α_s(CIPT)

Sébastien Descotes-Genon

Laboratoire de Physique Théorique
CNRS & Université Paris-Sud 11, 91405 Orsay, France

α_s *workshop* - February 9 2011

Introduction

Motivation

- Improved perturbative computation of Adler function
  [P. Baikov et al. 2008]
- ALEPH τ spectral functions with some improved branching ratios and V/A separation [BABAR and Belle 2008]

M. Davier et al, Eur. Phys. J. C 56 (2008) 305 [hep-ph/0803.0979]

- Reanalysis of τ spectral moments
- Extraction of $\alpha_s(m_\tau^2)$ using Contour Improved Perturbation Theory

Tau spectral functions

Unitarity: $R_{\tau,V/A} = \mathcal{B}_{V^-/A^-\nu_\tau}/\mathcal{B}_e$ can be expressed as

$$R_{\tau,V/A}(s_0) = 12\pi S_{\rm EW}|V_{ud}|^2 \int_0^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2\left[\left(1+2\frac{s}{s_0}\right){\rm Im}\Pi^{(1)}(s) + {\rm Im}\Pi^{(0)}(s)\right]$$

$S_{\rm EW} = 1.0198 \pm 0.0006$ short-dist. electroweak

in terms of 2-pt correlator of axial/vector $U_{ij} = \bar{q}_i\gamma_\mu(\gamma_5)q_j$

$$i\int d^4x\, e^{iqx}\langle 0|T[U^\mu_{ij}(x)U^\nu_{ij}(0)^\dagger]|0\rangle = [-g^{\mu\nu}q^2 + q^\mu q^\nu]\Pi^{(1)}_{ij,U}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij,U}(q^2)$$

with cut along real positive axis : spectral functions of given spin

${\rm Im}\Pi^{(1,0)}_{\bar{u}d(s),V/A}(s) = \frac{1}{2\pi} v_1/a_{1,0}(s)$, experimentally accessible

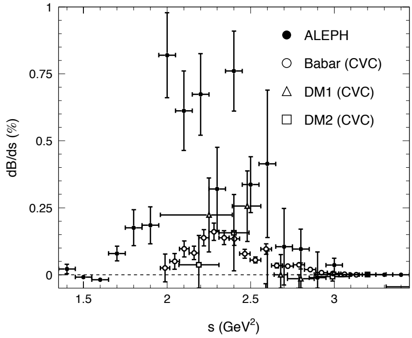
$$v_1(s)/a_1(s) = \frac{m_\tau^2}{6\,|V_{ud}|^2\, S_{\rm EW}} \frac{\mathcal{B}_{V^-/A^-\nu_\tau}}{\mathcal{B}_e} \frac{dN_{V/A}}{N_{V/A}\, ds}\left[\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$a_0(s)$ id without $(1+2s/m_\tau^2)$

V/A separation

- V/A separation simple for π channels due to G-parity
- Main limitation used to be from strange channel $K\bar{K}\pi$

ALEPH 1999	e^+e^- DM1 and DM2	$f_{A,\mathrm{CVC}} = 0.94^{+0.06}_{-0.08}$
CLEO 2000	a_1 in $\tau \to \nu_\tau \pi^- 2\pi^0$ + unitar.	$f_{A,a_1} = 1.30 \pm 0.24$
CLEO 2004	partial wave $K^-K^+\pi^-$	$f_{A,K\bar{K}\pi} = 0.56 \pm 0.10$



