α_s in Electroweak Physics

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I. $\alpha_{\mathbf{s}}$ from Z Decays

II. Impact of α_s on $\delta\rho$

I. $\alpha_{\mathbf{s}}$ from Z Decays

(Chetyrkin, JK, Kwiatkowski, Phys.Rep.277,189;

Baikov, Chetyrkin, JK; . . .)

1. Qualitative considerations
2. The massless case
   - non-singlet
   - singlet
3. Mass effects $(\mathbf{m_b})$
4. Mixed, non-factorizable terms
   - $Z \to q\overline{q}$, $Z \to b\overline{b}$

1. Qualitative considerations

Remember :

$\Gamma_{\rm had} \approx 69.91(6)\%$ of $2495.2(2.1)$ MeV $\quad \Rightarrow \quad 1744.2(2.0)$ MeV

$\delta\alpha_s = 3 \cdot 10^{-3}$ corresponds to $\delta\Gamma_{\rm had} = 1.7$ MeV

Two distinctly different observables :

- $R_Z = \Gamma_{\rm had}/\Gamma_\mu \approx 20.8,$ counting of events
  δR_Z dominated by statistics
- $\sigma_{\rm Peak}(\rm had) \sim \frac{\Gamma_e\,\Gamma_{\rm had}}{(\Gamma_{\rm lept}+\Gamma_{\rm had})^2}$ [or $\sigma_{\rm Peak}(\rm lept)$]
  $\delta\sigma$ dominated by luminosity (Bhabba-scattering)
  Reduction of sensitivity : $\frac{\delta\sigma(\rm had)}{\sigma(\rm had)} \sim -0.4\,\frac{\delta\Gamma_{\rm had}}{\Gamma_{\rm had}}$

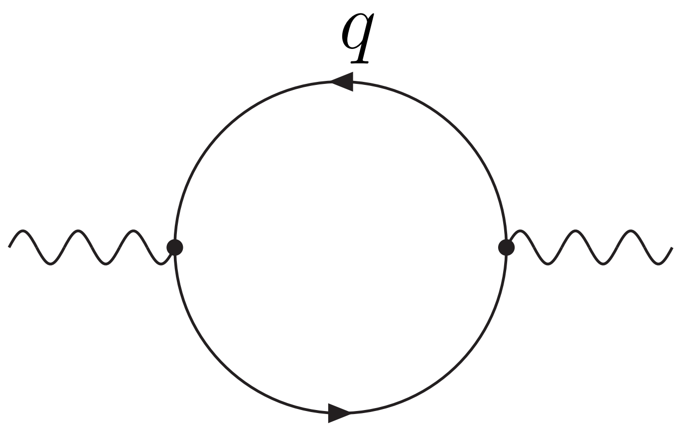
Perspectives : GIGA-Z

α_S from R_Z : $\delta\alpha_s = 5 - 7 \times 10^{-4}$

(Drawback : slight implicit dependence of $\Gamma_{\rm had}/\Gamma_{\rm lept}$ on electroweak physics : M_H ; SUSY)

2. The massless case

- non-singlet terms : dominant



previously :

$$\Gamma \propto \sum (v_q^2 + a_q^2) \left(1 + \frac{\alpha_s}{\pi} + 1.409 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.767 \left(\frac{\alpha_s}{\pi}\right)^3\right)$$

scale variation ($\mu = (0.5 - 2) M_Z$)

$\Rightarrow \delta\alpha_s \approx 0.6 \cdot 10^{-3}$

electroweak working group

$\alpha_s = 0.1185 \pm 0.0026$ (exp)

$\boxed{\text{recent result :}}$ $\quad -79.98 \left(\frac{\alpha_s}{\pi}\right)^4$

