Running and decoupling of α_s

(at low scales)

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Facts about α_s

- $\alpha_s^{(5)}(M_Z) = 0.1184 \pm 0.0007$ [Bethke'09,PDG]
- $\overline{\mathrm{MS}}$ scheme
- running
- decoupling

β^{QCD}

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s}{\pi} = \beta = -\left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 + \frac{\alpha_s}{\pi}\beta_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \beta_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \beta_3 + \ldots\right]$$

β_0 [Gross,Wilczek'73; Politzer'73]

β_1 [Caswell'74; Jones'74; Egorian,Tarasov'74]

β_2 [Tarasov,Vladimirov,Zharkov'80; Larin,Vermaseren'93]

β_3 [van Ritbergen,Vermaseren,Larin'97; Czakon'05]

n_f	β_0	β_1	β_2	β_3
3	2.25	4.00	10.06	47.23
5	1.92	2.42	2.83	18.85

β_i obtained from renormalization constants for α_s:

$$0 = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s^{\text{bare}} = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} (Z_{\alpha_s} \alpha_s)$$

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SFB TR9

Solutions of $\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s}{\pi} = \beta$

$$\ln \frac{\mu^2}{\Lambda^2} = \int \frac{da}{\beta(a)} = \frac{1}{\beta_0}\left[\frac{1}{a} + b_1 \ln a + (b_2 - b_1^2)a + \left(\frac{b_3}{2} - b_1 b_2 + \frac{b_1^3}{2}\right) a^2\right] + C$$

$$L = \ln(\mu^2/\Lambda^2)\,, \quad b_i = \frac{\beta_i}{\beta_0}\,, \quad a = \frac{\alpha_s}{\pi}$$

$$\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_0 L} - \frac{b_1 \ln L}{(\beta_0 L)^2} + \frac{1}{(\beta_0 L)^3}\left[b_1^2(\ln^2 L - \ln L - 1) + b_2\right]$$
$$+ \frac{1}{(\beta_0 L)^4}\left[b_1^3\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3 b_1 b_2 \ln L + \frac{b_3}{2}\right]$$
$$+\mathcal{O}(1/L^5)$$

μ	Λ	$1/L^5$
M_Z	213 MeV	$4 \cdot 10^{-6}$
M_τ	361 MeV	0.003

$$\alpha_s(M_\tau) = 0.332 \pm 0.005_{\rm exp} \pm 0.015_{\rm th}$$

[Baikov,Chetyrkin,Kühn'08]

incl.	$\delta\alpha_s(M_\tau)$	$\delta\alpha_s(M_Z)$
$\mathcal{O}(1/L^4)$	$+0.004$	$+0.0004$
$\mathcal{O}(1/L^5)^{(1)}$	-0.005	-0.0009
$\mathcal{O}(1/L^5)^{(2)}$	-0.004	-0.0004

(1): $\beta_4 = 0$; (2): $\beta_4 = 200$

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Solutions of $\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}\frac{\alpha_s}{\pi} = \beta$ (2)

Preferable (to "Λ")

- numerical solution of $\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}\frac{\alpha_s}{\pi} = \beta$ with truncated $\beta(\alpha_s)$
- "resummed formulae": (see, e.g., [Huber,Lunghi,Misiak,Wyler])

$$\begin{aligned}
\tilde{\alpha}_s(\mu) &= \frac{\tilde{\alpha}_s(\mu_0)}{v} - \left(\frac{\tilde{\alpha}_s(\mu_0)}{v}\right)^2 \frac{\beta_1}{\beta_0}\ln v \\
&+ \left(\frac{\tilde{\alpha}_s(\mu_0)}{v}\right)^3 \left[\frac{\beta_2}{\beta_0}(1-v) + \left(\frac{\beta_1}{\beta_0}\right)^2 \left(\ln^2 v - \ln v + v - 1\right)\right] \\
&+ \left(\frac{\tilde{\alpha}_s(\mu_0)}{v}\right)^4 \left\{\frac{\beta_3}{\beta_0}\frac{(1-v^2)}{2} + \frac{\beta_2\beta_1}{\beta_0^2}\left[(2v-3)\ln v + v^2 - v\right]\right. \\
&\quad \left. + \left(\frac{\beta_1}{\beta_0}\right)^3 \left[-\ln^3 v + \frac{5}{2}\ln^2 v + 2(1-v)\ln v - \frac{1}{2}(v-1)^2\right]\right\}
\end{aligned}$$

$$\tilde{\alpha}_s = \alpha_s/4\pi; \quad v = 1 + \tilde{\alpha}_s\beta_0\ln(\mu^2/\mu_0^2)$$

Heavy quark thresholds

- decoupling (matching, threshold corrections)
- match ($\overline{\mathrm{MS}}$ scheme)
  QCD with $n_f = n_l + 1$ flavours to
  QCD with n_l flavours
- $\alpha_s^{(n_l)}(\mu) = \zeta_{\alpha_s}(\mu)\alpha_s^{(n_f)}(\mu)$ [Bernreuther,Wetzel'81; Larin,van Ritbergen,Vermaseren'94]

[Chetyrkin,Kniehl,Steinhauser'97]

$$\zeta_{\alpha_s} = 1 + \sum_{n\geq 1} \zeta_{\alpha_s}^{(n)} \left(\frac{\alpha_s^{(n_f)}}{\pi}\right)^n$$

[Schröder,Steinhauser'06; Chetyrkin,Kühn,Sturm'06]

⇨ $\alpha_s(\mu)$ is not a continuous function of μ

⇨ $\zeta_{\alpha_s}(\mu)$: $\mu = \mu_{\mathrm{dec}}$
unphysical scale, not fixed by theory $\longrightarrow$ dependence on μ_{dec} should become weaker if the order of perturbation theory is increased

running & decoupling

N-loop running $\leftrightarrow$ $(N-1)$-loop decoupling

$$A(\alpha_s^{(n_f)}(\mu_h)) \longrightarrow A(\alpha_s^{(n_f-1)}(\mu_l))$$

	A avail. to order	running	decoupling
LO:	$\alpha_s(\mu)$	"β_0"	$\zeta_{\alpha_s} = 1$
NLO:	$\alpha_s(1 + X_1\alpha_s)$	"β_1"	$\zeta_{\alpha_s} = 1 + \ldots \frac{\alpha_s}{\pi}$
NNLO:	$\alpha_s(1 + X_1\alpha_s + X_2\alpha_s^2)$	"β_2"	$\zeta_{\alpha_s} = 1 + \ldots \frac{\alpha_s}{\pi} + \ldots \left(\frac{\alpha_s}{\pi}\right)^2$

...

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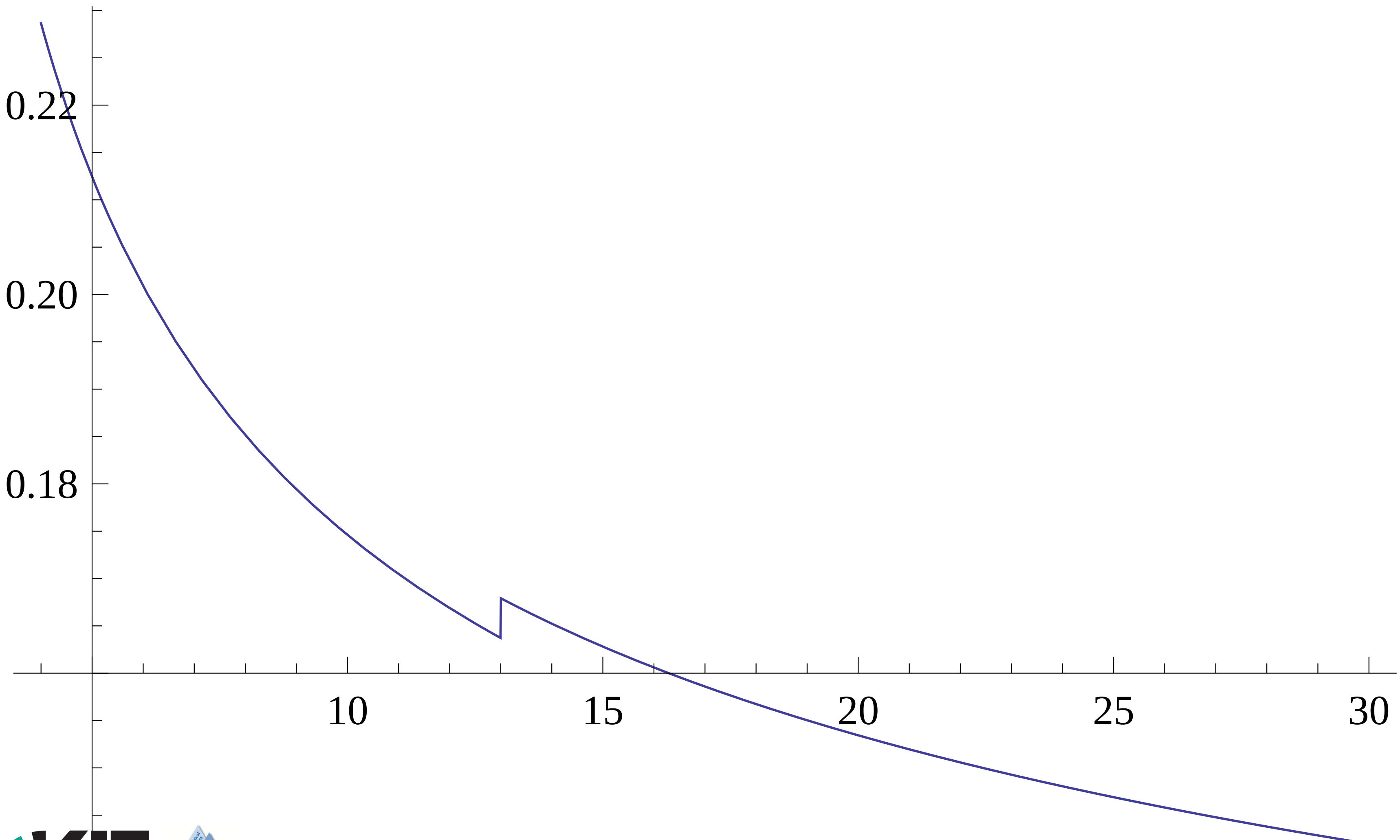
$\mu_{\rm dec}$

