Duality violations in hadronic tau decays

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Workshop on Precision Measurements of alpha_S
Feb. 9-11, München, Germany

Theory:

$$\int_0^{s_0} ds\, w(s)\, \rho_{V,A}(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, w(s)\, \Pi_{V,A}(s)\ , \quad \rho(s) = \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)$$

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Take $w(s) = (1 - s/s_0)^2 w'(s)$ ($w'(s)$ polynomial) doubly pinched; replace

$\Pi_{V,A}(s) \to \Pi^{OPE}_{V,A}(s) = \Pi^{PT}_{V,A}(s) + \frac{c_4}{s^2} - \frac{c_6}{s^3} + \dots$ (Braaten, Narison, Pich, '92)

and fit $\alpha_s,\ c_4,\ c_6\ \dots$ at $s_0 = m_\tau^2$ using a collection of weights $w'(s)$.

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Pinching assumed to suppress

$$-\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\left[\Pi_{V,A}(s) - \Pi^{OPE}_{V,A}(s)\right] = -\int_{s_0}^{\infty} ds\, w(s)\, \rho^{DV}_{V,A}(s)$$

where $\rho^{DV}_{V,A}(s)$ is the “duality-violating” part of the spectral function.

Theoretical errors for precision α_s:

(Pivovarov, '92, Le Diberder, Pich, '92)

1) Resummation of perturbation theory ("CIPT" vs. "FOPT")
   Still significant difference even with availability of $O(\alpha_s^4)$
   (Baikov, Chetyrkin, Kuhn, 08; Beneke, Jamin, '08, Menke, '09, Caprini, Fischer, '09
   Descotes-Genon, Malaescu, '10)

2) Truncation of the OPE
   Important: given a weight $w(s)$ of degree n,
   include dominant OPE terms to order $1/s^{n+1}$ (Maltman, Yavin, '08)

3) Duality violations: (exponentially) suppressed, but not necessarily
   negligible at or below the tau mass (Catà, Golterman, Peris, '08)

Ignoring any of these leads to uncontrolled systematics in value of α_s !

Assumption for DVs: *ansatz*

(Catà, Golterman, Peris, '05)

Take $\rho^{DV}_{V,A}(s) = \theta(s - s_{min})\left[\kappa_{V,A}\, e^{-\gamma_{V,A} s}\, \sin(\alpha_{V,A} + \beta_{V,A} s)\right]$

- based on Regge-like meson spectrum in each channel with finite widths asymptotic behavior of spectral function consistent with a model with the correct analytical properties (Blok *et al.*, Bigi *et al.*, '98-'99)

- four new parameters in each channel: need to vary s_0 in order to fit data; setting $\kappa_{V,A} = 0$ also a -- probably very poor! -- model

- can use unpinched or singly-pinched weights, like $w(s) = 1\ ,\quad 1 - s/s_0$

$$\int_0^{s_0} ds\, w(s)\, \rho_{V,A}(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\, \Pi^{OPE}_{V,A}(s) \;-\int_{s_0}^{\infty} ds\, w(s)\, \rho^{DV}_{V,A}(s)$$

sum rules probe DVs on interval $[s_0, \infty)$

Fitting strategies:

$$\qquad\qquad\qquad\qquad c_4 \qquad\qquad c_6$$
$$\qquad\qquad\qquad\qquad \downarrow \qquad\qquad \downarrow$$

Use weights $w_{n=0,1,\dots} = 1 \ , \ 1 - s/s_0 \ , \ (1 - s/s_0)^2 \ , \ \dots$

Moments $M_n(s_0) = \int_0^{s_0} \frac{ds}{s_0} w_n(s) \, \rho(s) \ , \quad s_0 \in [s_{min}, m_\tau^2]$ strongly correlated,

hence correlation matrices for any combination of moments poorly conditioned: machine-precision zero eigenvalues with combination of two or more moments

Strategies:

0) Use only simplest moment, $M_0(s_0)$: no issues with correlations

1) Combine moments in "uncorrelated" fits; find errors by propagation or MC

2) Fit $\alpha_s, \ \kappa, \ \gamma, \ \alpha, \ \beta$ to $M_0(s_0)$, use results as priors for
(re)fit $\alpha_s, \ c_4, \ \kappa, \ \gamma, \ \alpha, \ \beta$ to $M_1(s_0)$, use results as priors for
(re)fit $\alpha_s, \ c_4, \ c_6, \ \kappa, \ \gamma, \ \alpha, \ \beta$ to $M_2(s_0)$, *etc.*
each step is a correlated fit

Data and fits:

Two sets of experimental data: ALEPH and OPAL
ALEPH: publicly available correlation matrices miss correlations due to unfolding and thus are incomplete $\Rightarrow$ these data cannot be safely used until fixed

Therefore, we are at the moment limited to OPAL data (which have strong experimental correlations reflected in available covariance matrix).

Plots: similar fits for both CIPT and FOPT, non-strange vector channel

Strategy 0 $w(s) = 1$ moments with DVS and without DVs

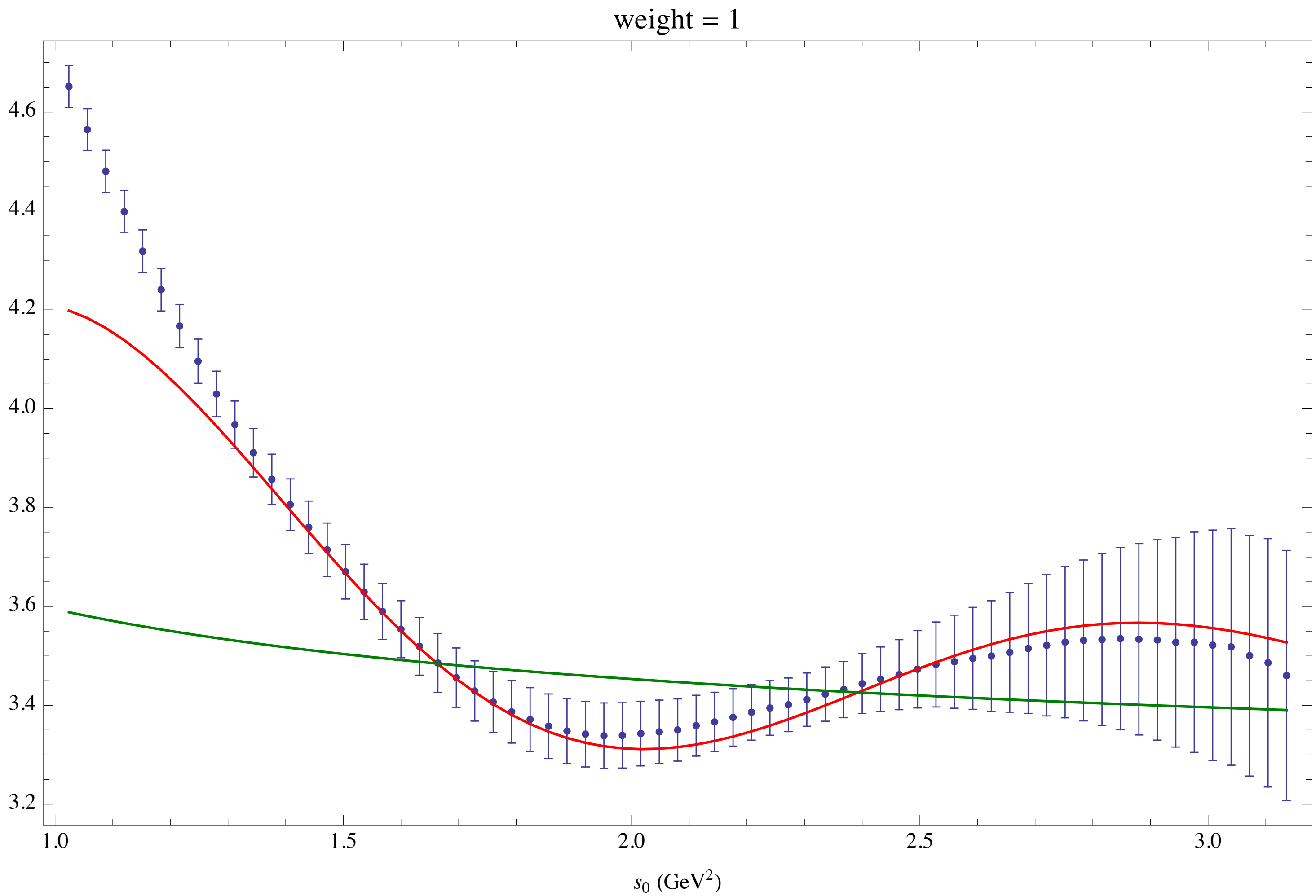
Strategy 1 (uncorr., etc.) $w(s) = 1$ and $w(s) = (1 - s/s_0)^2$ with/without DVs