$\Rightarrow$ shift of α_s by $+0.5 \cdot 10^{-3}$

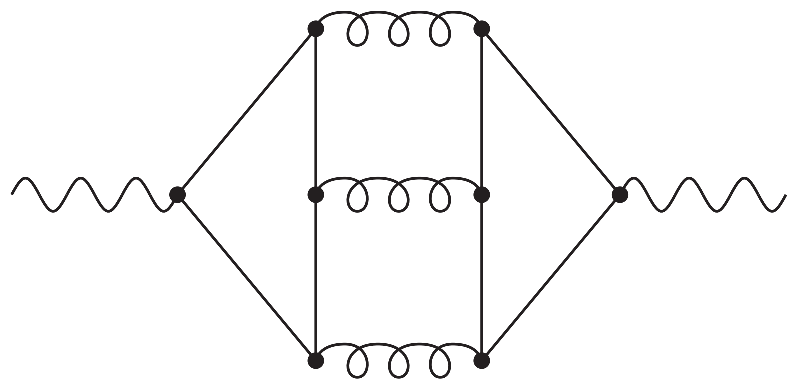
(Baikov, Chetyrkin, JK)

$\alpha_s = 0.1190 \pm 0.0026$

evaluated by considering shift in non-singlet term only

confirmed by Z-fitter etc.

- singlet terms : vector current



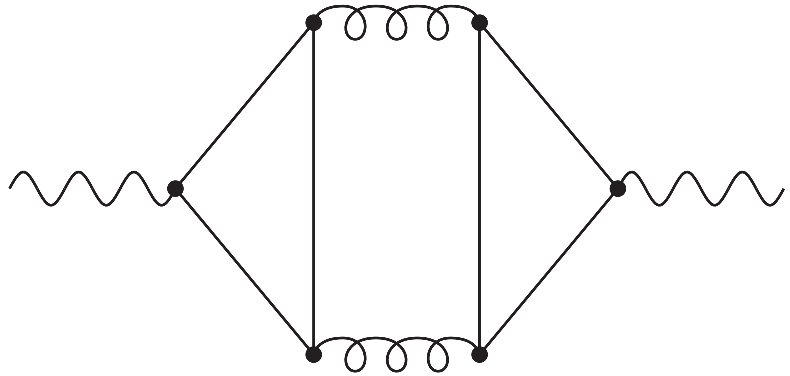
starting $\mathcal{O}(\alpha_s^3)$, small QCD-coefficients,

large cancellations : $\sim \left(\sum v_q\right)^2 \left[-1.240 \left(\frac{\alpha_s}{\pi}\right)^3\right]$

$$\left(\sum v_q\right)^2 = \sum_q \left(I_3^q - 2Q_q \sin^2\theta_W\right)^2 \approx 0.43$$

α_s^4-term not yet available (and irrelevant)

- singlet terms : axial current



no naive decoupling of top;

dominant term $\mathcal{O}(\alpha_s^2) \cdot \mathcal{O}\left(\ln\, m_t/m_Z\right)$ (Kniehl, JK)

result available to $\mathcal{O}(\alpha_s^3)$ (Chetyrkin, Tarasov)

$\Gamma_{\rm singlet} = -1.82\,$MeV (remember : $\delta\Gamma_{\rm had} = 2\,$MeV)

$\big(\mathcal{O}(\alpha_s^3)$ term $\sim 0.24\,$MeV , $\mathcal{O}(\alpha_s^4)$ term : work in progress $\big)$

3. *Mass Effects :* m_b *!*

axial rate

Born : $\sqrt{1-4m_b^2/M_Z^2}^{\,3} \approx 1-6m_b^2/M_Z^2$

which m_b ?

large logarithmic terms resummed :

$\overline{m}_b(M_Z^2) = 2.83(2)$ GeV

correction available up to order

$\alpha_s^4 m_b^2/M_Z^2$ and $\alpha_s^3 m_b^4/M_Z^4$

$\Rightarrow \Delta\Gamma_{m_b} = -1.5$ MeV

Born approximation with $m_b = m_{\text{Pole}} = 4.7$ GeV

would lead to $\Delta\Gamma_{m_b} = -4$ MeV in total :

most α_s^4 corrections are available

remaining (small) singlet terms : soon

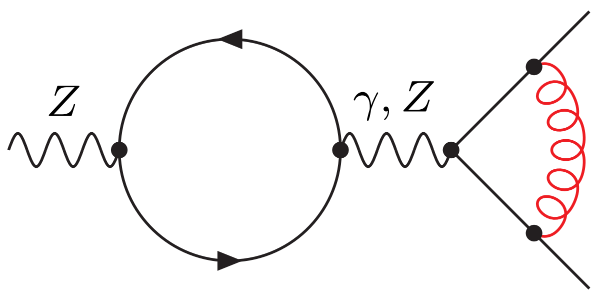
4. Mixed corrections

QCD-electroweak non-factorizable terms :