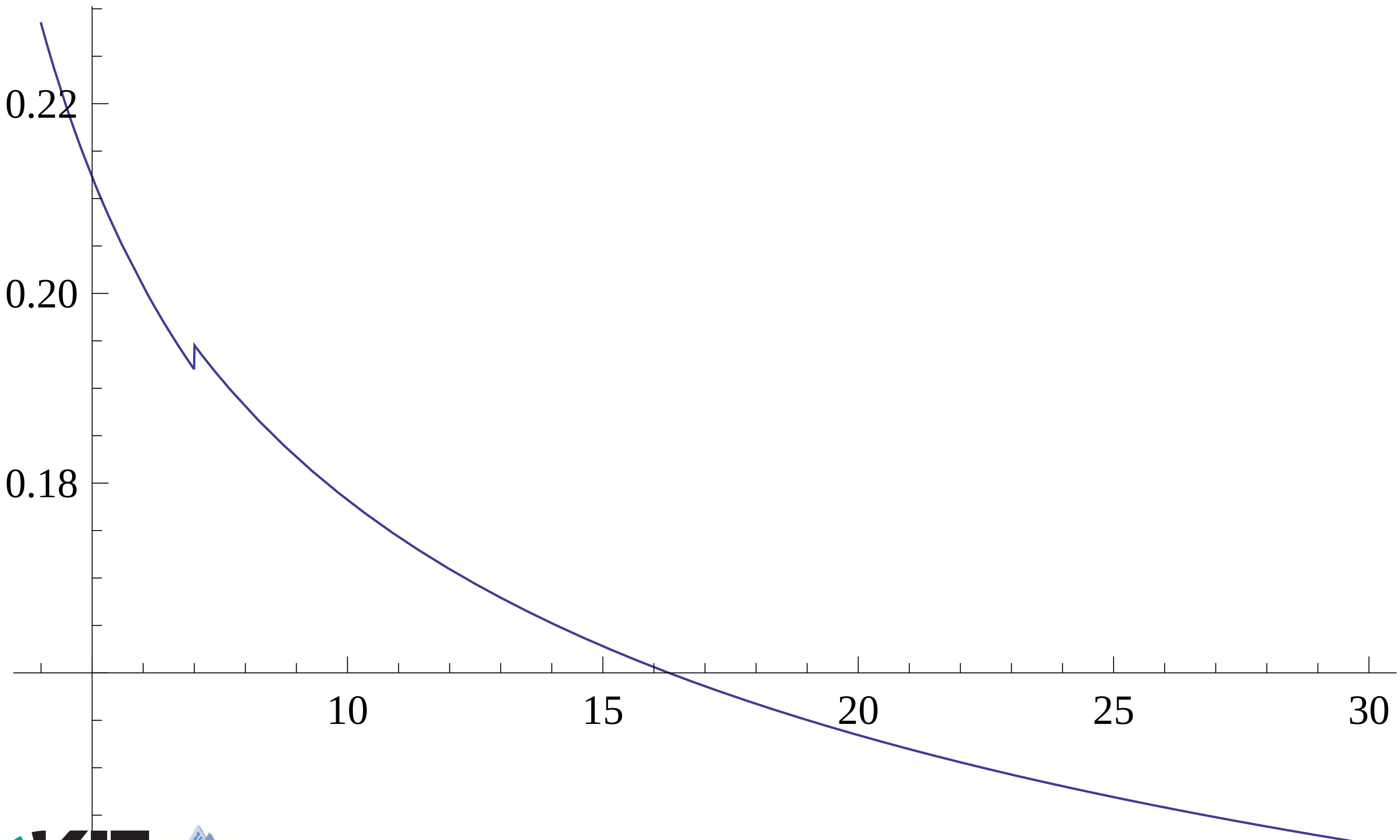
$\alpha_s(\mu)$ for 4 GeV $\leq \mu \leq$ 30 GeV; 4-loop analysis
$\mu_{\rm dec} = 13$ GeV



$\mu_{\rm dec}$

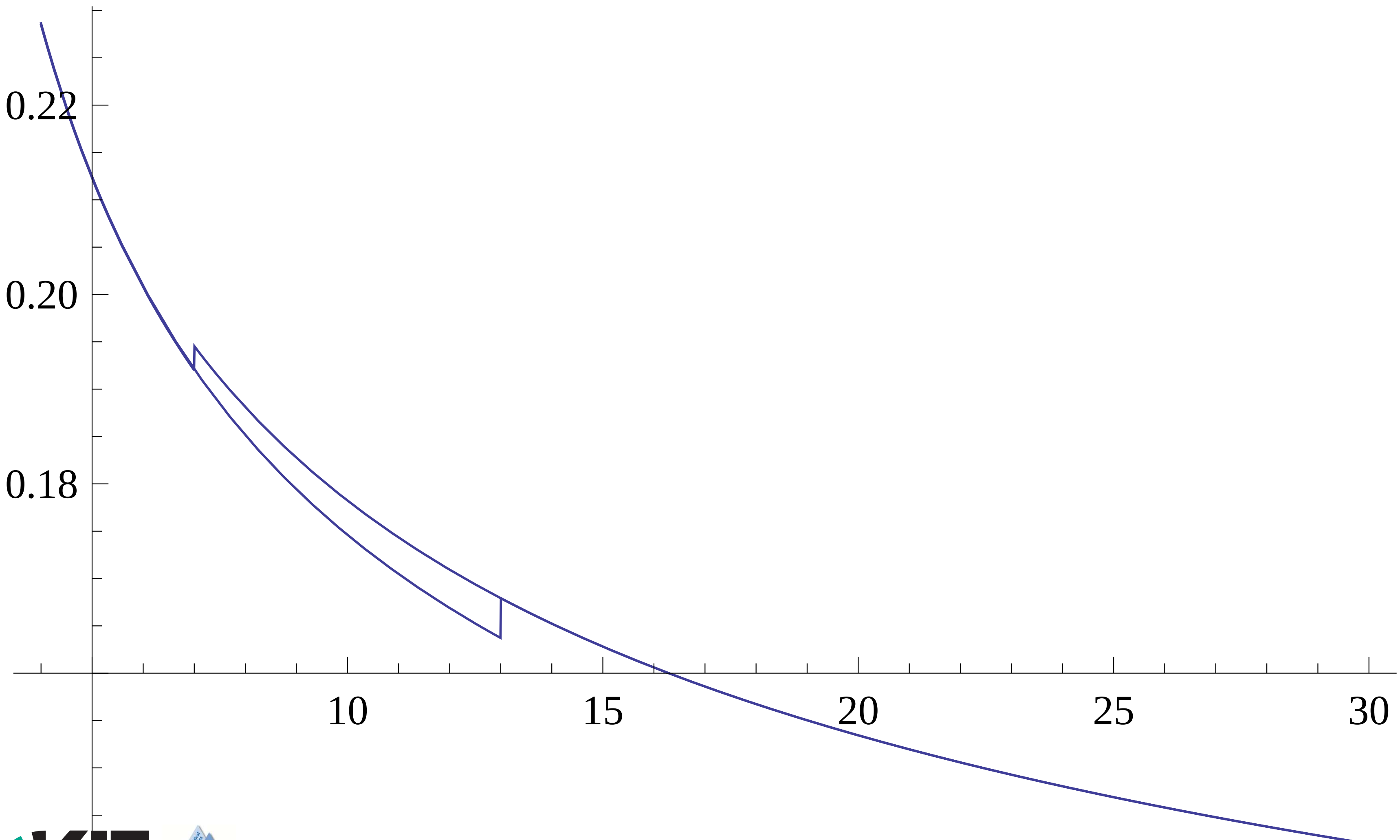
$\alpha_s(\mu)$ for 4 GeV $\leq \mu \leq$ 30 GeV; 4-loop analysis

$\mu_{\rm dec} = 7$ GeV



μ_{dec}

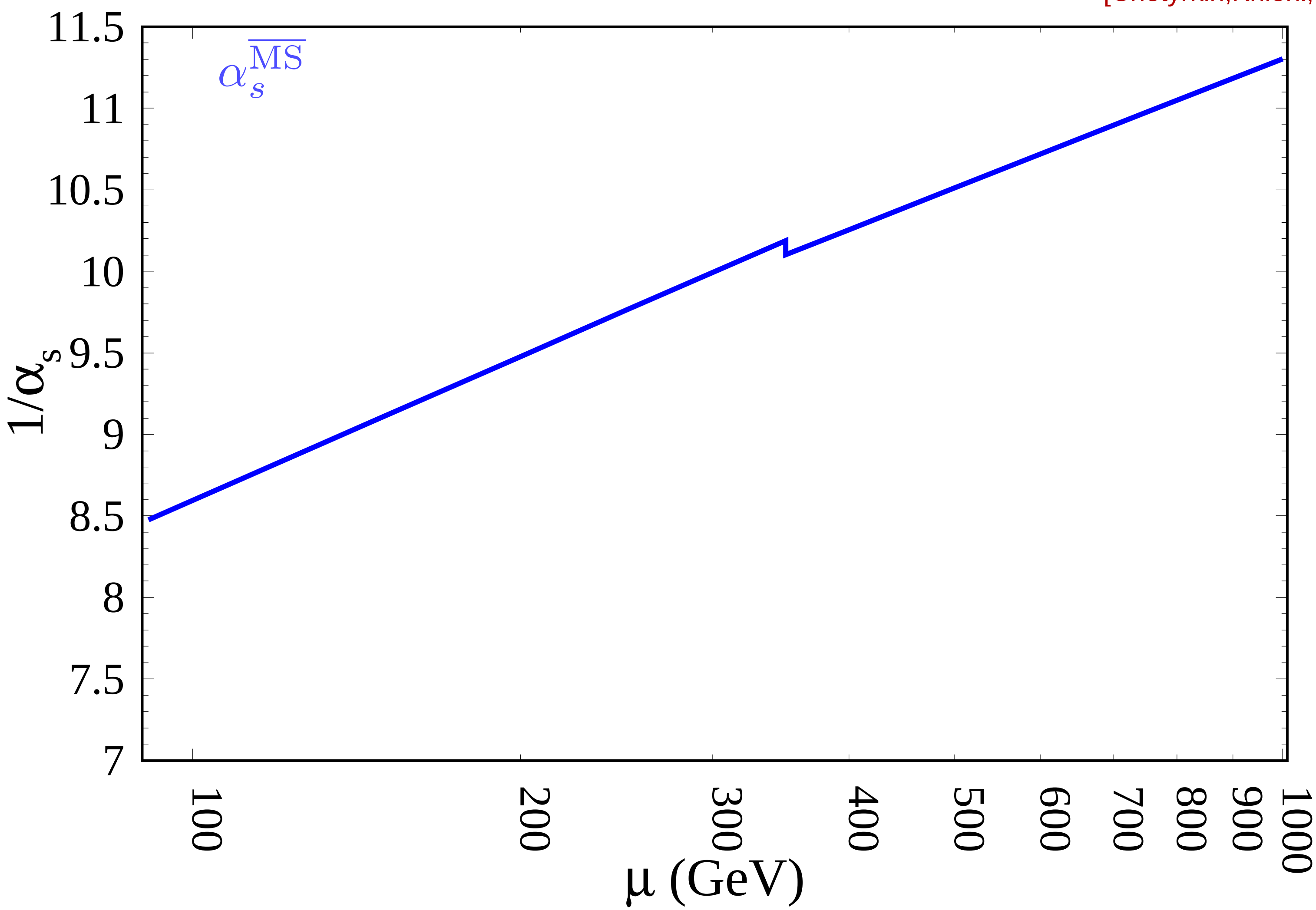
$\alpha_s(\mu)$ for 4 GeV $\leq \mu \leq$ 30 GeV; 4-loop analysis
$\mu_{\text{dec}} = 7$ GeV and $\mu_{\text{dec}} = 13$ GeV



