cdot \chi^{\dagger}\boldsymbol{\sigma}T^{a}\psi, \qquad \mathcal{P}_{1}(^{3}S_{1}) = \frac{1}{2}\left[\psi^{\dagger}\boldsymbol{\sigma}\chi \cdot \chi^{\dagger}\boldsymbol{\sigma}(-\frac{i}{2}\overrightarrow{\mathbf{D}})^{2}\psi + \text{h.c.}\right],$ $\mathcal{O}_{8}(^{3}P_{0}) = \frac{1}{3}\psi^{\dagger}(-\frac{i}{2}\overrightarrow{\mathbf{D}}\cdot\sigma)T^{a}\chi\chi^{\dagger}(-\frac{i}{2}\overrightarrow{\mathbf{D}}\cdot\sigma)T^{a}\psi$

$$R_{\gamma} \equiv \frac{\Gamma(\Upsilon(1S) \to \gamma X)}{\Gamma(\Upsilon(1S) \to X)} = \frac{36}{5} \frac{e_b^2 \alpha}{\alpha_s} \frac{N}{D}$$

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The matching coefficients are known in the literature: $C_{O_8(^1S_0)} = 81/(8\pi^2 - 72)$ $C_{\mathcal{P}_1(^3S_1)} = -(19\pi^2 - 132)/(12\pi^2 - 108)$

$$C_{O_8(^3S_1)} = 81n_f/(20\pi^2 - 180)$$

$$C_{\gamma O_8(^1S_0)} = 27/(4\pi^2 - 36)$$

 $C_{gg\gamma} = -1.71 \text{ (for } n_f = 4)$

 $C_{\mathcal{P}_1(^3S_1)} = -(19\pi^2 - 132)/(12\pi^2 - 108)$ $C_{O_8(^3P_0)} = \frac{567}{(8\pi^2 - 72)}$ $C_{\gamma O_8(^3P_0)} = \frac{189}{(4\pi^2 - 36)}$ $C_{ggg} = 3.79 \pm 0.54 \text{ (for } n_f = 4)$

$$\begin{pmatrix} N = 1 + C_{gg\gamma} \frac{\alpha_s}{\pi} + C_{\mathcal{P}_1(^3S_1)} \mathcal{R}_{\mathcal{P}_1(^3S_1)} + \\ + \frac{\pi}{\alpha_s} C_{\gamma O_8(^1S_0)} \mathcal{R}_{O_8(^1S_0)} + \frac{\pi}{\alpha_s} C_{\gamma O_8(^3P_0)} \mathcal{R}_{O_8(^3P_0)} + \mathcal{O}_N(v^3) \end{pmatrix}$$

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Octet matrix elements:
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• $\mathcal{O}_8({}^1S_0)$ and $\mathcal{O}_8({}^3P_0)$ have been estimated in the continuum (weak coupling)

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• $\mathcal{O}_8({}^3S_1)$ and $\mathcal{O}_8({}^1S_0)$ have been calculated on the lattice BODWIN, LEE, SINCLAIR OF

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• $\mathcal{O}_8({}^3S_1)$ and $\mathcal{O}_8({}^1S_0)$ have been calculated on the lattice BODWIN, LEE, SINCLAIR OF

The continuum and lattice calculation of $\mathcal{O}_8({}^1S_0)$ are compatible

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• we include all contributions up to $O(v^2)$ in our counting: radiative, relativistic, octet

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-->accurate at NLO in v^2 and $lpha_s$

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• we include all contributions up to $O(v^2)$ in our counting: radiative, relativistic, octet

-->accurate at NLO in
$$v^2$$
 and $lpha_s$

• the nonperturbative contributions are the same in N and D apart from $\mathcal{R}_{O_8(^3S_1)}$ that turns out to be small Two different extractions C (for continuum)

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Common features of C and L:

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• Results are rather insensitive to the values of $O_8({}^1S_0)$ $O_8({}^3P_0)$

they would be sensitive to $O_8(\,{}^3S_1)\,$ that however is very small from the lattice evaluation (smaller than NRQCD power counting)