$\mathcal{O}(\alpha_s \cdot \alpha_{\text{weak}}) \sim \mathcal{O}(\alpha_s^3)$

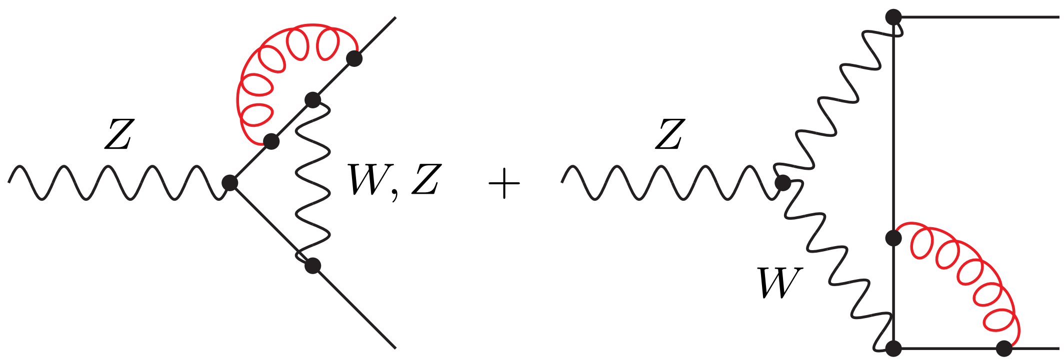
$$\begin{aligned} \Gamma &= \Gamma_{\text{BORN}}\left(1+\delta_{\text{weak}}+\delta_{\text{strong}}\right) \\ &\stackrel{?}{=} \Gamma_{\text{BORN}}\left(1+\delta_{\text{weak}}\right)\left(1+\delta_{\text{strong}}\right) \end{aligned}$$

valid for subclass ("oblique" one loop corrections)



$+ \ldots$

not valid for vertex corrections



$+ \ldots$

instructive example : QCD $\otimes$ QED (Kataev)

$$1+\frac{\alpha_s}{\pi}+Q_q^2\frac{3}{4}\frac{\alpha}{\pi}-\frac{\alpha_s}{\pi}Q_q^2\frac{1}{4}\frac{\alpha}{\pi}$$ correct result

$$\neq\left(1+\frac{\alpha_s}{\pi}\right)\left(1+Q_q^2\frac{3}{4}\frac{\alpha}{\pi}\right)$$ factorization

QCD $\otimes$ QED strategy :

evaluate difference between result and factorized corrections

$\Rightarrow$ vertex corrections only!

light (u,d,c,s) quarks : (Czarnecki, K.)

expansion in M_W^2/M_Z^2 and $M_W^2/(4M_Z^2)$

$$\delta\Gamma(\text{QCD}\otimes\text{EW}) \quad - \quad \underbrace{\frac{\alpha_s}{\pi}\delta\Gamma(\text{1 Loop EW})}_{\text{factorization}} \quad = \quad -0.55\,\text{MeV}$$

$$\widehat{=} \quad \delta\alpha_s \approx 1\cdot 10^{-3}$$

$\boxed{b\bar{b}}$

$\alpha_s G_F m_t^2$ (Fleischer et al.)

$\alpha_s \alpha_{\text{weak}} \ln m_t^2$ (Kwiatkowoski, Steinhauser)

$\alpha_s \alpha_{\text{weak}} \left(\text{const} + 1/m_t^2\right)$ (Harlander, Seidensticker, Steinhauser)