Intermezzo: α_s in $\overline{\rm MS}$ and MOM scheme

Appelquist-Carazzone-Theorem: automatic decoupling in MOM scheme

[Chetyrkin,Kniehl,Steinhauser'09]



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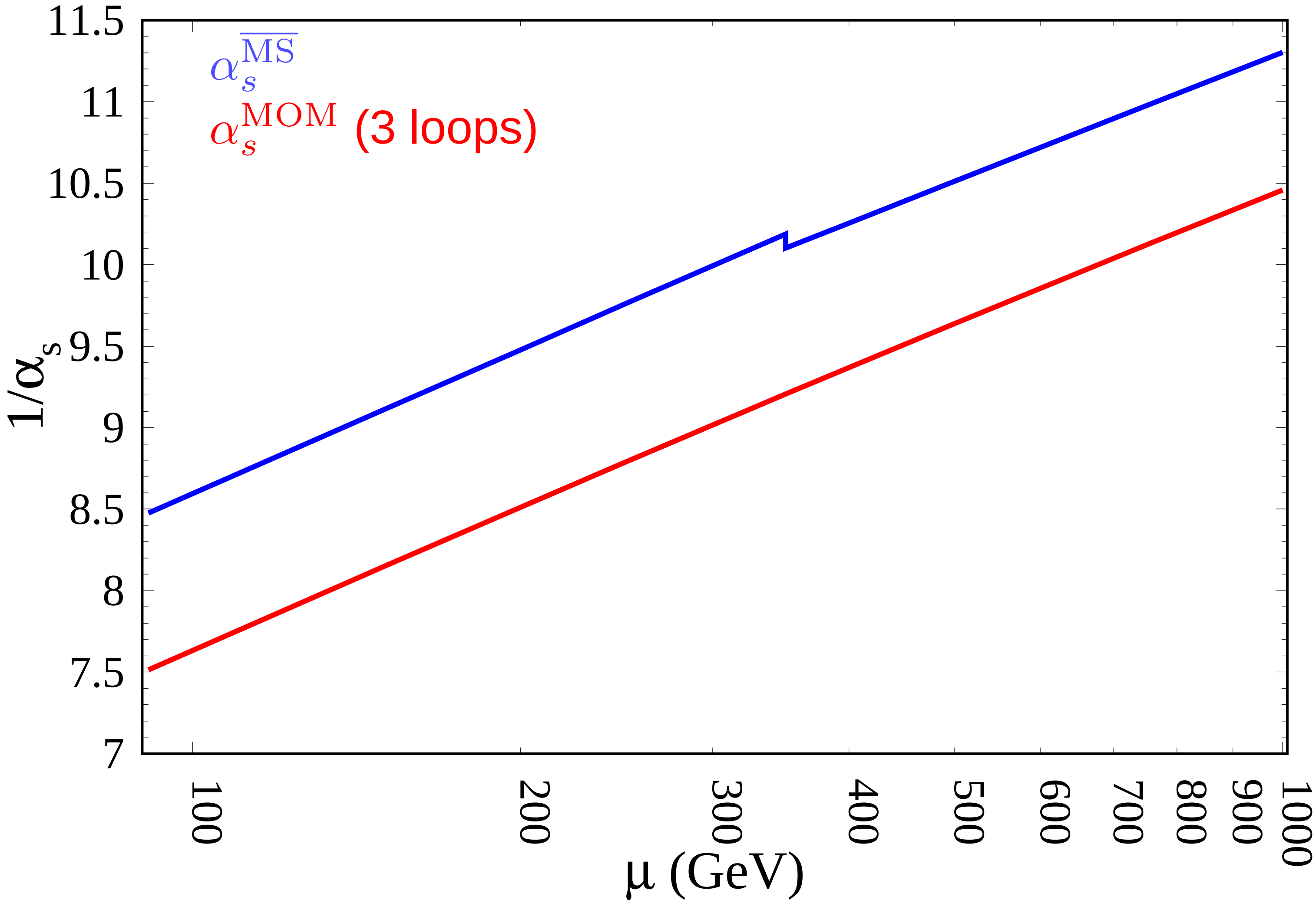

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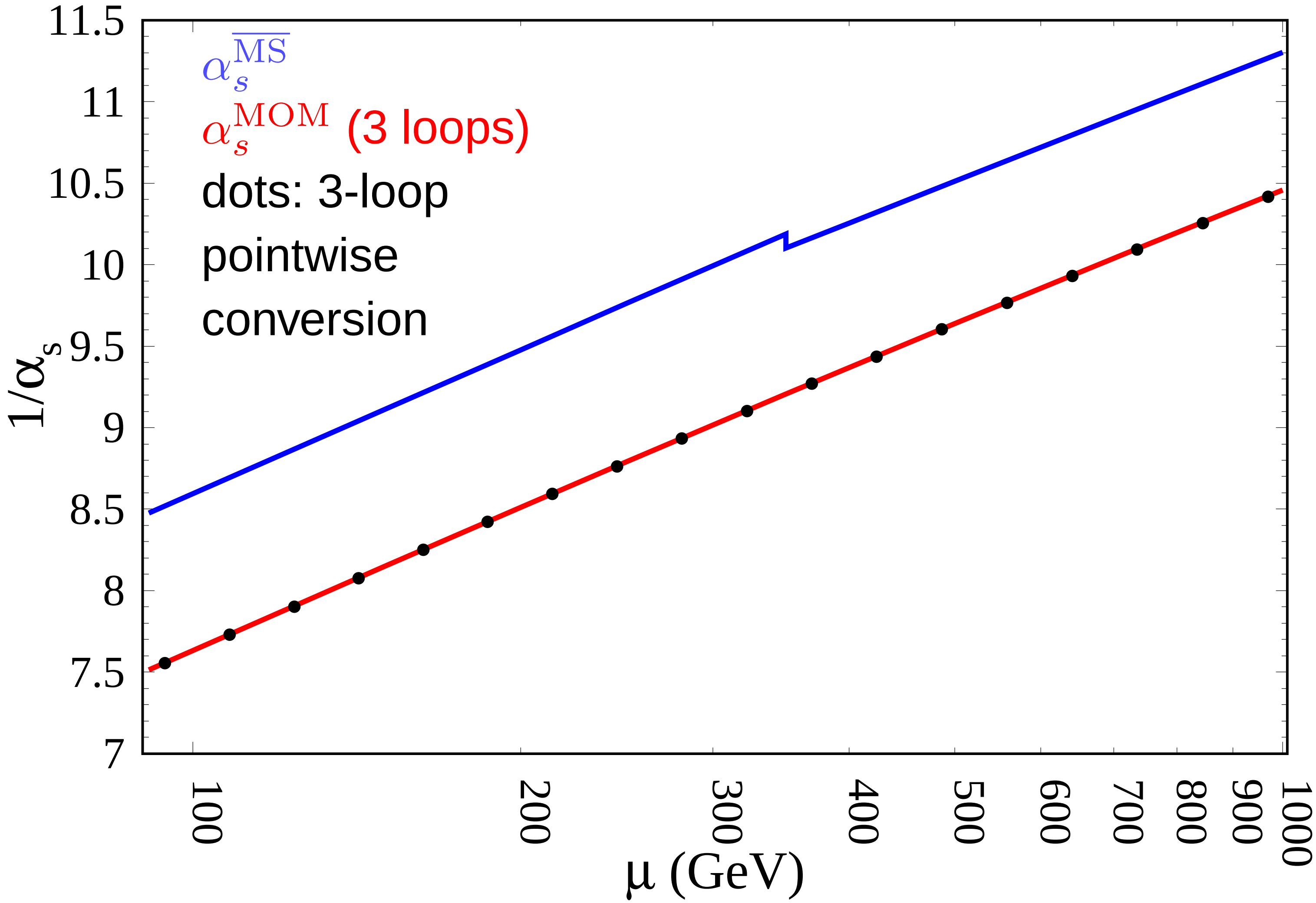
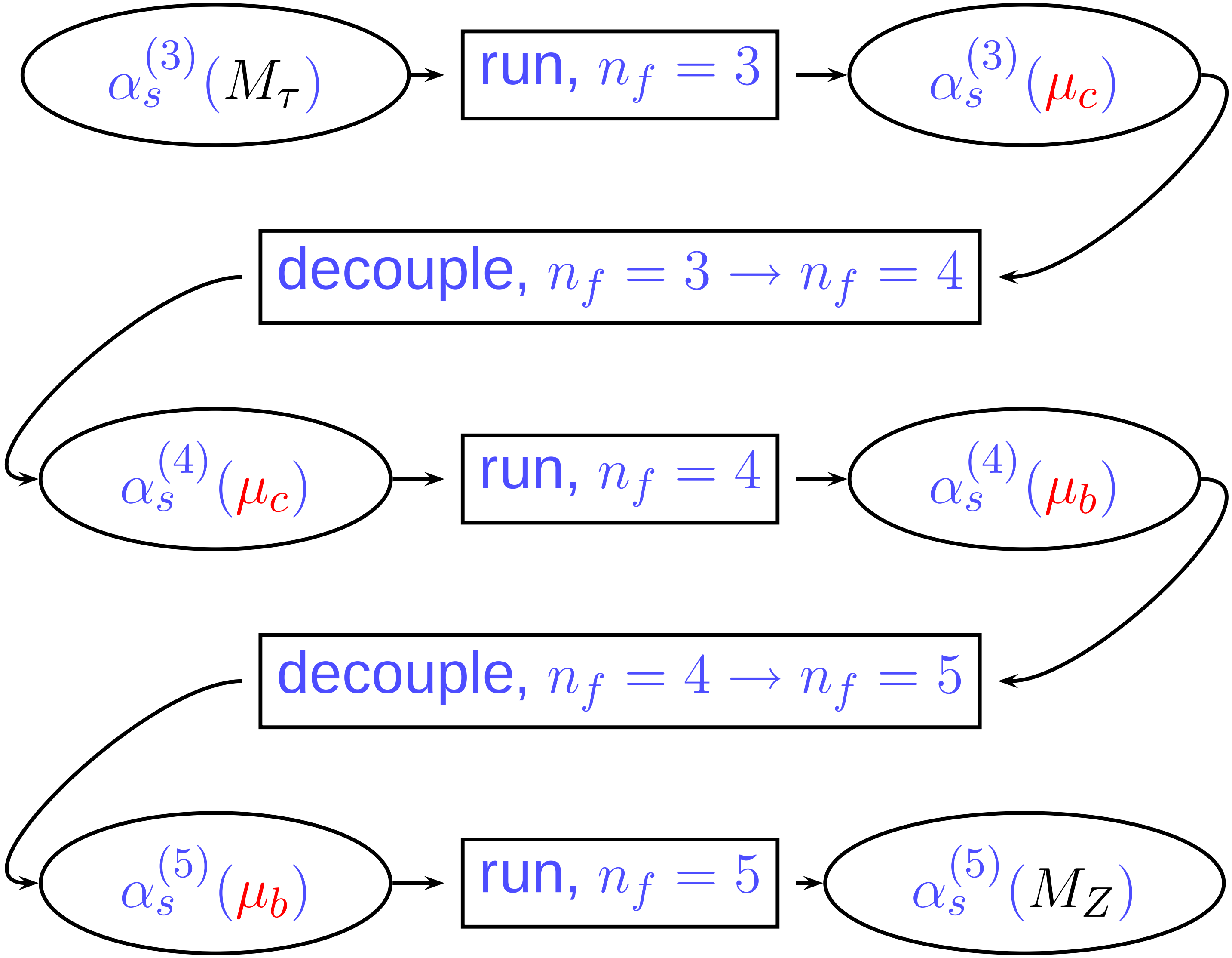