Strategy 2 (priors) $w(s) = 1$ and $w(s) = (1 - s/s_0)^2$ with/without DVs

Fits without DVs for strategies 1 and 2 use only $w(s) = (1 - s/s_0)^2$

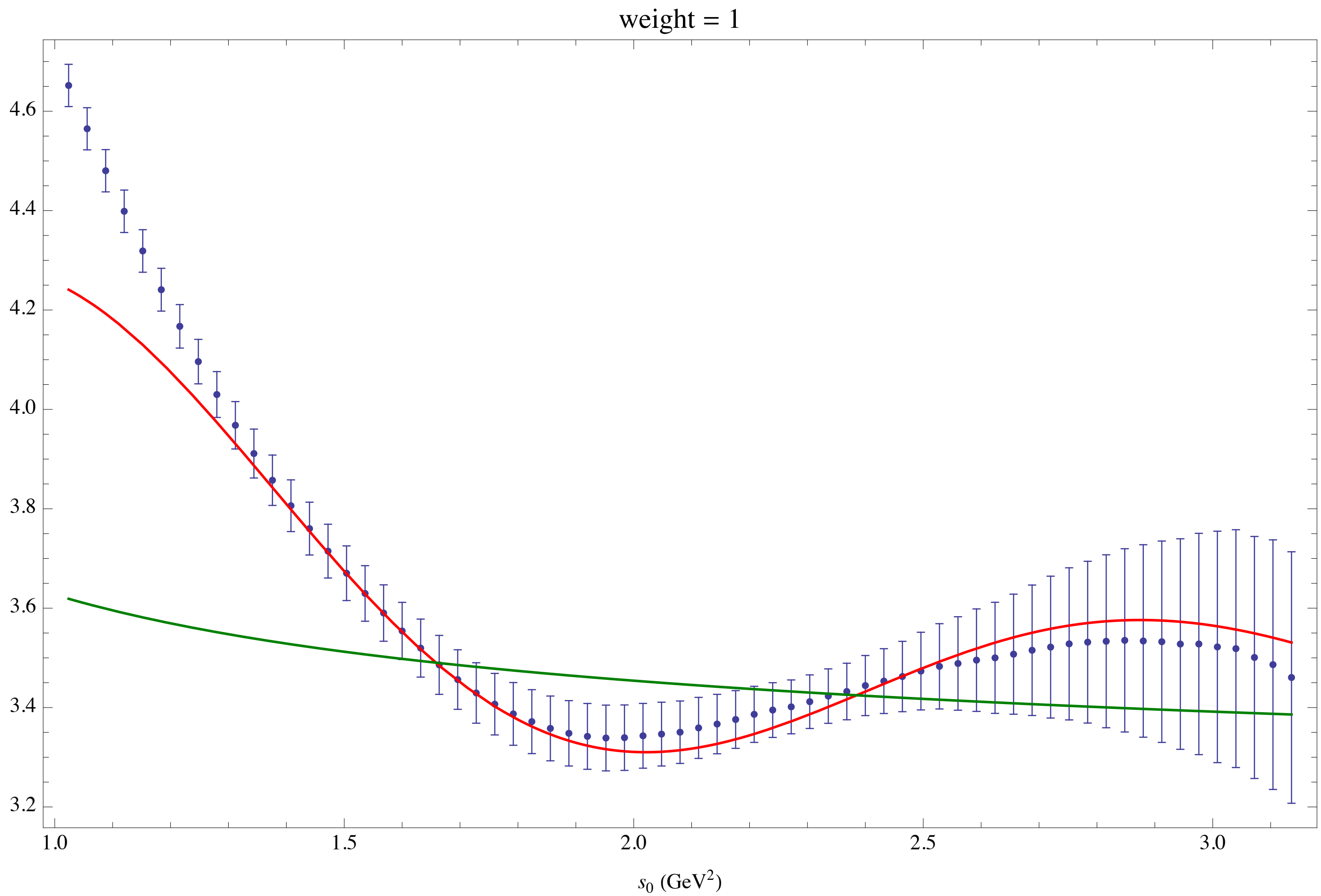
check spectral functions for all

w(s) = 1 , CIPT, DVs $s_{min} = 1.5\ \mathrm{GeV}^2$ (red), no DVs $s_{min} = 1.8\ \mathrm{GeV}^2$ (green)



Strategy 0 using moment $w(s) = 1$

w(s) = 1 , FOPT, DVs $s_{min} = 1.5\ \mathrm{GeV}^2$ (red), no DVs $s_{min} = 1.8\ \mathrm{GeV}^2$ (green)