expansion in M_Z^2/m_t^2 and $\left(M_Z^2/(2M_W^2)\right)^2$

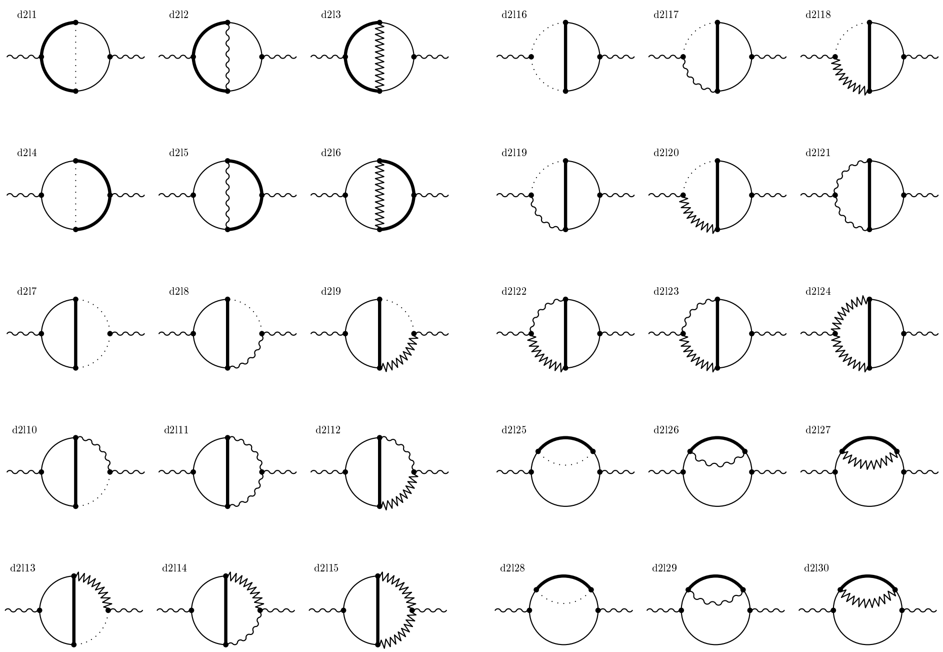
diagrams (see next slide)

result :

$$\delta\Gamma\left(Z \to b\bar{b}\right) - \delta\Gamma\left(Z \to d\bar{d}\right) =$$

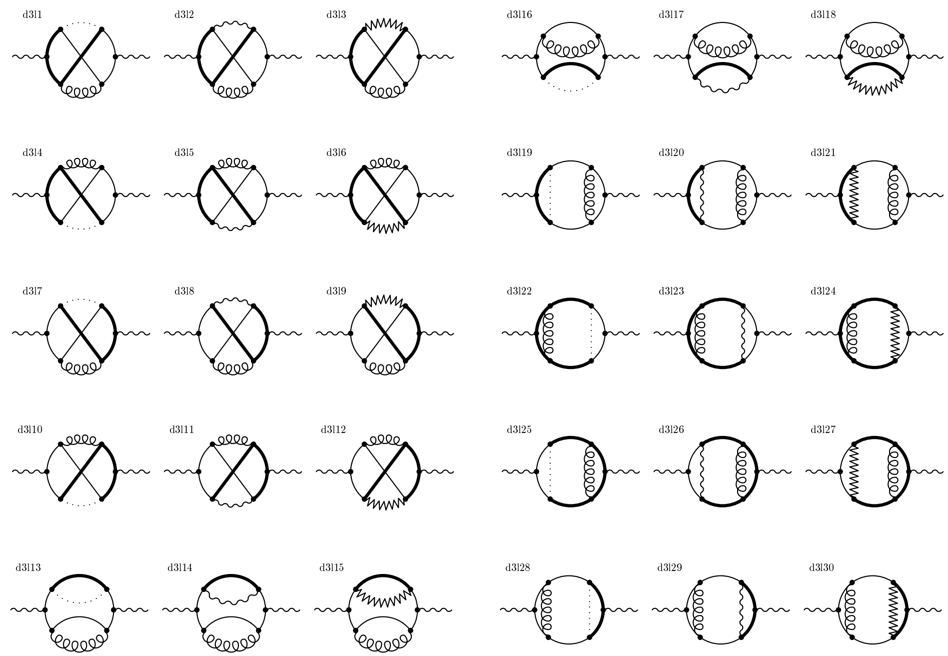
$$\Big(\underbrace{\overset{m_t^2}{-5.6} \quad \overset{\text{subleading}}{-0.79}}_{\mathcal{O}(\alpha_{\text{weak}})} \quad \underbrace{\overset{m_t^2}{+0.50} \quad \overset{\text{subleading}}{+0.06}}_{\mathcal{O}(\alpha_s\alpha_{\text{weak}})} \Big) \text{ MeV}$$