$\Longrightarrow$ Previous analyses:

$$f_A(K\bar{K}\pi) = 0.75 \pm 0.25$$

Analysis of Babar e^+e^- + CVC

$$f_{A,\mathrm{CVC}}(K\bar{K}\pi) = 0.833 \pm 0.024$$

Update of inclusive hadronic τ widths

- New measurements of τ strange decays ($K\pi^0$, $K_S\pi^-$, and $K^-\pi^+\pi^-$) [Babar, Belle]

$$R_{\tau,S} = 0.1615 \pm 0.0040 = R_\tau - (R_{\tau,V} + R_{\tau,A})$$

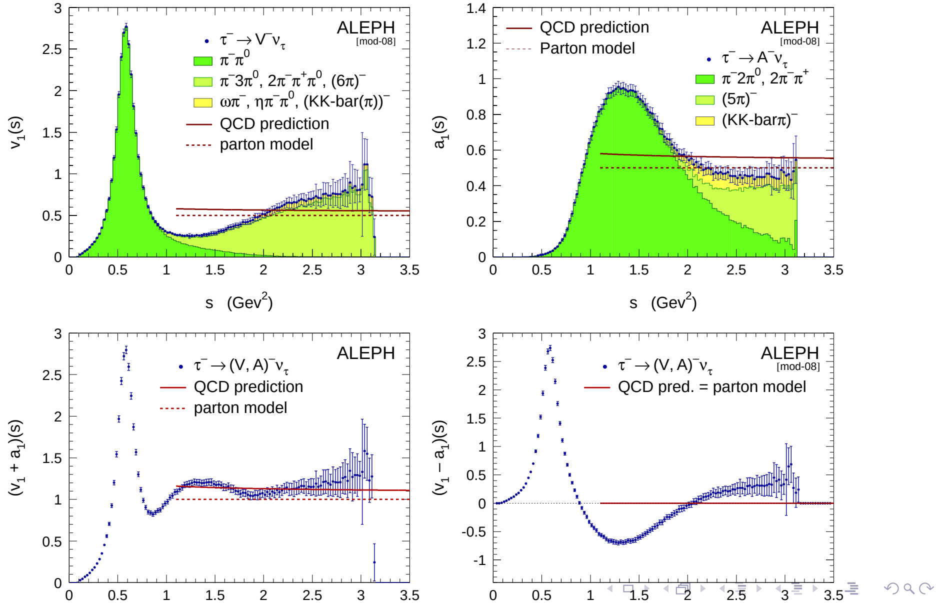
- New axial fraction for $K\bar{K}\pi$

$$R_{\tau,V} = 1.783 \pm 0.011 \pm 0.002 \qquad R_{\tau,A} = 1.695 \pm 0.011 \pm 0.002$$

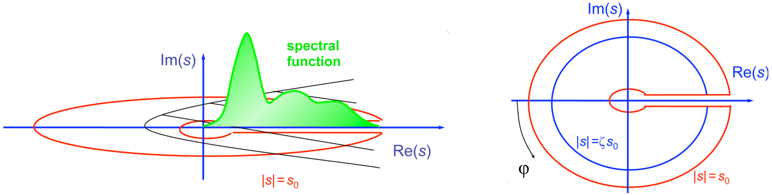
$$R_{\tau,V+A} = 3.479 \pm 0.011 \qquad R_{\tau,V-A} = 0.087 \pm 0.018 \pm 0.003$$

- 1st error : experimental
- 2nd error : V/A separation, dominated by $K\bar{K}\pi\pi$

Update of the ALEPH spectral functions



Contour integral and OPE



- Convert integral along cut $0 < s - i\epsilon < s_0$ into a contour $|s| = s_0$
- OPE assuming quark-hadron duality holds even close to real cut

$$R_{\tau,V/A} = \frac{3}{2} S_{\rm EW} |V_{ud}|^2 \left(1 + \delta^{(0)} + \delta'_{\rm EW} + \delta^{(2,m_q)}_{ud,V/A} + \sum_{D=4,6,\ldots} \delta^{(D)}_{ud,V/A}\right)$$

with $\delta^{(D)}_{ud,V/A} = \sum_{{\rm dim}\mathcal{O}=D} C'_{V/A}(s_0,\mu) \frac{\langle \mathcal{O}_D(\mu)\rangle_{V/A}}{s_0^{D/2}}$ and EW corrections

- $R_{\tau V/A}$ dominated by perturbative contribution, related to $\alpha_s(m_\tau^2)$