➪ Equivalence of MOM and $\overline{\rm MS}$ scheme

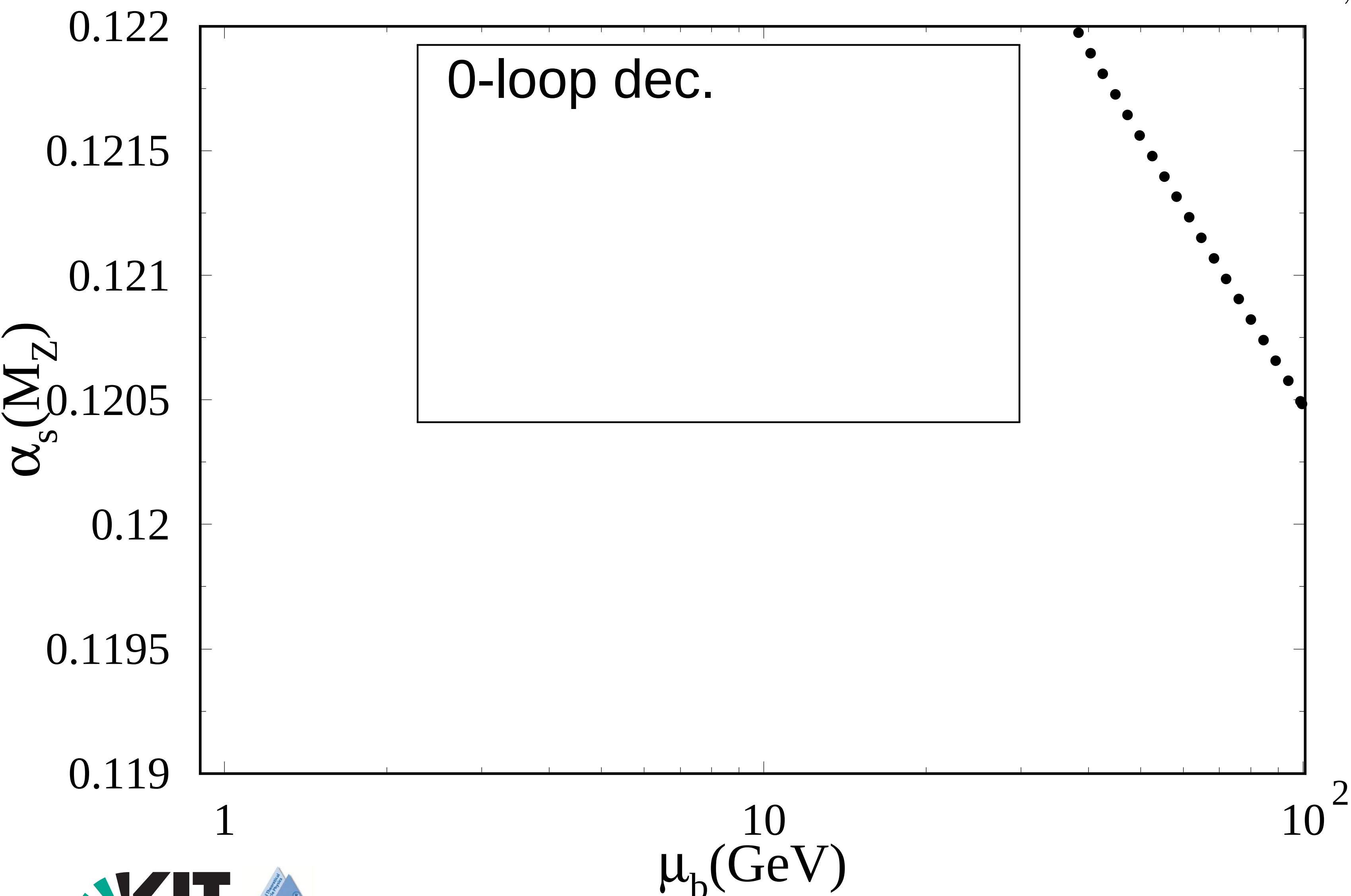
$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$



$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

$\mu_{c,\mathrm{dec}} = 3$ GeV

$\mu_{b,\mathrm{dec}} \equiv \mu_b$



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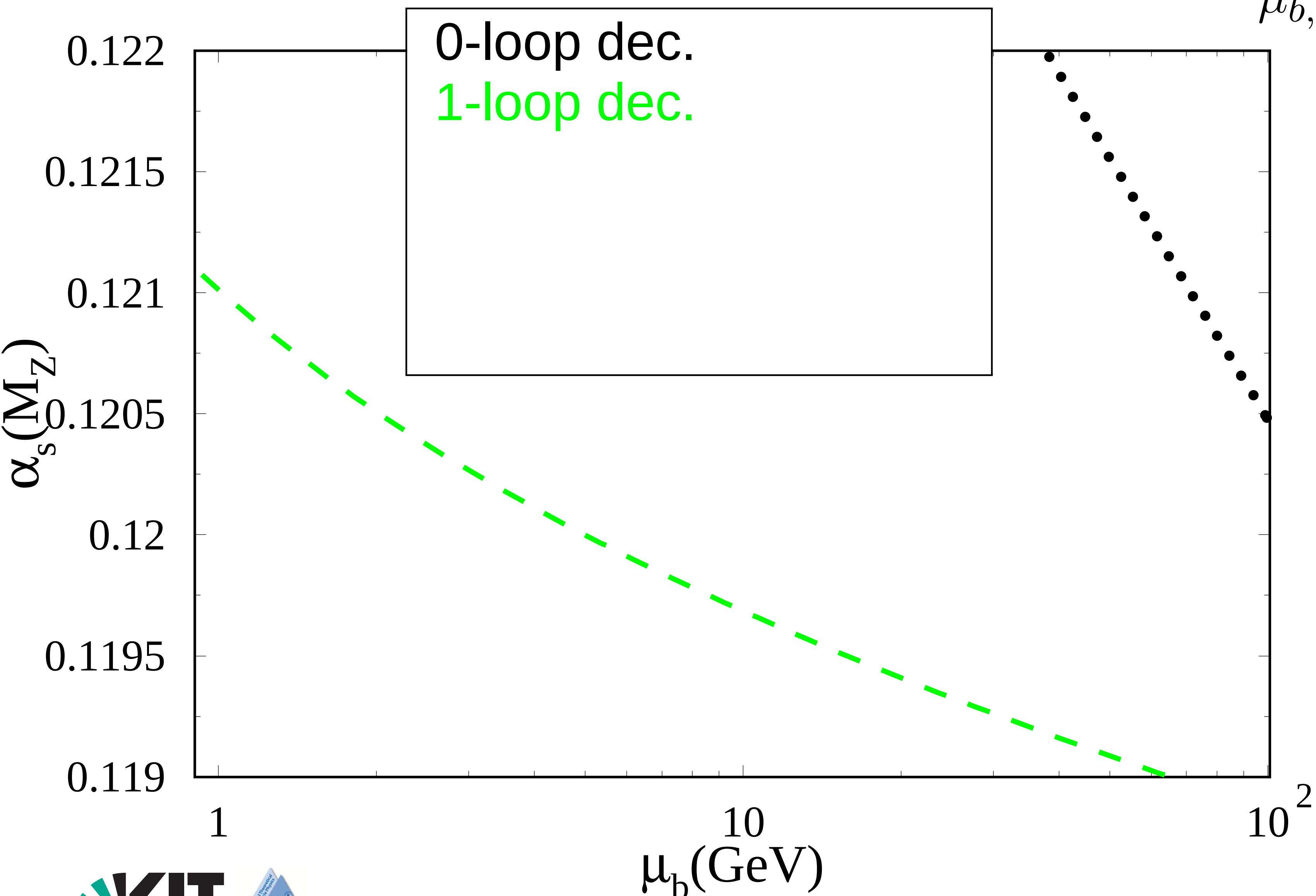