Two–Loop

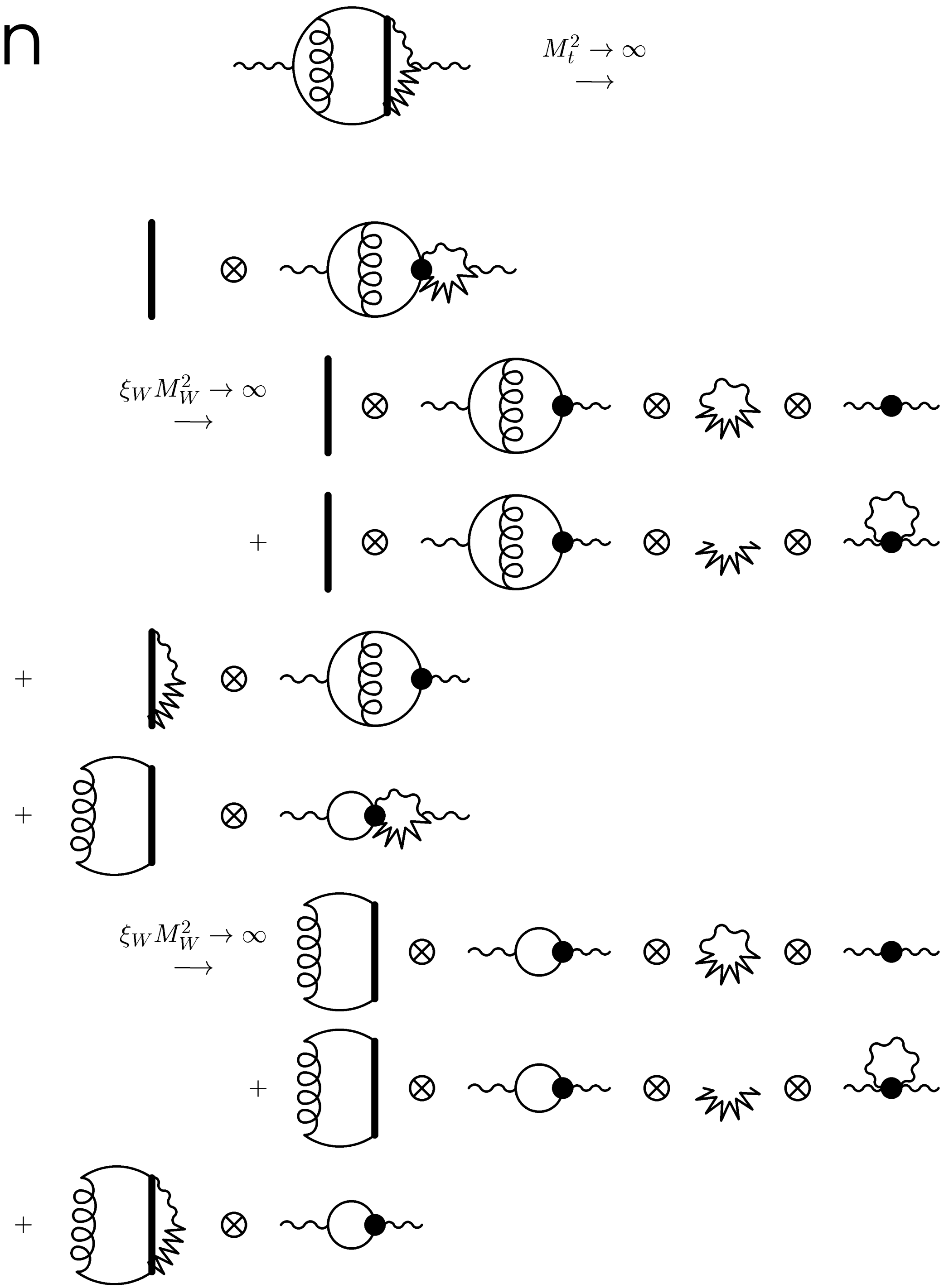


Three–Loop

(306 diagrams)



Expansion



large cancellations among subleading terms (const, $\log 1/m_t^2$) for one- and two-loop contributions!