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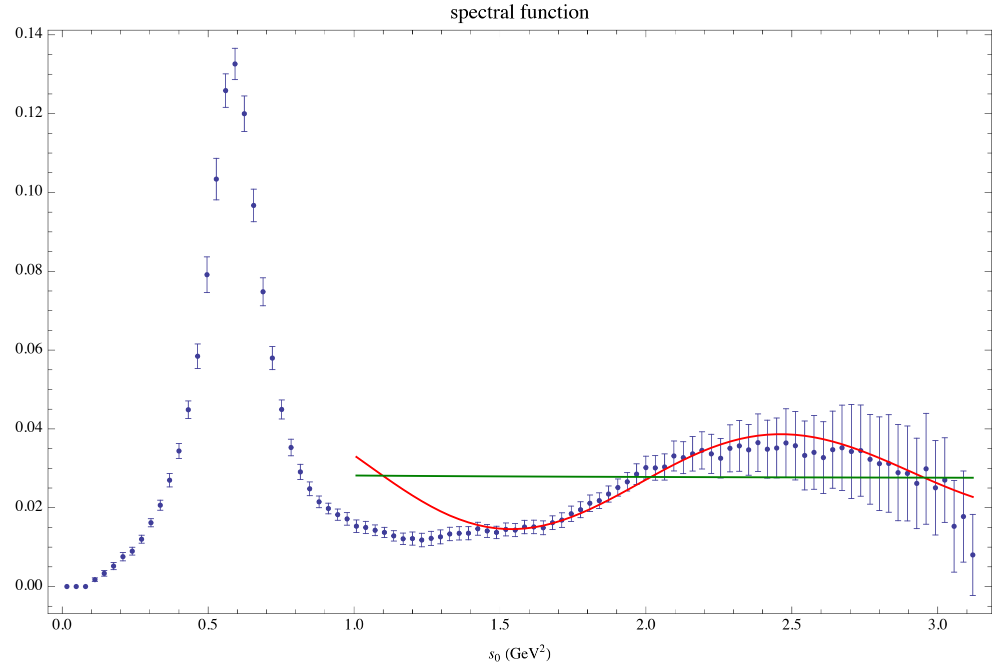
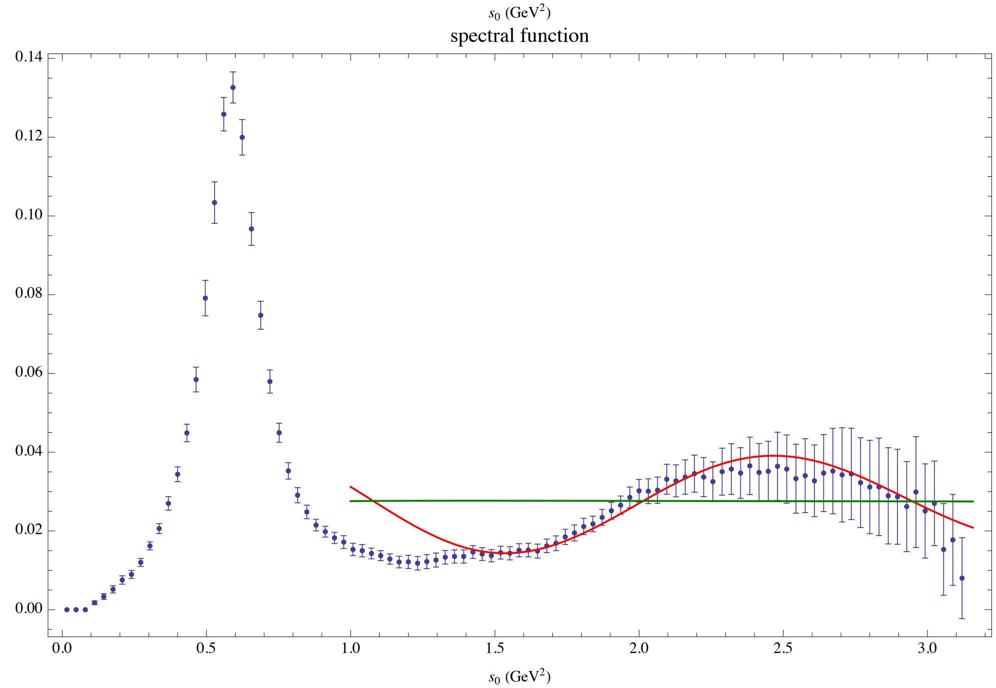
spectral functions:

CIPT



FOPT



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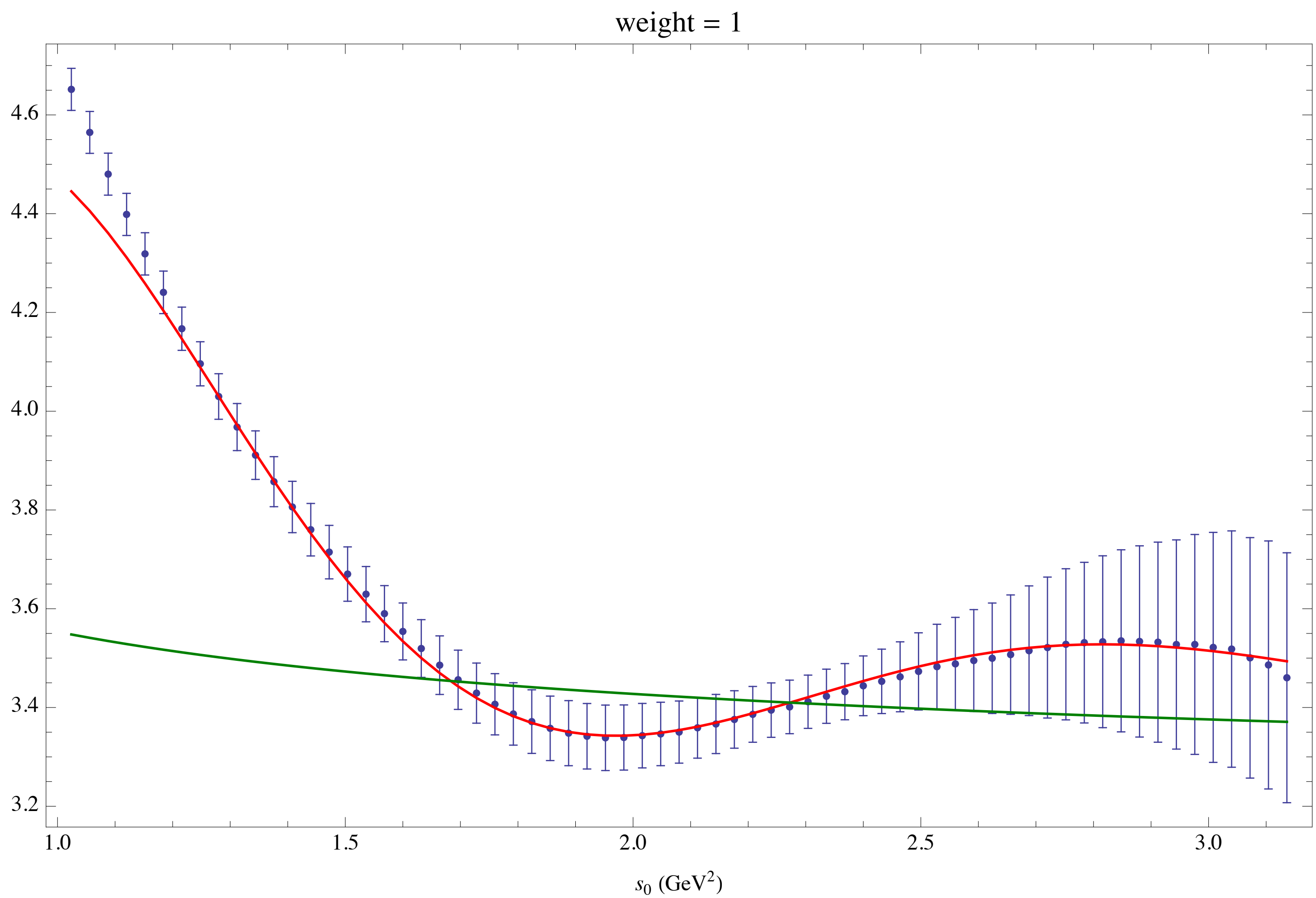
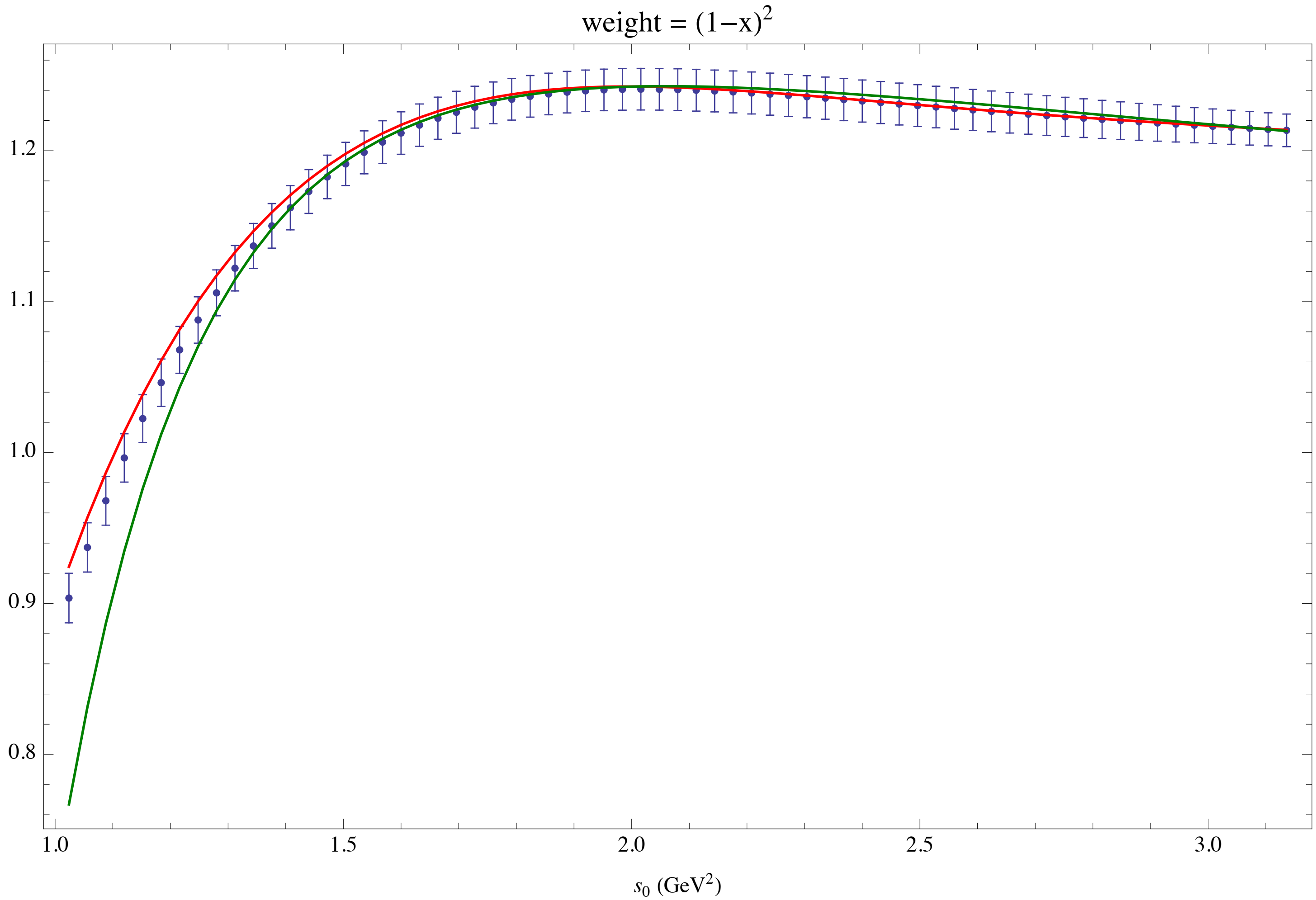