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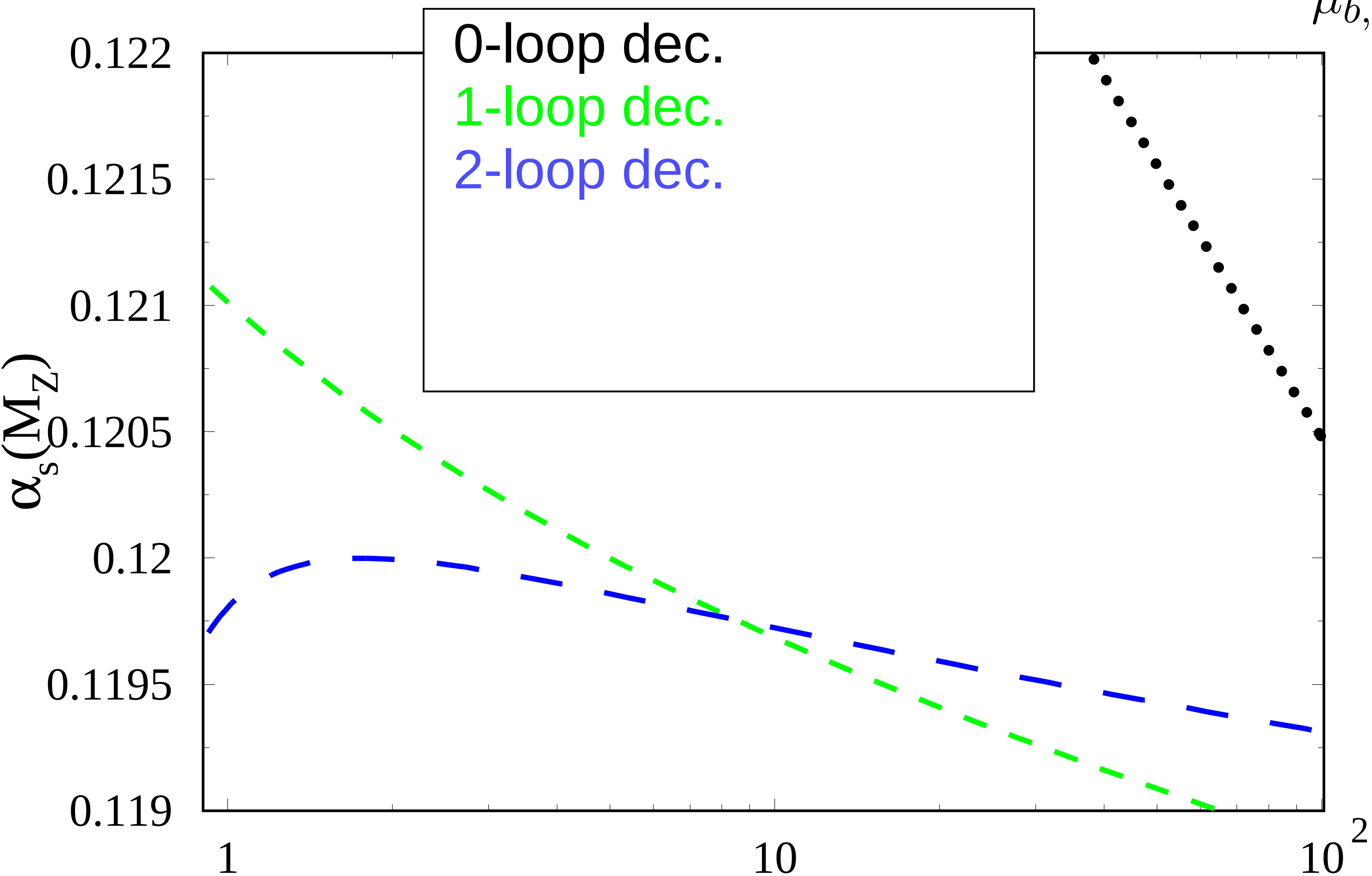

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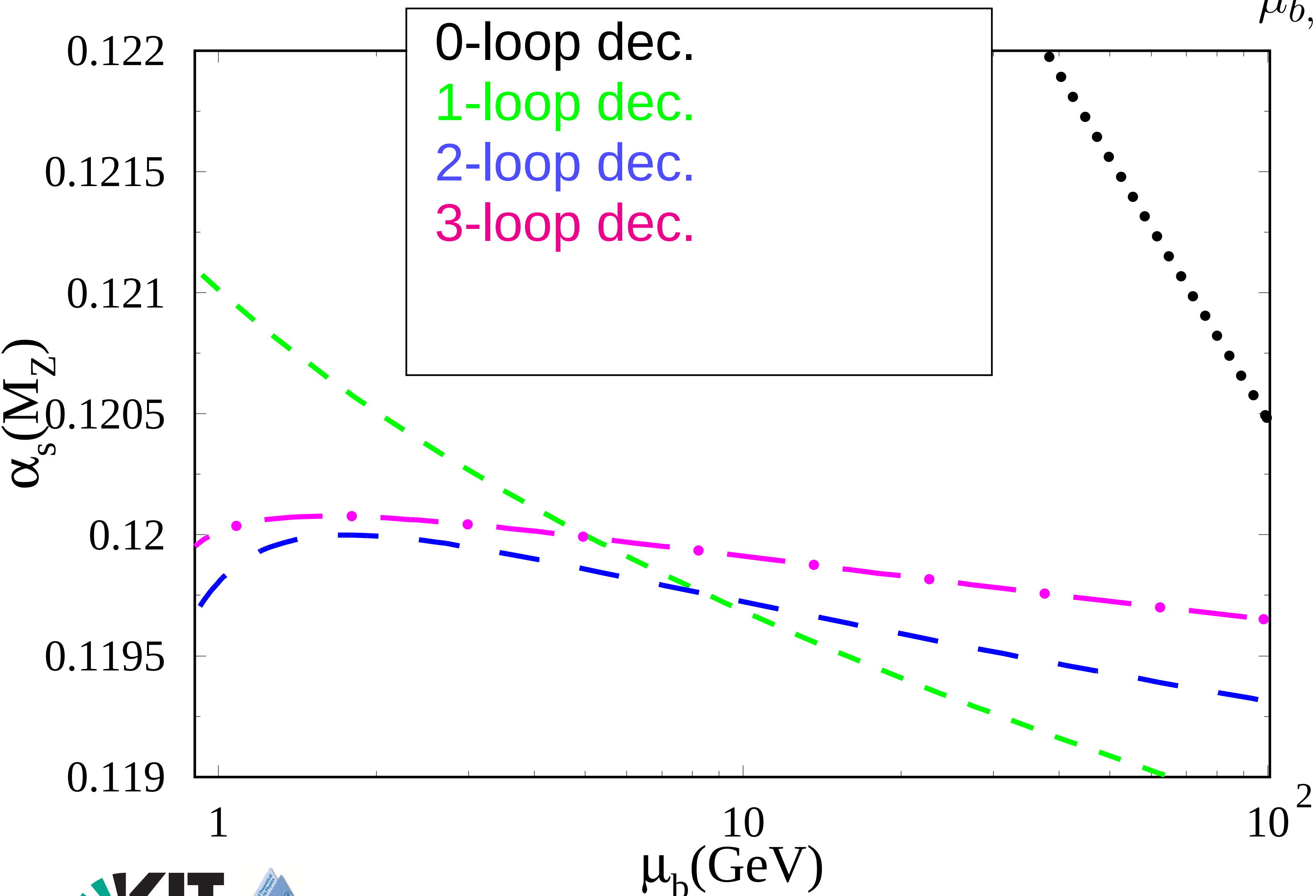

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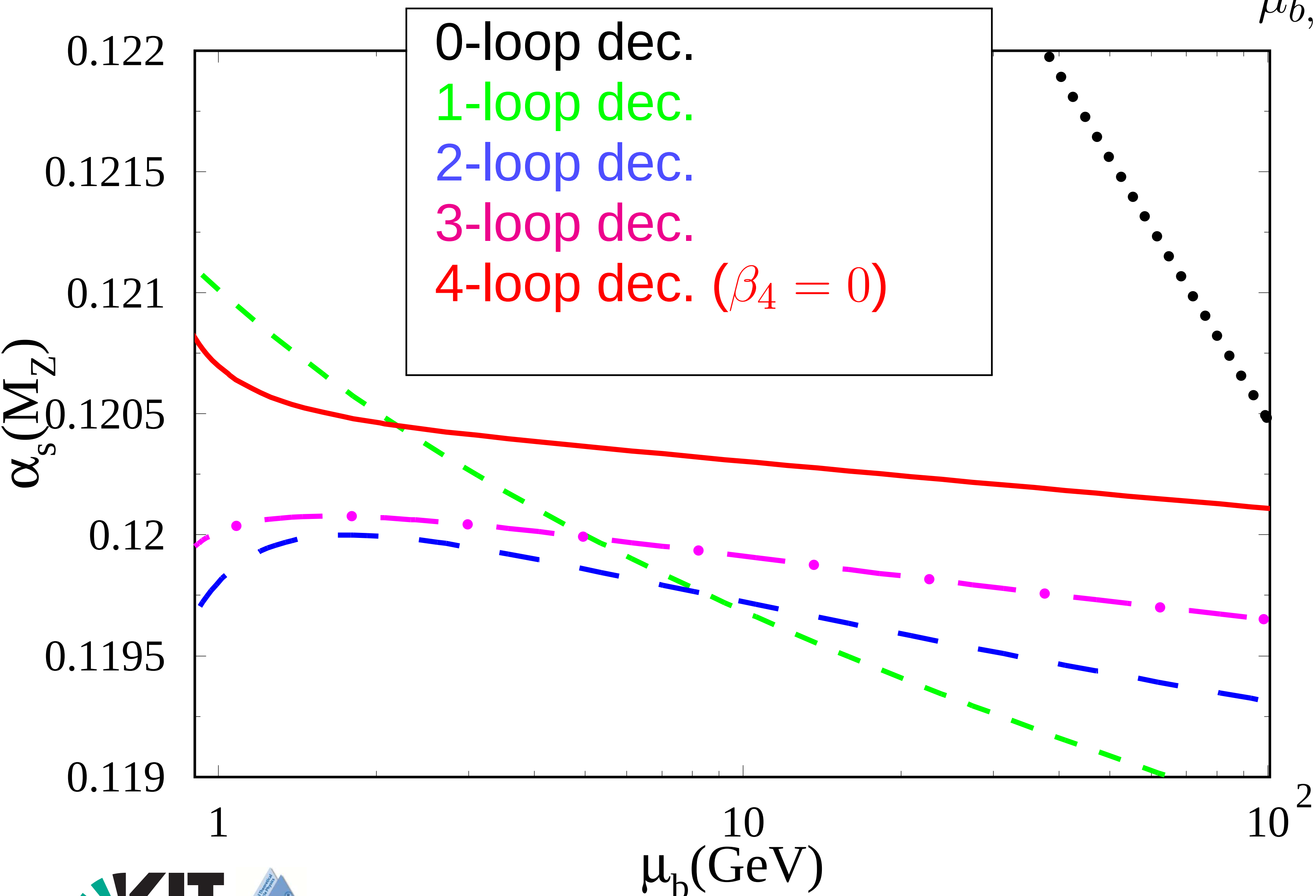