RGE for α_s

$$1+\delta^{(0)} = -2\pi i \oint_{|s|=s_0} \frac{ds}{s}\left[1-2\frac{s}{s_0}+2\frac{s^3}{s_0^3}-\frac{s^4}{s_0^4}\right]D(s) \qquad D(s)=\frac{1}{4\pi^2}\sum_{n=0}^{\infty}\tilde{K}_n(\xi)a_s^n(-\xi s_0)$$

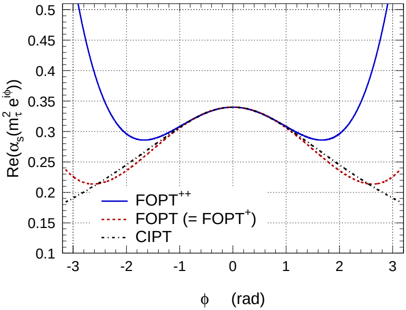
- $K_{0\ldots4}$ known for Adler function $D(s) = -s d\Pi/ds$ [P. Baikov, 2008]
- $a_s(s) = \alpha_s(s)/\pi$ on the contour obtained by RGE, expansion in $a_s = a_s(s_0)$ and $\eta = \log(s/s_0)$

$$\begin{aligned}
a_s(s) - a_s = \\
& a_s^2\beta_0\eta \\
& +a_s^3[-\beta_1\eta + \beta_0^2\eta^2] \\
& +a_s^4[-\beta_2\eta + \tfrac{5}{2}\beta_0\beta_1\eta^2 - \beta_0^3\eta^3] \\
& +a_s^5[-\beta_3\eta + (\tfrac{3}{2}\beta_1^2 + 3\beta_0\beta_2)\eta^2 - \tfrac{13}{3}\beta_0^2\beta_1\eta^3 + \beta_0^4\eta^4] \\
& +a_s^6[-\beta_4\eta + (\tfrac{7}{2}\beta_1\beta_2 + \tfrac{7}{2}\beta_0\beta_3)\eta^2 - (\tfrac{35}{6}\beta_0\beta_1^2 + 6\beta_0^2\beta_2)\eta^3 + \tfrac{77}{12}\beta_0^3\beta_1\eta^4 - \beta_0^5\eta^5] + \ldots
\end{aligned}$$

- unknown $\beta_{n\geq5}$ (set to zero) involved at higher orders
- central value for β_4 assuming geometrical growth

α_s along the contour

- CIPT = step-wise integration along contour with previous exp
- FOPT = one-step determination with RGE cut at η^5, a_s^6
- FOPT^{++} = id., but including all known orders up to η^5 [including higher-order contributions $a_s^7 \to a_s^{10}$]



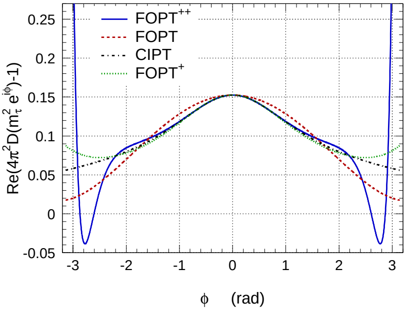
[Ref value: $\alpha_s(m_\tau^2) = 0.34$]

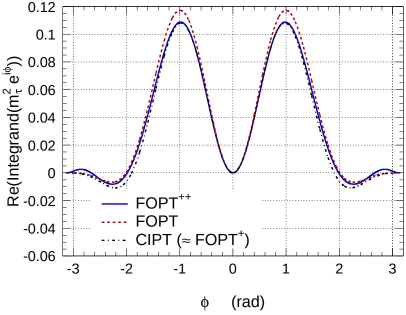
$Re[\alpha_s(m_\tau^2 e^{i\phi})]$

- FOPT/FOPT^{++} : $\eta^{k\geq5}$ dominate when included, large deviations near cut
- CIPT$\simeq$ CIPT^{++}
- jth step of CIPT with N steps up to and including a_s^n leads to error $O(j/N^{n+1})$