arbitrary units:

$$
\begin{array}{lll}
\dfrac{\alpha_s}{\pi} & \Big[1.16 & m_t^2 \\
 & +(1.21-0.49) & m_t^0(\ln + \text{const}) \\
 & +(0.30-0.65) & m_t^{-2}(\ln + \text{const}) \\
 & +(0.02-0.21+0.01) & m_t^{-4}(\ln^2 + \ln + \text{const}) \\
 & +\text{small}\Big] & \\
= & \frac{\alpha_s}{\pi}\big[1.16+0.13\big] & \widehat{=}\, 0.68 \text{ MeV from} \\
 & & \text{non-factorizable terms}
\end{array}
$$

$$m_t^2 + \ln m_t^2 \text{ misleading !}$$

Outlook on α_s

$\boxed{\alpha_s^4 :}$ dominant terms available

(non-singlet, and mass terms)

singlet vector and axial vector : soon (tiny!)

$\alpha_s^2 \alpha_{\mathrm{weak}}$ small (could be done for GIGA-Z)

$\boxed{\text{GIGA-Z :}}$ $\delta\alpha_s = 5 - 7 \cdot 10^{-4}$ from experiment

theoretically robust result

$\boxed{\text{near future :}}$ R below B-threshold or below $\Upsilon(1S)$

Lumi : $8 \cdot 10^{35}$ cm^{-2}s^{-1} $\Rightarrow$ $2 \cdot 10^8$ evts/day

$\Rightarrow$ statistical precision : $\delta\sigma/\sigma \sim 10^{-4}$

assume $\delta\sigma/\sigma \sim 10^{-3}$ at 10 GeV

$\Rightarrow$ $\delta\alpha_s(10\,\mathrm{GeV}) = 3.5 \cdot 10^{-3}$ $\Rightarrow$ $\delta\alpha_s(M_Z) = 1.6 \cdot 10^{-3}$!

dedicated analysis

II. Impact of $\alpha_{\mathbf{s}}$ on $\delta\rho$

$\boxed{\text{leading term :} \quad \Delta\rho = 3\frac{\sqrt{2}G_F m_t^2}{16\pi^2}}$ (Veltman)

large difference between $\overline{\text{MS}}$ and OS mass :

$$m_t(\text{OS}) - m_t(\overline{\text{MS}}) \approx 10\,\text{GeV}$$

$\alpha_s(\widehat{=}\text{two loop})$		(Djouadi; Kniehl, JK, Stuart; Fleischer + ...)
$\alpha_s^2(\widehat{=}\text{three loop})$	required and relevant already	(Chetyrkin + ...)
$\alpha_s^3(\widehat{=}\text{four loop})$	kept in reserve	(Chetyrkin + ...; Czakon + ...)

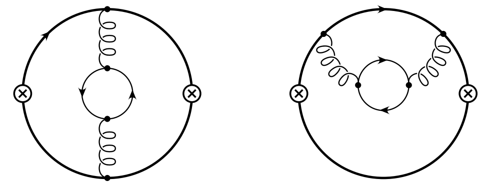
Δr, $\Delta\kappa$ are complicated functions of m_t, M_Z and M_W