Strategy 1 using moments $1\ ,\quad 1 - s/s_0\ ,\quad (1 - s/s_0)^2$

$(1\text{-}x)^2$, CIPT, DVs $s_{min} = 1.5\ \mathrm{GeV}^2$ (red), no DVs $s_{min} = 1.8\ \mathrm{GeV}^2$ (green)



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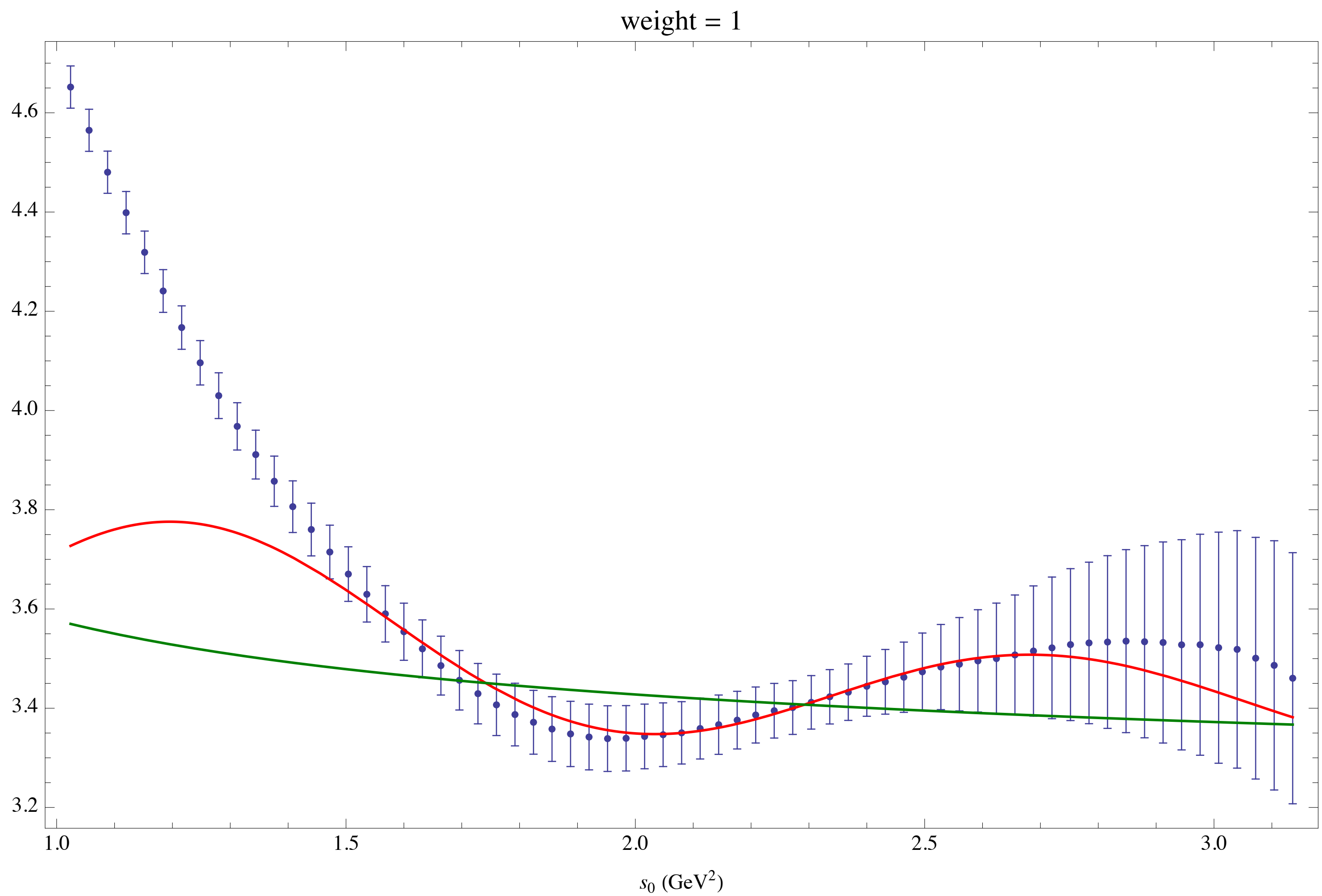
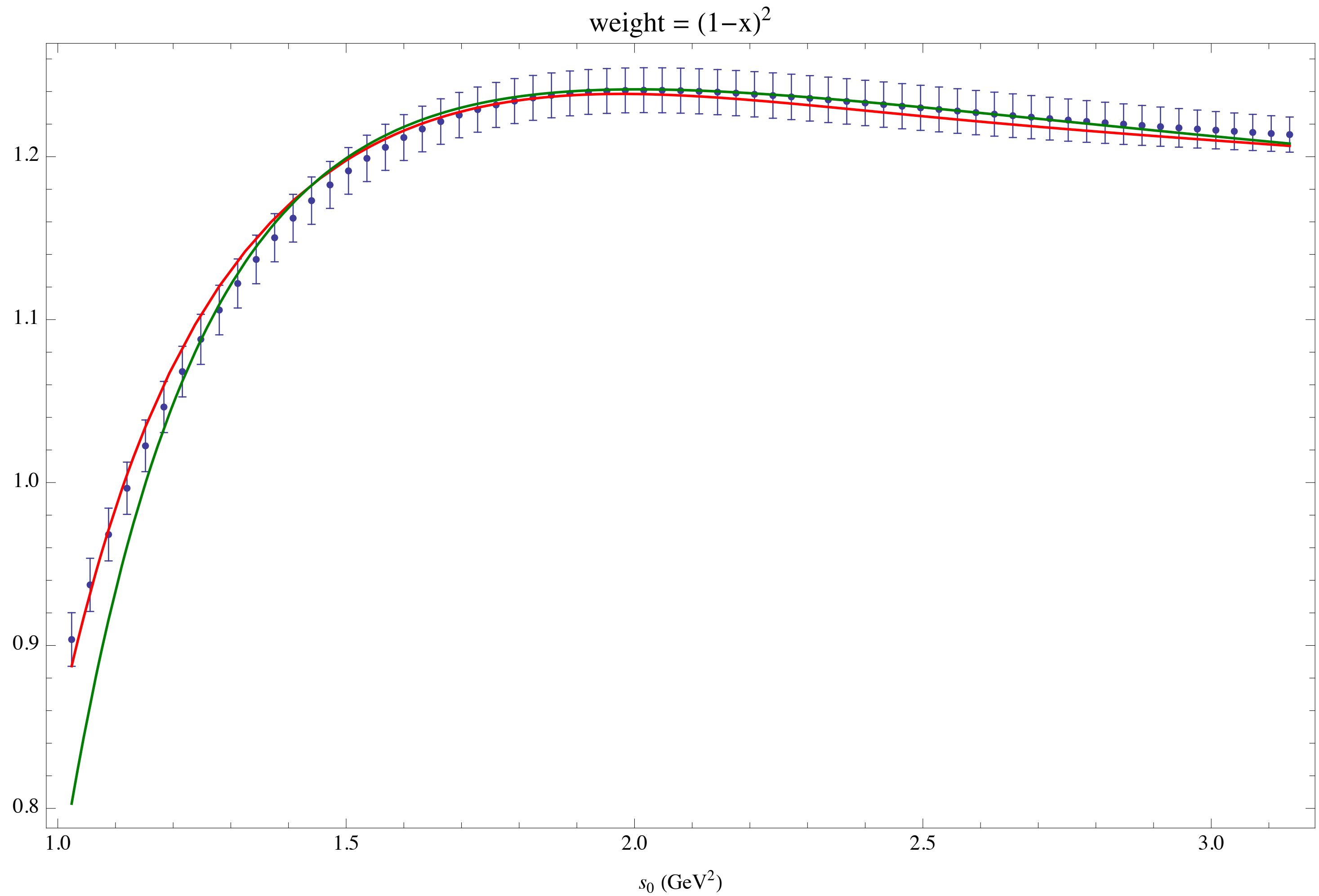