$\Longrightarrow$CIPT error $1/N^n$-suppr. compared to FOPT

Fixed order truncation





$Re[4\pi^2 D(m_\tau^2 e^{i\phi}) - 1]$

Real part integrand for $1 + \delta^{(0)}$

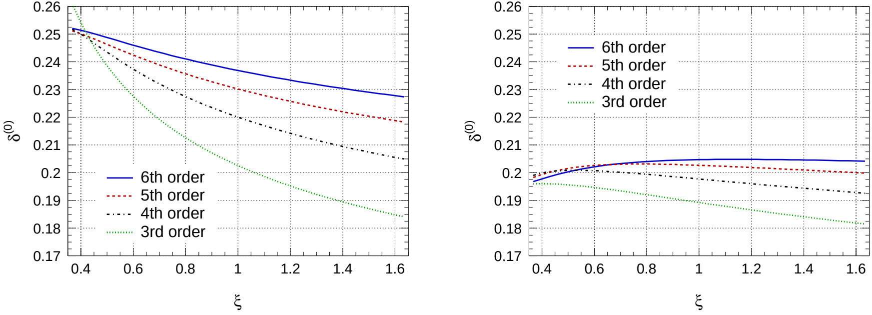
- FOPT+ : truncation for a_s, but not for the integral yielding $\delta^{(0)}$
- FOPT+/CIPT similar : truncation after integration makes difference for FOPT/CIPT
- Crucial role of the pinching weight to suppress cut region

Dependence on the renormalisation scale

Scale for perturbative computation of Adler function $O(m_\tau^2)$

$$D(s) = \frac{1}{4\pi^2} \sum_{n=0}^{\infty} \tilde{K}_n(\xi) a_s^n(-\xi m_\tau^2)$$

$\Longrightarrow$signficant uncertainty for $\delta^{(0)}$ and thus α_s



FOPT CIPT

Numerical tests

- Computation of $\delta^{(0)}$ for $\alpha_s(m_\tau^2) = 0.34$
- Unknown $K_{5,6}$ and β_4 by geometric growth (higher set to 0)
- Variation of unknown coefficients $\delta\beta_4, K_5, K_6 \pm 100\%$
- Variation of scale $\delta\xi \pm 0.63$ (i.e. scale $m_\tau^2 \pm 0.63$)

	$n=1$	$n=2$	$n=3$	$n=4$	$(n=5)$	$(n=6)$	$\sum_{n=1}^{4}$	$\sum_{n=1}^{5}$	$\sum_{n=1}^{6}$
FOPT	0.1082	0.0609	0.0334	0.0174	0.0101	0.0067	0.2200	0.2302	0.2369
$\delta(\beta_4)$	0	0	0	0	0	± 0.0006	0	0	± 0.0006
$\delta(K_5)$	0	0	0	0	± 0.0056	± 0.0108	0	± 0.0056	± 0.0164
$\delta(K_6)$	0	0	0	0	0	± 0.0047	0	0	± 0.0047
$\delta(\xi)$	–	–	–	–	–	–	${}^{+0.0317}_{-0.0151}$	${}^{+0.0209}_{-0.0119}$	${}^{+0.0152}_{-0.0095}$
CIPT	0.1476	0.0295	0.0121	0.0085	0.0049	0.0020	0.1977	0.2027	0.2047
$\delta(\beta_4)$	∓ 0.0003	∓ 0.0001	∓ 0.0001	∓ 0.0001	∓ 0.0001	∓ 0.0001	∓ 0.0006	∓ 0.0007	∓ 0.0008
$\delta(K_5)$	0	0	0	0	± 0.0049	0	0	± 0.0049	± 0.0049
$\delta(K_6)$	0	0	0	0	0	± 0.0020	0	0	± 0.0020
$\delta(\xi)$	–	–	–	–	–	–	${}^{+0.0032}_{-0.0051}$	${}^{+0.0005}_{-0.0044}$	${}^{+0.0001}_{-0.0079}$