L (for lattice)

 $\begin{array}{rcl} 0 \leq & \mathcal{R}_{O_8(^1S_0)} & \leq 4.8 \times 10^{-3} \\ & 0 \leq & \mathcal{R}_{O_8(^3S_1)} & \leq 1.6 \times 10^{-4} \\ -2.4 \times 10^{-4} \leq & \mathcal{R}_{O_8(^3P_0)} & \leq 2.4 \times 10^{-4} \\ & -0.052 \leq & \mathcal{R}_{\mathcal{P}_1(^3S_1)} & \leq -0.035 \end{array}$



$$\begin{array}{|c|c|c|c|c|} \label{eq:linear_li$$

In weak coupling the matrix elements can be calculated, the same central value used to fit the photon spectrum in Garcia-Soto 04 is used

 $\langle \Upsilon(1S) | O_1(^3S_1) | \Upsilon(1S) \rangle$ is needed at NNLO

 $\mathcal{R}_{\mathcal{P}_1(^3S_1)} = -0.015,$ $\mathcal{R}_{O_8(^1S_0)} = 0.0012,$

 $\mathcal{R}_{O_8(^3S_1)} = 8 \times 10^{-5}$.

 $\mathcal{R}_{O_8(^3P_0)} = 0.0011$.

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the up	oper limit	of $O_8({}^3S_1)$	estimate of the octets

corresponds to twice the lattice value

The uncertainty on α_s induced by a given parameter is evaluated by varying it in the range and keeping all other parameters at their central values. We obtain

$$\begin{split} \delta_{C_{ggg}} \alpha_{s} &= 0.0009, \\ \delta_{\alpha_{s}(m_{b}v)} \alpha_{s} &= ^{+0.0006}_{-0.0064}, \\ \delta_{\alpha_{s}(m_{b}v^{2})} \alpha_{s} &= ^{+0.0083}_{-0.0076}, \\ \delta_{\mathcal{R}_{O_{8}(^{3}S_{1})}} \alpha_{s} &= 0.0016, \\ \delta_{\mathcal{R}_{O_{N}(v^{3})}} \alpha_{s} &= ^{+0.0035}_{-0.0034}, \\ \delta_{\mathcal{R}_{\mathcal{O}_{D}(v^{3})}} \alpha_{s} &= ^{+0.0026}_{-0.0025}, \\ \delta_{R_{\gamma}^{exp}} \alpha_{s} &= 0.01. \end{split}$$

We assume these errors to be independent and sum them up quadratically, obtaining

$$\begin{aligned} \alpha_{\rm s}(M_{\Upsilon(1S)}) &= 0.185^{+0.014}_{-0.015} \,. \\ \alpha_{\rm s}(M_Z) &= 0.120^{+0.005}_{-0.006} \,. \end{aligned}$$

Errors Discussion

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THE MAIN UNCERTAINTY OF OUR DETERMINATION COMES FROM THE SYSTEMATIC EXPERIMENTAL ERROR

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with respect to PDG06: HINCHCLIFFE, MANOHAR

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 $\frac{\Gamma(\Upsilon(1S) \to \gamma X)}{\Gamma(\Upsilon(1S) \to l^+ l^-)}$

claiming it is cleaner. But the uncertainty increases due to the an increased sensibility to the octets. Moreover the leptonic width suffer from large corrections in alphas. We obtain the final result (14)

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$$\alpha_{\rm s}(M_Z) = 0.119^{+0.006}_{-0.005}$$





Future Prospects for $lpha_s$ determinations from quarkonia

- More precise data for R_γ for $\Upsilon(1S)$ would allow a more precise extraction of $\alpha_s(M_{\Upsilon(1S)})$
- A new measurement of the inclusive photon spectrum at BESIII would provide the possibility to extract $\alpha_s(M_{J/\psi})$ GARCIA, SOTO 2007
- Previous to its discovery the η_b was indicated as the place where a very precise determination of α_s a could be obtained from the hyperfine separation

KNIEHL, PENIN. PINEDA, SMIRNOV, STEINHAUSER 2007

BACKUP