two loop : analytic result

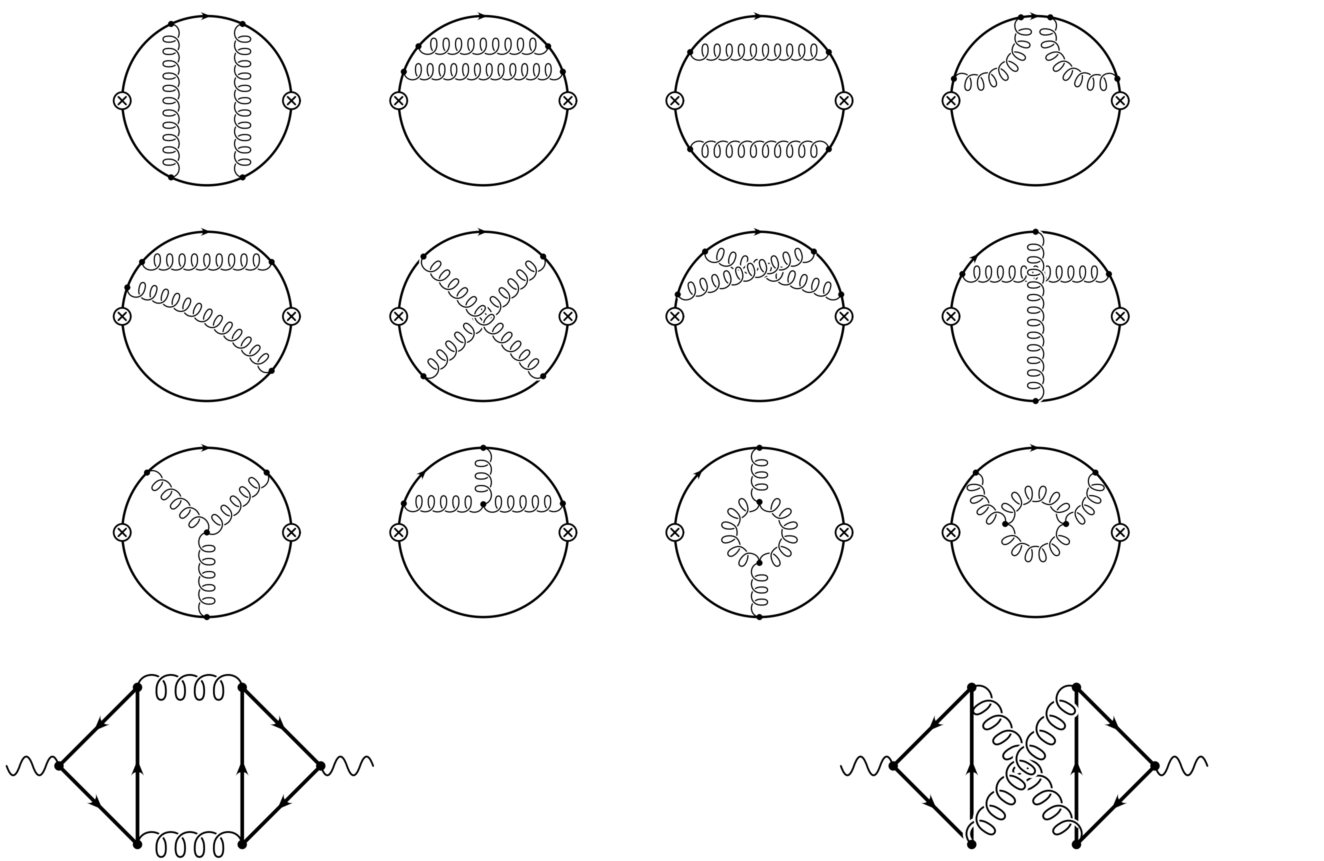
three loop ; four loop

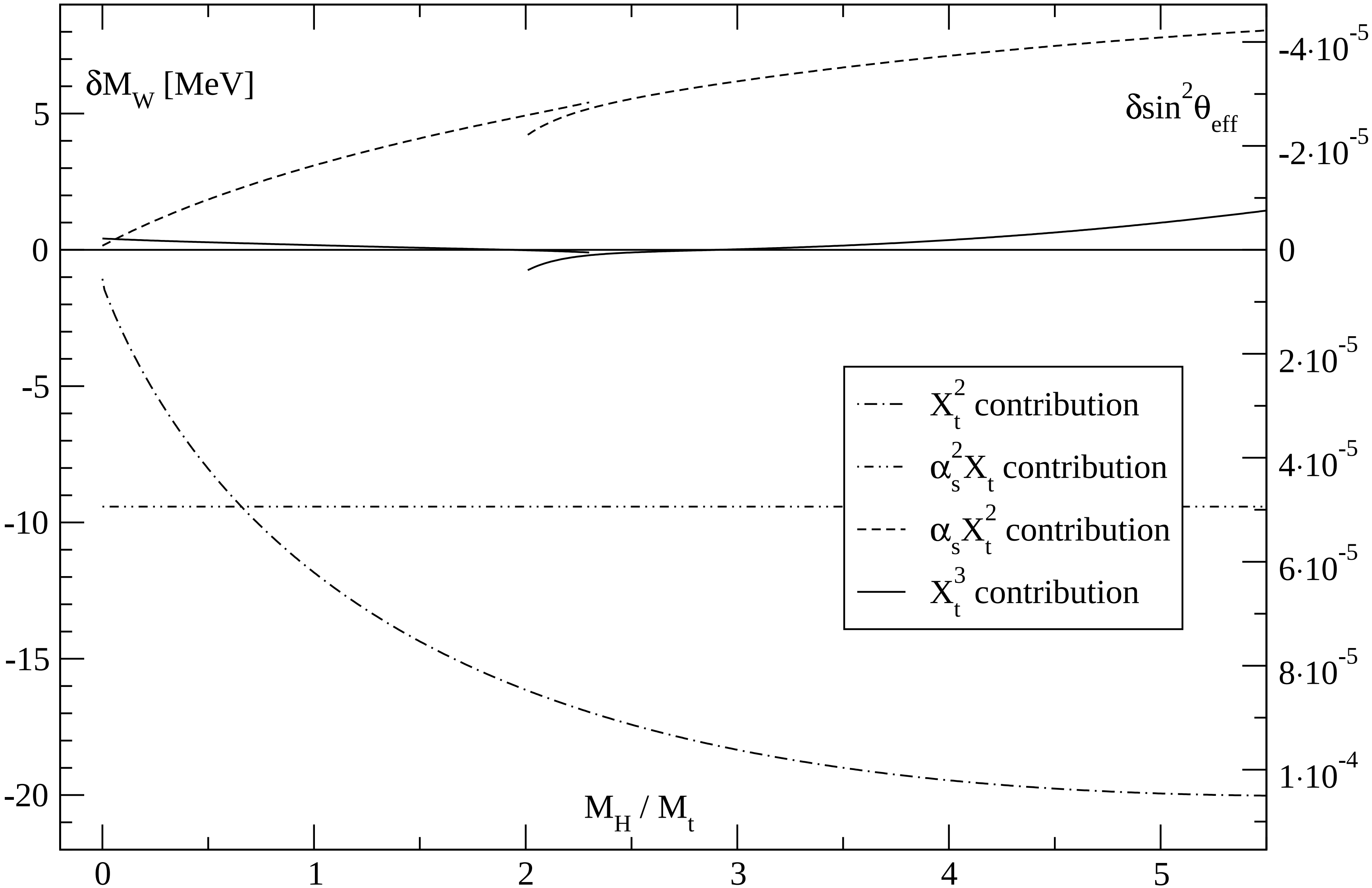
- expansion in $(M_{W,Z}/m_t)^2$ to arbitrary order
- analytic fct of expansion parameter!
- tadpole integrals

Three–Loop Diagrams



Purely gluonic contribution to $\mathcal{O}(\alpha_s^2)$





δM_W [MeV]
δsin²θ_eff
5
0
-5
-10
-15
-20
-4·10^-5
-2·10^-5
0
2·10^-5
4·10^-5
6·10^-5
8·10^-5
1·10^-4
X_t² contribution
α_s²X_t contribution
α_sX_t² contribution
X_t³ contribution
M_H / M_t
0
1
2
3
4
5

Result : δM_W in MeV

	α_s^0	α_s^1	α_s^2	α_s^3	$\alpha_s\alpha_{\rm weak}$
m_t^2	611.9	-61.3	-10.9	-2.1	2.5
log+const	136.6	-6.0	-2.6	-	-
$1/m_t^2$	-9.0	-1.0	-0.2	-	-
$\sum$	739.5	-68.3	-13.7	-2.1	2.5

$\boxed{\alpha_s^2\text{-term :} \quad 13.7\,\text{MeV} \;\widehat{=}\; \delta m_t = 2\,\text{GeV} \quad \text{(TEVATRON)}}$

$\boxed{\alpha_s^3\text{-term :} \quad 2.1\,\text{MeV} \;\widehat{=}\; \delta m_t = 0.3\,\text{GeV} \quad \text{(ILC)}}$

Conversely : $M_{\rm Pole}$ fixed

$\delta\alpha_s = 2 \cdot 10^{-3} \quad \Rightarrow \quad \delta M_W = 1.7\,\text{MeV}$

$\Rightarrow$ irrelevant in near future

Conclusions

- α_s from Z decays is theoretically robust
- α_s^4-term moves Z- and τ-result closer together
- GIGA-Z would be nice to have
- R from B-factory :
  statistically powerfull! systematics?
- QCD corrections to ρ-parameter are important
  and well under control
- present knowledge of α_s sufficient
- ILC?