Convergence near the cut, scale dependence $\Longrightarrow$CIPT preferred

[FOPT/CIPT : M. Beneke ; S. Menke ; I Caprini and J. Fischer 2010]

Models of quark-duality violation (1)

- in spite of pinched weight, contribution near cut where OPE fails
- soft contributions $\exp(-\lambda Q^k)/Q^l$ for $Q^2 < 0$ missed by OPE
- translated into oscillatory behaviour for $Q^2 > 0$

Models of duality violation, providing oscillatory behaviour for v, a

[M. Shifman 2000]

- Instanton background field of size ρ ($\sin(\rho\sqrt{s})$ oscillation)

$$\Delta\Pi^{(I)} = \frac{C_I}{Q^2} K_1(Q\rho) K_{-1}(Q\rho)$$

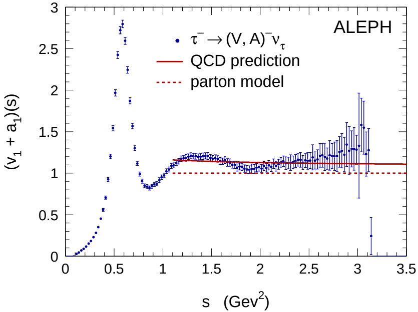
- Comb of resonances with growing width ($\sin(s/\sigma)$ oscillation)

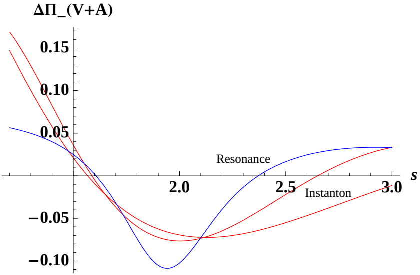
$$\Delta\Pi^{(II)}(Q) = C_{II}\left[-\frac{\psi(z)+1/z}{4\pi^2(1-B/3\pi)} - \Pi^{(OPE)}(Q)\right] \qquad z = \left(\frac{Q^2}{\sigma^2}\right)^{1-\frac{B}{(3\pi)}}$$

Models of quark-duality violation (2)

Parameters determined to vanish around $s = 1.6$ GeV^2
and to match smoothly $v + a$ from s ≥ 2 GeV^2

- Instanton : $2.4 \leq \rho \leq 4.4$ GeV^{-1} $\Longrightarrow \Delta\delta^{(0)} \leq 5 \times 10^{-3}$
- Resonances : $1.65 \leq \sigma^2 \times 2$ GeV^2, $0.3 < B < 0.6$ $\Longrightarrow \Delta\delta^{(0)} < 7 \leq 10^{-4}$





$$\Delta\delta^{(0)} = -2\pi i \oint_{|s|=s_0} \frac{ds}{s} \left[1 - 2\frac{s}{s_0} + 2\frac{s^3}{s_0^3} - \frac{s^4}{s_0^4}\right] \Delta D(s)$$

[M. Golterman's talk]

Spectral moments

Joint fit of $(k,\ell) = (1,0), (1,1), (1,2), (1,3)$ for $D^{k\ell}_{\tau,V/A} = R^{k\ell}_{\tau,V/A}/R_{\tau,V/A}$

$$R^{k\ell}_{\tau,V/A} = \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^\ell \frac{dR_{\tau,V/A}}{ds} \qquad R^{00}_{\tau,V/A} = R_{\tau,V/A}$$

expressed through OPE in terms of

- $D = 0$: α_s
- $D = 2$: quark masses (PDG)
- $D = 4$: quark condensate $\langle \bar{q}q \rangle$ ($F_\pi^2 M_\pi^2$ via Gell-Mann-Oakes-Renner) and gluon condensate $\langle \alpha_s GG \rangle$
- $D = 6$: four quark operators down to $\rho \langle \bar{q}q \rangle^2$ by vacuum saturation
- $D = 8$: single (effective) condensate
- $D \geq 10$: contributions neglected, even for higher moments