Strategy 1 using moments $1\ ,\quad 1 - s/s_0\ ,\quad (1 - s/s_0)^2$

(1-x)^2 , FOPT, DVs $s_{min} = 1.5\ \mathrm{GeV}^2$ (red), no DVs $s_{min} = 1.8\ \mathrm{GeV}^2$ (green)



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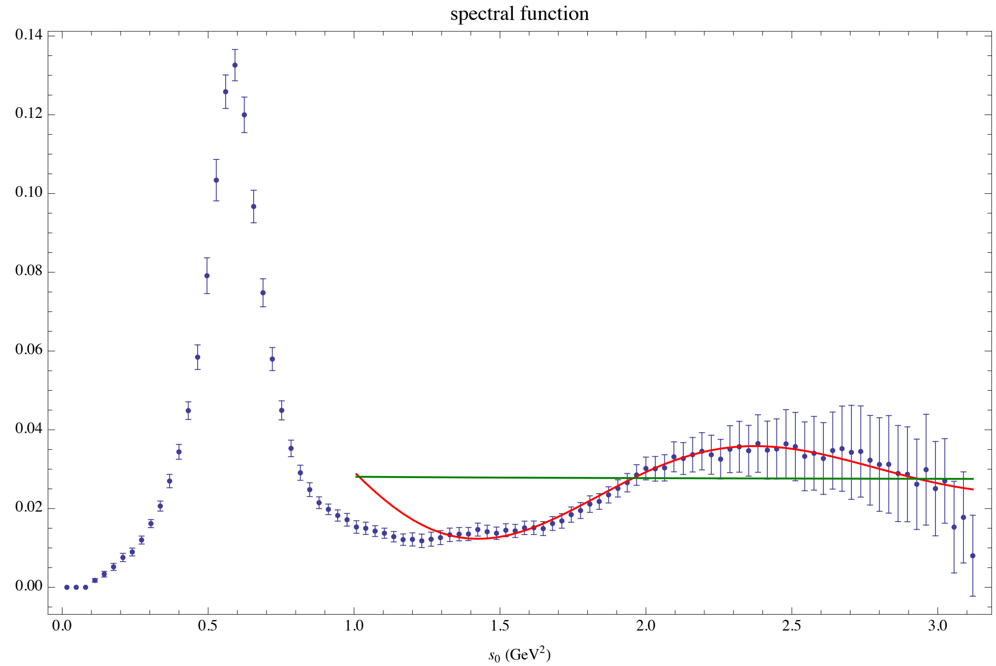
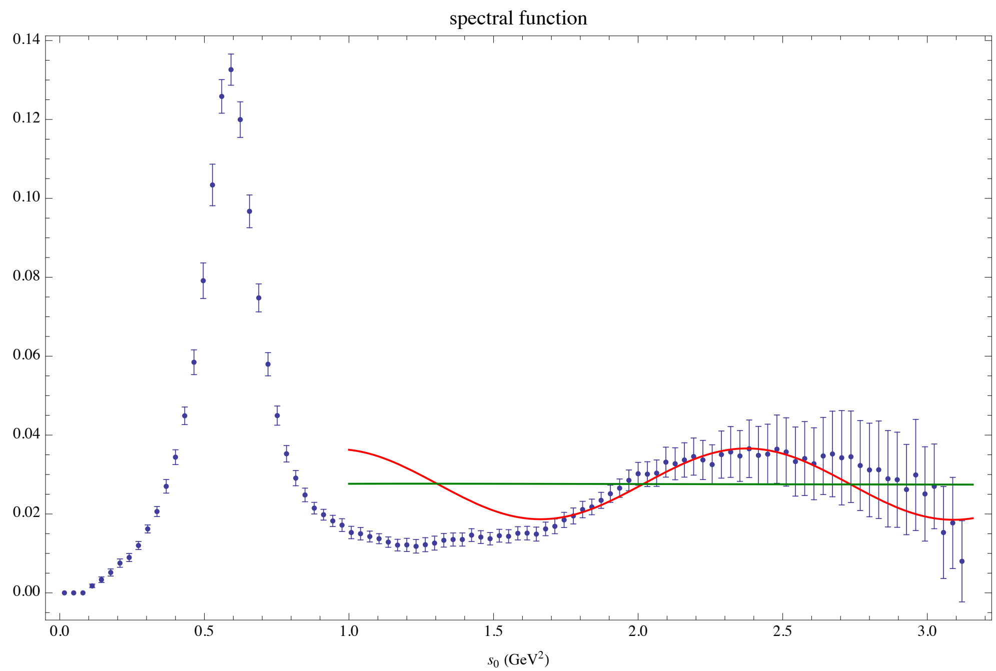
spectral functions:

CIPT



FOPT



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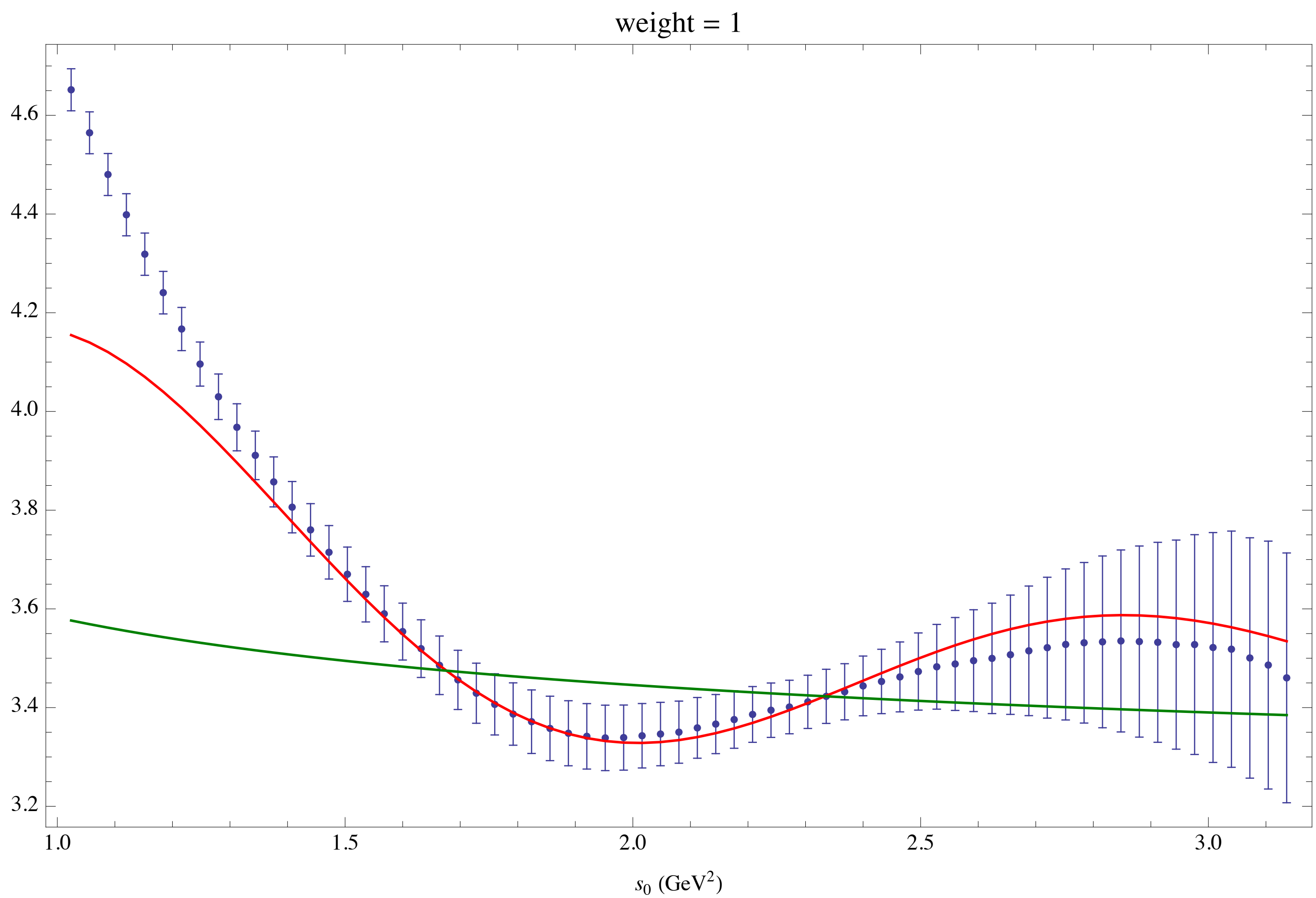
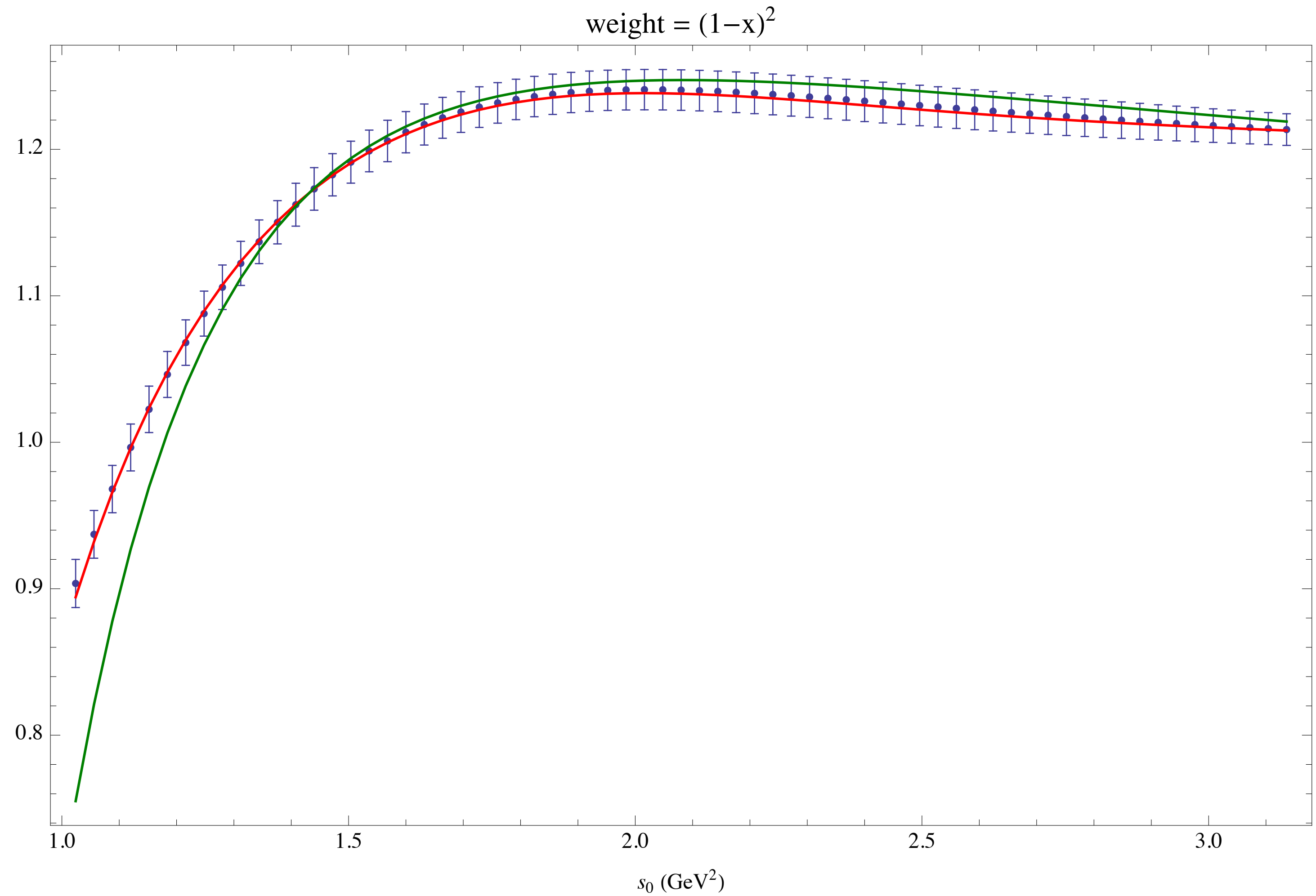