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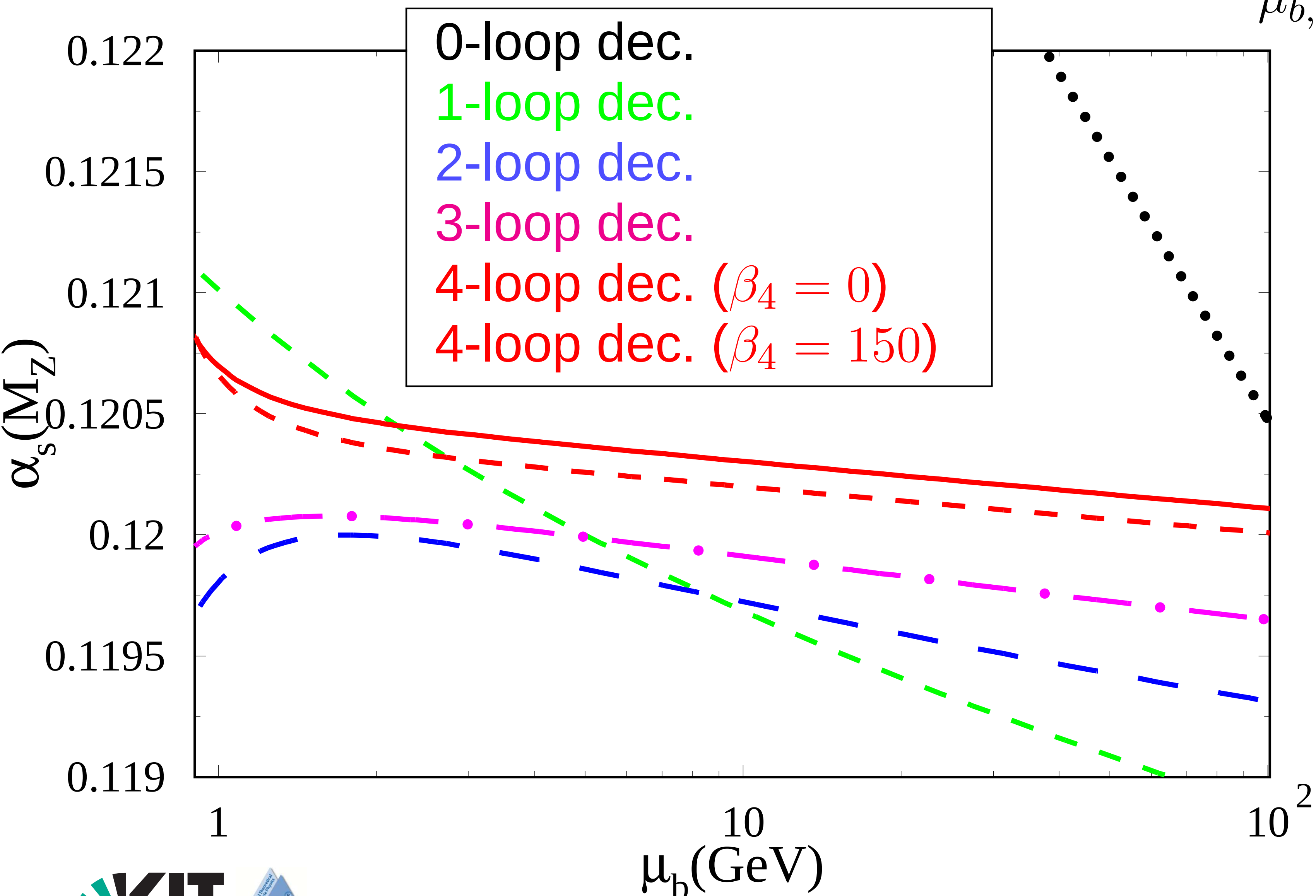

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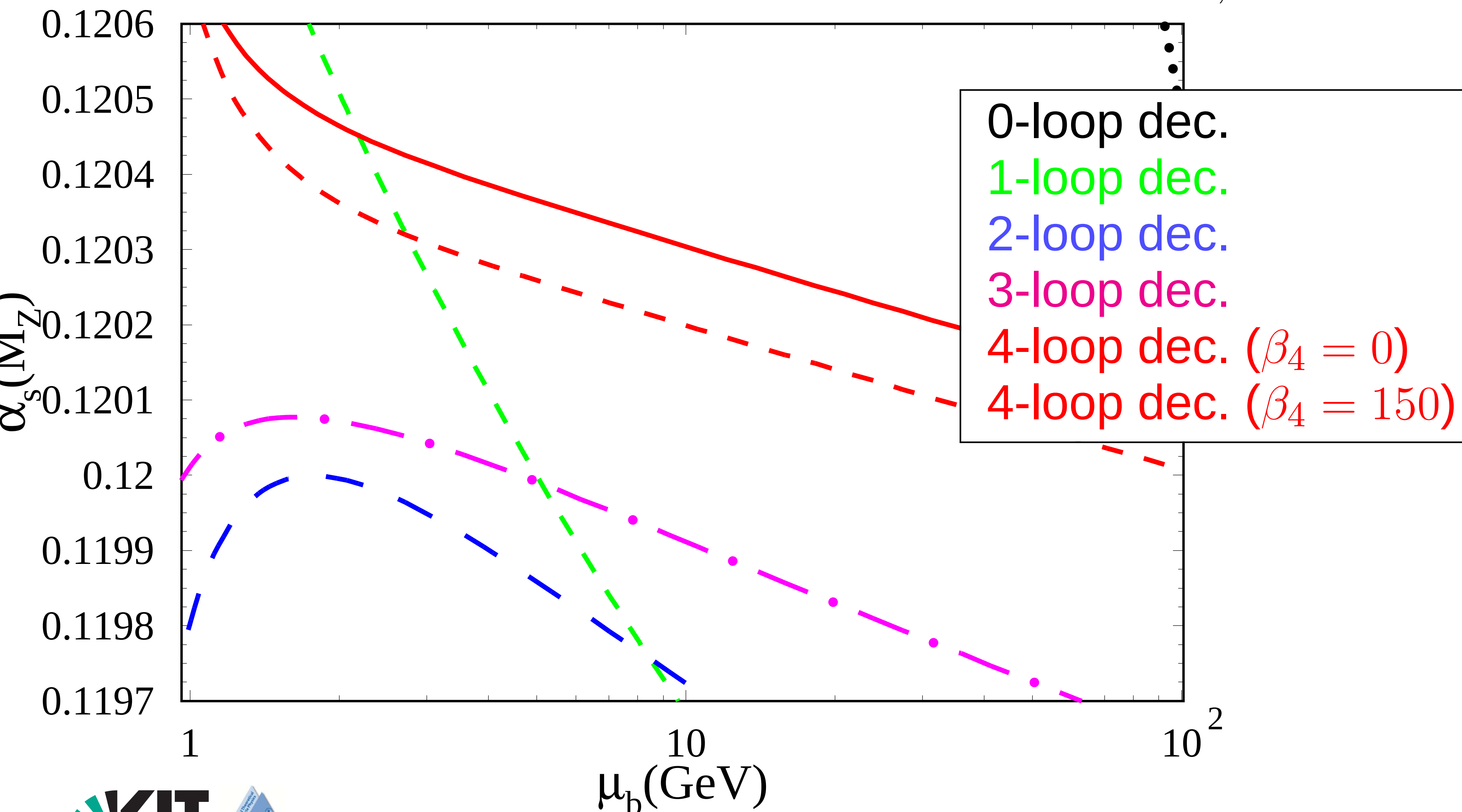

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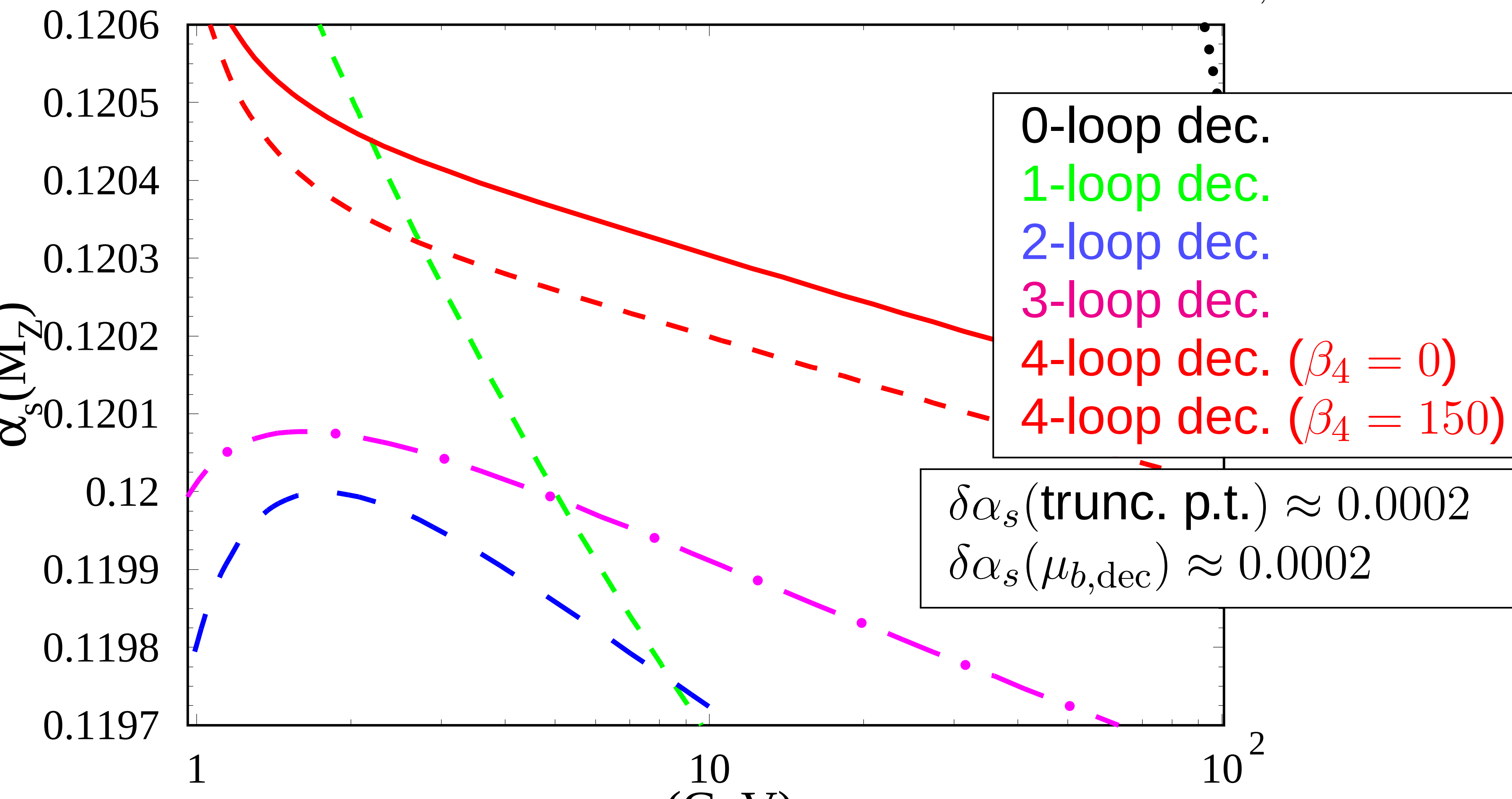


$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$ — numbers

$$\alpha_s^{(3)}(M_\tau) = 0.332 \pm \delta^\tau_{\rm exp} \pm \delta^\tau_{\rm th}$$

$\Rightarrow$ $\alpha_s^{(5)}(M_Z) = 0.1200 \pm \delta^Z_{\rm exp} \pm \delta^Z_{\rm th} \pm \delta_{\rm evol}$ $\qquad$ $\delta_{\rm trunc.p.t.} \approx 0.0002$

$$(\delta_{\rm evol})^2 = (\delta_{\mu_{c,\rm dec}})^2 + (\delta_{\mu_{b,\rm dec}})^2 + (\delta_{\rm trunc.p.t.})^2$$