- $R^{00}_{\tau,V/A}$: contributions from $D/2 = 0, 1, 3, 4$
- $R^{1\ell}_{\tau,V/A}$: contributions from $D/2 = 0, \ell+1, \ell+2, \ell+3, \ell+4, \ell+5$

Correlations

V	$D^{10}_{\tau,V}$	$D^{11}_{\tau,V}$	$D^{12}_{\tau,V}$	$D^{13}_{\tau,V}$
$R_{\tau,V}$	-0.287	0.153	0.274	0.302
$D^{10}_{\tau,V}$	1	-0.821	-0.981	-0.993
$D^{11}_{\tau,V}$	–	1	0.899	0.824
$D^{12}_{\tau,V}$	–	–	1	0.988

A	$D^{10}_{\tau,A}$	$D^{11}_{\tau,A}$	$D^{12}_{\tau,A}$	$D^{13}_{\tau,A}$
$R_{\tau,A}$	-0.255	0.013	0.178	0.272
$D^{10}_{\tau,A}$	1	-0.746	-0.963	-0.978
$D^{11}_{\tau,A}$	–	1	0.866	0.646
$D^{12}_{\tau,A}$	–	–	1	0.938

V+A	$D^{11}_{\tau,V+A}$	$D^{12}_{\tau,V+A}$	$D^{13}_{\tau,V+A}$
$D^{10}_{\tau,V+A}$	-0.722	-0.974	-0.987
$D^{11}_{\tau,V+A}$	1	0.801	0.662
$D^{12}_{\tau,V+A}$	–	1	0.975

- $(1,0)$ dominated by low-energy peaks (ρ or π, a_1)
- $(1,\ell>0)$ more and more sensitive to higher energies
  $\Longrightarrow$increasing negative correl. with $(1,0)$ [positive among them]
- Negligible correlation between $R_{\tau,V+A}$ and $D^{ij}_{\tau,V+A}$
- Point-by-point correlelations ? Impact on R, D matrix ?

[D. Boito et al., 2010]

Outcome of the fit (1)

Param.	Vector (V)	Axial-Vector (A)	$V + A$
$\alpha_s(m_\tau^2)$	$0.3474 \pm 0.0074^{+0.0063}_{-0.0074}$	$0.3345 \pm 0.0078^{+0.0063}_{-0.0074}$	$0.3440 \pm 0.0046^{+0.0063}_{-0.0074}$
$\delta^{(0)}$	0.2093 ± 0.0080	0.1988 ± 0.0087	0.2066 ± 0.0070
$\delta^{(2)}$	$(-3.2 \pm 3.0) \cdot 10^{-4}$	$(-5.1 \pm 3.0) \cdot 10^{-4}$	$(-4.3 \pm 2.0) \cdot 10^{-4}$
$\langle a_s GG \rangle$	$(-0.8 \pm 0.4) \cdot 10^{-2}$	$(-2.2 \pm 0.4) \cdot 10^{-2}$	$(-1.5 \pm 0.3) \cdot 10^{-2}$
$\delta^{(4)}$	$(0.1 \pm 1.5) \cdot 10^{-4}$	$(-5.9 \pm 0.1) \cdot 10^{-3}$	$(-3.0 \pm 0.1) \cdot 10^{-3}$
$\delta^{(6)}$	$(2.68 \pm 0.20) \cdot 10^{-2}$	$(-3.46 \pm 0.21) \cdot 10^{-2}$	$(-3.7 \pm 1.7) \cdot 10^{-3}$
$\delta^{(8)}$	$(-8.0 \pm 0.5) \cdot 10^{-3}$	$(9.5 \pm 0.5) \cdot 10^{-3}$	$(8.1 \pm 3.6) \cdot 10^{-4}$
Total $\delta_{\rm NP}$	$(1.89 \pm 0.25) \cdot 10^{-2}$	$(-3.11 \pm 0.16) \cdot 10^{-2}$	$(-5.9 \pm 1.4) \cdot 10^{-3}$
χ^2/DF	0.07	3.57	0.90

- Small negative gluon condensate
- Opposite-sign contributions $\delta^{(6)}$ and $\delta^{(8)}$ for V and A