Strategy 2 using moment $(1 - s/s_0)^2$ with priors from 1 , $1 - s/s_0$

$(1\text{-}x)^2$, CIPT, DVs $s_{min} = 1.5\ \mathrm{GeV}^2$ (red), no DVs $s_{min} = 1.8\ \mathrm{GeV}^2$ (green)



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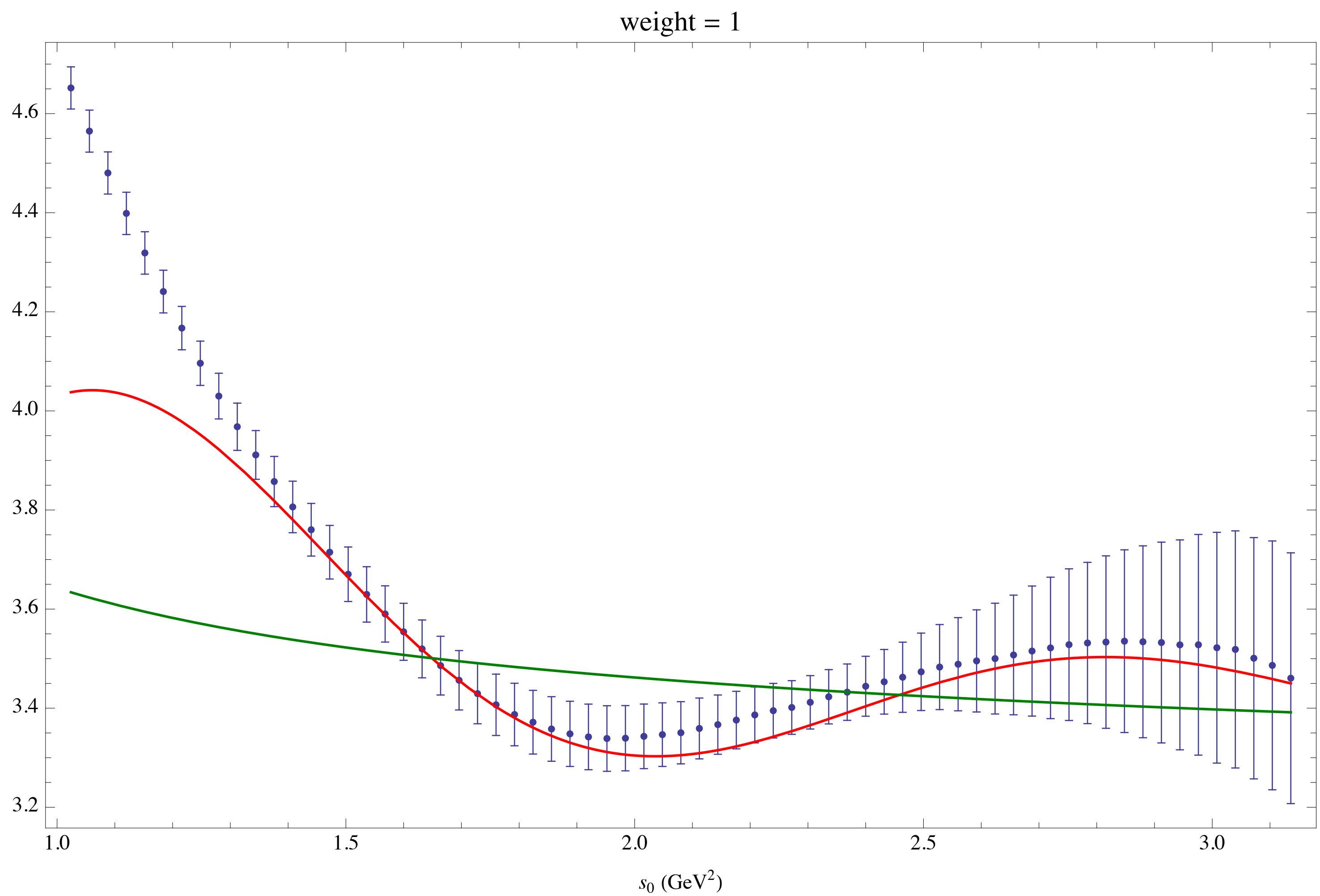

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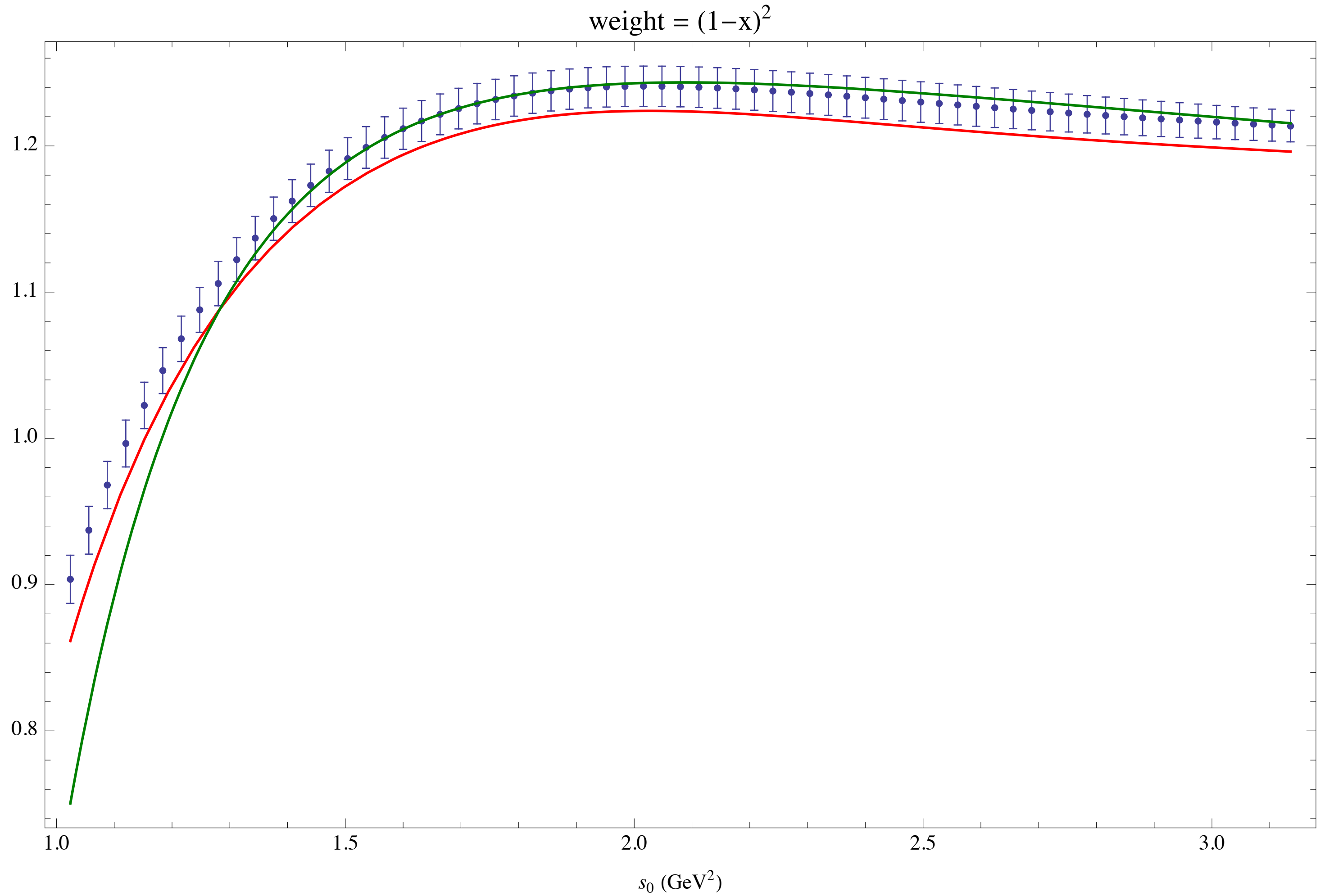