$\mu_{c,\rm dec}$	$\mu_{b,\rm dec}$	$\alpha_s^{(5)}(M_Z)$
3 GeV	$m_b(m_b)$	0.1200
3 GeV	$0.5\, m_b(m_b)$	0.1200
3 GeV	$5\, m_b(m_b)$	0.1199
3 GeV	$10\, m_b(m_b)$	0.1198
$m_c(m_c)$	$m_b(m_b)$	0.1202
$5\, m_c(m_c)$	$m_b(m_b)$	0.1199
$10\, m_c(m_c)$	$m_b(m_b)$	0.1198

$\Rightarrow$ $\delta_{\rm evol} \approx 0.0004$

$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$ — numbers (2)

$\Rightarrow \delta_{\rm evol} \approx 0.0004$

$\alpha_s^{(3)}(M_\tau) = 0.332 \pm 0.005_{\rm exp} \pm 0.015_{\rm th}$ [Baikov,Chetyrkin,Kühn'08]

$\Rightarrow$

$\alpha_s^{(5)}(M_Z) = 0.1202 \pm 0.0006_{\rm exp} \pm 0.0018_{\rm th} \pm 0.0003_{\rm evol}$

$\alpha_s^{(3)}(M_\tau) = 0.3156 \pm 0.0030_{\rm exp} \pm 0.0051_{\rm th}$ [Beneke,Jamin'08]

$\Rightarrow$

$\alpha_s^{(5)}(M_Z) = 0.11795 \pm 0.00038_{\rm exp} \pm 0.00063_{\rm th} \pm 0.00020_{\rm evol}$

Single-step decoupling of b and c

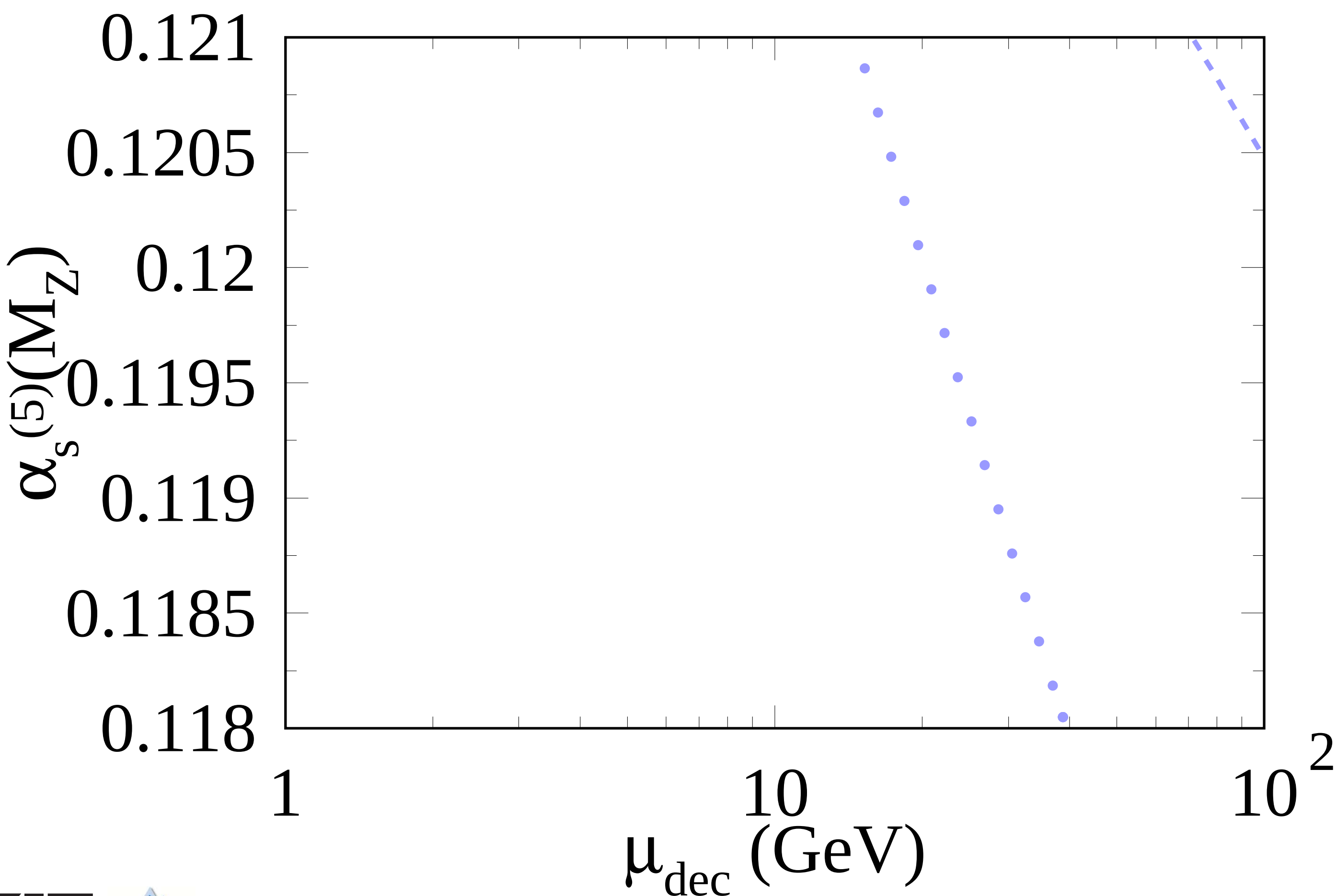
$m_c \approx 1.3$ GeV, $m_b \approx 4.2$ GeV

[Grozin,Höschele,Hoff,Steinhauser]

preliminary

➪ $m_c \ll m_b$ justified ??

Idea: decouple b and c quark simultaneously



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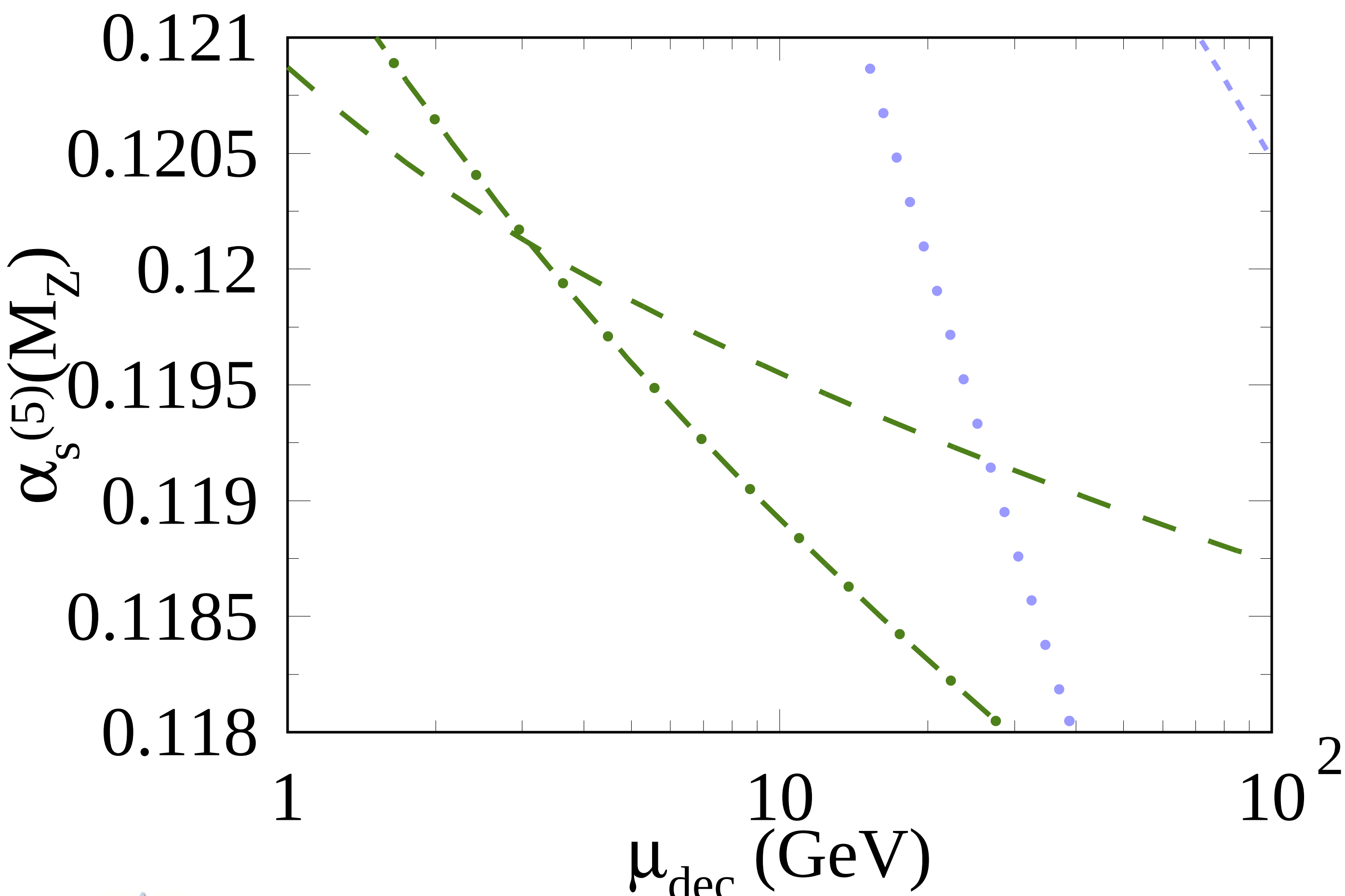