Outcome of the fit (2)

Experimental spectral moments and theoretical counterparts (after fit)

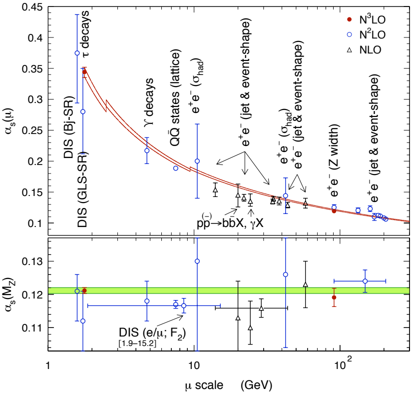
	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
$D^{1\ell}_{\tau,V}$	0.71668	0.16930	0.05317	0.02254
$D^{1\ell\,(\mathrm{theo})}_{\tau,V}$	0.71568	0.16971	0.05327	0.02265
$\Delta^{\mathrm{exp}} D^{1\ell}_{\tau,V}$	0.00250	0.00043	0.00054	0.00041
$D^{1\ell}_{\tau,A}$	0.71011	0.14903	0.06586	0.03183
$D^{1\ell\,(\mathrm{theo})}_{\tau,A}$	0.71660	0.14571	0.06574	0.03130
$\Delta^{\mathrm{exp}} D^{1\ell}_{\tau,A}$	0.00182	0.00063	0.00036	0.00025
$D^{1\ell}_{\tau,V+A}$	0.71348	0.15942	0.05936	0.02707
$D^{1\ell\,(\mathrm{theo})}_{\tau,V+A}$	0.71668	0.15767	0.05926	0.02681
$\Delta^{\mathrm{exp}} D^{1\ell}_{\tau,V+A}$	0.00159	0.00037	0.00033	0.00025

$\Longrightarrow$Difficulties for axial fit for $(k,\ell) = (1,0), (1,1)$ (low s)

$\alpha_s(CIPT)$

$$\alpha_{s,CIPT}(m_\tau^2) = 0.344 \pm 0.005_{exp} \pm 0.007_{th}$$

with th. errors : $K_5[0.0062], \xi[^{+0.0007}_{-0.0040}], S_{EW}[0.0007], |V_{ud}|[0.0005]$



$\alpha_{s,CIPT}(M_Z) =$
$0.1212 \pm 0.0005_{exp}$
$\pm 0.0008_{th} \pm 0.0005_{evol}$

where evolution adds in quadrature uncertainties from

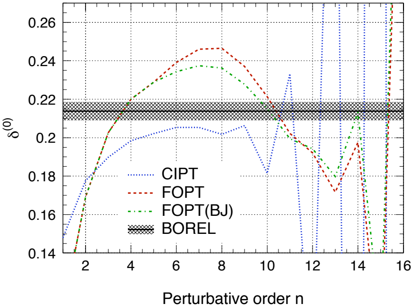
- b, c-quark masses
- matching scales
- truncation in matching
- truncation in RGE

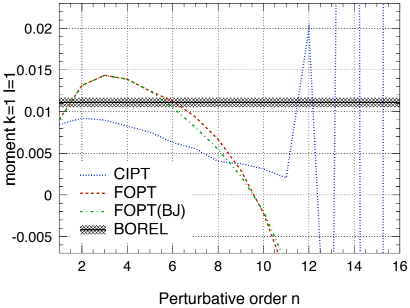
Back-up

CIPT/FOPT and higher orders

Model for higher-order PT for D (1 UV and 2 IR renormalons) ajusted to match known orders [M. Beneke and M. Jamin 2009 ; M. Beneke]

- FOPT converges more slowly than CIPT, but it hits ”exact” value given by Borel sum $\Longrightarrow$FOPT favoured w.r.t CIPT
- Statement to consider w.r.t model and integration weight [I. Caprini and J. Fischer 2010 ; SDG and B. Malaescu 2010]





Different low-order structure $w^{(k=1,\ell=1)}$