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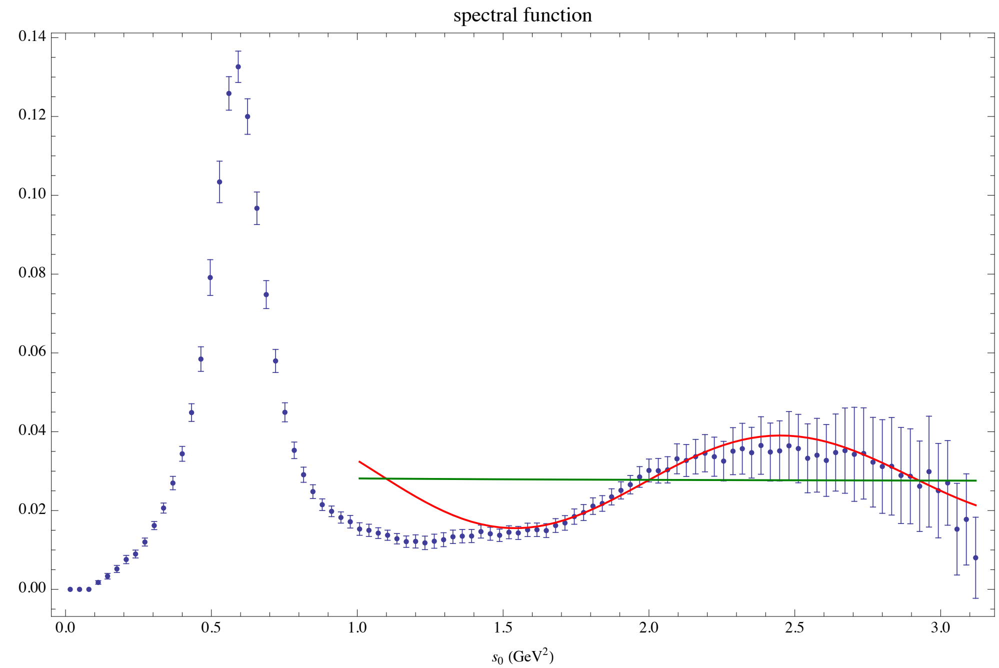
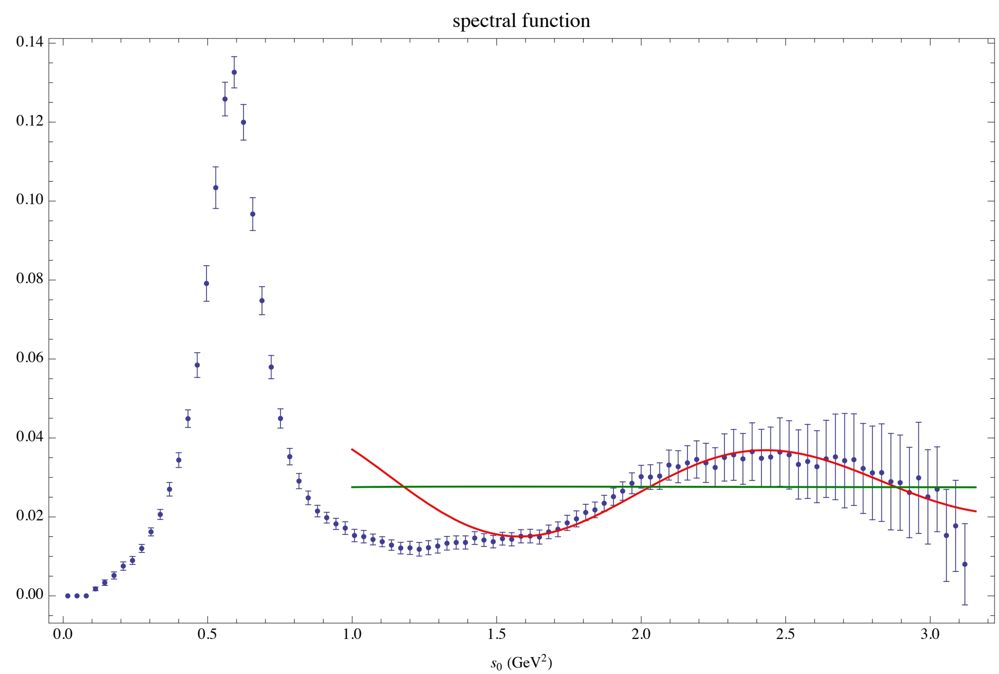
spectral functions:

CIPT



FOPT



Some numbers for α_s : (vector channel)

DVs	FOPT	CIPT	Strategy
Yes	0.307(18)	0.322(25)	0
No	0.271	0.279	1
Yes	0.287(19)	0.326(70)	
No	0.284	0.288	2
Yes	0.289(8)	0.327(13)	

(errors with Strategy 2 possibly underestimated)

- omitting DVs appears to have significant effect; need to be modeled in order to check

- fits without DVs very sensitive to s_{min}, more so than those with DVs

Fits to vector and axial channels

Use spectral functions, and all nth degree moments to $1 - \frac{3}{2}\frac{s}{s_0} + \frac{1}{2}\left(\frac{s}{s_0}\right)^3$ with strategy 1 . No errors yet, but look for stability:

- get OPE coefficients up to c_{2n+2}
- same, but use input for α_s , c_4 , and DV parameters from $n = 1$
- compare

$n = 3$, $s_{min} = 1.6\ \mathrm{GeV}^2$, CIPT

	values for step 1	values for step 2
α_s	0.316	0.316
c_{6V} (GeV^6)	-0.0066	-0.0066
c_{6A} (GeV^6)	0.00014	0.00016
c_{8V} (GeV^8)	0.010	0.010
c_{8A} (GeV^8)	0.0041	0.0041

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- compare

$n = 3$, $s_{min} = 1.6$ GeV^2, FOPT

	values for step 1	values for step 2
α_s	0.289	0.302
c_{6V} (GeV^6)	-0.0092	-0.0058
c_{6A} (GeV^6)	-0.0023	0.0010
c_{8V} (GeV^8)	0.017	0.0099
c_{8A} (GeV^8)	0.0091	0.0031

Conclusions

- High precision determination of α_s from tau decays requires understanding of Duality Violations; pinched weights do not suppress DVs sufficiently

- From exploration of OPAL data, fits including about 8 “nuisance” parameters for DVs are possible

- Assuming our *ansatz* for DVs, we obtain, from vector channel with $w = 1$ preliminary values

$$\alpha_s(M_\tau) = 0.322(25) \Rightarrow \alpha_s(M_Z) = 0.1188(29) \quad \text{(CIPT)}$$
$$\alpha_s(M_\tau) = 0.307(18) \Rightarrow \alpha_s(M_Z) = 0.1169(24) \quad \text{(FOPT)}$$

- More sophisticated fitting strategies may help reduce the errors (strong correlations!); more weights with consistent treatment of OPE