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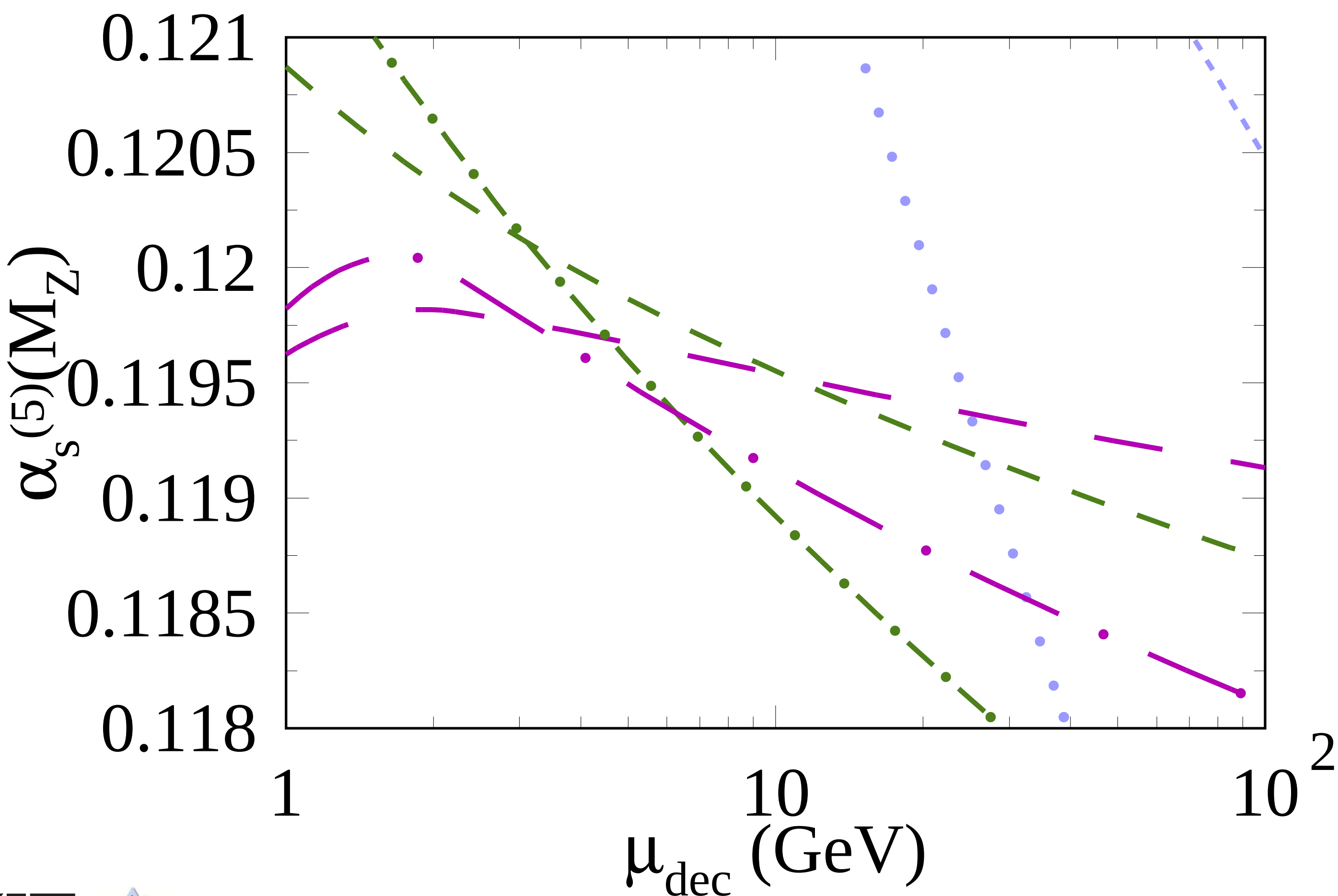

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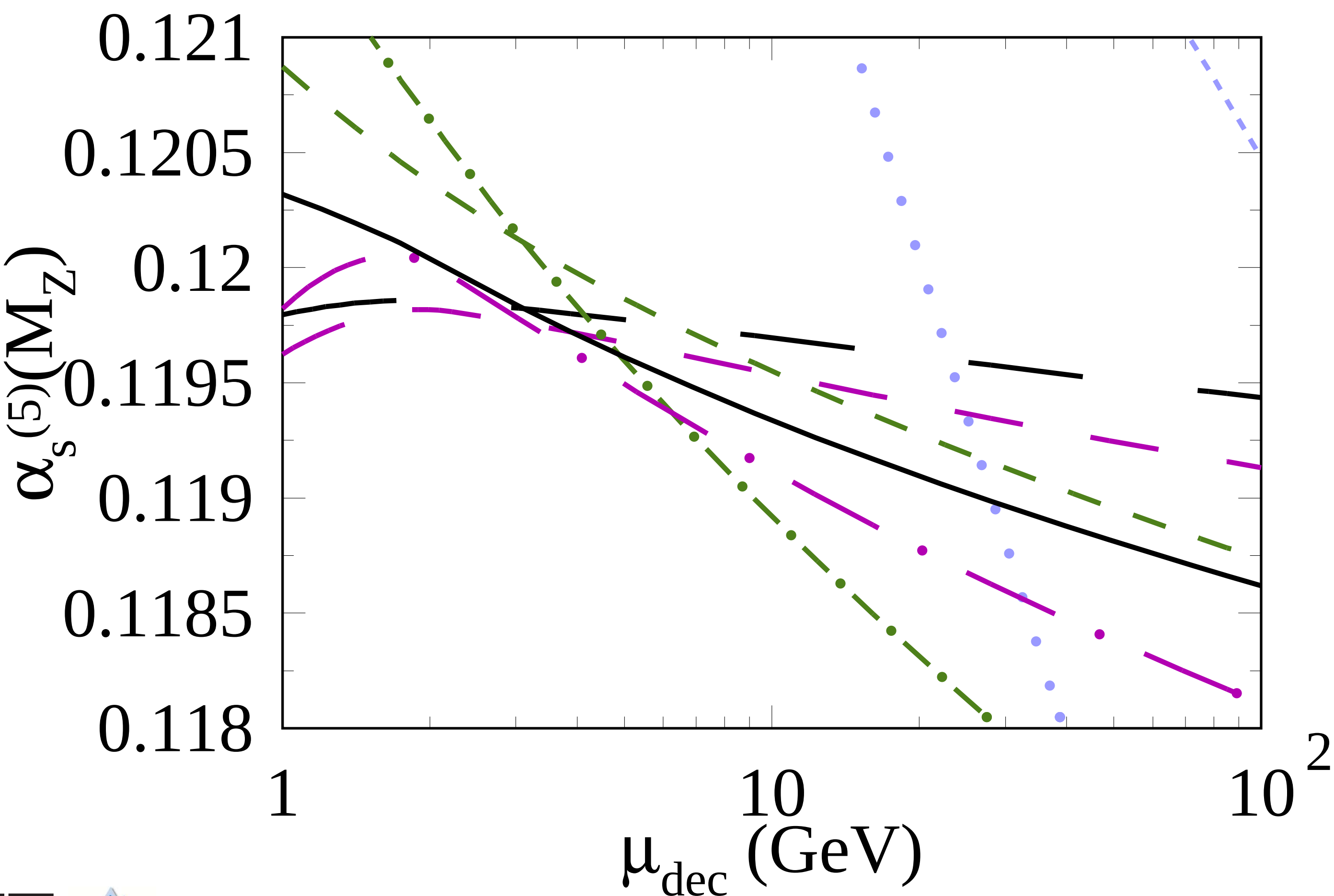

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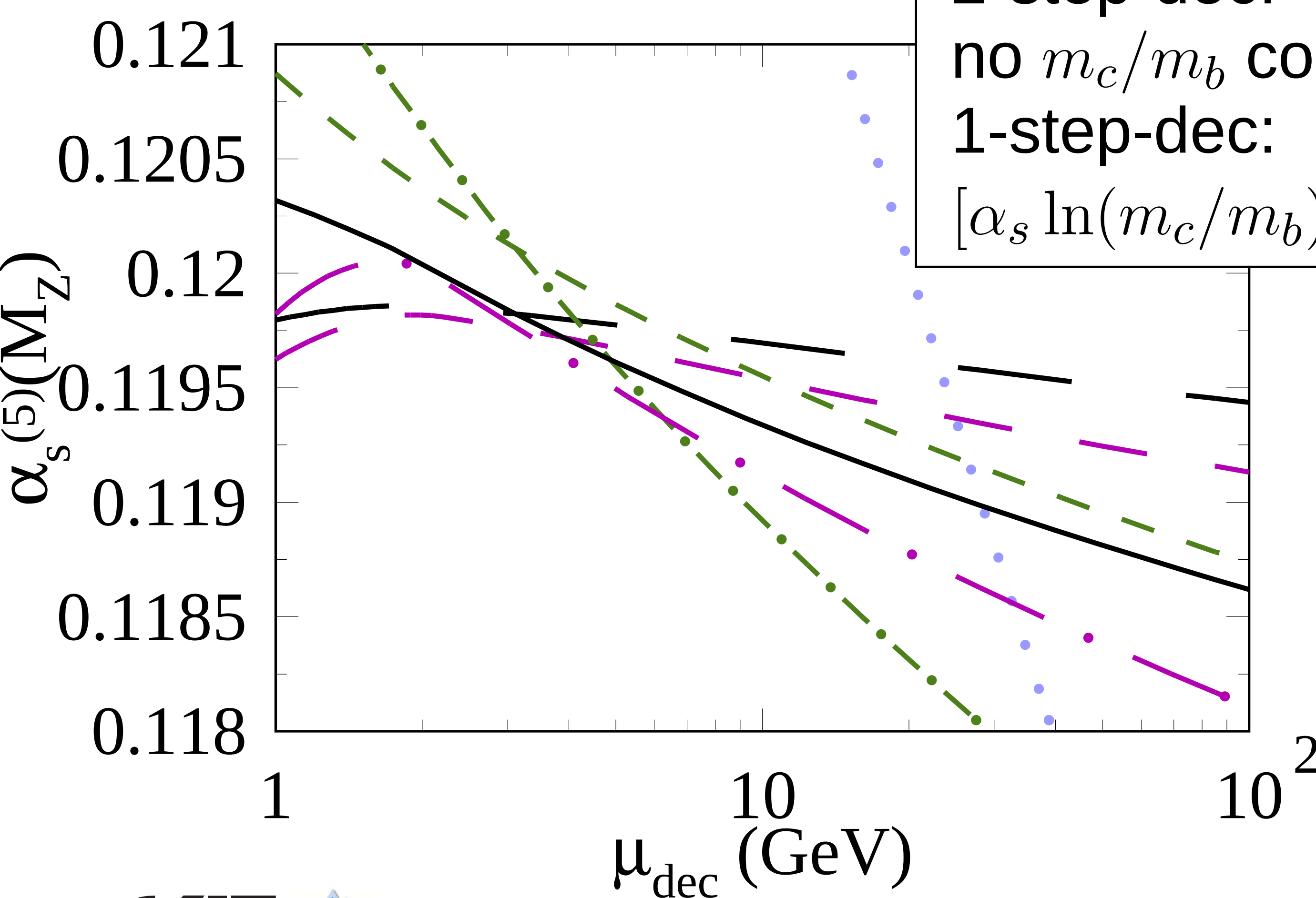

2-step-dec:
no m_c/m_b corrections
1-step-dec:
$[\alpha_s \ln(m_c/m_b)]^N$ not resummed

Conclusions

- do not use $\alpha_s(\Lambda)$ (at least not for small scales)
- $\delta\alpha_s = 0.0007$ ➪ 3-(maybe also 4-)loop decoupling relevant
- comparison MOM $\leftrightarrow$ $\overline{\mathrm{MS}}$
- RunDec [Chetyrkin,Kühn,Steinhauser'00]

  `www-ttp.particle.uni-karlsruhe.de/Progdata/ttp00/ttp00-05`
- 1-step decoupling